Instituto de Física
Universidade de São Paulo

ISOSPIN AND DENSITY DEPENDENCES OF NUCLEAR MATTER SYMMETRY ENERGY COEFFICIENTS

BRAGHIN, FL

Instituto de Física, Universidade de São Paulo, CP 66.318
05315-970, São Paulo, SP, Brasil

Publicação IF – 1569/2001

UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

1569

Isospin and density dependences of nuclear matter symmetry energy coefficients

Fábio L. Braghin *

Nuclear Theory and Elementary Particle Phenomenology Group,

Instituto de Física, Universidade de São Paulo, C.P. 66.318; CEP 05315-970; São Paulo - SP, Brazil.

May 7, 2003

Abstract

Symmetry energy coefficients of explicitly isospin asymmetric nuclear matter at variable densities (from $.5\rho_0$ up to 2 ρ_0) are studied as generalized screening functions. An extended stability condition for asymmetric nuclear matter is proposed. We find the possibility of obtaining stable asymmetric nuclear matter even in some cases for which the symmetric nuclear matter limit is unstable. Skyrme-type forces are extensively used in analytical expressions of the symmetry energy coefficients derived as generalized screening functions in the four channels of the particle hole interaction producing alternative behaviors at different ρ and b (respectively the density and the asymmetry coefficient). The spin and spin-isospin coefficients, with corrections to the usual Landau Migdal parameters, indicate the possibility of occurring instabilities with common features depending on the nuclear density and n-p asymmetry. Possible relevance for high energy heavy ions collisions and astrophysical objects is discussed.

PACS numbers: 21.30.-x, 21.65.+f, 26.50.+x, 26.60.+c

Key-words: Symmetry energy, nuclear density, isospin asymmetry, screening functions, isospin, spin, incompressibilities.

IF- USP - 2001/ 2003

To be published in Int. Journ. of Mod. Phys. E

*e-mail: braghin@if.usp.br

1 Introduction

The symmetry energy coefficients and their dependences on the density and isospin symmetry are of relevance, for example, for the description of macroscopic nuclear properties as well as for proto-neutron and neutron stars. In the case of the neutron-proton (n-p) symmetry energy coefficient (s.e.c) of nuclear matter (a_τ) it represents the tendency of nuclear forces to have greater binding energies (E/A) for symmetric systems - equal number of protons and neutrons. It contributes as a coefficient for the squared neutron-proton asymmetry in usual macroscopic mass formula in the parabolic approximation:

$$E/A = H_0(A,Z) + a_\tau (N-Z)^2/A^2, \tag{1}$$

where H_0 does not depend on the asymmetry, Z, N and A are the proton, neutron and mass numbers respectively. Higher orders effects of the asymmetry (proportional to $(N-Z)^n$ for $n \neq 2$ [1]) are usually expected to be less relevant for the equation of state (EOS) of nuclear matter based on such parameterizations [2, 3]. a_τ is also the parameter which measures the response of the system to a perturbation which tends to separate protons from neutrons. It is given by the static polarizability of the system (the inverse isospin screening function) which also should depend on the asymmetry of the medium. In the scalar channel one can define the (dipolar) incompressibility which is related to the nuclear matter incompressibility [4]. These two coefficients (isovector and scalar) and their dependences on the neutron-proton asymmetry were initially investigated in a previous work [4]. Other symmetry coefficients may also be defined in nuclear matter, for instance, the spin a_σ and spin-isovector $a_{\sigma\tau}$ ones. The former would correspond to the difference in the binding energy due the inclusion of a polarized nucleon in the medium whereas the second involves the distinction between neutrons and protons as well. A calculation for the dynamical and static polarizabilities - proportional to the inverse of such symmetry coefficients in asymmetric matter - was done using Skyrme effective forces in [4, 5, 6]. The spin channel is relevant for the study of the neutrino interaction with matter because it couples with axial vector current together with the scalar channel [7, 8, 9]. A suppression of the spin susceptibility (in this work we will be dealing rather with its inverse, the spin symmetry coefficient - $A_{1,0}$, as shown in sections 2 and 3, corresponding also to an increase of the Landau parameter G_0) lead to the suppression of Gamow Teller transitions which are of interest for the supernovae mechanism [9]. Calculations with Skyrme interactions usually result in smaller G_0 (corresponding to smaller spin symmetry coefficients) than microscopic calculations leading to instabilities associated to ferromagnetic polarized states [10, 9]. In the present work we show this conclusion about Skyrme interactions is not necessarily correct. The spin-isospin channel has been associated with instabilities which would lead to pion condensation. In the language of Migdal-Landau parameters one would have the parameter $G_0' < -1$ for the formation of such condensates.

In the present work we show that these coefficients may provide us with a way of checking the stability of asymmetric nuclear matter with respect to the explicit isospin asymmetry. Another aim is to articulate the idea that different Skyrme-type forces may be appropriate to the description of diverse phenomena at variable ranges of the nuclear density and asymmetry by extending the previous studies of s.e.c. Some Skyrme interactions will be used to assess the possible behavior of these functions. The parameters of one of the forces we use (SLyb) were fitted from results of neutron matter properties obtained from microscopic calculations in [11]. Other forces (SkSC4, SkSC6 and SkSC10), which have slightly different density dependencies, had their parameters fixed by adjusting a large amount of nuclear masses [12, 13].

This paper reviews and extends the works presented in [4, 6] and it is organized as follows. In the next section we remind an argument previously developed to investigate generalized static polarizabities which allow for the study of the nuclear matter stability. In section 3 the static polarizabilities in the four channels of the particle-hole interaction with several Skyrme forces is studied in asymmetric nuclear matter at variable densities as derived from the linear response method. Next a new stability condition with respect to neutron-proton fluctuations asymmetry is proposed. In the last section possible consequences are pointed out and the results are summarized.

