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Spontaneous Symmetry Breakings in an Extended Linear Sigma Model at Finite Density

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Abstract

The linear sigma model invariant under $O(4)$ (with pions and a sigma) coupled to fermions (two component baryons) and to a (massive) gauge vector field is considered for the description of a system at finite (baryonic) density, which may be bound. The Gaussian variational approximation with truncations is considered to compute quantum fluctuations for the spin zero particles. Nearly exact solutions for the stability equation of a bound homogeneous system, which are solutions of equations of mouvement, are found within a proposed general prescription. The "incompressibility" modulus K_∞ can have any value to be adjusted as a boundary condition for a differential equation, eventually assuming the correct nuclear matter value. The sigma and the pions have non zero expected classical values at finite density corresponding to (condensates of) the "chiral" and isospin dynamical symmetry breakings, respectively. The sigma expectation value, expected to be identified with the QCD scalar condensate $\bar{\sigma} \propto < \bar{q}q >$, seems to decrease as density increases although there may have solutions in which it increases. A generalized symmetry radius is defined at a given stable density and the in medium sigma and pion masses are analyzed. The two components of in medium fermions, neutron and proton for example, acquire different masses due to the isospin symmetry breaking and these states may even oscillate in the baryonic medium. A non trivial solution is proposed of an extended Euler-Lagrange equation for the (massive) vector field - which is not quantized so far, and which may eventually be a kind of classical "dressed photon". This vector field may be a "condensate" and it seems to indicate the occurrence of a gauge dynamical symmetry breaking at finite density typical of a superconductor.

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1 Introduction

When a Lagrangian theory is invariant under a transformation group and the ground state of this theory is not, this symmetry is spontaneously (or dynamically) broken (SSB). The strong interacting realization of such implementation of a theory was discussed by Nambu [1]. In this case there appears a non zero expected value of field(s) (composite or not, one or more) in the ground state, the so-called condensate(s). The energy is lowered and the system is re-arranged [2]. This is the Nambu picture in contrast to the Wigner-Weyl one in which there appears states multiplets with opposite parity what is not observed in the low energy hadronic phenomenology.

Quantum Chromodynamics (QCD) is the fundamental theory for the Strong Interactions, at least up to a certain level. It has intrincated flavor and color non abelian structures and strong coupling constants for processes mainly at low and intermediary energies. At not high energies quarks and gluons, whose color charges respect the SU(N=3) algebra, are confined such that no colored states are exptected to be observed. Due to these features it is very difficult to obtain exact solutions what is then to be mostly accomplished in finite lattices where space-time is discretized. One therefore constructs effective models which respect the main properties and symmetries of the QCD for the range of energy densities for the process of interest. It is interesting to note that one may have different effective theories for certain ranges of these variables which can be different if one consider different systems such as the vacuum or high energy density formed in relativistic heavy ions collisions as AGS, RHIC and CERN. In the vacuum, the lightest strong interacting particles are known to respect, at least approximatedly, chiral symmetry - whose group is $SU_L(2) \times SU_R(2)$ - which is spontaneously broken down to $SU(2)$. QCD vacuum would have therefore a preferencial direction in chiral/flavor space. Vacuum would acquire a non trivial structure due to the formation of scalar quark–anti-quark condensate $< \bar{q}q >$, which would be the order parameter of the Chiral SSB [3].

These features can be taken into account in sigma models ($O(4)$ invariant) which, in the linear realization with mesons, implement chiral symmetry $SU(2) \times SU(2)$ (two flavors QCD) in the Nambu realization with two fields: the (pseudo-scalars) pions and the (scalar) sigma - which is called the chiral partner of the pion. Pseudo scalar pions have small masses in the hadronic scale and they are almost omnipresent in hadronic and nuclear physics. The (ground state) expected value of sigma is considered to be, in this model, the order parameter of chiral SSB, $< \sigma > \propto < \bar{q}q >$. Long time ago S. Weinberg pointed difficulties in the linear sigma model for describing hadronic processes [4] which are being rediscussed nowadays. and proposed the non linear realization of chiral symmetry in which the sigma field is eliminated. There are actually strong evidences for the existence of the sigma [5, 6, 7, 8] which would be a Higgs field for hadrons, though a composite one. Both implementations of chiral symmetry have been extensively developped since the works of Schwinger, Levy–Gell-Mann, Nambu and Weinberg among others [1, 4, 3].

The linear realization of chiral symmetry exhibits several advantadges over the non linear realization, for example, for the description of finite density systems [9]. At finite density QCD is known/expected to have a complex phase diagram with the appearance of other effects [3] with an eventual restoration of the chiral symmetry when $< \bar{q}q >$ would be close to zero at high energy densities. While the sigma acquires a classical value the pion field (ground state) expected value are to be zero in the vacuum, $\bar{\pi} =< vac|\pi|vac >= 0$. As far as we know, two ways of obtaining general properties of normal finite density baryonic states using a linear implementation of chiral symmetry were studied in the past. The linear sigma model with a vector meson and "vacuum polarization" of nucleons has been studied and applied to nuclear matter and, in some cases, partially appropriate description of some properties have been obtained. Besides the normal solution, "abnormal" bound states were also found but faced as problem for the description of finite nuclei; the vector meson mass was also considered to be generated by a Higgs mechanism from the coupling to the sigma field what yielded other difficulties [10]. A second way was to consider quantum fluctuations for the baryonic and spin zero bosonic fields, but they introduced unaccetable behaviors or ghost poles eventually associated with imaginary effective potential [11]. This however seems to be rather an issue of most approximations used to deal with quantum fluctuations and may eventually be related to the asymptotic freedom of the asymmetric phase of the $\lambda\phi^4$ model which is quite similar to the linear sigma model (LSM) [12]. Nevertheless it is worth to emphasize that in these works some of the QCD believed properties were not taken into account (for example in the analysis of [13]) as we propose in this article and forthcoming ones. Furthermore, in all these cases the pion expected value in the finite density ground state from a condensation ("classical" value), like a "pion condensate", has not been taken into account which does not seems to be the same of that developped during the 70's [14]. The full solution of the nuclear systems and the "self consistency", in a general way, was truncated at an approximated level [15].

