Instituto de Física
Universidade de São Paulo

EXTERNAL CHROMO MAGNETIC FIELD IN COLOR SUPERCONDUCTIVITY WITH A TEST MODEL

BRAGHIN, FL Ñ é Docente

Instituto de Física, Universidade de São Paulo, CP 66.318
05315-970, São Paulo, SP, Brasil

Publicação IF – 1580/2003

UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

External chromo magnetic field in color superconductivity with a test model

Fábio L. Braghin

Instituto de Física, Universidade de São Paulo

- Understanding strong interactions: phase diagram
- Matter at high densities: other states (color/flavor degrees of freedom)
- Neutron stars and dense astrophysical objects

- Superconductivity (idea)
- in Quantum Chromodynamics (idea)
- External chromo magnetic field
- Test model: interaction due to scalar boson exchange
- Preliminar results

APRESENTADO NA / PRESENTED IN
1º JORNADA SOBRE TEORIAS DE GAUGE E
ASSUNTOS RELACIONADOS, AGOSTO 2002 - IF-USP
SÃO PAULO, BRAZIL

This work has been initiated with the investigation of the effect of classical vector fields (for example as those considered in relativistic model of nuclear systems) which leads naturally to a redefinition of chemical potentials for finite density systems. The external chromofield was done, in part but not only, due to a sort of mathematical generalization. Modifications of the phase diagram does appear due to this fields like exemplified in this seminar and will be discussed with more details in a forthcoming paper.

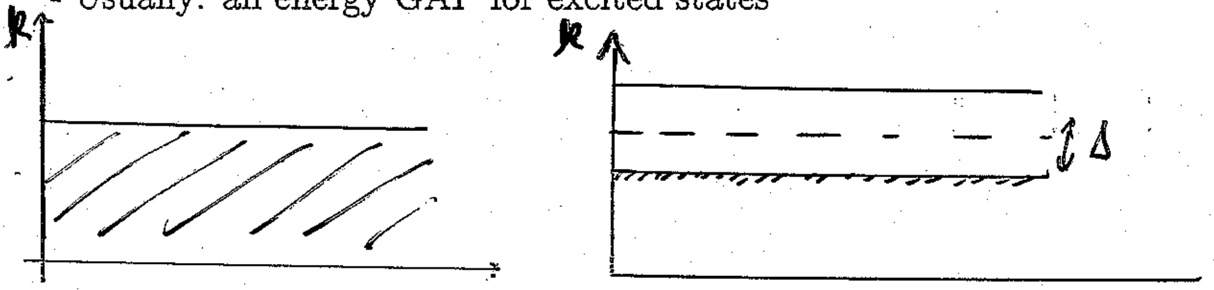
The work was started at the Theory Group of the Physics Department of Brookhaven National Laboratory, USA, in December 2001 and continued in Sao Paulo, Brazil. The author wishes to thank the warm discussions held with R.P. Pisarski, J. Lenaghan, D. Rischke, L. McLerran, C. Bertulani, C.L. Lima and S. Padula. The work was supported by FAPESP, Brazil.

1 Superconductivity: idea

- Fermi system at finite density (usual Bardeen-Cooper-Schrieffer)
  - Very low temperatures: zero voltage difference with finite current
  - There appears a condensate (pairing between charged particles) ATTRACTIVE INTERACTION DESTABILIZE FERMI SURFACE

$<\psi_i\psi_j> \equiv \Delta_{i,j} \neq 0$: it "breaks (spontaneously) a gauge symmetry"

  - Usually: an energy GAP for excited states



- In principle: $<\Phi|\bar{\psi}\bar{\psi}\psi\psi|\Phi> \to \underline{<\Phi|\bar{\psi}\bar{\psi}|\Phi>}<\Phi|\psi\psi|\Phi>$ $|\Phi\rangle = \sum_i a_i |N_i\rangle$

particle-particle cond. $=$ anti-particle-anti-particle cond. unless for a phase

- However $\bar{\psi}$ are far from Fermi surface: $\underline{<\bar{\psi}\bar{\psi}>} \to 0$
- Δ constant (exclusion of magnetic field)
- $\Delta = \Delta(k)$, eg. Larkin-Orchinnikov-Fulde-Ferrell (crystaline)

when used with care, the notion of spontaneous gauge symmetry breaking can be extremely convenient fiction

