Instituto de Física
Universidade de São Paulo

SPONTANEOUS SYMMETRY BREAKDOWN IN NUCLEAR MATTER

BRAGHIN, FL

Instituto de Física, Universidade de São Paulo, CP 66.318
05315-970, São Paulo, SP, Brasil

Publicação IF – 1570/2001

UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

Spontaneous Symmetry Breakdown in Nuclear Matter

Fábio L. Braghin *

Nuclear Theory and Elementary Particle Phenomenology Group,

Instituto de Física, Universidade de São Paulo

C.P. 66.318; CEP 05315-970; São Paulo - SP, Brazil.

Abstract

The linear sigma model coupled to a massive vector (gauge) meson, V_μ, is considered for the description of nuclear matter properties. The field equations (of pions $\vec{\pi}$, sigma σ, nucleons N, V_μ) are analyzed in the homogeneous limit in a nearly self consistent way which yields the saturation density at the correct point. Solutions are sought semi-analytically. The pion condensate is found to be non zero at finite density corresponding to a dynamical symmetry breaking. The solution of the vector meson mean field (which can be viewed as another condensate) indicates the occurrence of another dynamical symmetry breaking at finite density. The scalar condensate, related to the QCD scalar condensate $< \bar{q}q >$, seems to decrease -as usually expected. The chiral radius at the saturation density as well as the pion and sigma masses are analyzed. The non zero pion condensate causes a splitting between the neutron and proton effective masses as well as oscillations between these two isospin states in the medium. These results indicates the existence of another QCD condensate at finite density: a pseudo-scalar $< \bar{q}q >_{ps}$.

PACS numbers: 11.15.Ex; 11.30.Qc, 11.30.Rd, 12.39.Fe, 21.30.-x, 21.65.+f

Key-words: Spontaneous symmetry breaking, nuclear matter, Goldstone bosons, isospin, saturation, gauge symmetry, chiral symmetry, condensates, oscillation.

IF- USP - 2001

*e-mail: braghin@if.usp.br

1 Introduction

Dynamical symmetries play an essential role in systems described by Strong Interactions and regulate many aspects of the structure and dynamics of strong interacting particles. Whenever a theory invariant under a continuous transformation group presents a non invariant ground state the corresponding symmetry is said to be spontaneously (or dynamically) broken. In this case the theory is expected to present zero energy collective Goldstone modes [1].

Being constituted by Hadrons, atomic nuclei should be expected to be described by the theory of the strong interactions. Given that the fundamental theory for these systems, the Quantum Cromodynamics (QCD), has a very complex non abelian structure and strong coupling constants at low energies it is very difficult to obtain a description of nuclear systems from it. In the vacuum, the lightest strong interacting particles are known to respect, approximately at least, chiral symmetry $SU_L(2) \times SU_R(2)$ which is spontaneously broken down to $SU(2)$. Pions, whose masses are small in the hadronic scale, are viewed as the Goldstone bosons of such SSB. At the same time the vacuum acquires a non trivial structure due to the formation quark-anti quark condensate $< \bar{q}q >$, the order parameter of the Chiral SSB. For low and intermediary energy processes one is lead to construct effective models which respect these main properties and symmetries of the QCD. These features can be taken into account via sigma models which, in the linear realization, implement chiral symmetry with two fields: the (pseudo-scalars) pions and the (scalar) sigma. The scalar field acquires a non zero expected value in the vacuum $< \sigma >$ due to the Chiral SSB which lowers the energy density of the vacuum. Although in the usual picture of hadronic physics the (small) pion mass is considered to break explicitly the chiral symmetry in [2] it is shown that the massive character of pions can be understood in a chiral invariant fashion if the quantum fluctuations are taken into account.

In this work we argue that the Linear Sigma Model (LSM) with a vector meson yields a suitable frame for the description of finite density nuclear matter properties and eventually of nuclei. In spite of limitations found before [3] we show new insights and we argue that those limitations may be due to the methods used so far. The self consistency of the field equations is taken into account with particular prescriptions. We found that there is a (isospin) spontaneously broken symmetry generating non zero expected value for the pion field. Correspondingly we suggest that isovector collective modes in nuclear matter are the Goldstone bosons. In the next section the linear sigma model with an Abelian massive gauge vector boson is presented and their dynamical equations are shown taking into account quantum fluctuations of the sigma and pion fields in a truncated version of a variational approach. Their solutions are discussed and some other possible consequences are analyzed.

2 Linear sigma model at finite density

The Lagrangian density of the Linear Sigma Model with nucleons $N(\mathbf{x})$, sigma and pions $(\sigma, \vec{\pi})$ coupled to a (gauge) vector meson V_μ is given by:

$$\begin{aligned} \mathcal{L} = \bar{N}(\mathbf{x})\left(i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma - i\gamma_5 \vec{\tau}.\vec{\pi})\right) N(\mathbf{x}) + \frac{1}{2}\left(\partial_\mu \sigma . \partial^\mu \sigma + \partial_\mu \vec{\pi} . \partial^\mu \vec{\pi}\right) + \\ -\frac{\lambda}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4}\left((\sigma)^2 + (\vec{\pi})^2 - v^2\right)^2, \end{aligned} \tag{1}$$

where the covariant gauge derivative is: $\mathcal{D}^\mu = \partial^\mu - i g_V V^\mu$, the gauge invariant tensor is: $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. g_V, g_S and λ are the coupling constants and $v = f_\pi$ in the vacuum. If the pion mass were considered to break the chiral symmetry explicitly we add another term $\mathcal{L}_{sb} = c\sigma$. We consider the possible existence of condensates of the sigma as well as of the pions and look for solutions. The coupling of the nucleon to these (scalar and pseudo scalar) mesonic fields yields the effective masses for N. From expression (1) the Hamiltonian can be calculated.