2 Generalized Symmetry Energy Coefficients

We review a qualitative argument from [14, 4] for exploring them. We consider a small amplitude (ϵ) external perturbation which acts, through the third Pauli isospin matrix τ_3, in nuclear matter separating nucleons with isospin up and down [1]. This originates fluctuations $\delta\rho = \delta\rho_n - \delta\rho_p \equiv \beta$ of the nucleon densities. The energy of the system can be written as:

$$H = H_0 + A_{0,1}\frac{(\rho_n - \rho_p)^2}{\rho} + \epsilon\beta, \tag{2}$$

[1]This argument is valid for the four channels. It is enough to consider other external perturbations: σ_3, $\sigma_3\tau_3$ and $\mathbf{1}$ for the spin, spin-isospin and scalar channels respectively.

where $\mathcal{A}_{s,t}$ is the isovector symmetry coefficient ($s = 0, t = 1$ - spin, isospin) usually denoted by a_τ, J or β. For the other channels (s, t) one defines different symmetry coefficients. We can re-write the n-p density difference in terms of β as: $\rho_n - \rho_p = (\rho_{n,(0)} - \rho_{p,(0)} + \beta)$, where the subscript $_{(0)}$ indicates static densities, without the external source. Although one is dealing with an infinite system, the arguments are to be valid for finite nuclei.

The polarizability is defined as the ratio of the density fluctuation to the amplitude of the external perturbation and it can be written as [4, 14]:

$$\Pi^{s,t} \equiv \frac{\beta}{\epsilon} = -\frac{\rho}{2\mathcal{A}_{s,t}(b,\beta)}, \tag{3}$$

where we consider the general channel (s,t) as done in [5]. Note that $\mathcal{A}_{s,t}$ is a function of b and β and these parameters may be related, as argued below. The occurrence of these functional dependences of $\mathcal{A}_{s,t}$ can be found just by the stability condition with respect to the fluctuation from expression (2):

$$\frac{dH}{d\beta} = 0. \tag{4}$$

2.1 Isospin dependence of $\mathcal{A}_{s,t}$

The neutron proton asymmetry used in the present work is defined by the neutron and proton densities ρ_n, ρ_p as:

$$b = \frac{\rho_n}{\rho_p} - 1. \tag{5}$$

It varies from $b = 0$, in symmetric nuclear matter, up to infinity, in neutron matter. The coefficient b is related to a frequently used asymmetry coefficient:

$$\alpha = \frac{(2\rho_{0n} - \rho_0)}{\rho_0}, \tag{6}$$

by the expression:

$$b = 2\alpha/(1-\alpha). \tag{7}$$

As stated above $\mathcal{A}_{s,t}$ is a function of the density fluctuation β. Although β is not the explicit n-p asymmetry itself (given by b) we will consider that it depends on it (as it was also argued in [4, 6]). We consider, as shown below, these parameters are related to each other and therefore we will write $\mathcal{A} = \mathcal{A}(\beta)$ shortly. In [5, 6] different prescriptions were discussed for $\beta = \beta(b)$ in the calculation of the response function of asymmetric nuclear matter. We have used (and it was shown to be a reasonable prescription for the dynamical response function [5]) the one which leads to the following relation between the fluctuation β and the explicit n-p asymmetry b:

$$\beta = \delta\rho_n \left(\frac{2+b}{1+b}\right), \tag{8}$$

Where $\delta\rho_n$ is the neutron density fluctuation. In the n-p symmetric limit $\beta = 2\delta\rho_n$ and in another limit, in neutron matter, $\beta = \delta\rho_n$. The above prescription (expression (8)) is based on the assumption that the density fluctuations are proportional to the respective density of neutrons and protons, i.e., $\delta\rho_n/\beta = \rho_n/\rho$, being ρ the total density. In spite of being rather appropriated for the isovector channel, this kind of assumption can be considered as a starting point for the other channels (spin, scalar) in asymmetric nuclear matter. Prescription (8) is therefore model-dependent and different choices for it yield other forms for the (asymmetric) static screening functions. The dynamic response functions are less sensitive to this prescription [5].

¿From the solution of the polarizability (3) we calculate the first derivative with relation to b:

$$\frac{d\beta}{db} = \frac{\epsilon\rho}{2\mathcal{A}_{0,1}^2}\frac{d\mathcal{A}^{0,1}}{db} = -\frac{\beta}{\mathcal{A}_{0,1}}\frac{d\mathcal{A}^{0,1}}{db}. \tag{9}$$

Another expression can be obtained from the relation between b and β of (8). It yields:

$$\frac{d\beta}{db} = -\frac{\beta}{(2+b)(1+b)}. \tag{10}$$

Equating these two last equations we obtain:

$$-\mathcal{A}^{0,1}\frac{\beta}{(2+b)(1+b)} = -\beta\frac{d\mathcal{A}^{0,1}}{db}, \tag{11}$$

¿From which it is possible to derive the following relation between the isospin s.e.c. and the n-p asymmetry [4]:

$$\mathcal{A}^{0,1} = \mathcal{A}^{0,1}_{sym}\frac{2+2b}{2+b}. \tag{12}$$

In this expression $\mathcal{A}_{sym} = a_\tau \simeq 30MeV$ is the s.e.c. of symmetric nuclear matter ($b=0$). For $b=2$ (neutron density three times larger than the proton density) we obtain $\mathcal{A} = 1.5\mathcal{A}_{sym}$. In the limit of neutron matter $\mathcal{A}(b \to \infty) = 2\mathcal{A}_{sym}$. The prescription of expression (8) was the relevant information for this calculation. Any other relation between b and β will induce different asymmetry dependence of the symmetry energy coefficient [6]. Another assumption for deriving expression (12) was that ρ is independent of b. This would be non trivially different if one considers a complete self consistent calculation with the equation of state of a proto-neutron star, for example.

3 Generalized Screening functions with Skyrme forces

In this section in in the next one we nearly extend the analysis of [4]. A nearly exact expression for the dynamical polarizability of a non relativistic hot asymmetric nuclear matter at variable densities was derived with Skyrme interactions in [5]. The approximations were: (i) to equate the asymmetry coefficient defined for the momentum density to the density asymmetry coefficient (the variation of the corresponding term yields differences in the response function of less than 1%), (ii) to choose a particular (discussed and justified [5]) prescription for the asymmetry density fluctuations - expression (8).