In this work we find results suggesting that the Linear Sigma Model (LSM) with nucleons and a (massive) gauge vector field yields in an interesting picture considering pion "condensate" - which has not been considered in the way it is proposed here so far. This is in agreement to what has been proposed by Brown and Rho [9]. We show new insights (for example, considering the pion "condensate") and we argue that previous limitations may be eliminated. Moreover strong evidences of the existence of the sigma, the chiral partner of the pion claim for new developments. This scalar particle composed, in principle, by a pair of quark-antiquark can be described by a field whose classical component corresponds to $< \bar{q}q >$. This scalar condensate would be the main component for masses of light sector of QCD spectrum in a sort of mixed superfluid and Higgs picture, for QCD-hadronic Physics respectively, which also can accomodate the quark masses [16]. This paper is organized as follows. In the next section the linear sigma model (LSM) with nucleons and a massive vector abelian gauge boson for finite density systems is presented. In the following section the Gaussian variational approximation for the sigma and pions is performed and truncations of the corresponding non-perturbative quantized effective potential are performed. The vector field equation is a sort of modified Hamilton equation as a proposal to obtain a solution which takes into account more non linearities than usually done for a Fermi liquid picture. This corresponds to a new variational method which yields an extended Hamilton Jacobi equation, in principle expected to be valid at least for finite density systems. In section 4 the variational and stability equations for a bound homogeneous limit are solved with particular prescriptions. We show there may take place a (isospin) spontaneous symmetry breaking (SSB) generating non zero expected value for the pion field at finite density besides the usual chiral SSB. The bosonic averaged values in the ground state will be referred to as "condensates" [17]. The stability condition for the normal nuclear density is satisfied and the incompressibility modulus is calculated resulting an excellent agreement with observations [18]. Some symmetry properties for the *in medium* observables are found. We mostly found solutions for which that the scalar condensate (associated with the chiral order parameter) decreases with density although there may exist others with which it may increase [19, 20] whose meaning is less appealing. The mass of the vector field is considered to be either non-zero at finite density, and it eventually reduces to zero in the vacuum what may suggest to relate it to a "dressed photon" or it maybe explained to correspond to one of the lighest vector meson by means of a vector meson dominance in the medium [15]. Numerical results for some equations as well as some other possible pictures and consequences are analyzed in section 5. In final part there is a summary.

2 Gauged Linear sigma model for finite baryonic density systems

The $O(4)$ invariant Lagrangian density of the Linear Sigma Model with two-component baryons, $N_i(\mathbf{x})$, sigma and pions (σ, π) covariantly coupled to a gauge vector field V_μ may be given by:

$$\begin{array}{c}\mathcal{L} = \bar{N}_i(\mathbf{x})\left(i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma + i\gamma_5 \vec{\tau}.\vec{\pi})\right) N_i(\mathbf{x}) + \frac{1}{2}\left(\partial_\mu\sigma.\partial^\mu\sigma + \partial_\mu\vec{\pi}.\partial^\mu\vec{\pi}\right) + \\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{4}\left((\sigma)^2 + (\vec{\pi})^2 - v^2\right)^2 + \frac{1}{2}m_V^2 V_\mu V^\mu,\end{array} \tag{1}$$

where the covariant derivative is: $\mathcal{D}^\mu = \partial^\mu - ig_V V^\mu$, the gauge invariant tensor is: $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. g_V, g_S and λ are the coupling constants and $v = F_\pi \simeq 88$ MeV, the pion decay constant, in the vacuum in the so called chiral limit. Pion mass is known/expected to break the chiral symmetry usually explicitly, in such case one should add another (small) term $\mathcal{L}_{sb} = c\sigma$, where c is a constant proportional to the mass. We however consider the pion mass in such a way it is generated by non perturbative quantum fluctuations of spin zero particles themselves [21]. Basically this can be seen with a shift of the chiral condensate from F_π to $f_\pi \simeq 92$ MeV as it is expected [22, 21] due to the massive character of the pions. This now is due to quantum fluctuations. This is of no qualitative relevance for our main results [16]. A shift of the coupling of the temporal component of the vector field g_V to the nucleons is equivalent to a (re)definition of a chemical potential whose introduction can be done adding the Lagrange multiplier $\delta\mathcal{L} = -\bar{N}\gamma_0\mu_0 N$ to the above Lagrangian.

We will develop the model in a Hamiltonian formalism within which we consider the existence of non zero ("classical") expected values (condensates) for all the bosonic fields and look for solutions for their equations within variational approximations. In this work nearly the whole baryon masses (as explained below) come from the the coupling to the scalar mesonic field by a Higgs mechanism ($M^* = g_S\bar{\sigma}$). This seems to be present in quark effective models interacting with bosonic fields which undergoes superfluidity generating a bound scalar isoscalar particle (σ, as a composite Higgs) [16, 23]. As at least $\bar{\sigma}$ must depend (strongly) on the density so M^* does, the effective nucleon mass. An explicit mass term for the baryons in the Lagrangian (due to another quark and gluon effects and different mechanism(s), for instance a gluon condensate is expected to contribute for some hadron masses such as the rho and nucleon masses [24]) does not seem to change the qualitative results of this work although it would break chiral symmetry explicitely. This would correspond to an *in medium* (effective) mass, for example such as: $M^* = M \pm g_S\bar{\sigma}$ which breaks explicitely chiral symmetry.

In the same way, the mass of the vector field can be generated by the scalar condensate for gauge invariant covariant couplings to the other bosons (substituting ∂_μ by a covariant derivative in the derivative terms). Besides this possibility we consider an explicit mass term given by: $\delta\mathcal{L} = 1/2\ m_V^2 V_\mu V^\mu$

which may be considered to be zero only at zero baryonic density (ρ_B) - corresponding to a different Lagrangian density due to the finite density. Therefore, at least two non excludent possibilities arise for the vector boson (effective) mass origin and behavior. Explicitely writing the resulting terms due to the vector field covariant coupling to the other bosons we have:

$$\mathcal{L}_{V-B} = \frac{1}{2}\left(\partial_\mu + ig_V V_\mu\right)\sigma\left(\partial^\mu - ig_V V^\mu\right)\sigma + \frac{1}{2}\left(\partial_\mu + ig_V V_\mu\right)\pi\left(\partial^\mu - ig_V V^\mu\right)\pi + \frac{1}{2}m_V^2 V_\mu V^\mu, \qquad (2)$$

where we consider only the component V_0 to be non zero. The non derivative terms are masses terms of the sigma and pion AND/OR of the vector field. While in the latter picture the usual non renormalizability issue arises, there is the former possibility, sigma and pion masses due to a classical component V_μ, which has not been considered so far. Although they are non excludent ideas, in this work we take the second (usual) one to obtain a zero vector field mass in the vacuum (non zero mass in the medium) and leave a further development for another work. We will be concerned, at this level, with an homogenous system in which case the other (spatial) components of the vector field V_i may be set to zero as usually considered in nuclear systems calculations. The expression for in medium m_V^2 at finite density can be given by:

$$\tilde{m}_V^2 = m_V^2 - g_V^2(\bar{\sigma}^2 + \bar{\pi}^2). \qquad (3)$$

The other terms of (massive and coupled) V_μ (spatial $V_i, i = 1,2,3$) would introduce time dependence and spatial inhomogeneities, therefore they were not considered. Since the vector field acquires mass, at the classical level, the other components may contribute with another coupled equation. They would basically produce shifts in the numerical results.