The nucleon field is quantized in terms of creation and annihilation operators. Its wave function can be written as superposition of spinor (χ), isospinor (η) and coordinate components $u(\mathbf{p})$, $v(\mathbf{p})$. It generates non zero scalar, baryonic and pseudo-scalar densities (ρ_s, ρ_B and ρ_{PS}). We will not explicitly evaluate here all these quantities but only make use of ρ_B. The resulting expressions for the quantized fermionic energy density ρ_f and for the baryonic density ρ_B are given respectively by:

$$\rho_f = \gamma \int^{k_F} \frac{d^3k}{(2\pi)^3} \sqrt{\mathbf{k}^2 + (M^*)^2}, \qquad \rho_B = \gamma \int^{k_F} \frac{d^3k}{(2\pi)^3}. \tag{2}$$

In these expressions $\gamma = 4$ in symmetric nuclear matter, k_F is the nucleon momentum at the Fermi surface and the effective mass M^*. In fact we will show that M^* is a matrix which depends on the isospin (and spin) of the nucleons. But for the sake of the main argument it will be considered to be a number as usually done: $M^* = g_S \bar{\sigma}$. We will see that this is the leading contribution.

To take into account the quantum fluctuations of the sigma and pion fields we consider a truncated version of the variational approach using a Gaussian trial wave-functional in the Schroedinger picture [4]. This variational principle states that a maximum bound for the energy density of the vacuum can be obtained by calculating the averaged energy with trial wave-functions ($\mathcal{H}_\phi =< \Phi|\hat{H}|\Phi >$) whose (trial) parameters are fixed when the energy density is minimum with relation to its parameters. The averaged value of the Hamiltonian is calculated with trial (Gaussian) wave-functionals for the scalar and pseudo-scalar fields: $< \tilde{\Psi}[\sigma, \vec{\pi}]|H(\sigma, \vec{\pi})|\tilde{\Psi}[\sigma, \vec{\pi}] >$. For the sigma we can write:

$$\Psi\left[\sigma(\mathbf{x})\right] = Nexp\left\{-\frac{1}{4}\int d\mathbf{x}d\mathbf{y}\delta\sigma(\mathbf{x})G_S^{-1}(\mathbf{x},\mathbf{y})\delta\sigma(\mathbf{y})\right\}, \tag{3}$$

Where $\delta\sigma(\mathbf{x}) = \sigma(\mathbf{x}) - \bar{\sigma}$; the normalization is N, the variational parameters are the condensate $\bar{\sigma} =< \Psi|\sigma|\Psi >$, the quantum fluctuations represented by the width of the Gaussian $G_S(\mathbf{x},\mathbf{y}) =< \Psi|\sigma(\mathbf{x})\sigma(\mathbf{y})|\Psi >$. An analogous expression for the pion sector is considered with variational parameters given by: $\bar{\vec{\pi}}$ and $G_P^{a,b}$, which is a matrix that can be considered as diagonal as a particular case along this work ($G_P^{a,a} = G_P$). This reduces the corresponding functional space. We will assume that these quantum fluctuations only intervene for the meson masses, as shown below, as well as for a shift the respective fields:

$$\tilde{\vec{\pi}}^2 = \bar{\vec{\pi}}^2 + G_P, \qquad \tilde{\sigma}^2 = \bar{\sigma}^2 + G_S. \tag{4}$$

This corresponds to a truncation on the self consistency with a particular renormalization energy point. More exact calculations are being done for this model as well as in the absence of pions for non linear scalar couplings in [5, 6].

The minimizations of the averaged energy with respect to the Gaussian variational parameters yield the GAP equations which define the minimum of the potential for these fields. The following set of equations is obtained for the sigma sector:

$$\begin{aligned}&\lambda\left(\tilde{\sigma}^2 + G_S + \tilde{\vec{\pi}}^2 - v^2\right) + 2\frac{d\rho_f}{d\tilde{\sigma}^2} = 0;\\ &\frac{d\rho_f}{dG_S} - \frac{G_S^{-2}}{8} - \frac{\Delta}{2} + \frac{\lambda}{4}\left(6\tilde{\sigma}^2 + 2\tilde{\vec{\pi}}^2 - 2v^2\right) = 0.\end{aligned} \tag{5}$$

From this last expression we can write that:

$$\mu_S^2 = \frac{\lambda}{2}\left(3\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2 + ...\right) \tag{6}$$

This last expression gives the sigma mass. For the sake of clearness we truncate the possible complete self consistency considering that:

$$\frac{d\rho_f}{dG_i} = 0, \qquad G_P = G_S = G \simeq constant. \tag{7}$$

The corresponding GAP equations for the pion field are:

$$\begin{aligned}&\lambda\left(\tilde{\vec{\pi}}^2 + \tilde{\sigma}^2 - v^2\right) + 2\frac{d\rho_f}{d\tilde{\vec{\pi}}^2} = 0;\\ &\mu_P^2 = \frac{\lambda}{2}\left(3\tilde{\vec{\pi}}^2 + \tilde{\sigma}^2 - v^2 + ...\right)\end{aligned} \tag{8}$$

As discussed above, these expressions (6,8) for the meson masses, with the shift of the fields (4), are the only effects of the quantum fluctuations in the present work. It is also assumed the possibility that quantum fluctuations generate a mass for the pion without break the chiral symmetry explicitly. This is implemented considering that $\bar{\sigma} \simeq 89MeV$ which is the value attributed to the chiral limit of the pion decay constant [7]. The quantum fluctuations, G, yield the missing value for $\tilde{\sigma} = f_\pi$. This is discussed in a more consistent frame extensively elsewhere [5]. Assuming the usual hypothesis of a explicit symmetry breaking ($\mathcal{L}_{sb} = c\sigma$) to generate the pion mass the results of this work are almost unchanged.