The generalized static screening functions $A_{s,t}$ (or symmetry energy coefficients according to the discussed before) in asymmetric nuclear matter at finite temperature can be written as [5]:

$$A_{s,t} = \frac{\rho}{N}\Big\{1 + 2\overline{V_0^{s,t}}N_c + 6V_1^{s,t}M_p^*(\rho_c+\rho_d) + 12M_p^*V_1^{s,t}\overline{V_0^{s,t}}\,(N_c\rho_d - \rho_c N_d) + (V_1^{s,t})^2\left(36(M_p^*)^2\rho_c\rho_d - 16M_p^*M_cN_d\right)\Big\}. \tag{13}$$

The densities ρ_v, M_v and N_v are given respectively by:

$$\rho_v = v\rho_n + (1-v)\rho_p, \quad M_v = vM_n + (1-v)M_p \quad N_v = vN_n + (1-v)N_p,$$

where v stands for n-p asymmetry coefficients (c, d) defined below (a measure of the fraction of neutron densities). The above densities at finite temperature are defined by:

$$(N_q, \rho_q, M_q) = \frac{2M_p^*}{\pi^2}\int df_q(k)(k, k^3, k^5).$$

In these expressions $f_q(k)$ are the fermion occupation numbers for neutrons $(q=n)$ and protons $(q=p)$ which will be considered only for the zero temperature limit. In this case: $df_q(k) = \delta(k-k_F^q)dk_q$, which makes the above integrals trivial. $\overline{V_0}$ and V_1 are functions of the Skyrme forces parameters (see in [5]) and $M_p^* = m_p^*/(1+a/2)$ is a modified effective mass for the proton. Besides that, the four asymmetry coefficients are:

$$a = \frac{m_p^*}{m_n^*} - 1, \quad b = \frac{\rho_{0n}}{\rho_{0p}} - 1, \quad c = \frac{1+b}{2+b}, \quad d = \frac{1}{1+(1+b)^{\frac{2}{3}}}. \tag{14}$$

3.1 Results for Skyrme interactions

In this section we present results for the s.e.c. using Skyrme forces with the expression (13) in all the four channels of the particle-hole interaction. The scalar, isovector and spin channels were initially discussed in a previous works [4, 6] and those analysis is extended here. The present analysis on the spin and spin-isospin channels was not done before.

In Figure 1 we show the generalized isovector symmetry energy coefficient $A_{0,1}$ as a function of the ratio of the nuclear density to the saturation density (ρ/ρ_0) for Skyrme interactions SLyb [11], SkSC4, SkSC6 and SkSC10 [12, 13]. Since Skyrme forces are not necessarily expected to describe Physics at high densities we investigate the behavior of the symmetry energy coefficients (s.e.c., which can also be called generalized screening functions for each channel) from 0.5 ρ_0 up to 2 ρ_0. The symmetry energy coefficient for some of the forces in Figure 1 reach a maximum value when $\rho_0 < \rho \leq 1.7\rho_0$ and then decrease. This last behavior is typical of non relativistic calculations. There are two forces which yield different behaviors: SkSC4 makes the slope of $A_{0,1}$ much less negative reaching an instability point ($A_{0,1} < 0$) at smaller densities than the other forces and SkSC10 for which the slope is larger and positive. This last behavior is present in relativistic calculations. The point in which the symmetry energy coefficient is zero would correspond to a phase transition of nuclear matter to an asymmetric state of neutron or proton matter. Although the usual Skyrme forces are not well suited for high densities there is a tendency of a decrease of this screening function (s.e.c.) until a phase transition in a dense nuclear system. The dependence on b, on the other hand, leads to a more repulsive behavior as seen in Figures 2 and 3 eventually compensating the attractiveness of low or high densities.

In Figure 2 the same parameter, $A_{0,1}$, is shown as a function of ρ/ρ_0 for the interactions SLy, SkSC4 and SkSC6 for non zero n-p asymmetries, i.e., for b=.25 and b=.54. The latter corresponds to a n-p asymmetry of the nucleus of ^{208}Pb. The absolute values of the isovector symmetry energy coefficient (s.e.c) increase with relation to the symmetric nuclear matter but the behavior with varying ρ is nearly the same in general. However, for the force SkSC6, the isovector screening function has a higher slope in this range of densities. The same coefficient as a function of the n-p asymmetry b is shown in Figure 3 at the densities $0.5, 1$ and $2\rho_0$. Again we note different behaviors depending on the force. For the forces SLy and SkSC4 the s.e.c. increases, as noted above, for higher densities and n-p asymmetries but for $\rho = 2\rho_0$ the values are smaller at higher asymmetries. For the force SkSC6 the higher the density the higher the slope with b. The tendency is that the interaction becomes more repulsive. Whereas the dependence of $A_{0,1}$ on b is a new possibility its dependence on ρ has been studied quite extensively and a distinction between the behavior of relativistic and non relativistic models can be done which still are nearly observed here. While relativistic calculations lead to a continuous increase of the symmetry energy coefficient [15], non relativistic (microscopic or not) calculations usually yield a saturation at densities of the order of $2.5\rho_0$ with a decreasing value at higher densities [15, 16]. Although these are the more common results there is a non relativistic variational calculation with three body forces [16] which presents an increasing symmetry energy coefficient for higher densities. All these calculations adopt the parabolic approximation for the symmetry energy as discussed by [17] [2]. We have just shown, however, that it is possible to obtain the behavior typical from relativistic calculations with the Skyrme force SkSC10.

The spin symmetry energy coefficient is plotted in Figures 4, 5 and 6 as a function of ρ/ρ_0 and b respectively. In Figure 4 one sees that, for increasing densities, $A_{1,0}$ may have different slopes. For the interactions SkSC4, SkSC6 and SkSC10 the spin- symmetry energy coefficient is already negative at low densities and decrease still more with ρ. This feature is clear in Figure 5 where the same spin s.e.c. is shown for non zero n-p asymmetries. These forces exhibit a behavior typical of a ferromagnetic matter with stronger spin alignment as density increases. With the force SLy, on the other hand, the spin symmetry coefficient indicates that no polarized state occurs the range of densities under study.

In Figure 6 the spin symmetry energy is shown as a function of b with forces SLy, SkSC4 and SkSC6 for some different densities. The common trend is the increase of $A_{1,0}$ with b, i.e., at very asymmetric matter the spin interaction tends to become more repulsive. However the particular behavior of the spin s.e.c. with b is different for each effective force at a given density. This means that whereas for the SLy force the increase of n-p asymmetry makes $A_{1,0}$ to increase (proportionally) at each density considered, for the forces SkSC4 and SkSC6 the slope is higher for higher densities. This makes the curves of $A_{1,0}$ for different densities to cross at certain n-p asymmetry. We can compare our results to the ratio of spin susceptibility of interacting neutron matter (for which $b \to \infty$) to the non interacting Fermi gas obtained by Fantoni, Sarsa and Schmidt [18] by means of the auxiliary field diffusion Monte Carlo method. This ratio is proportional to the polarizability as obtained in expression (13) and therefore inversely proportional to the spin symmetry coefficient $A_{1,0}$. First of all we note that, in most cases, contrarily to what we find, the values they find are all positive for the range of densities considered by them, from $0.75\rho_0$ up to $2.5\rho_0$. However the slope seems to be nearly the same as that we obtain for low values of the n-p asymmetry. Consequently they may obtain instabilities for higher density neutron matter whereas we do not observe this result in our calculations with Skyrme forces (the comparison is meaningful for neutron matter: b is very large). Other comments about this are

[2]We leave for a forthcoming work a discussion about corrections to the parabolic approximation in the frame of a relativistic model, following ideas contained in section 2 of the present work and in [4].

drawn in section 5.

