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MODIFIED LINEAR SIGMA MODEL AT FINITE FERMIONIC DENSITY WITH DYNAMICAL SYMMETRY BREAKING

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We consider the linear sigma model invariant under $O(4)$ transformations coupled to baryons and to a massive vector gauge meson is considered for the description of a finite baryonic density system. The Euler Lagrange equations are considered such that a stability equation for a bound system is satisfied with particular prescriptions. All the bosonic components are found to have non zero expected classical values (condensates) at finite density corresponding to dynamical symmetry breakings.

1 Introduction

Quantum Cromodynamics (QCD) has a very complex non abelian structure and strong coupling constants at low energies being very difficult to obtain exact solutions. One is therefore lead to construct effective models which respect the main properties and symmetries of the QCD for those energy ranges. In the vacuum, the lightest strong interacting particles are known to respect, approximately at least, chiral symmetry $SU_L(2) \times SU_R(2)$ which is spontaneously broken down to $SU(2)$. Pions, whose masses are small in the hadronic scale, are viewed as the Goldstone bosons of such SSB. The vacuum acquires a non trivial structure due to the formation quark-anti quark condensate $< \bar{q}q >$, the order parameter of the Chiral SSB. These features can be taken into account via sigma models which, in the linear realization with mesons, implement chiral symmetry with two fields: the (pseudo-scalars) pions and the (scalar) sigma [1]. At finite density, QCD is known, and expected, to have a very complex phase diagram with the appearance of other condensates [2]. With different approaches finite density QCD and effective models have been intensively studied in the last years.

In this work we study the O(N) Linear Sigma Model (LSM) coupled to baryons which form an infinite bound stable system with a massive vector meson corresponding to a particular case of that developped in [3]. The exact field equations are truncated to allow for analytical solutions by considering particular prescriptions for the stability condition of the system.

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2 Linear sigma model with vector meson at finite density

The Lagrangian density of the chirally symmetric Linear Sigma Model for nucleons $N(\mathrm{x})$, sigma and pions $(\sigma, \vec{\pi})$ covariantly coupled to a massive gauge vector meson V_μ is:

$$\mathcal{L} = \bar{N}(\mathrm{x}) \left(i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma + i\gamma_5 \vec{\tau}.\vec{\pi}) - \mu_0\gamma_0 \right) N(\mathrm{x}) + \frac{1}{2} \left(\partial_\mu\sigma.\partial^\mu\sigma + \partial_\mu\vec{\pi}.\partial^\mu\vec{\pi} \right) + \\ -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{4} \left((\sigma)^2 + (\vec{\pi})^2 - v^2 \right)^2 + \frac{1}{2} m_V^2 V_\mu V^\mu, \tag{1}$$

where the covariant gauge derivative is: $\mathcal{D}^\mu = \partial^\mu - ig_V V^\mu$, the gauge invariant tensor is: $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. The chemical potential is μ_0, g_V, g_S and λ are the coupling constants and $v = f_\pi$ is the pion decay constant in the vacuum. The coupling of the temporal component of the vector meson to the baryons is equivalent to the definition of a chemical potential. We consider the possible existence of classical components (condensates) for all the mesonic fields and look for solutions.

The baryonic degrees of freedom are incorporated by means of the densities: scalar, baryonic and pseudo-scalar densities (ρ_s, ρ_B and ρ_{PS}). The fermionic density can be approximatedly written in terms of the baryonic density as [4]:

$$\rho_f = 4 \int^{k_F} \frac{d^3k}{(2\pi)^3} \sqrt{\mathbf{k}^2 + (M^*)^2}, \qquad \rho_B = 4 \int^{k_F} \frac{d^3k}{(2\pi)^3}. \tag{2}$$

In these expressions k_F is the nucleon momentum at the Fermi surface.