The baryon (nucleon) field is quantized in terms of creation and annihilation operators. Its wave function is usually written as superposition of spinor , isospinor and coordinate components . It generates non zero scalar, baryonic and eventually pseudo-scalar densities (ρ_s, ρ_B and ρ_{PS}). We will not explicitly evaluate here all these quantities but only indicate some usual calculations of ρ_B and the quantized fermionic energy density ρ_f. From the Landau's Fermi liquid theory, they are respectively given by the expressions:

$$\rho_B = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}, \qquad \rho_f = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + (M^*)^2}. \qquad (4)$$

In these expressions k_F is the baryon momentum at the Fermi surface. This picture should be expected to be limited and, to go further, these two quantities are considered in this work as functions of the bosonic degrees of freedom with functional forms which are to be determined such as the expected observables are obtained in agreement with experimental values. This means that in the averaged Hamiltonian from which the Hamilton and variational equations are extracted these densities are to depend on the bosons parameters. The (interacting) bosonic properties and effects produce these quantities by means of their interactions with baryons. This way we expect to take into account higher order non linear and interacting effects with relation to the usual picture with Euler-Lagrange equations which lead to a Fermi liquid theory. The effective mass - $M^* = g_S\bar{\sigma}$ - and the nucleon momentum at the Fermi surface are the parameters used in the present work for the determination of baryons properties. Latter we will show that M^* may be defined as a matrix which depends on the isospin (and spin) of the baryons when $\bar{\pi} \neq 0$. For the moment $M^* = g_S\bar{\sigma}$ is a number.

3 Gaussian Variational Approximation for Spin Zero Bosons

To take into account the quantum fluctuations of the sigma and pions we consider the variational approach using Gaussian trial wave-functionals in the Schroedinger picture [25] and perform truncations. With the variational principle an estimation for the energy density of the ground state can be obtained by calculating the functional averaged energy with trial wave-functionals $|\Psi[\sigma,\boldsymbol{\pi}]>$. The trial parameters of the wave-functional are fixed to yield a minimum (total) energy density with relation to them,

$$\mathcal{H}_{tot} \equiv <\tilde{\Psi}_{s.z.}[\sigma,\boldsymbol{\pi}]\,|H[\sigma,\boldsymbol{\pi},V_\mu,N]\,|\tilde{\Psi}_{s.z.}[\sigma,\boldsymbol{\pi}]>$$

where the subscript (s,z) stands for spin zero components. The averaged energy density $\mathcal{H}_{tot}$ is considered to be the total averaged Hamiltonian, including the contributions of the other bosons and baryons. The trial Gaussian wave-functional for the spin zero bosons is decomposed into two Gaussians $|Psi[\sigma,\boldsymbol{\pi}]>= |\Psi_S[\sigma]>\times|\Psi_P[\boldsymbol{\pi}]>$. For the sigma component we can write:

$$\Psi_S[\sigma(\mathbf{x})] = \mathcal{N}_S exp\left\{-\frac{1}{4}\int d^3\mathbf{x}d^3\mathbf{y}\delta\sigma(\mathbf{x})G_S^{-1}(\mathbf{x},\mathbf{y})\delta\sigma(\mathbf{y})\right\}, \quad (5)$$

Where, in the translational invariant limit, $\delta\sigma(\mathbf{x}) = \sigma(\mathbf{x}) - \bar{\sigma}$; the normalization is $\mathcal{N}_S$ such that $\int\mathcal{D}[\sigma]\Psi_S^*\Psi_S = 1$ (omitting space coordinates), the variational parameters are the condensate $\bar{\sigma} =< \Psi_S|\sigma|\Psi_S >$, the quantum fluctuations, represented by the width of the Gaussian which is the two-point function in 3 dimensional space,

$$G_S(\mathbf{x},\mathbf{y}) =< \Psi_S|\delta\sigma(\mathbf{x})\delta\sigma(\mathbf{y})|\Psi_S > .$$

This function is the Feynman Green's function of an explicitely covariant formalism [2]. An analogous expression for the pions is considered with variational parameters given by: $\bar{\boldsymbol{\pi}} =< \Psi_P|\boldsymbol{\pi}|\Psi_P >$ and

$G_P^{a,b}(\mathbf{x},\mathbf{y})$, which is a matrix in isospin space that can be considered to be diagonal as a particular case along this work ($G_P^{a,a} = G_P$). This reduces the corresponding functional space and it guarantees the explicit "chiral and isospin" invariances.

It will be assumed that the quantum fluctuations of spin zero bosons - through the two two-point Green's functions G_S and G_P - have two effects only:

(1) they produce and change the resulting meson masses - shown below - as well as

(2) they cause shifts of the respective condensates with respect to their values at the tree level. This is considered such that the modified condensates ($\tilde{\sigma}$ and $\tilde{\pi}$) are related to the tree level ones ($\bar{\sigma}$ and $\bar{\pi}$, respectively) by:

$$\tilde{\pi}^2 = \bar{\pi}^2 + G_P, \qquad \tilde{\sigma}^2 = \bar{\sigma}^2 + G_S. \tag{6}$$

We are omitting the coordinate dependence of functions $G_i = G_i(\mathbf{x},\mathbf{x})$, which contain ultraviolet (UV) divergences and are local. Quantum fluctuations always re-arrange the model parameters determined in the tree level. Due to the two above hypothesis it is not needed to evaluate the G_i functions explicitely because fixing the values of $\bar{\sigma}$ and $\tilde{\sigma}$, which are given by $F_\pi = 88$ MeV and $f_\pi = 92$ MeV, as discussed in section 2 and in [21], it has a finite value already. Chiral radius and scalar condensate are not always equal to the pion decay constant [8, 26]. These two approximations (1 and 2) correspond to a truncation of the full (self-consistent) Gaussian effective potential of spin zero fields, neglecting few non linear terms in G of the effective potential. This prescription is however nearly exact. As the model is an effective model this should be valid and it should correspond nearly to a fixed energy scale from the renormalized theory [19, 20]. Furthermore the variational approximation, such as it is used, does not introduce different couplings (a third order coupling in the σ potential, for example) as it occurs in the usual procedure of shifting fields after a SSB. More exact calculations are being done for this and other theories and will be shown elsewhere.