The spin-isospin symmetry energy coefficient for symmetric matter ($b = 0$) as a function of ρ/ρ_0 is shown in Figure 7. For all the forces, $A_{1,1}$ are decreasing functions of the density and they become negative between $\rho \simeq 1.8\rho_0$ and $2.1\rho_0$. The only significant difference comes from the value of $A_{1,1}$ at low densities, being that SLy force implies a larger value than the SkSC forces. In Figure 8 the same analysis holds for asymmetric matter for $b = .25$ and $.54$. However, with increasing asymmetry, the instability point is changed differently for each of the forces. This is very interesting as well and very similar to the spin channel (of figure 5). The SLy force presents the spin-isospin coefficient which changes much less than the SkSC forces. For these Skyrme interactions instabilities associated to pion condensation would be found already at low densities, $A_{1,1} < 0$. Therefore, for the Skyrme type interactions we use, we expect a there is a connection between the appearance of spin instabilities (leading to ferromagnetic states) and spin-isospin instabilities (for pion condensation), even for not very asymmetric nuclear matter (up to $b \simeq 0.54$).

In Figure 9, $A_{1,1}$ is plotted as a function of b for different Skyrme forces. In this Figure the behavior of the spin-isospin s.e.c. with the n-p asymmetry parameter b is shown for different nuclear matter densities (below, equal to and above the saturation density). It can be noted that for the force SLy an increasing n-p asymmetry (b) makes the spin-isovector interaction continuously more repulsive instead of allow for attractiveness while the opposite behavior is found for SkSC6. One would find the instabilities associated to pion condensation at roughly any nuclear density for SkSC6 at nearly any n-p asymmetry, which does not seem to be realistic [19]. The other forces SkSC also reproduce this last behavior with different slopes. We are lead to conclude that Skyrme interactions may be able to reproduce roughly well any kind of dependence of the (generalized) symmetry energy coefficients in a particular range of density and/or n-p asymmetry.

In Figures 10 and 11 the scalar s.e.c. is shown as a function of ρ/ρ_0 for b=0 and b=0.25 and .54 respectively. These Figures show that, roughly speaking, all the interactions are attractive or weakly repulsive at low densities and more repulsive as density increase. This may be related to the instabilities found, for example, in [20]. However it is interesting to see in Figure 11 that, depending on the interaction, with the increase of the n-p asymmetry the scalar symmetry energy coefficient is reduced. This occurs for lower densities but not necessarily at higher ρ. This is well seen in Figure 12 for $A_{0,0}$ as a function of b. This may be understood as a general tendency of resulting instabilities for (very) asymmetric nuclear matter with a curious exception for force SLy at a density $2\rho_0$. The behavior of the incompressibility with increasing n-p asymmetries, at the saturation density, for the force SLy is nearly the same as that found in other calculations with Skyrme forces or relativistic calculation [21, 22] and microscopic variational study at finite temperatures [23]. For a higher density ($2\rho_0$), however, this scalar coefficient increases with b for the same SLy force. A comparison of this kind was explicitly done in [4]. The other forces exhibit smaller slopes and make $A_{0,0}$ to decrease faster. This last kind of behavior, more pronounced, was also found in other works (for the bulk incompressibility modulus K_∞ depending on the used interaction [22, 23]. Therefore the behavior of the incompressibilities (K_∞ and $A_{0,0}$ - the "dipolar incompressibility") with n-p asymmetry depend strongly on the used interaction.

4 Stability of Asymmetric nuclear matter

Consider that the binding energy can be minimized with respect to β yielding an equilibrium state for nuclear matter. This means that: $d(E/A)/d\beta = 0$, yielding a differential equation for $\mathcal{A}_{s,t}$ as a function of β or $\Pi_{s,t}$ by expression (3). In the case ρ is not dependent on β the solution for this resulting differential equation is given by [4]:

$$\frac{\mathcal{A}_{s,t}}{\rho_0} = \frac{C}{\Pi_{s,t}^2} - \frac{1}{\Pi_{s,t}}, \tag{15}$$

where C is a constant. Inverting the above expression we find:

$$\Pi_{s,t} = -\frac{\rho_0}{2\mathcal{A}_{s,t}} \pm \sqrt{\frac{4C_{s,t}\rho_0}{\mathcal{A}_{s,t}} + \frac{\rho_0^2}{4\mathcal{A}_{s,t}^2}}, \tag{16}$$

which, in principle, can be either real or complex. Furthermore, there may appear two branches of solutions for $b \neq 0$. We want to emphasize that the relation between this expression and those obtaining from the linear response with Skyrme forces is not completely understood. As a boundary condition one requires the usual expression for the symmetric limit obtaining:

$$C^{(s,t)} = -\frac{\rho_0}{4\mathcal{A}_{s,t}^{sym}}. \tag{17}$$

Such that the corresponding polarizability of symmetric nuclear matter $\Pi_0^{s,t}$ by: $\Pi_0^{s,t} = -\rho_0/(2\mathcal{A}_{s,t}^{sym})$ [14].

To really be a stable minimum of the binding energy the second derivative of the energy must be positive. Using the form given by expression (3), and the approximation that ρ_0 does not depend on

β, we can write this second derivative as:

$$\frac{d^2H}{d\beta^2} = -\frac{1}{\Pi} + \frac{4\Pi}{\rho_0}\frac{d\mathcal{A}}{d\Pi} + \frac{\Pi^2}{\rho_0}\frac{d^2\mathcal{A}}{d\Pi^2}. \tag{18}$$

With the solution given by expression (15) we find, in the (s,t) channel, that:

$$\frac{d^2H}{d\beta^2_{s,t}} = -\frac{\mathcal{A}_{s,t}}{\rho} - \frac{C}{\Pi^2_{s,t}} = \pm\frac{2\sqrt{\rho^2 + 4C\rho_0\mathcal{A}_{s,t}}}{\rho\, \Pi^{s,t}} > 0. \tag{19}$$

The constant C may be negative (stable symmetric nuclear matter according to expression (17)) or positive and $\mathcal{A}$ and Π also may have negative or positive signs. It is possible to have a case where the symmetric nuclear matter is unstable, $\mathcal{A}_{sym} < 0$, but a stable asymmetric nuclear matter is obtained, with positive $d^2H/d\beta^2$. This is a curious result. It seems to suggest that the stability line of nuclei can occur, at least with its qualitative real behavior, without introducing explicitly electromagnetic forces.