In the homogeneous case the Euler Lagrange equations define the minimum of the potential for these fields. The following set of equations is obtained for the sigma, pions and vector meson fields:

$$\begin{aligned} &\bar{\sigma}\lambda \left(\bar{\sigma}^2 + \bar{\vec{\pi}}^2 - v^2 \right) + \frac{d\rho_f}{d\bar{\sigma}} = 0; \\ &\lambda\bar{\pi}_a \left(\bar{\vec{\pi}}^2 + \bar{\sigma}^2 - v^2 \right) + \frac{d\rho_f}{d\bar{\pi}_a} = 0; \\ &g_V \left(\rho_B + V_0 \frac{d\rho_B}{dV_0} \right) - m_V^2 V_0 = 0. \end{aligned} \tag{3}$$

In this way, self consistent solutions of the interacting fields can be found analytically by solving partial differential equations. We could impose still further self consistency considering mutual (implicit) dependences, as for example, by taking simultaneously: $\rho_f = \rho_f(\sigma, \vec{\pi}, V_0)$ and $V_0 = V_0(\rho_B, \sigma, \vec{\pi})$ and so on. The system of coupled equations become hard to solve.

The equation for the classical value of the pion field (condensate) has non zero solutions whenever the derivative of the fermionic density with respect to the pion field is non zero. This occurs because:

$$\frac{d\rho_f}{d\bar{\pi}_a} = \frac{d\rho_f}{d\bar{\sigma}}\frac{d\bar{\sigma}}{d\bar{\pi}_a} \neq 0. \tag{4}$$

Therefore the complete self consistency of the equations makes the pion classical field be non zero. Besides that the effective mass is explicitely modified with the pion condensate causing a splitting between the neutron and proton masses and the possibility of oscillations between these two isospin states [3]. The physical masses for each of the bosonic quanta (ϕ_i) are found by considering the usual deviations around the minimum of each effective potential: $\phi_i \to <\phi_i> + \delta_i$. The pion mass can be set to zero establishing a relationship between $\bar{\sigma}(vac)$ and v.

The Euler- Lagrange equation for V_0 is faced as a differential equation of the baryonic density ρ_B as a function of V_0 we obtain the following solution:

$$V_0(\rho_B) = \frac{-g_V\rho_B \pm \sqrt{g_V^2\rho_B^2 - 2C_V\rho_B m_V^2}}{m_V^2}, \tag{5}$$

where C_V is a negative constant. This constant will be the only contribution of the vector meson sector to the energy density $\mathcal{H}_V = C_V\rho_B$. In the limit of zero density we get $V_0 \to 0$.

The stability equation for nuclear matter is solved by separating the total binding energy $E/A = \mathcal{H}/\rho_B$ into three parts (fermionic, for the vector meson and for the spin zero fields) which are considered to be approximatedly independent in the stability equation: $\mathcal{H} = \rho_f + \mathcal{H}_V + \mathcal{H}_{LSM}$. For this expression we therefore consider:

$$\begin{aligned} \frac{d\mathcal{H}_V}{d\rho_B} &= \frac{\mathcal{H}_V(\rho_0)}{\rho} \\ \frac{d\rho_f}{d\rho_B} &= \frac{\rho_f(\rho_0)}{\rho} \\ \frac{d\mathcal{H}_{LSM}}{d\rho_B} &= \frac{\mathcal{H}_{LSM}(\rho_0)}{\rho} \end{aligned} \tag{6}$$

The solutions for these equations can also be solutions for the equations of motion depending on the constants to be fixed when solving first order differential equations. The full reliabity of this approximation is under investigation. Within this approximation the incompressibility modulus of the medium is analytically calculated [3].

With the present calculation we found non zero pion and sigma classical fields at finite baryonic (stable) density. From equations (3) an approximate value for the squared classical value of the pion can be given by:

$$\bar{\vec{\pi}}^2 = \frac{\sigma^2(\sigma^2 - v^2)}{4(-\frac{\sigma^2}{2} \pm v^2)}. \tag{7}$$

As boundary conditions we use that in the vacuum $\bar{\sigma} = v$ and $\bar{\vec{\pi}}^2 = 0$ representing the chiral symmetry spontaneously broken and exact isospin symmetry. We found that the values of the classical values $\bar{\sigma}$ can increase or diminish with relation to the value of the vacuum, although the most widespread and reasonable solution is a tendency towards zero, the restoration of chiral SSB. On the other hand the pion classical field (condensate) becomes non zero breaking isospin symmetry. Extensive numerical studies will be reported elsewhere [3].

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