The minimizations of the total averaged energy, from which the equation of motion for the vector field is also derived, with respect to the Gaussian variational parameters yield the GAP and condensate equations which define the minimum of the potential for these parameters. The following two expressions are obtained for the sigma parameters:

$$\begin{aligned} \frac{\delta \mathcal{H}^{tot}}{\delta \bar{\sigma}} &= 0 \to \bar{\sigma}\lambda\left(\bar{\sigma}^2 + 3G_S + \bar{\pi}^2 + G_P - v^2\right) + \frac{d\rho_f}{d\bar{\sigma}} + W_S = 0; \\ \frac{\delta \mathcal{H}^{tot}}{\delta G_S} &= 0 \to \frac{d\rho_f}{dG_S} - \frac{G_S^{-2}}{8} - \frac{\Delta}{2} + \frac{\lambda}{4}\left(6\bar{\sigma}^2 + 2\bar{\pi}^2 + 6G_S + 2G_P - 2v^2\right) + W_P = 0, \end{aligned} \tag{7}$$

where the W_i stands for variations of other terms with respect to $\bar{\sigma}$ and G_S. The second expression can be written in a compact and transcendental form for G_S:

$$G_S = G_S(\mathbf{x},\mathbf{x}) =< \mathbf{x}|\frac{1}{\sqrt{\Delta + \mu_S^2}}|\mathbf{x} > \tag{8}$$

where the Laplacian Δ was diagonalized and the sigma mass is μ_S obtained from equation (7), expressing the averaged value of G_S with the ("physical") mass [27] $\mu_s^2 = \lambda(3\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2) + ...$ For the sake of clearness we perform usual approximations from what would be the exact solution considering that: the fermionic density does not depend explicitely on the fluctuations G_i and fluctuations of both particles, sigma and pions, are not too large and are nearly equal ($G_P \simeq G_S = G$). This can only be obtained by fixing different cutoffs because $\mu_S \neq \mu_P$.

The corresponding equations for the pion, without the intermediary step shown above, are:

$$\begin{aligned} &\lambda\bar{\pi}_a\left(\bar{\pi}^2 + 3G_P + \bar{\sigma}^2 + G_S - v^2\right) + \frac{d\rho_f}{d\bar{\pi}_a} = 0; \\ &\mu_P^2 = \lambda\left(3\tilde{\pi}^2 + \tilde{\sigma}^2 - v^2 + ...\right) \end{aligned} \tag{9}$$

As far as the fermionic density depends on the pion field it may produce a variable pseudo scalar density $d\rho_f/d\bar{\pi} \propto \rho_{ps}$. The baryonic and fermionic densities were considered to depend on $\tilde{\pi}$ which is chosen to be non zero. This condensate is to represent kind of "long range pionic correlations" which should be of relevance to the actual fermionic wave-function and densities leading to a non zero derivative in the above equation. As discussed above, the expressions for the meson masses ($\mu_S^2 \simeq 600MeV$ and $\mu_P^2 \simeq 140$ MeV), and the shift of the fields (6) are the only effects of the quantum fluctuations in the present work in which some parameters of the model are adjusted with experimental data. Since we will not explicitly calculate the two point function G_i, there will be no concern with Ultra Violet divergences here.

We can face the two equations of the sigma and pion condensates as partial differential equations for $\rho_f = \rho_f(\tilde{\sigma}) = \rho_f(\tilde{\pi}^2)$. Furthermore, these equations are isomorfic and show an equal dependence of ρ_f with each of these "classical fields" - apart from the fact that the condensates are expected to have very different values in the vacuum at least. They can be written as:

$$\frac{d\rho_f}{d\tilde{\pi}^2} \simeq \frac{d\rho_f}{d\tilde{\sigma}^2} \simeq \frac{\lambda}{2}(\tilde{\pi}^2 + \tilde{\sigma}^2 - v^2), \tag{10}$$

Total energy density and vector field equation

The resulting total averaged energy yields an extense expression as functions of the four variational parameters, two condensates and two Green's functions, model parameters, plus nucleonic densities and vector field variables and coupling constants. The truncated averaged energy density can be written as:

$$\mathcal{H}^{tot} = \rho_f + g_V V_0 \rho_B - \frac{1}{2}\tilde{m}_V^2 V_0^2 + \frac{\lambda}{4}(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)^2, \tag{11}$$

Where $\tilde{m}$ is an "effective mass" for the vector field (expression (3)). From now on, ρ_B (and eventually ρ_f) are to depend on the classical $\tilde{\sigma}$, $\tilde{\vec{\pi}}$ and V_0, as variational parameters.

As discussed previously, to take into account further correlations and many-body effects of the fermions for the vector field solution we will consider that the energy density is varyed with respect to the classical vector field, which is not quantized so far. Since we do not calculate the explicit dependence of the fermions wave function on V_0 we consider the baryonic density to depend on it. This is expected to account, variationally, for further correlations, many body and eventual quantum fluctuations effects. The modified (variational) equation is given by:

$$g_V \left(\rho_B + V_0 \frac{d\rho_B}{dV_0} \right) - \tilde{m}_V^2 V_0 = 0. \tag{12}$$

Where V_0 is now a sort of variational parameter for a more exact density $\rho_B = \rho_B[V_0]$, and eventually in a still more complete self consistent fashion $\rho_f[\rho_B(V_0)]$.

4 Stability and Solutions

In this section solutions for the above equations (7,9,12) are searched such that the main properties of a stable finite density system are consistently described. The stability condition of the bound system can be written as:

$$\frac{d\mathcal{H}}{d\rho_B} = \frac{\mathcal{H}}{\rho_B}\bigg|_{\rho_B=\rho_0} < 0, \tag{13}$$

where ρ_0 is the stability density. To satisfy this expression we consider some prescriptions for the variables such as to consider the dependence of them on the baryonic density. In particular, the expression for the energy density (11) within condition (13) is separated into three equations such that they obey the stability condition individualy:

$$\begin{aligned} \frac{d\rho_f}{d\rho_B} &= \frac{\rho_f}{\rho_B}, \\ \frac{d(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)}{d\rho_B} &= \frac{(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)}{2\rho_B}, \\ \frac{d\mathcal{H}_V}{d\rho_B} &= \frac{\mathcal{H}_V}{2\rho_B}. \end{aligned} \tag{14}$$

In this last expression $\mathcal{H}_V$ is the energy density with contributions of the vector field in expression (11). A detailed comparison of this prescription with the exact result will be shown elsewhere. The solutions for these (stability) equations do obey the equations of motion, being this approximation therefore probably reliable.