For the sake of comparison we quote the usual stability condition for a Fermi liquid in each channel of the interaction. It is given by:

$$a_{s,t} = N_0(1 + J_0^{s,t}) > 0. \tag{20}$$

where $J_0^{s,t}$ stands for any of F_0, F_0', G_0, G_0' respectively for the scalar $(s=0, t=0)$, isovector $(s=0, t=1)$, spin $(s=1, t=0)$ and spin-isovector $(s=1, t=1)$ channels [24]. These conditions correspond to the denominators of the response function of *symmetric* nuclear matter at zero temperature and at saturation density.

5 Further discussion and Summary

The expressions we have derived and used for the (isospin) symmetry energy coefficient (with and without effective Skyrme forces) break isospin symmetry because the results for an excess of neutron (density) is different from those of higher proton fraction. In principle this is expected although usually considered to be small in mass formulae for finite nuclei [3]. With the expressions we derived the effects of this symmetry breaking can be expected to be larger eventually related to other effects [25].

Recently it has been proposed that in the core of dense (neutron) stars there may occur a phase of matter in which color degrees of freedom are deconfined generating di-quark condensates in channels in which the interaction is attractive. Several possibel scenarios for this color superconducting phase of matter can be drawn, among them the color-flavor locked phase [26]. We would like to point out that with the increase of the isospin symmetry energy coefficient, for instance, the resulting fraction of protons, after the supernovae stage, would be higher hindering the emergency of a pure neutron matter in the death of the star. As a consequence the final proportion of up and down quarks would be different than that from a pure neutron star. This may modify the color superconducting phase. These remarks may apply to relativistic or high energy heavy ions collisions which generates high baryonic density.

The relevance of the isovector symmetry energy coefficient and its dependence on the nuclear density has been intensively studied due to its relevance to the determination of the nuclear matter equation of state which is very important for the comprehension of high energy ions collisions and eventually the dynamics of astrophysical objects. An interesting recent study was done by Bao An Lin [27] who however only is concerned about the parabolic approximation without a more general form of the asymmetry term. This extension would be of high interest for the field.

Summarizing, the dependence of the s.e.c. on the n-p asymmetry was studied extending the results of ref. [4, 6]. Generalized symmetry energy coefficients of asymmetric (non relativistic) nuclear matter at variable densities were investigated. The density and n-p asymmetry dependences of the s.e.c. in the different channels were analyzed for different Skyrme forces. They may yield very different behaviors including the possibility (or not) of nuclear matter to undergo phase transitions. These forces can describe different behaviors of the symmetry energy coefficients. Although one should not believe that only one Skyrme force parametrization could account for the description of all nuclear observables at different densities, temperatures and n-p asymmetries it is acceptable the idea that several parametrizations could hopefully describe different ranges of the dependence of nuclear observables (as the s.e.c.) with these variables. Whereas the isovector, scalar and spin channels were rather an extension of the work presented in [4, 6] the results for spin and spin-isospin channels are scarce in the literature. We showed that, for the Skyrme forces under consideration (SLy, SkSC4, 6, 10) there is a kind of correlation between the two channels and the possibility of occurring simultaneous ferromagnetic instabilities and/or pion condensation at high densities and n-p asymmetries. Therefore, in principle, different values can be expected for the (bulk) symmetry energy coefficients in nuclei and nuclear matter, with different n-p asymmetries, and neutron stars. In the spin channels it is possible to expect spin polarized asymmetric matter yielding magnetic fields in neutron stars, as discussed in [10], according to the results in figures 4,5 and 6 for forces the SkSC. However with the increase of

b we find that the spin interaction may be rather repulsive, hindering this magnetization effect with the use of these Skyrme forces in the range of densities analyzed here. A similar behavior for the spin-isospin channel (which may indicate instabilities for pion condensation) was found with the same Skyrme interactions. A new stability condition for an asymmetric nuclear medium was proposed. It is given by:

$$\frac{d^2H}{d\beta_{s,t}^2} = -\frac{\mathcal{A}_{s,t}}{\rho} - \frac{C}{\Pi_{s,t}^2} = \pm\frac{2\sqrt{\rho^2 + 4C\rho_0\mathcal{A}_{s,t}}}{\rho\,\Pi^{s,t}} > 0.$$

Where C is a constant and $\Pi^{s,t} = \rho/(2\mathcal{A}_{s,t})$ for each channel.

Acknowledgements

This work was supported by FAPESP, Brazil.

References

[1] J. Jänecke and E. Comay, Nucl. Phys. **A 436** (1985) 108.

[2] C.-H. Lee *et al*, Phys. Rev. **C 57** (1998) 3488.

[3] P. Moeller, J.R. Nix, W.D. Myers and W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995); Y. Aboussir, J.M. Pearson, A.K. Dutta and F. Tondeur, At. Data Nucl. Data Tables **61**, 127 (1995).

[4] F.L. Braghin, Nuc. Phys. **A 696** (2001) 413; and F.L.B., *Erratum* Nuc. Phys. **A** to be published.

[5] F.L. Braghin, Phys. Lett. **B 446**, (1999) 1; *idem ibid: Erratum* submitted to P.L. B.; Nucl. Phys. **A 665**, (2000) 13

[6] F.L. Braghin, Brazilian Journal of Physics, accepted to publication, *Proceedings of the Brazilian Meeting of Nuclear Physics*, September 2002, São Paulo, ed. D.P. Menezes.

[7] R.F. Sawyer, Phys. Rev. **C 11**, 2740 (1975), N. Iwamoto and C.J. Pethick, Phys. Rev. **D 25**, 313 (1982).

[8] J. Navarro, E.S. Hernandez, D. Vautherin, Phys. Rev. **C 60** (04)5801 (1999).

[9] S. Reddy, M. Prakash, J.M. Lattimer, J.A. Pons, Phys. Rev. **C 59**, (1999) 2888.