Rajagopal and Wilczek (about color superconductivity),

2 Color superconductivity in QCD

At finite densities: $\Delta \sim < q_i^T q_j > \neq 0$ (color antitriplet channel)

- Barrois (1977) / Bailin-Love (1984) $\Delta \sim 1 MeV$

- Alford *et al*, Rapp *et al* 1998, $\Delta \sim 100 MeV$

  Attractive interaction in color anti-triplet channel: $< q_i^T \Gamma q_j >$

(Effective) Interactions between quarks usually considered:

1) Instanton induced interactions ('t Hooft 1978) (or similar Nambu-Jona-Lasinio like int.) generates $< qq >$

$$\mathcal{L} = \frac{G}{4(N_c^2-1)} \left\{ \frac{1}{4N_c} (\bar{\psi} \sigma_{\mu\nu} \tau_\alpha^- \psi)^2 + \frac{(2N_c - 1)}{2N_c} \left((\bar{\psi} \tau_\alpha^- \psi)^2 + (\bar{\psi} \gamma_5 \tau_\alpha^- \psi)^2 \right) \right\} \quad (1)$$

eg. Rapp *et al* 1998

2) One gluon exchange: very high densities, asymptotic freedom, leading processes with g weak

- extensively studied: eg. Schafer and Wilczek / Pisarski and Rischke (1999/2000)
- $T_C \simeq .567 \Delta_0$ $\quad \Delta(T)/\Delta(T=0)$

  the same as in BCS-type superconductivity
- but $\Delta \sim exp(-\pi/2\bar{g})$ in contrast to $\Delta \sim exp(-a/g^2)$ in usual BCS

- At the end: Δ_{QCD} seems to be gauge invariant (Pisarski,Rischke)

- QCD vacuum: condensates

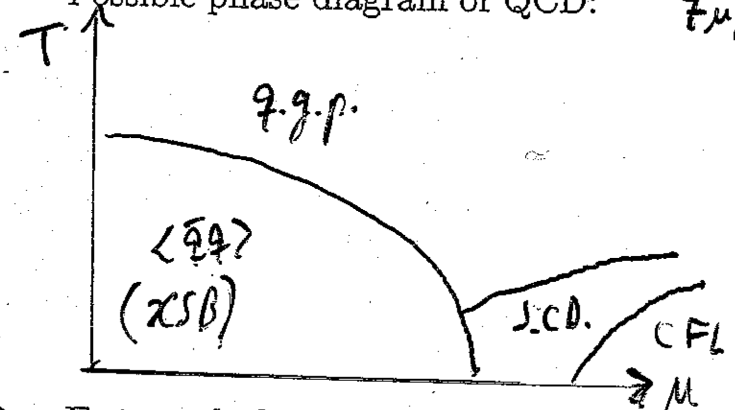
1) scalar $< 0|\bar{q}^i_r q^i_l|0 >$ (chiral SSB)

2) $< 0|F^a_{\mu\nu} F^{\mu\nu}_a|0 >$ (trace anomaly)

- Chromo electric and magnetic classical fields: $E^a_i \equiv F^a_{i,0} \quad \epsilon_{i,j,k} B^a_k \equiv F^a_{i,j}$ where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$

- As (fermionic/baryonic) density increases $< \bar{q}q >$, $< F_{\mu\nu} F^{\mu\nu} >$ change and there appears $< qq >$: non trivial phase diagram

- Possible phase diagram of QCD: q_u, q_d $(m \to 0)$ + q_s $(m_s \sim 200\,MeV)$



$\longrightarrow \langle q^{i\alpha}_a q^{j\beta}_b \epsilon^{ij} \rangle \propto \epsilon_{abC} \epsilon^{\alpha\beta C}$

3 External chromo magnetic field

- Ebert and Volkov (PLB 272, (1991)): in a Nambu-Jona-Lasinio type model $< F_{\mu\nu} F^{\mu\nu} >$ can be mimicked by means of external classical A^a_μ