From the first of the differential equations (14) we find a solution for the dependence of ρ_f on the baryonic density ($\rho_f = \rho_f(\rho_B)$) which is in agreement with that resulting from the integration of expression (4) in the range of densities not too far from ρ_0. A solution for the above prescription (14) is given by:

$$\rho_f = -K\frac{\rho_B}{9} Ln\left(\frac{\rho_B}{\rho_0}\right) + B\rho_B + K\frac{\rho_B^2}{9\rho_0}, \tag{15}$$

where B is a constant fixed to reproduce ρ_f according to expression (4) ($B \simeq 3.8 fm^{-1}$ for the values adopted in section 4) and K is the usual incompressibility modulus:

$$K = 9\rho_0^2 \left.\frac{d(E/A)}{d\rho_B^2}\right|_{\rho_0} > 0. \tag{16}$$

The agreement of this expression for $\rho_f(\rho_B)$ with the one written in (4) close to the stability density (because they have different slopes with k_F) garantees once again that the above prescriptions is still reliable.

From the second expression in (14) we find a solution which seems to represent a sort of "constraint" which defines a symmetry radius in the medium:

$$(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2) = \tilde{C}\sqrt{\rho_B}. \tag{17}$$

The above expression is, in principle, valid at zero density and at ρ_0 but seems to be also valid at different ρ_B. $\tilde{C}$ is a constant to be fixed by this boundary condition. Therefore, in the vacuum at tree level we have $\tilde{\pi} = 0$, $\bar{\sigma}^2 = F_\pi^2$ as discussed above. We stress that $\tilde{\sigma} = f_\pi \neq \bar{\sigma}$ in the vacuum, i.e., quantum fluctuations re-arrange the scalar condensate and other characteristics of the symmetries. The curious "constraint" between the values of the condensates yields the saturation density (or are fixed by it) and seems to have some meaning by means of a dimensionful proportionality constant to be calculated microscopically. The presence of the $\tilde{\pi}$ which should be present at experimental conditions may alter several usual interpretations of the observations which lead unambiguously to a direct (and fast) reduction of the scalar condensate.

The "condensate" equations from expressions (7,9), for $\tilde{\sigma}$ and $\tilde{\pi}$, can be faced as differential equations for ρ_f as written in equation (10). They can be written as:

$$(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2) \simeq \frac{2}{\lambda}\frac{d\rho_f}{d\tilde{\sigma}^2} \simeq \frac{2}{\lambda}\frac{d\rho_f}{d\tilde{\pi}^2} \tag{18}$$

Those equations are isomorfic and show an equal dependence of ρ_f with each of these two variables. We envisage three ways of finding solutions.

(1) An approximated solution for the two condensate equations can be found as if ρ_f were a function of these fields independently. These solutions for ρ_f, labeled for each of the above equations with a and b can be inverted and written as:

$$\begin{aligned} \tilde{\sigma}^2 &= \frac{v^2}{2} - \tilde{\vec{\pi}}^2 \pm \sqrt{\left(\frac{v^2}{2} - \tilde{\sigma}^2\right)^2 + \frac{4}{\lambda}\rho_f^a}, \\ \tilde{\vec{\pi}}^2 &= \pm\left(-\frac{v^2}{2} \pm \sqrt{\frac{\rho_f^b}{\lambda} + \frac{v^4}{4}}\right). \end{aligned} \tag{19}$$

Eliminating ρ_f from the second expression ($\rho_f^a = \rho_f^b$) we find the following approximated value for the pion condensate (if $|\tilde{\vec{\pi}}|^2 << v^2$):

$$\tilde{\vec{\pi}}^2 \simeq \frac{\tilde{\sigma}^2(\tilde{\sigma}^2 - \tilde{v}^2)}{4(-\frac{\tilde{\sigma}^2}{2} \pm \tilde{v}^2)}. \tag{20}$$

With $g_S = 10$ and $M^* = 0.7M$, where $M = 940$ MeV, we find the values $\tilde{\vec{\pi}}^2 \simeq 0.47 fm^{-2}$ and $\tilde{\vec{\pi}}^2 \simeq -0.034 fm^{-2}$. Only the second value seems consistent with the approximation done for expression (20). For the sake of comparison we remind that $v^2 \simeq f_\pi^2 \simeq 0.22 fm^{-2}$. In these solutions, as well as in others more exact, $\tilde{\vec{\pi}}^2$ may be either positive or negative.

(2) An alternative way can also be done by adding the two differential equations (18) which can be seen as partial differential equations. This yields $\rho_f = \rho_f^{(1)}(\tilde{\sigma})$ and $\rho_f = \rho_f^{(2)}(\tilde{\vec{\pi}}^2)$ with constants fixed for the boundary $\rho_B = 0$ when $\tilde{\vec{\pi}} = 0$ and $\tilde{\sigma} = v$. The constant obtained in the solution of the differential equations resulting from the addition of both equations for ρ_f are fixed by requiring that in the vacuum $\tilde{\vec{\pi}} = 0$ and $\tilde{\sigma} = v$. We find the solutions:

$$\begin{aligned} \rho_f^{(1)} &\simeq \frac{\lambda}{2}\tilde{\sigma}^2\left(v^2 - \tilde{\sigma}^2\right) + C_f(\tilde{\sigma}^2 - v^2), \\ \rho_f^{(2)} &= \frac{\lambda}{2}\tilde{\vec{\pi}}^2(v^2 - \tilde{\vec{\pi}}^2) - C_f\tilde{\vec{\pi}}^2, \end{aligned} \tag{21}$$

Where C_f is a constant.

(3) A more consistent way of solving these equations is to face the second of the above equations (18) as a differential equation for $\rho_f = \rho_f(\tilde{\vec{\pi}}^2)$ in which $\tilde{\sigma}^2$ is given by the "preliminar solution" of the first equation, which is given by the first expression in (19). We change the variables to: $t^2 = z^2 - 4\rho_f/\lambda - v^2 v^2$, and $z = \tilde{v}^2 - \tilde{\vec{\pi}}^2$ and obtain the following equation:

$$\frac{dt}{dz} = \frac{z}{t} \mp 1. \tag{22}$$

A solution for this differential equation is given by:

$$z\sqrt{t^2 - zt - z^2} = C_{\pi} exp\left(-\frac{\sqrt{5}}{5} arctanh\left(\frac{\sqrt{5}(z-2t)}{5z}\right)\right), \tag{23}$$

Where C_{π} is a constant determined by the boundary condition: $\bar{\pi}(\rho_f = 0) = 0$. This is a transcendental solution which may yield several solutions for only one fermionic density and sigma classical field $\bar{\sigma}$ which was eliminated by the first of expressions (21). Numerical values are showed in the next section. The opposite reasoning is valid for determining $\tilde{\sigma}^2$. Nevertheless in the present work this variable was fixed by a reasonable value of the baryonic effective mass as discussed above.