[10] M. Kutschera and W. Wójcik, Phys. Lett. **B 223**, 11 (1989)

[11] E. Chabanat *et al*, Nucl. Phys. **A 627**, (1997) 710.

[12] A.D. Dutta *et al*, Nucl. Phys. **A 458**, 77 (1986); F. Tondeur *et al*, Nucl. Phys. **A 470**, 93 (1987), Y. Aboussir *et al*, Nucl. Phys. **A 549**, 155 (1992).

[13] M. Onsi, H. Przysiezniak and J.M. Pearson, Phys. Rev. **C 50**, (1994) 460.

[14] F.L. Braghin, D. Vautherin and A. Abada, Phys. Rev. **C 52**, (1995) 2504.

[15] M. Prakash, T.L. Ainsworth, J.M. Lattimer, Phys. Rev. Lett. **61** 2518 (1988)

[16] M. Baldo, I. Bombaci, G.F. Burgio, Astron. Astrophy. **328** 274 (1997).

[17] H. Huber, F. Weber, M.K. Weigel, Phys. Rev. **C 51**, 1790 (1995).

[18] S. Fantoni, A. Sarsa, K.E. Schmidt, Phys. Rev. Lett. **87**, 181101 (2001)

[19] A.B. Migdal, E.E. Saperstein, M.A. Troitsky, D.N. Voskresensky, Phys. Rep. **192** 179 (1990).

[20] S. Ayik, M. Colonna, Ph. Chomaz, Phys. Lett. **B 353**, (1995) 417.

[21] E. Baron, J. Cooperstein, S. Kahana, Phys. Rev. Lett. **79** (1997) 609.

[22] S. Yoshida. H. Sagawa and N. Takigawa, Phys. Rev. **C 58**, 2796 (1998)

[23] M. Modarres and G.H. Bordbar, Phys. Rev. **C 58**, 2781 (1998)

[24] D. Pines and P. Nozieres, *The theory of Quantum Liquids*, W.A. Benjamin Inc., New York, 1966.

[25] For example: L.A. Barreiro, A.P. Galeão, G. Krein, Phys. Lett. **B 358** 7 (1995); W. Broniowski, W. Florkowski, Phys. Lett. **B 440** 7 (1998).

[26] For example: M. Alford, J.A. Bowers, F. Rajagopal, Phys. Rev. **D 63** 074016 (2001).

[27] Bao- An Lin, Phys. Rev. Lett. **88**, (2002) 192701; Nucl. Phys. **A 708**, (2002) 365.

Figure captions

Figure 1 Neutron-proton symmetry energy coefficient $A_{0,1} = \rho/(2\Pi_R^{0,1})$ of symmetric nuclear matter as a function of the ratio of density to the saturation density ($u = \rho/\rho_0$) for interactions SLyb (dotted-dashed line), SkSC4 (solid), SkSC6 (dotted), SkSC10 (dashed).

Figure 2 The same as Figure 1 for interactions SLy with b=0.25 (thick dotted-dashed line), b=.54 (thin dotted-dashed line), SkSC6 with b=.25 (thick dotted), b=.54 (thin dotted), SkSC4 with b=.25 (thick solid) and b=.54 (thin solid).

Figure 3 Neutron-proton symmetry energy coefficient $A_{0,1} = \rho/(2\Pi_R^{0,1})$ as a function of the asymmetry coefficient b at different densities: solid lines for SkSC4 (very thick: $\rho = .5\rho_0$, thick:$\rho = \rho_0$, thin:$\rho = 2\rho_0$,), dotted lines for SkSC6 (same conventions as SkSC4) and dotted-dashed lines for SLy.

Figure 4 Spin symmetry energy coefficient $A_{1,0} = \rho/(2\Pi_R^{1,0})$ as a function of the ratio of density to density at saturation ($u = \rho/\rho_0$) for interactions SLy, SkSC4, SkSC6 and SkSC10 with the same conventions of Figure 1.

Figure 5 Spin symmetry energy coefficient $A_{1,0} = \rho/(2\Pi_R^{1,0})$ as a function of the ratio of density to density at saturation ($u = \rho/\rho_0$) for interactions SLy, SkSC4, SkSC6 and SkSC10 with the same conventions of Figure 2.

Figure 6 Spin symmetry energy coefficient $A_{1,0} = \rho/(2\Pi_R^{1,0})$ as a function of the asymmetry coefficient b at different densities and for the different forces: with the same conventions of Figure 3.

Figure 7 Spin-isospin symmetry energy coefficient $A_{1,1} = \rho/(2\Pi_R^{1,1})$ as a function of the ratio of density to density at saturation ($u = \rho/\rho_0$) for interactions SLy, SkSC4, SkSC6 and SkSC10 with the same conventions of Figure 1.

Figure 8 Spin-isospin symmetry energy coefficient $A_{1,1} = \rho/(2\Pi_R^{1,1})$ as a function of the ratio of density to density at saturation ($u = \rho/\rho_0$) for the different interactions: with the same conventions of Figure 2.

Figure 9 Spin-isospin symmetry energy coefficient $A_{1,1} = \rho/(2\Pi_R^{1,1})$ as a function of the asymmetry coefficient b at different densities and with the different forces: using the same conventions of Figure 3.

Figure 10 Scalar symmetry energy coefficient $A_{0,0} = \rho/(2\Pi_R^{0,0})$ as a function of the ratio of density to density at saturation ($u = \rho/\rho_0$) for the different interactions with the conventions of Figure 1.

Figure 11 Scalar symmetry energy coefficient $A_{0,0} = \rho/(2\Pi_R^{0,0})$ as a function of the ratio of density to density at saturation ($u = \rho/\rho_0$) for the different interactions: with the conventions of Figure 2.

Figure 12 Scalar symmetry energy coefficient $A_{0,0} = \rho/(2\Pi_R^{0,0})$ as a function of the asymmetry coefficient b at different densities and with the different forces, the conventions of Figure 3.