The Euler Lagrange equation for the vector meson was calculated for a gauge in which only the component V_0 is non zero and homogeneous. This is the case which is usually studied. The equation is given by:

$$g_V\left(\rho_B + V_0\frac{d\rho_B}{dV_0}\right) - m_V^2 V_0 = 0. \tag{9}$$

This equation, is as relevant for the description of nuclear matter properties (and eventually of the nuclei) as the others.

The total averaged energy density can be written as:

$$\mathcal{H} = \rho_f + g_V V_0 \rho_B - \frac{1}{2}m_V^2 V_0^2 + \frac{\lambda}{4}(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - \tilde{v}^2)^2. \tag{10}$$

The binding energy per nucleon is: $E/A = \mathcal{H}/\rho_0$.

3 Particular Truncated Self Consistent Solutions

In this section solutions for the above equations are given such that the main properties of the nuclear matter are consistently described. The stability condition of nuclear matter at the saturation density can be written as:

$$\left.\frac{d\mathcal{H}}{d\rho_B}\right|_{\rho_B=\rho_0} = \left.\frac{\mathcal{H}}{\rho_B}\right|_{\rho_B=\rho_0}, \tag{11}$$

where ρ_0 is the density of saturation. To guarantee that these expressions are satisfied we consider some prescriptions for the dependence of the variables involved in on the baryonic density. Namely:

$$\begin{array}{l} \dfrac{d\rho_f}{d\rho_B} = \dfrac{\rho_f}{\rho_B}, \\ \dfrac{d(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)}{d\rho_B} = \dfrac{(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)}{\rho_B}, \\ \dfrac{d\mathcal{H}_V}{d\rho_B} = \dfrac{\mathcal{H}_V}{\rho_B}. \end{array} \tag{12}$$

In this last expression $\mathcal{H}_V$ is the energy density contribution of the vector meson.

From the first of equations (12) we find a solution for the dependence of ρ_f on the baryonic density which is in excellent agreement with that resulting from the integration of expression (2). It is given by:

$$\rho_f = -K\frac{\rho_B}{9}Ln\left(\frac{\rho_B}{\rho_0}\right) + B\rho_B - K\frac{\rho_B^2}{9\rho_0}, \tag{13}$$

where B is a constant to be found and K is the incompressibility modulus:

$$K = 9\rho_B^2\frac{d(E/A)}{d\rho_B^2}. \tag{14}$$

From the second expression in (12) we find a constraint which can be considered as defining a "chiral radius" (encompassing an "isospin radius") in the medium:

$$(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2) = \tilde{C}\sqrt{\rho_B} + \tilde{A}. \tag{15}$$

In this expression $\tilde{C}$ and $\tilde{A}$ are constants. Whereas the first of these constants is really necessary we choose $\tilde{A} = 0$ for the sake of conciseness. Therefore in the vacuum: $\tilde{\sigma}^2 = v^2 = f_\pi^2$ as discussed above.

The GAP equations (5, 8), for the $\tilde{\sigma}$ (which is a GAP equation of the "in medium" chiral SSB) and for the $\tilde{\vec{\pi}}$ can be faced as differential equations for ρ_f:

$$(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - \tilde{v}^2) + \frac{2}{\lambda}\frac{d\rho_f}{d\tilde{\sigma}^2} = 0, \qquad (\tilde{\vec{\pi}}^2 + \tilde{\sigma}^2 - \tilde{v}^2) + \frac{2}{\lambda}\frac{d\rho_f}{d\tilde{\vec{\pi}}^2} = 0. \tag{16}$$

Where $\tilde{v}^2 = v^2 - G$. These equations are isomorfic and show an equal dependence of ρ_f with each of these chiral fields. Assuming that the main component of the nucleon effective mass is provided by the sigma expectation value $\tilde{\sigma}$ we use M^* to fix it choosing a constant g_S in the medium. However we can face these two expressions as differential equations for ρ_f. An approximated solution for the two GAP equations as if ρ_f were a function of these fields independently and neglecting the $\tilde{\vec{\pi}}^4$ term which was found to be small. The constants obtained in the solution of these equations of ρ_f are fixed by requiring that in the vacuum $\tilde{\vec{\pi}} = 0$ and $\tilde{\sigma} = v$. These solutions can be written as:

$$\begin{aligned} \tilde{\sigma}^2 &\simeq \frac{v^2}{2} - \tilde{\vec{\pi}}^2 \pm \sqrt{\left(\frac{v^2}{2} - \tilde{\sigma}^2\right)^2 + \frac{4}{\lambda}\rho_f}, \\ \tilde{\vec{\pi}}^2 &= \pm\left(-\frac{v^2}{2} \pm \sqrt{\frac{\rho_f}{\lambda} + \frac{v^4}{4}}\right). \end{aligned} \tag{17}$$

Eliminating the ρ_f from the first expression with the second we find the following approximated value for the pion condensate:

$$\tilde{\vec{\pi}}^2 \simeq \frac{\tilde{\sigma}^2(\tilde{\sigma}^2 - \tilde{v}^2)}{4(-\frac{\tilde{\sigma}^2}{2} \pm \tilde{v}^2)}, \tag{18}$$

With $g_S = 10$ and $M^* = 0.7M$ we find the values $\bar{\sigma}^2 \simeq 0.47 fm^{-2}$ and $\bar{\sigma}^2 \simeq -0.034 fm^{-2}$. Only the second value is consistent with the approximation done for expression (18).