- In a dense medium (relat. heavy ions collisions) A^a_μ may differ considerably from vacuum values (however high T destroys $< qq >$ as well as other condensates)

- Inequivalent vector potential yield same (chromo) magnetic field

- D. Ebert *al* with NJL type interaction - external $\vec{H}^a$ may enhance Δ (even at $\mu = 0$)

- several works, eg. Alford, Berges, Rajagopal, Gorbar, Ebert *et al*

4 Superconductivity due to a scalar boson exchange with an external chromo magnetic field

Scalar boson (ϕ) exchange generates difermions $< q_a q_a >$

Extension of the Approach of Pisarski-Rischke Phys. Rev. **D 60** 094013 (1999)

$$\mathcal{L} = \overline{\Psi}\left(i\gamma_\mu D^\mu + \gamma_0\mu - g\phi\right)\Psi + \frac{1}{2}D_\mu\phi D^\mu\phi - \frac{1}{2}M^2\phi^2 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2}$$

Where $D_\mu = \partial_\mu - g' A^a_\mu \lambda_a/2$.

Scalar field in chromo electromagnetic field:

$$\begin{aligned} &Tr\mathcal{D}^\mu\phi(\mathrm{x})\mathcal{D}_\mu\phi(\mathrm{x}) \to \partial^\mu\phi(\mathrm{x})\partial_\mu\phi(\mathrm{x}) + Tr\frac{g'^2}{4}A^a_\mu A^\mu_a\phi^2(\mathrm{x}) \\ &M^2 \to \tilde{M}^2 \end{aligned} \tag{3}$$

Integrating out the scalar field:

$$I\left[\bar{\psi},\psi\right] = \int d^dx d^dy\left(\bar{\psi}(\mathrm{x})(G_0^+)^{-1}(\mathrm{x},\mathrm{y})\psi(\mathrm{y}) + \frac{g}{2}\bar{\psi}(\mathrm{x})\psi(\mathrm{x})D(\mathrm{x},\mathrm{y})\bar{\psi}(\mathrm{y})\psi(\mathrm{y})\right) \tag{4}$$

where $D(\mathrm{x},\mathrm{y})$ is the scalar field propagator

In the mean field approach:

$$\begin{aligned} &\psi\bar{\psi} \to \psi\bar{\psi} - <\psi\bar{\psi}>; \quad (\psi\bar{\psi})^\dagger \to (\psi\bar{\psi})^\dagger - <\psi\bar{\psi}>^\dagger, \\ &\psi\psi \to \psi\psi - <\psi\psi>; \quad (\psi\psi)^\dagger \to (\psi\psi)^\dagger - <\psi\psi>^\dagger. \end{aligned} \tag{5}$$

For which the quartic interaction may be

$$g_a \frac{g^2}{2} \bar{\psi}(\mathbf{x})\psi(\mathbf{y}) D(\mathbf{x},\mathbf{y}) \bar{\psi}(\mathbf{x})\psi(\mathbf{y}) + g_b \frac{g^2}{2} \bar{\psi}(\mathbf{x})\psi(\mathbf{x}) D(\mathbf{x},\mathbf{y}) \bar{\psi}(\mathbf{y})\psi(\mathbf{y}) \tag{6}$$

g_a for the formation of $\boxed{<\psi\bar{\psi}>}$ cond. // g_b for $\boxed{<\psi\psi>}$ cond.

The action in the frame of the mean field approach:

$$I = \frac{1}{2}\int d\mathbf{x}d\mathbf{y} \left(\bar{\Psi}(\mathbf{x}) A(\mathbf{x},\mathbf{y}) \Psi + \bar{\Psi}(\mathbf{x}) B(\mathbf{x},\mathbf{y}) \Psi_C + \bar{\Psi}(\mathbf{x})_C C(\mathbf{x},\mathbf{y}) \Psi + \bar{\Psi}(\mathbf{x})_C D(\mathbf{x},\mathbf{y}) \Psi_C \right),$$

$$\Psi = \begin{pmatrix} \psi \\ \psi_c \end{pmatrix} \qquad \psi_c = C\,\bar{\psi}^T(-k) \tag{7}$$