Figure 1

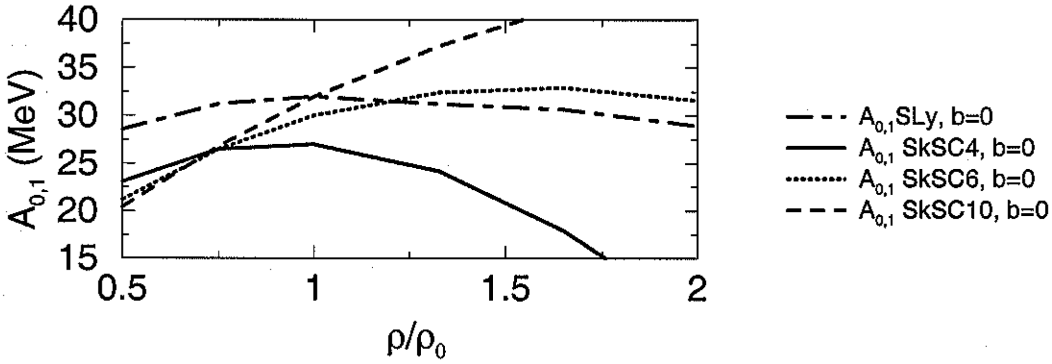

Figure 2

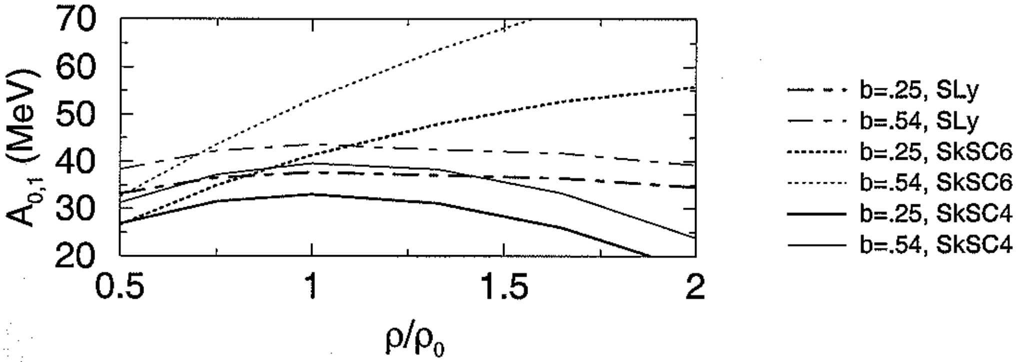

Figure 3

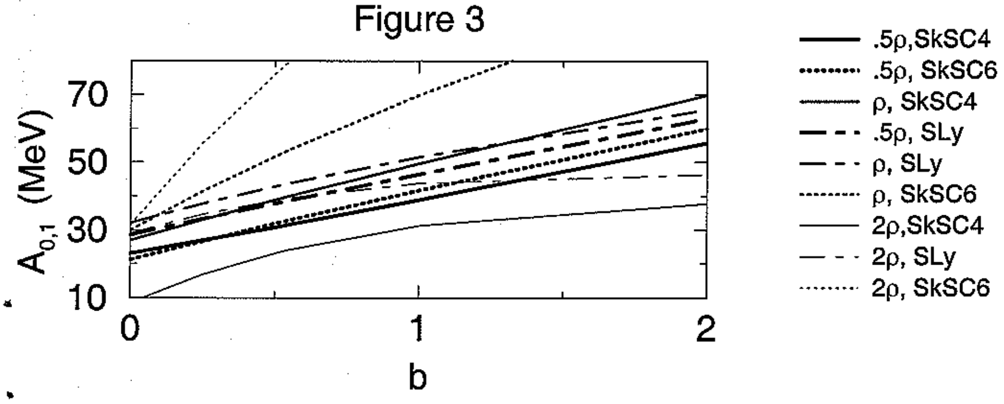

Figure 4

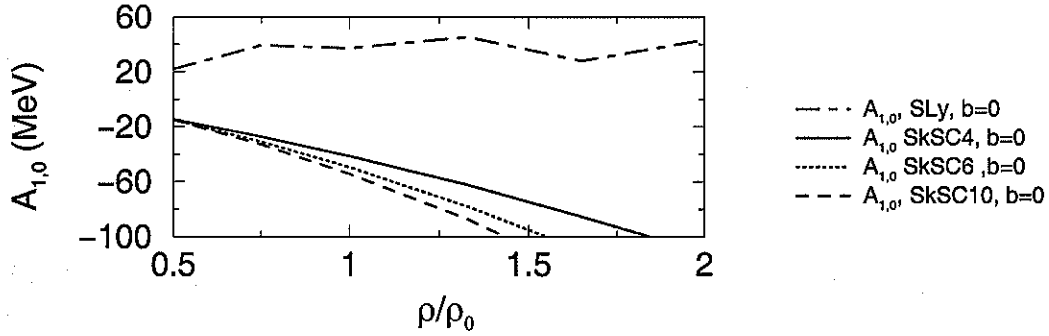

Figure 5

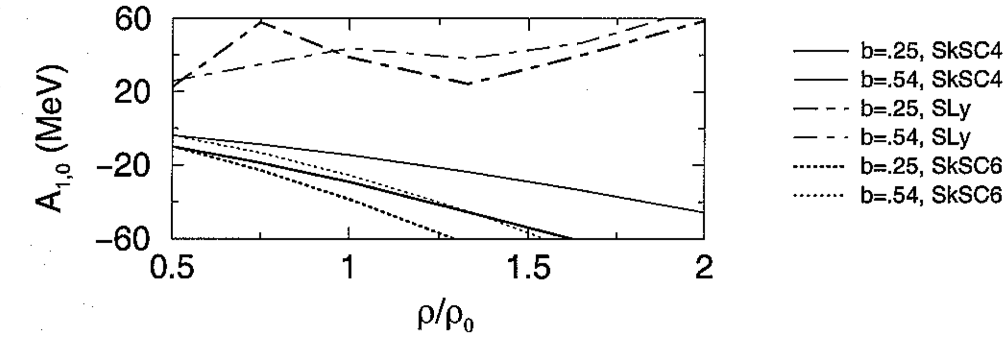

Figure 6

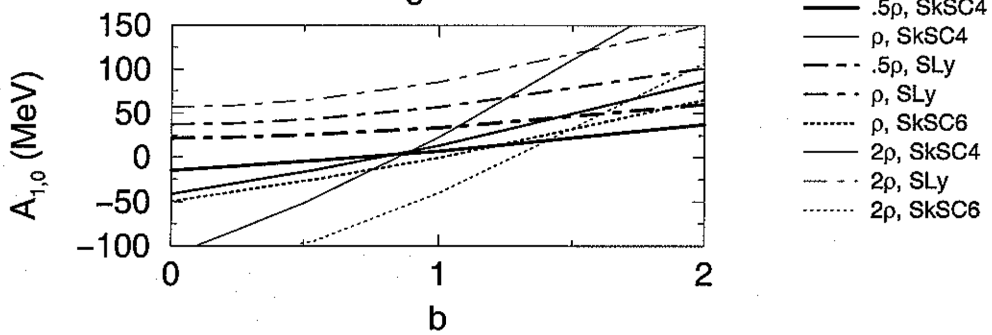