Finally, considering the extended Hamilton (or extended Euler-Lagrange) equation for V_0 - expression (12) - as a differential equation of the baryonic density ρ_B as a function of V_0 we obtain the following solution:

$$V_0(\rho_B) = \frac{-g_V \rho_B \pm \sqrt{g_V^2 \rho_B^2 - 2C_V \rho_B m_V^2}}{m_V^2}, \tag{24}$$

where C_V is a (negative) constant which is fixed to provide non-complex values of V_0. In spite of the existence of two solutions for V_0 they are degenerated and this constant will be proportional to the only contribution of the vector field to the energy density within this approach: $\mathcal{H}_V = C_V \rho_B$. Furthermore, in the limit of zero density *and/or* zero mass, $\tilde{m}_V^2$, we get $V_0 \to 0$. It is still required that the mass of the vector field is proportional to the density, i.e., it is an *in medium* effect, as required. This seems to be consistent with the supposition of equivalence of the redefinition of V_0 and the introduction of the chemical potential. It is seen that the baryonic density generates a non zero value of V_0 - which may be viewed as a "condensate" from at least another SSB, of a gauge symmetry.

The constant C_V is found fixing the nucleon binding energy. If the solution of the vector field component, V_0, corresponds to a value which minimizes the baryonic density at the saturation density ρ_0, i.e., requiring the baryonic density to be stable with relation to variations on V_0, we find that: $d\rho_B/dV_0 = 0$, $\quad d^2\rho_B/dV_0^2 > 0$. At this point we would have: $\bar{V}_0 = g_V \rho_0 / m_V^2$ which is the usual solution for the vector field associated to the lightest vector meson. However the contribution to the energy density can be written as $< \mathcal{H}' >_v = g_V^2 \rho_B^2 / (2 m_V^2)$ which has thus quite different form.

The massive vector field is then characteristic from a superconductive state which is expected to occur at finite density. This seems to suggest the existence of still another QCD condensate at finite density. The question of (non) renormalizability of the LSM and (couple to) massive gauge theory (moreover forming a finite density medium) will not be addressed in detail here. It is nevertheless worth to emphasize that this model is an effective model for QCD.

5 Numerical Results

Considering all the terms in the averaged density energy for the calculation of binding energy ($-E/A \equiv \mathcal{H}/\rho_0$) we adopt the values of $K \simeq 1$ fm^{-1}, $-E/A = 16.0$ MeV and $\rho_0 = 0.16$fm^{-3} in expressions (11,15,17,23,24). The value of the coupling constant λ is varied to obtain for the solutions of $\tilde{\sigma}^2$. We take $M^* \simeq 0.7M$ and $g_S = 9$. These values fix $\bar{\sigma} = M^*/g_S$. We would like to stress that the effective mass in nuclear matter is usually consider to be higher than in (most part of the) finite nuclei, which it depends on r. However considering smaller or larger effective mass these calculations would still present numerical solutions for the classical fields as presented below.

In figure 1 the solutions of expression (23) of the equation of the pion "condensate" square as a function of the coupling λ are shown - there may have several values for only one value of λ. The dots (crosses) correspond the minimum (maximum) values which the square pion "condensate" may acquire with the above parameters for each value of λ, i.e., the classical pion field, $\bar{\pi}^2$, may have values between the dots and crosses. We see that the pion condensate may be imaginary as well as it may assume relatively large absolute values with relation to $f_\pi^2 \simeq .22$ fm^{-2}. There is an intriguing behavior in this figure in two points: the discontinuities of the values when $\lambda \simeq 16$ and $\lambda \simeq 43$. These discontinuities are not yet very well understood. The values found in expression (20) can be, in some points, quite consistent with these shown in the figure 1. The solutions of the expression (20) do not depend on λ however.

The corresponding maximum and minimum values for the Symmetry Radius $\tilde{C}$ are obtained from expression (17) with the results of Figure 1 for fixed $\tilde{\sigma}$ and they can be seen in Figure 2 as a function of λ at the stabitily density. The same behavior of figure 1 is found because $\tilde{\sigma}$ was kept nearly constant (the most reasonable and believed value). These results are in agreement with the usual idea of symmetry restoration as precluded by Brown and Rho [9]. However a more extensive comparison will be left for another work. As showed above, the generalized symmetry radius $\tilde{C}$ is related to the values which the "pion condensate" may assume. Depending on these values we also found solutions for which the chiral symmetry behavior with density is the opposite. This may appear by using the same way of calculating $\bar{\pi}^2$ (item **3** above) to calculate the $\tilde{\sigma}$, i.e., from a similar expression to that shown for the sigma, for deriving equation (23). This means the possibility, although not very usual nor appealing, of further chiral symmetry breaking with $\tilde{\sigma} > v$ at finite density [19, 20]. At high densities asymptotic freedom makes QCD coupling constants to be weak and therfore quarks to deconfine eventually restoring chiral symmetry such that $\bar{\sigma} \to 0$. This is in fact what happens inside the nucleon [8]. It is interesting to note that the quantities usually identified with the chiral order parameter *in medium* may be in fact including the square pion condensate. In other words, we can ask: is the usual parameter associated to the experimental behavior of the nuclear matter at variable densities only $\tilde{\sigma}$ or does it take into account somehow the $\tilde{\pi}$ behavior which is not taken into account in the usual models?

For some of the solutions of figures 1 and 2 we show results for the vector field constant C_V which is found fixing the binding energy in expression (11) for fixed pion and sigma classical values and choosing the resulting $\tilde{C}$. Some values are shown below (for $M^* = 0.7M$ and $g_S = 9$):

$$\begin{aligned} \lambda \simeq 16.0 \to \sqrt{-\tilde{C}\sqrt{\rho_0}} \simeq 25MeV \to C_V \simeq -3.6fm^{-1} + M, \\ \lambda \simeq 40.0 \to \sqrt{+\tilde{C}\sqrt{\rho_0}} \simeq 55MeV \to C_V \simeq -3.9fm^{-1} + M, \end{aligned} \tag{25}$$

where M is the nucleon (effective) mass. Although the resulting C_V are similar there are some differences between these two solutions. In the first the value of the pion condensate is small (being more reasonable) ($\tilde{\pi}^2 \simeq -0.05fm^{-2}$), the constant $\tilde{C}$ is small and negative indicating that $\tilde{\sigma}^2 + \tilde{\pi}^2 < v^2$ at the saturation density and making $\tilde{\sigma}$ closer to f_π. In the second case $\tilde{C}$ is large and positive due to the large value of the pion condensate $\tilde{\pi}^2 \simeq 0.14fm^{-2}$ and then $\tilde{\sigma}^2 + \tilde{\pi}^2 > v^2$. The values of C_V are nearly consistent with estimates from the expressions deduced above for the vector field parameters, to be of the order of the usual ones, i.e., $m_V \simeq 780$ MeV and $g_V \simeq -5$.