Where:

$$A = (G^+_{0,ff})^{-1} \equiv -i \left(i\gamma_\mu(\partial^\mu - ig' A^\mu_a \lambda_a/2) + \gamma_0\mu + g_a g \int D(\mathbf{x},\mathbf{y}) <\psi\bar{\psi}(\mathbf{y})> \right) \delta(\mathbf{x}-\mathbf{y})$$
$$D = (G^-_{0,ff})^{-1} \equiv -i \left(i\gamma_\mu(\partial^\mu - ig' A^\mu_a \lambda_a/2) - \gamma_0\mu + g_a g \int D(\mathbf{x},\mathbf{y}) <\psi\bar{\psi}(\mathbf{y})> \right) \delta(\mathbf{x}-\mathbf{y})$$
$$B = \Delta^+ = g_b g^2 <\Psi_C\bar{\Psi}> D(\mathbf{x},\mathbf{y})$$
$$C = \Delta^- = g_b g^2 \gamma_0 (\Delta^+(\mathbf{x},\mathbf{y}))^\dagger \gamma_0. \tag{8}$$

DOES [redacted]

The condensate $<\bar{\psi}\psi>$ (break[redacted]chiral symmetry): quark mass ?

By performing a Fourier transformation (restricting possible configurations)

$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi exp \int_{|\mathbf{k}|>0} d^3\mathbf{k} (\bar{\psi}\bar{\psi}_C) S^{-1}(\mathbf{k}) \left(\psi\psi_C\right)^T = \mathcal{N} det \left(S^{-1}\right)^{\frac{1}{2}} \tag{9}$$

The full propagator:

$$S = \begin{pmatrix} G^+ & -G_0^+\Delta^- G^- \\ -G_0^-\Delta^+ G^+ & G^- \end{pmatrix}, \tag{10}$$

where

$$G^{\pm} = ((G_0^{\pm})^{-1} - \Delta^{\mp} G_0^{\mp} \Delta^{\pm})^{-1} \tag{11}$$

Using identity:

$$< \Psi(k)\bar{\Psi}(k) >= -S(k) \tag{12}$$

The GAP equations are:

$$\begin{aligned}
\Delta^{\pm}(k) &= g^2 T \int_p D(k-p) G_0^{\mp}(p)\Delta^{\pm}(p)\left((G_0^{\pm})^{-1}(p) - \Delta^{\mp}(p)G_0^{\mp}(p)\Delta^{\pm}(p)\right) \\
< \psi\bar{\psi}(q) > &= \int_p D(k-p)\left((G_0^{+})^{-1}(p) - \Delta^{-}(p)G_0^{-}(p)\Delta^{+}(p)\right)^{-1}, \\
< \bar{\psi}\psi(q) > &= \int_p D(k-p)\left((G_0^{-})^{-1}(p) - \Delta^{+}(p)G_0^{+}(p)\Delta^{-}(p)\right)^{-1}.
\end{aligned} \tag{13}$$

where

$$(G_{\bar{f}f}^{\pm})_0^{-1}(k) = -\left(\gamma_\mu(k^\mu - igA_a^\mu\lambda_a/2) \mp \gamma_0\mu + g_a g \int_k D(k) < \psi\bar{\psi}(k) >^{\pm}\right). \tag{14}$$

To compare with one gluon exchange:

$$\Delta(k) = g^2 \int \frac{d^4p}{(2\pi)^4}\Gamma_a^\mu D_{\mu,\nu}^{a,b}(k-p)G_0(p)\Delta(p)\left((G_0^{\pm})^{-1} - \Delta^{\mp}G_0^{\mp}\Delta^{\pm}\right)^{-1}\Gamma_b^\nu \tag{15}$$