Figure 7

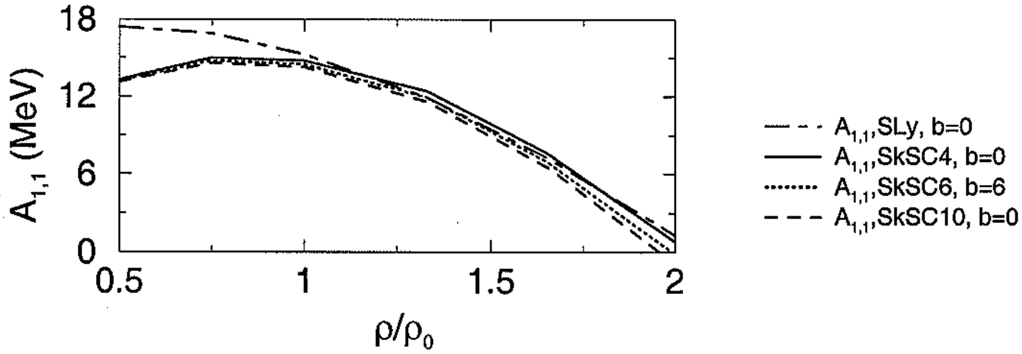

Figure 8

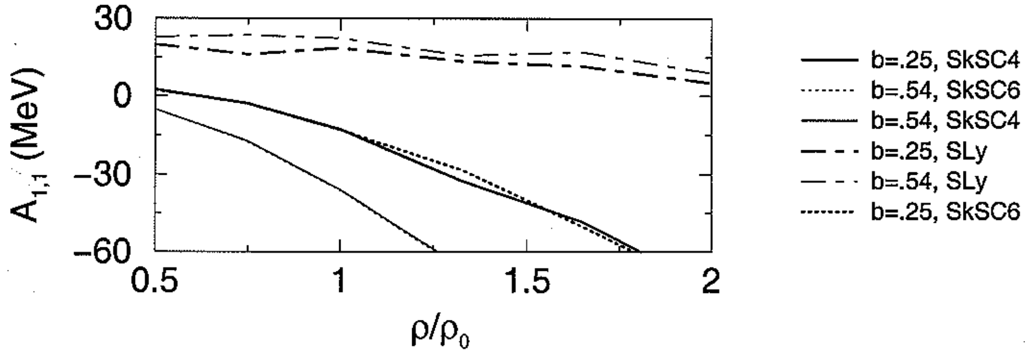

Figure 9

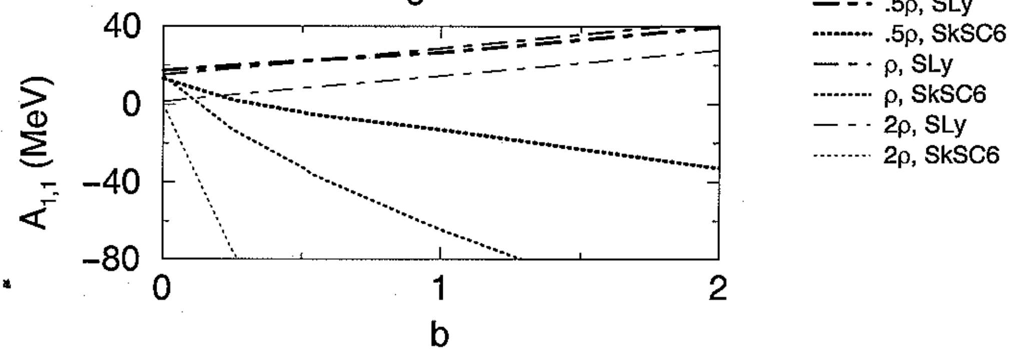

Figure 10

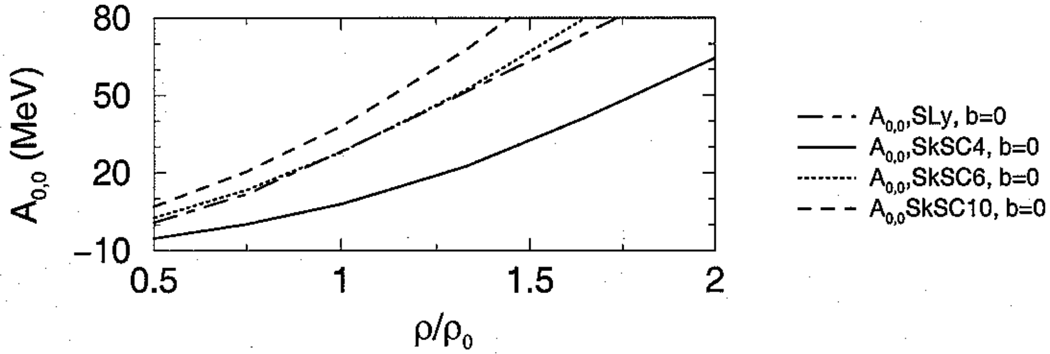

Figure 11

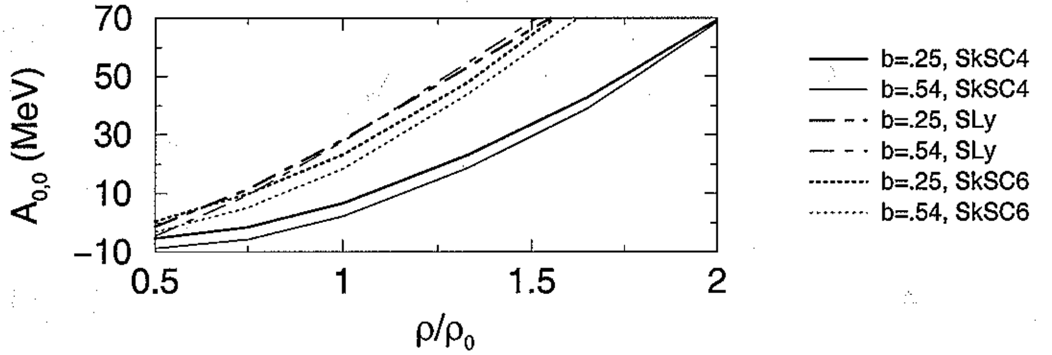

Figure 12