A more consistent approach for determining the pion condensate is to seek it by facing the second of the above equations (16) as differential equation for $\rho_f = \rho_f(\tilde{\vec{\pi}}^2)$ in which $\tilde{\sigma}^2$ is given by the first with expression (17). This differential equation for $\rho_f(\tilde{\vec{\pi}}^2)$ (with $\tilde{\sigma}^2$ eliminated) yields a transcendental equation as solution. The numerical *self consistent* values are discussed in the next section. We can also estimate the pseudo-scalar density to be of the order of $\rho_{PS} \simeq \bar{\vec{\pi}}/\bar{\sigma}$.

Alternatively we can consider the condensates to be nearly independent of each other and face these GAP equations (16) as partial differential equations for which we find other solutions:

$$\begin{aligned} \rho_f &\simeq \frac{\lambda}{2}\tilde{\sigma}^2\left(\tilde{v}^2 - \tilde{\sigma}^2\right) + C_f(\tilde{\sigma}^2 - \tilde{v}^2), \\ \rho_f &= \frac{2}{\lambda}\tilde{\vec{\pi}}^2(\tilde{v}^2 - \tilde{\vec{\pi}}^2) - C_f\tilde{\vec{\pi}}^2, \end{aligned} \tag{19}$$

Where C_f is a constant. Together with the above expressions for ρ_f and the symmetry radius $\tilde{C}\sqrt{\rho_B}$ (15) we obtained a consistent basis for the study of the dependence of the condensates with density. Expecting that these solutions are valuable in some range of ρ_f as well as ρ_B we can equate expressions (19) to obtain the following constraint equation for the condensates:

$$C_f\tilde{C}\sqrt{\rho_0} = \frac{\lambda}{2}\left((\tilde{\sigma}^2 - \frac{\tilde{v}^2}{2})^2 - (\tilde{\vec{\pi}}^2 - \frac{\tilde{v}^2}{2})^2\right). \tag{20}$$

This also expresses the dynamical symmetry breaking which occur in the medium.

From these solutions and the expressions of ρ_f in terms of the baryonic density (2 or 13) we can study the dependence of the condensates with the nuclear density. This will be extensively done in [5].

We found that, at finite density, there exist a non zero pion condensate solution since the fermionic density depends on $\bar{\vec{\pi}}$ generating a pseudo scalar density. To explicit this feature we have to re-write the nucleon effective mass. In fact, from the averaged value of the energy we should consider that M^* is a matrix which depends on the isospin (and eventually spin) of the nucleons:

$$M^*_{a,b;s} = g_S < \Psi[\sigma,\pi]|. < N_{a,s}|(\sigma + i\gamma_5\vec{\tau}.\vec{\pi})|N_{b,s} > .|\Psi[\sigma,\pi] >= g_S(\bar{\sigma} + i\tilde{M}^d_{(a,b)}\bar{\vec{\pi}}_d). \tag{21}$$

In this expression a, b stands for neutrons/protons, and $\tilde{M}$ is a non diagonal isospin matrix. This matrix also includes a dependence on the nucleon spin state. The nucleonic mass can be therefore obtained the averaged value $\bar{\sigma}$ plus a contribution from the averaged value of the pion field. This allows for the possibility of different values of M^* and even for oscillations between the states of proton and neutron.

The quantities calculated with it -mainly the densities (ρ_f in particular)- will be considered to be the usual ones, i.e., with a constant diagonal effective mass - unless explicitly considered: $M^*_{a,b}(\vec{\pi}) = M^*$.

Finally, considering the equation of V_0 of (9) as a differential equation of the baryonic density ρ_B as a function of the baryonic density we obtain the following solution:

$$V_0(\rho_B) = \frac{-g_V\rho_B \pm \sqrt{g_V^2\rho_B^2 - 2C_V\rho_B m_V^2}}{m_V^2}, \tag{22}$$

where C_V is a constant. This constant will be the only contribution of the vector meson sector to the energy density $\mathcal{H}_V = C_V\rho_B$. In the limit of zero density $V_0 \to 0$ as expected. It is seen that the baryonic density generates a non zero value of V_0- a condensate. This may be seen as a dynamical symmetry breaking of this gauge symmetry. Requiring the baryonic density to be stable with relation to variations on V_0 we find that: $C_V = 2g_V\bar{V_0}$. Where $\bar{V_0}$ is the value of V_0 at the saturation density. In this point we have $m_V^2 = -2g_V\rho_0/\bar{V_0}$ which can be identified to the omega meson mass. As we have developed above, this solution makes the density stable, i.e., $d\rho_B/dV_0 = 0, \quad d^2\rho_B/dV_0^2 > 0$. This also indicates that this solution is similar to a dynamical symmetry breakdown: the saturating baryonic density is a minimum point in terms of the condensate V_0. This seems to suggest the existence of still another QCD condensate at finite density. These results are different from those usually considered and they can be compared to the contribution of the vector meson in the frame of other relativistic models [3]. The contribution to the energy density in the usual models can be written as $< \mathcal{H}' >_v = g_V^2\rho_B^2/(2m_V^2)$ which is thus four times smaller in modulus.

3.1 Numerical estimates and discussion

Considering all the terms in the averaged density energy (i.e., in the binding energy) we fix the values of $K = 200$ MeV, $-E/A = 16.0$ MeV, $\rho_0 = 0.16 fm^{-3}$. The value of the coupling constant λ is changed to search for the solutions. We take $M^* \simeq 0.7M$ and $g_S = 9$, although meaningful deviations may be considered. These values fix $\bar{\sigma} = m^*/g_S$.