The external field equation is:

$$\partial_\mu F_a^{\mu\nu} = g' < \bar{\psi}(\mathrm{x})\gamma^\nu\lambda_a\psi(\mathrm{y}) > \tag{16}$$

dynamical/non homogeneous external field: vector condensates

Spinors

- $$\Delta = \underline{\Delta_1\gamma_5} + \underline{\Delta_2\vec{\gamma}.\vec{k}}\gamma_0\gamma_5 + \Delta_3\gamma_0\gamma_5 + \underline{\Delta_4} + \underline{\Delta_5\vec{\gamma}.\vec{k}\gamma_0} + \Delta_6\vec{\gamma}.\vec{k} + \Delta_7\vec{\gamma}.\vec{k}\gamma_5 + \Delta_8\gamma_0 \tag{17}$$

However:

1. $< \bar{q}q >$ plays the role of mass for the quarks in the equations
2. quark masses mix states of chiralities and duplicate the number of possible different di-quark condensates

In a first analysis $< \bar{q}q >$ neglected, resulting in four possible states of helicity and chirality:

$$\Delta(k) = \phi^+_{r,+}(k)P^+_{r,+}(k) + \phi^+_{l,-}(k)P^+_{l,-}(k) + \phi^-_{r,-}(k)P^-_{r,-}(k) + \phi^-_{l,+}(k)P^-_{l,+}(k) \tag{18}$$

Using projectors for quarks with helicities $\pm$ and chiralities r, l:

$$P_\pm(k) = \frac{1 \pm \gamma_5\gamma_0\vec{\gamma}.\hat{k}}{2} \qquad P_{r,l} = \frac{1 \pm \gamma_5}{2} \tag{19}$$

For each of the condensates $\phi_{r,l;\pm}$ there is an equation such as:

$$\phi^a_{r,+}(k) = \frac{g^2}{2}\int \frac{d^4p}{(2\pi)^4} D(k-p, M)\left(\frac{A - \hat{k}.\hat{p}}{{p_0}^2 - \epsilon^+_a\left[\phi_{r,+}(p)\right]^2}\phi^a_{l,-} + \frac{B + \hat{k}.\hat{p}}{{p_0}^2 - \epsilon^-_a\left[\phi_{l,+}(p)\right]^2}\phi^a_{l,+}\right) \tag{20}$$

a is for color (always diagonal: $< q_a q_a >$)

For zero external field $\epsilon^\pm_a[\phi_{r,+}] = \sqrt{(|\vec{p}| \mp \mu)^2 + |\phi|^2}$

A, B are also dependent on the choice of the external field

GAP equations in the form:

$$\phi^+_{r,+} = \phi^+_{l,-}(F^+_0(\phi_{l,-}) - F^+_1(\phi_{l,-})) + \phi^-_{l,+}(F_0(\phi_{l,+} + F_1(\phi_{l,+})), \tag{21}$$

For non zero: $\phi^+_{r,+}$, $\phi^-_{r,-}$, $\phi^+_{l,-}$, $\phi^-_{l,+}$

Integrals of the type:

$$(F^\pm_0, \quad F^\pm_1) = \frac{g^2}{2}\int\frac{dp_0}{2\pi}\int\frac{d\vec{p}}{(2\pi)^3}\frac{1}{(k-p)^2-\tilde{M}^2}(1\,, \quad \hat{k}\hat{p})\frac{1}{p_0^2-(\epsilon^\pm[\phi])^2} \tag{22}$$

(a) Poles from scalar boson propagator $\rightarrow$ if $\tilde{M}^2$ is small the approximation of punctual interaction (for which $\phi^- = \phi^+$) is not necessarily reliable

(b) Poles from fermions

For the cases of small k or zero momentum exchange: the integral $F_1 \rightarrow 0$

The F_0^+ integral:

$$\begin{aligned}
F^+_0 = \frac{g^2}{16\pi^2}\int_0^{\Lambda,\infty} dp\frac{p}{k\epsilon^+}\Bigg(-\frac{1}{2}\Bigg\{ln\left(\frac{\epsilon^+ - k_0 - \sqrt{(k-p)^2+M^2}}{\epsilon^+ + k_0 - \sqrt{(k-p)^2+M^2}}\right) + \\
- ln\left(\frac{\epsilon^+ - k_0 - \sqrt{(k+p)^2+M^2}}{\epsilon^+ + k_0 - \sqrt{(k+p)^2+M^2}}\right)\Bigg\} \\
+\frac{1}{2}\Bigg\{ln\left(\frac{\epsilon^+ + k_0 - \sqrt{(k-p)^2+M^2}}{\epsilon^+ - k_0 - \sqrt{(k-p)^2+M^2}}\right) - ln\left(\frac{\epsilon^+ + k_0 - \sqrt{(k+p)^2+M^2}}{\epsilon^+ - k_0 - \sqrt{(k+p)^2+M^2}}\right)\Bigg\} \\
+\frac{1}{4}ln\left(\frac{(\epsilon^+ + k_0)^2 - (k+p)^2 + M^2}{(\epsilon^+ + k_0)^2 - (k-p)^2 + M^2}\right) + \frac{1}{4}ln\left(\frac{(\epsilon^+ - k_0)^2 - (k+p)^2 + M^2}{(\epsilon^+ - k_0)^2 - (k-p)^2 + M^2}\right)\Bigg)
\end{aligned} \tag{23}$$

There are ultraviolet divergences, eliminated by:

$$F^\pm_0 \rightarrow F^\pm_0 + \frac{g^2}{4}\int^{\mu_R}\frac{d^3\mathbf{p}}{(2\pi)^2}\frac{1}{(k-p)^2}\frac{1}{|\mathbf{p}|} - F_{c.t.} \tag{24}$$

Example:

constant chromo magnetic field $A_i^a = \sqrt{H/g'}\delta_{i,a}$ (i=1,2,3),

else $A_i^a = 0$, leading to $H_i^a = \delta_i^a H$ = const.

- poles (for equal chemical potential):

$$(\epsilon^{\pm}[\phi_i])^2 = (\sqrt{\vec{p}^2} \mp \mu)^2 + \phi_i^2$$

$$(\epsilon^{\pm}[\phi_i])^2 = (\sqrt{\vec{p}^2} \underline{\pm A_1^1 g'/2 \mp \mu})^2 + \phi_i^2$$

$$(\epsilon^{\pm}[\phi_i])^2 = (\sqrt{(|\vec{p}| + g' A_3^3)^2} \underline{\pm A_2^2 g'/2 \mp \mu})^2 + \phi_i^2$$

Different "effective" chemical potential for each colored quark

- (Other external magnetic field may induce terms with role of quark mass) (?)
- We consider (approximatively): $|\phi|$ as constant reliable for small k, but $\phi = \phi(k)$
- Strong A_μ^a or ($\vec{H}$) may enhance $|\phi|$ (not simultaneously for all the colors)

- For punctual interaction $M^2 \to \infty$ (NJL): $\phi^+ = \phi^-$

- Limit of **weak coupling**: in principle $\phi << \mu$ ($\phi \sim \mu exp(-c/g^2)$)

$$F_0^-/F^+ \sim \frac{\epsilon^+}{\epsilon^-} << 1 \qquad \to \phi^- > \phi^+$$

- pairing dominated by fermions close to Fermi surface $\langle \Psi\Psi \rangle$
- H^a_{ext} induces $\tilde{\mu} < \mu$ enhancing ϕ

- Limit of **strong coupling**:

contributions far from Fermi surface

- increasing $|\phi|$
- if $\tilde{M}^2 \propto g'^2$, then $D \sim 1/M$ and $\phi^+ \sim \phi^-$ (BUT $\phi^- > \phi^+$)
- H^a_{ext}: non degeneracy in $|\phi^a|$

- Using approximations: massless fermions, momentum independent GAP

- Scalar boson exchange (effective degree of freedom of the system) as a test model for color superconductivity
along the lines proposed by Pisarski-Rischke

- Non zero energy exchange (non punctual interactions) modify GAP equations

- Detailed study of complete phase diagram $< qq >$, $< \bar{q}q >$ depend on considering mixed chirality/helicity states

- External chromo (electro) magnetic fields may lead to several different effects:
- changing fermion masses
- effective chemical potential (usually smaller) $\rightarrow \tilde{\mu} \sim \Delta \rightarrow < \bar{q}\bar{q} > \sim < qq >$
- may cause non degeneracy of ϕ_a

- Towards derivation of effective interactions for q-q which takes into account $< qq >$