In figure 3 we show the behavior of the pion mass in the medium (divided by its value in the vacuum) for some of the solutions shown in figures 1 and 2 as a function of the coupling λ - keeping $\tilde{\sigma}$ constant. We want to call the attention of the reader that the pion mass in the vacuum is non zero, generated by quantum fluctuations (expression (9)). A calculation with zero pion mass was equally done resulting in different results leading to a disappearance of the pion condensate for several values (but not all) of the coupling λ and will be shown elsewhere. By varying further the scalar condensate we can obtain different results. The behavior with varying density is more involved and deserves more developments. The increase of the value of μ_P may be associated to the tendency of the restoration of chiral SSB at finite density. A complete account of these possibilities will be shown elsewhere.

From the GAP equations of pion and sigma we can write the ratio of their *in medium* masses as:

$$\frac{\mu_P^2}{\mu_S^2} = \frac{2\tilde{\pi}^2 + \tilde{C}\sqrt{\rho_B}}{2\tilde{\sigma}^2 + \tilde{C}\sqrt{\rho_B}}. \tag{26}$$

This expression reduces to a non zero value in the vacuum according to the assumptions done for the pion mass because $G_i \neq 0$.

In Figure 4 we show values of the ratio of the *in medium* sigma mass ($\mu_S^2 = \mu_S^2(\tilde{\sigma}, \tilde{\pi}^2)$) to its value in the vacuum as a function of the λ for some of square pion condensate solutions of figure 1. This *in* *medium* mass, for lower values of the λ is lower than its value in the vacuum ($\mu_S^0 \simeq 482 MeV$ in the present work, smaller than usually considered) whereas for higher values of λ it becomes higher than μ_S^0. An increase of the μ_S^2 was also found in the quark meson coupling model with a similar calculation without pions [20] although the origins of this effect seem to be different here.

6 Further Discussions

The above solutions were found by fixing the scalar condensate to fit the *in medium* effective mass of the nucleon, the $\bar{\pi}$ contribution was assumed and found to be smaller for most values of the model parameters. This was done for a coupling $g_S = 9$ which is not necessarily the best value. The scalar condensate could then be smaller or greater than $\bar{\sigma}$ in the vacuum which would correspond to g_S smaller or greater by considering a (fixed) effective mass for the nucleon. The scalar and pseudo scalar condensates would compose the observables which are usually attributed uniquely to the value of $\bar{\sigma}$ without the pion condensate (it is only expected to appear at higher densities, even if this would contradict asymptotic freedom (and eventually deconfinement)) intervening in the same range of energies/densities. Recently quantum fluctuations of the sigma were included in the quark-meson-coupling model (QMC) without pions [20]. A further *in medium* symmetry breaking was found with increasing baryonic density (asymptotic freedom and the symmetry restoration inside nucleons as "bags" were not considered [28, 8]). These would be contradictory effects in principle although pions were not included in both models in the same way. In the present work, several pictures for restoration (or further breakdown) of isospin and chiral symmetries (and eventually the related gauge symmetry) appear. These results for μ_S and μ_P are not completely self consistent because $\bar{\sigma}$ and G_i are fixed at ρ_0. In principle renormalization of the ultraviolet divergences in a more complete calculation should not change this features because in an effective model the corresponding cutoff or renormalization energy scale are expected to be fixed finite. These solutions of decreasing and increasing $\bar{\sigma}$ with density would correspond respectively to the symmetry restoration -according qualitatively at least to the Brown Rho scaling [9] - or, in a less usual or appealing picture, to a further breaking of the chiral symmetry at finite baryonic density [19, 20]. This brings new possibilities for the phase diagram and structure of finite density QCD-like theories and models.

With the expressions for ρ_f (21) and the symmetry radius $\tilde{C}\sqrt{\rho_0}$ (from expression (17)) we obtained a consistent basis for the study of the dependence of the condensates with density. Expecting that the solutions for the densities (21,17) are good in the region close to ρ_0 we can equate expressions (21) to obtain another "constraint" for the condensates:

$$C_f \tilde{C} \sqrt{\rho_0} = \frac{\lambda}{2} \left((\tilde{\sigma}^2 - \frac{v^2}{2})^2 - (\tilde{\pi}^2 - \frac{v^2}{2})^2 \right). \quad (27)$$

This also expresses the dynamical symmetry breakings which occur in the medium due to the fact that C_f and $\tilde{C}$ are non zero. However if they were zero (or $\rho_B = 0$), for in the vacuum, we would also have non vanishing $\bar{\sigma}$ with the chiral SSB.

At finite baryonic densities therefore there seems to have a non zero expected classical pion field ("condensate", whose meaning is not necessarily the same as discussed in [14]). Departing from the calculation of M^* in the vacuum, using it for finite density situations [1], we propose in the following nucleonic effective masses [2]. This would explicit or manifest the fact that the nucleon densities depend on the pion condensate. From the averaged value of the Lagrangian we may consider a matrix for M^* which depends on the isospin (and spin) of the nucleons:

$$M^*_{a,b;s} = g_S < \Psi[\sigma,\pi]|. < N_{a,s}|(\sigma + i\gamma_5 \vec{\tau}.\pi)|N_{b,s} > .|\Psi[\sigma,\pi] >= g_S(\bar{\sigma} + i\tilde{M}^d_{(a,b),s}\bar{\pi}_d). \quad (28)$$

In this expression a, b, d stands for the isospin index (neutrons/protons), $\tilde{M}$ is a non diagonal isospin matrix and the final spin (s) structure is not written. This allows for the possibility of different values of baryonic masses - i.e. a mass splitting between neutron and proton states - but it also means possible oscillations between the isospin states (proton and neutron for example) in the medium associated to electroweak processes. This matrix includes a dependence on the baryon spins as well which is not discussed further here. The nucleonic mass would be therefore obtained by the averaged value $\bar{\sigma}$ plus a contribution from the averaged value of the pion field. However, the quantities calculated for the nucleons in the present work -the densities (ρ_f,ρ_B)- were the usual ones, i.e., with a constant diagonal effective mass due to $\bar{\sigma}$, i.e. $M^*_{a,b}(\bar{\pi}) \simeq M^*$, which is the leading term. Furthermore the mass splitting leads to different densities for nucleons what is expected to be observed in finite nuclei where the stability line shows already $N - Z \neq 0$ for non light nuclei. This non trivial solution corresponds to a non invariant ground state under an isospin transformation, although the Lagrangian is symmetric. This is a dynamical symmetry breaking of isospin symmetry.