In figure 1 self consistent solutions of the GAP equation of the pion condensate as a function of the coupling λ are shown (no consistent solution for negative $-60 \leq \lambda \leq 0$ was found). The dots (crosses) correspond the minimum (maximum) values which the square condensate may assume with the above parameters for each value of λ. This means that the squared pion field $\tilde{\vec{\pi}}^2$ may assume values between the dots and crosses. We see that the pion condensate may be imaginary as well as it may acquire relatively large values (remember that $f_\pi^2 \simeq .22 fm^{-2}$). There is an intriguing behavior in this figure which are the discontinuities of the values when $16 \leq \lambda \leq 43$ for the lower and upper values. Outside of this range of λ, i.e., for relatively weak and strong coupling, there are stronger constraints for its value. As a matter of fact, the values of the pion condensate are related to the values which the extended chiral radius $\tilde{C}$ may assume. The discontinuities in this figure are not yet very well understood. The corresponding maximum and minimum values for the Symmetry Radius $\tilde{C}$ from expression (15) can be seen in Figure 2 as a function of λ. The same behavior of figure 1 is found because $\tilde{\sigma}$ was kept constant. These results can be in agreement with the usual idea of symmetry restoration as precluded in other works [8]. However a more extensive comparison will be left for another work.

As discussed below the non zero pion condensate leads to a splitting in the nucleon masses, expression (21), in the medium. This is probably connected with the Nolen-Schiffer effect which relates the nucleon effective masses to the scalar QCD condensate $< \bar{q}q >$ [9]. Furthermore we can associate $\bar{\tilde{\pi}}$ to a pseudo-scalar condensate $< \bar{q}q >_{ps}$ that should be non zero at finite baryonic density. This mass splitting is nearly given by:

$$\Delta M^* = M_n^* - M_p^* \simeq 2ig_S|\bar{\tilde{\pi}}|. \tag{23}$$

Considering the imaginary solutions of $\bar{\tilde{\pi}}^2$ from figure 1 with $\lambda \simeq 60$ (only $\bar{\tilde{\pi}}^2 < 0$) we find that $\Delta M^* \simeq 40 g_S$ MeV which is seemingly too much large. The inverse reasoning can be done and then $g_S|\bar{\tilde{\pi}}|$ can be fixed to reproduce an expected ΔM^*.

The above solutions were found by fixing the scalar condensate to fit the effective mass of the nucleon, the $\bar{\tilde{\pi}}$ contribution was assumed and found much smaller. Results are consistent. However this was done for a coupling $g_S = 9$ which is not necessarily true. The scalar condensate could then be smaller or greater than $\tilde{\sigma}$ in the vacuum which would correspond to g_S smaller or greater by considering a fixed effective mass for the nucleon. These would correspond to the symmetry restoration or, in a less appealing scenario, to further breaking of the chiral symmetry in the medium.

For some of the solutions of figures 1 and 2 we check the binding energy of nuclear matter by means of expression (10). This fixes the constant C_V from the vector meson solution. Some values are shown below:

$$\begin{aligned} \lambda \simeq 16.0 &\rightarrow \sqrt{-\tilde{C}\sqrt{\rho_0}} \simeq 25MeV \rightarrow C_V/\rho_B \simeq -3.6fm^{-1}, \\ \lambda \simeq 40.0 &\rightarrow \sqrt{+\tilde{C}\sqrt{\rho_0}} \simeq 55MeV \rightarrow C_V/\rho_B \simeq -3.9fm^{-1}. \end{aligned} \tag{24}$$

Although C_V presents close values in these examples there is a large difference between these two solutions. In the first the value of the pion condensate is small ($\bar{\tilde{\pi}}^2 \simeq 0.05fm^{-2}$), the constant $\tilde{C}$ is small and negative indicating that $\tilde{\sigma}^2 + \bar{\tilde{\pi}}^2 < v^2$ at the saturation density. In the second case $\tilde{C}$ is large and positive due to the large value of the pion condensate $\bar{\vec{\pi}}^2 \simeq 0.14 fm^{-2}$ and then $\bar{\sigma}^2 + \bar{\vec{\pi}}^2 > v^2$. The values of C_V are consistent with estimates from the expressions of the last section taking $m_V \simeq 780 MeV$ and $g_V \simeq 4$.

In figure 3 we show the behavior of the ratio of the pion mass in the medium divided by its value in the vacuum for the solutions shown in figures 1 and 2 as a function of the coupling λ - still keeping $\bar{\sigma}$ constant. By varying the scalar condensate we can obtain different results - The increasing values may be associated to the restoration of chiral SSB. A complete account of these possibilities will be shown elsewhere [5].

In Figure 4 we show values of the ratio of the sigma mass in the medium to its value in the vacuum according to expression (6) as a function of the λ for squared pion condensate of figure 1. m_σ in the medium can be smaller or greater than its value in the vacuum. It is curious that the sigma mass in the medium, for lower values of the λ is lower than its value in the vacuum ($\mu_S^0 \simeq 482 MeV$ in the present work) whereas for higher values of λ it becomes higher than μ_S^0.

From the GAP equations of pion and sigma we can write the ratio of their masses as:

$$\frac{\mu_\pi^2}{\mu_\sigma^2} = \frac{2\bar{\vec{\pi}}^2 + \tilde{C}\sqrt{\rho_B}}{2\bar{\sigma}^2 + \tilde{C}\sqrt{\rho_B}}. \tag{25}$$

This expression reduces to a non zero finite value in the vacuum according to the assumptions done for the pion mass.

3.2 General remarks

The equation (8) for the pion mean field of is a GAP equation of the condensate $\bar{\vec{\pi}}$. This induced a mass splitting between neutrons and protons which makes the density ρ_f from expression (2) -and eventually ρ_B- to be different. The corresponding induced density difference $\rho_n - \rho_p$ can be also associated to the order parameter of this (isospin) spontaneously symmetry breaking as described by a Landau model [10]. This non trivial solution corresponds to a non invariant ground state under exchange of protons into neutrons transformations, although the Lagrangian is symmetric. The nuclear Ground State potential for symmetric ($N = Z$) nuclear systems is not invariant under transformations of protons into neutrons (and vice-versa). Zero energy (Goldstone) collective modes are therefore expected to occur. They can be found by means of the calculation of the response function of the nuclear matter [11, 12]. Non zero width can be obtaining by considering pairing effects which in fact destroys the zero energy character of the modes.

Other resonances -"spinorials"- are not necessarily collective motions but coherent at the saturation density and can become very collective, for example, in neutron stars and supernovae [12]. Scalar resonances are also observed as the monopolar and dipolar ones (a nearly exhaustive study with Skyrme type effective interactions was done in the second reference of [12]), and seem to be also manifestation of other dynamically broken symmetries [5]. From the 1970's onwards, several resonances were found to occur in nuclei, with respect to multipolarity as well as the corresponding quantum number (isospin and spin) [13]. They are actually found not only in nuclear ground states but also in excited states -"hot", fast rotating nuclei-, as predicted by Brink and Morinaga [14]. It is very plausible that the isovector ones correspond to the manifestation of the SSB found in the present work in nuclei. Their descriptions are more involved due to finite size and other effects like pairing which makes the zero energy character of the resonance disappears. The width of the Isovector Giant Dipole collective modes, for example, may disappear at nearly the same nuclear temperature at which the (so-called "liquid-gas") phase transition is observed in nuclei, i.e., nearly $T \simeq 4 \to 6MeV$, [15, 16]. This phase transition occurs at fragmentation densities, of the order of $\rho \simeq 0.15 - 0.3\rho_0$ which are of the same magnitude of those found for the appearance of scalar instabilities in nuclear matter with non relativistic Skyrme interactions [17]. It seems to correspond to the manifestation of the restoration of the SSB which leads to the formation of the pion condensate.

To what extend this SSB and chiral SSB as well as the underlying gauge symmetry for the vector meson - leading to finite saturation density with the right binding energy - are related is not yet clear.

Processes involving pions in the nuclear medium seem to provide valuable information. Let us take for granted that the Goldberger Treiman relation nearly holds at the saturation density. If we write it in such a way as to encompass quantum fluctuations with the rearrangement of the scalar condensate as considered in expression (4) we can write independently for protons or neutrons (which now would have non degenerated masses):

$$g_S \tilde{\sigma} = (M^* \pm \Delta M^*) g_A, \tag{26}$$

Where ΔM^* is given below expression . For in the vacuum (where $\Delta M^* = 0$, $\tilde{\sigma} = f_\pi$ and $M^* \to g_S \bar{\sigma}$) we obtain a small value $g_A \simeq 1.05$. It could not be expected to result a realistic value for g_A with the present arguments, but we can expect that the behavior at finite density would be reasonable.

The fact that the non linear models are usually accepted as more suitable for the nuclear observables may be a consequence of the fact that the calculations done with them take into account (explicitely) more properly nonlinearities which can also be present in the linear sigma models. This means that a truncation is always done in the self consistency of the coupled equations and the non linear models seem to take into account (effectively) more nonlinearities which would be present in more self consistent calculation for the LSM.

Acknowledgement

This work was supported by FAPESP, Brazil.

References

[1] J. Goldstone, Proc. Roy. Soc. (London) **239** (1957), 267.

[2] F.L. Braghin, *in preparation* .

[3] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16**, (1986) 1.

[4] P.M. Stevenson, Phys. Rev. **D 32**, 1389 (1985); W.A. Bardeen, M. Moshe, Phys. Rev. **D 28**, 1372 (1983); A.K. Kerman, C. Martin, D. Vautherin, Phys. Rev. **D 47**, 632 (1993) and references therein.

[5] F.L. Braghin, *in preparation.* F.L. Braghin, *accepted for publication in Phys. Rev. D*, hep-ph/0106338.

[6] F.L. Braghin, M. Bracco and M. Chiapparini, *in preparation.*

[7] J. Gasser and H. Leutwyler, Ann. of Phys. (NY) **158**, (1984) 142.

[8] G.E. Brown and M. Rho, Phys. Rep. **269**, (1996) 333.

[9] J.A. Nolen and J.P. Schiffer, Ann. Rev. Nucl. Sci. **19**, (1969) 471.

[10] J.W. Negele and H. Orland, *Quantum Many Particle Systems*, Addison Wesley, (1988).

[11] F. L. Braghin and D. Vautherin, Phys. Lett. **B 333** (1994) 289; F.L. Braghin, Doctoral thesis, Université de Paris XI, Orsay, France, 1995, unpublished; F.L. Braghin, D. Vautherin and A. Abada, Phys. Rev. **C 52** (1995) 2504.

[12] F.L. Braghin, Phys. Lett. **B 446** (1999) 1; F.L. Braghin, Nucl. Phys. **A 665** (2000) 13.

[13] M.N. Harakeh and A.E.L. Dieperink, Phys. Rev. **C 23**, (1981) 2329; B.F. Davis et al., Phys. Rev. Lett. **79** (1997) 609. F. Osterfeld, Rev. of Mod. Phys. **64** (1992) 491. C. Djalali et al., Phys. Rev. **59** (1999) 118.

[14] D. Brink, Doctoral Thesis, University of Oxford, UK, (1955). H. Morinaga, Phys. Rev. **101** (1956) 100.

[15] J. Pochodzalla *et al*, Phys. Rev. Lett. **75**, 1040.

[16] J. J. Gaardhoje, Ann. Rev. Nucl. Part. Sci. **42**, 483 (1992); N. Vinh Mau, *in* II TAPS, Guardamar -Spain, June (1993).

[17] F.L. Braghin, accepted to publication in Nucl. Phys. **A** (2001).

Figure caption

Figure 1 Squared pion condensate $\tilde{\vec{\pi}}^2$ (fm^{-2}) as a function of the coupling λ for $M^* = 0.7M$ and $g_S = 9$ found self consistently.

Figure 2 Symmetry radius $\tilde{C}$ $(fm^{-1/2})$ for the solutions of figure 1 as a function of λ.

Figure 3 Ratio of the squared pion mass in the medium divided by its value in the vacuum as a function of λ for the solutions of figure 1.

Figure 4 Ratio of the squared sigma mass in the medium divided by its value in the vacuum as a function of λ for the solutions of figure 1.

Figure 1

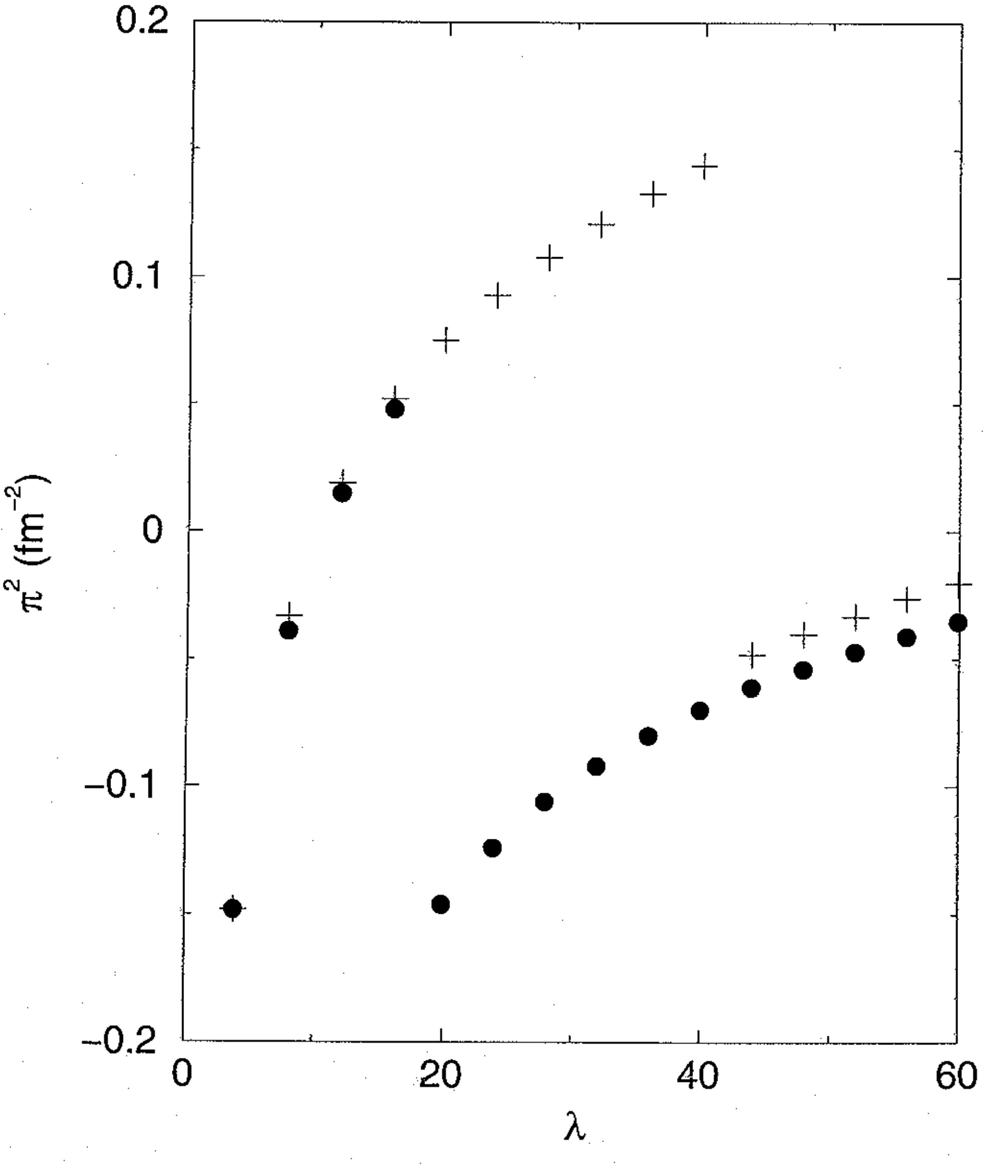

Figure 2

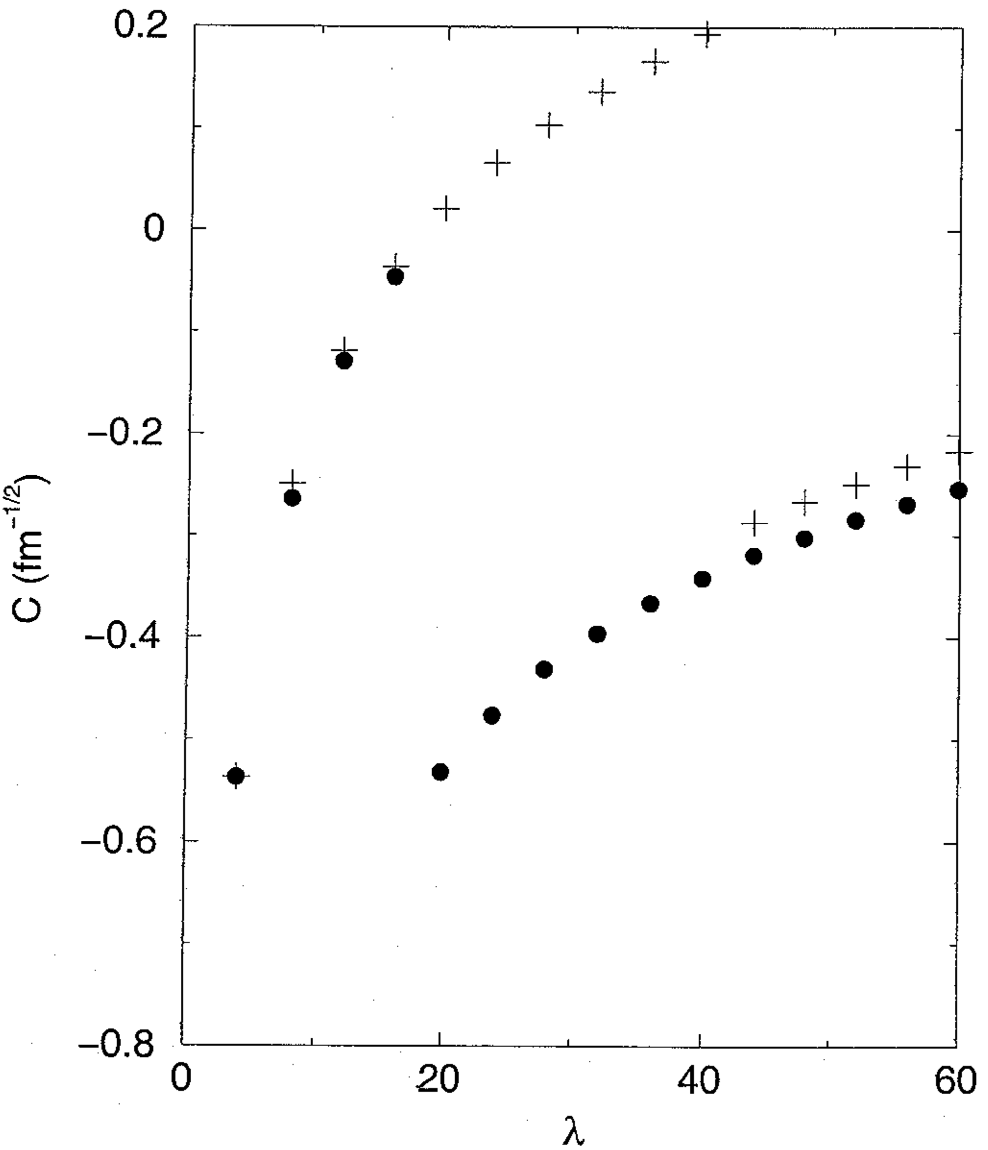

Figure 3

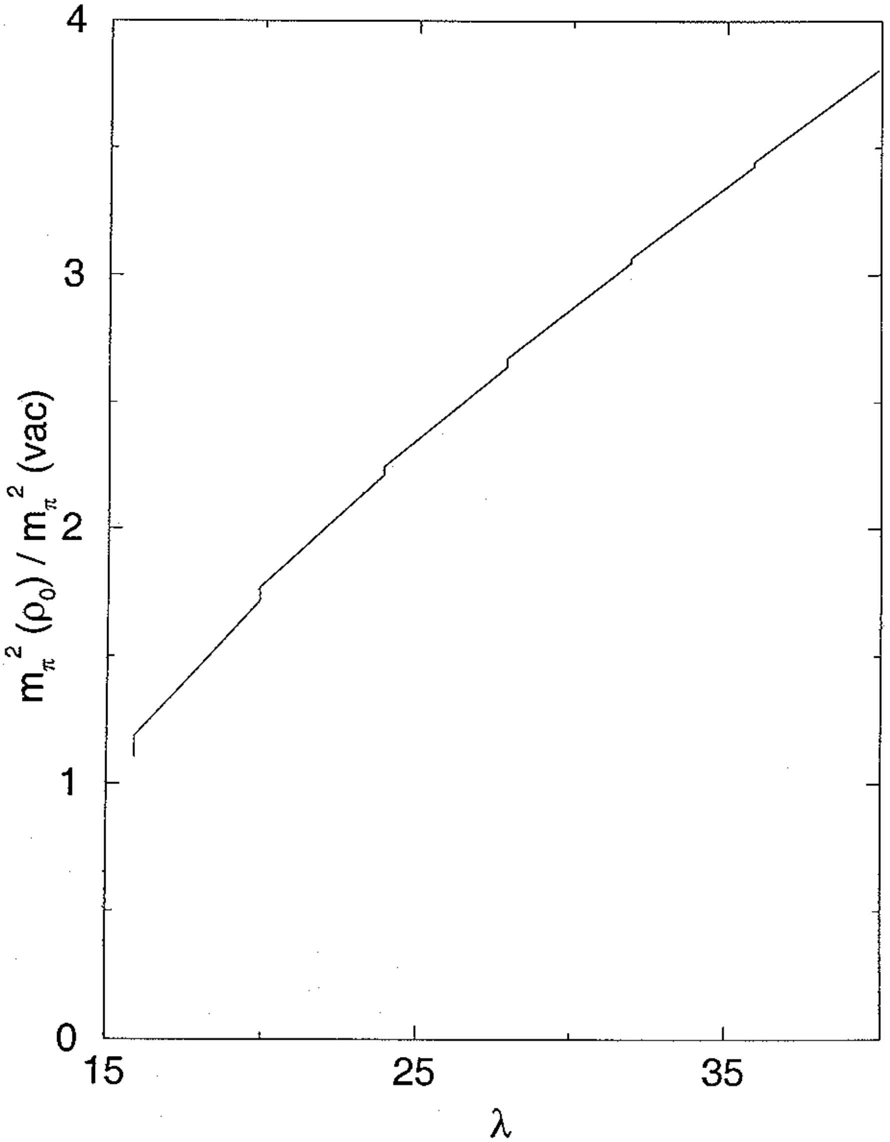

Figure 4