This mass splitting, which is probably connected with the Nolen-Schiffer effect [29], relates the nucleon effective masses to the scalar and pseudo scalar QCD condensate(s) $< \bar{q}q >, < \bar{q}\gamma_5(\tau)q >$. It is obtained

[1] Either from the Lagrangian density or from the equation of motion of the baryons

[2] A different expression seems to arise if M^* is calculated from the averaged energy density depending on the level of approximation for the baryon wavefunctional. This way a nucleon in the surface of a nucleus, where pairing is to be more important, would have not only a different value but also a slightly different "structure" than that of a bulk nucleon.

from expression (28) and it is given by:

$$\Delta M^* = M_n^* - M_p^* \simeq 2ig_S|\bar{\pi}_3|. \tag{29}$$

Considering one small imaginary solutions of $\bar{\pi}$ from figure 1 with $\lambda \simeq 60$ we find that $\Delta M^* \simeq 40 g_S$ MeV which is seemingly too much large for the coupling $g_S \simeq 9$. The inverse reasoning can be done and then $g_S|\bar{\pi}|$ could be fixed to reproduce an expected ΔM^*. This yields a smaller value. A more reasonable value can be found if we introduce the nucleon mass term in the Lagrangian for which case the coupling g_S would be smaller. In this case, one would have good hints from nuclear phenomenology of how to obtain better measures and manifestation of these pion "condensates".

For spontaneous symmetry breakings one usually expects zero energy collective modes to appear [30]. In nuclear matter calculations - usually zero sounds (damped or not) - they can be associated to giant resonances in nuclei [31]. In particular, the isovector channel seems to suggest the idea that the massive vector field is in fact a sort of "dressed (massive) photon" which would be characteristic of a superconducting state. In the isovector channel, which is excited by means of a photon from electromagnetic external interaction, there is a very collective resonant behavior (known as dipole isovector giant resonance in nuclei) that makes charged protons move in the opposite direction of neutrons although they are kept bound. The giant dipole resonances decay by photon emission with energies around $10 - 15$ MeV. These must be indicating a deeper relation between strong and electromagnetic interactions. Finally, the breakdown of isospin symmetry is connected to the charge conservation.

Processes involving pions in the nuclear medium provide valuable information. Let us take for grant that the Goldberger-Treiman relation nearly holds at the saturation density. If we write it in such a way as to encompass quantum fluctuations with the rearrangement of the scalar condensate as considered in expression (6) we can write, independently for protons or neutrons, (which now would have non degenerated masses):

$$g_S\bar{\sigma} = (M^* \pm \Delta M^*) g_A, \tag{30}$$

Where ΔM^* is given below expression (29) for protons and neutrons. For in the vacuum ($\Delta M^* \to 0$, $\bar{\sigma} = f_\pi$ and $M^* \to g_S\bar{\sigma}$) we obtain a small value $g_A \simeq 1.05$. It could not be expected to result a realistic value in the vacuum (although it is reasonable at the saturation density) for g_A with the present arguments, but we can expect that the behavior at varying density may be reasonable since it is expected to change with density in spite of new results [32]. Weak interactions should be considered for a full precise picture. Again we notice differences between the chiral radius and sigma (or QCD scalar) condensate (and eventually pion decay constant) at any density as precluded before [8].

The fact that the $\phi - \omega$ type models (cubic and quartic order scalar interaction terms) are usually accepted as more suitable for the nuclear observables [13, 15] seems to be a consequence of the fact that the calculations usually done with them take into account nonlinearities which may nearly be present (or which may be equivalent) in a more exact linear realization of the sigma model with "pion condensates" as we have studied (even if pion condensate is quite small and maybe difficult to measure experimentally, although not necessarily the same as studied before [14]). This also means that self consistency of these works ([13, 15]) may be enlarged: first of all, as we have shown, the usual vector field (not necessarily the omega meson) solution was extended to take into account more interactions and correlations with fermions. This means that a sort of truncation is done even in the coupled (Euler-Lagrange) equations for the non linear models - even more because this picture does not take into account all properties/observables of the system. We do not neglect the possibility that the scalar present in these non linear models is related to the sigma condensate in a non trivial way. However in our approach the connection with QCD is much clearer and more direct. Besides that the presence of the "pion condensates" indicates a stronger relationship to the Quantum Chromodynamics and secondly a richer qualitative basis to the description of nuclear systems. The next step seems to be the calculation of finite nuclei properties with the linear realization of chiral symmetry as we study here and compare to previous works.

7 Summary and final remarks

We have exhibited solutions for the linear sigma model coupled to a massive vector (gauge) boson and nucleons which are proposed to be the basic elements for the properties of a finite density hadronic system. This offers a suitable and beautiful frame for the study of zero and finite baryonic density strong interacting systems. The self consistent equations of the fields, at the level of approximations done, yield a non zero expected pion classical field ("condensate"). Pion "condensation" seems to occur already at not high density without need of further assumptions for the system, although our concept of condensate is not the same as considered before [14]. It may induce topological properties for the system. It was found to be directly related to the Chiral SSB which already occurs in the vacuum. Expressions of constraint for the fields at the saturation density were found defining a chiral-isospin radius given by an expression like (17), i.e.,

$$\tilde{C}\sqrt{\rho_B} \simeq \left(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2\right).$$

The non zero value of the pion condensate lead to a splitting of the neutron and proton masses in the medium with the possibility of oscillations between these isospin two states. Together with the pion condensates this garantees the charge conservation being eventually related to electroweak interactions issues which will be developped elsewhere. Some systems seems to be more appropriate to the experimetal observation of such effects.

A non zero solution for the massive (gauge) vector field - such as a "condensate" - at finite density was found from a modified dynamical equation and it seems to correspond to a dynamical breakdown of a gauge symmetry typical of superconductors. The classical vector field solution is a consistent solution which takes into account more effects than the usual solution considered in nuclear matter investigations. Its mass is considered to be non zero at finite density eventually reducing to zero in the vacuum suggesting it may be considered to be a *dressed photon* asking with arguments of the kind of vector meson dominance for identification with the omega meson. This may occur also due to a B.E.C. condensation of component(s) of this field. Phase diagram of QCD would probably include these pseudo scalar and vector "condensates". The present model will be considered for the description of (hadronic) nuclei eventually with inhomogeneous situations or in cases for which topological properties may arise.

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Figure caption

Figure 1 Squared pion condensate $\tilde{\pi}^2$ (fm^{-2}) as a function of the coupling λ for $M^* = 0.7M$ and $g_S = 9$ found self consistently.

Figure 2 Symmetry radius $\tilde{C}$ ($fm^{-1/2}$) for the solutions of figure 1 as a function of λ.

Figure 3 Ratio of the squared pion mass in the medium divided by its value in the vacuum as a function of λ for the solutions of figure 1.

Figure 4 Ratio of the squared sigma mass in the medium divided by its value in the vacuum as a function of λ for the solutions of figure 1.

Figure 1

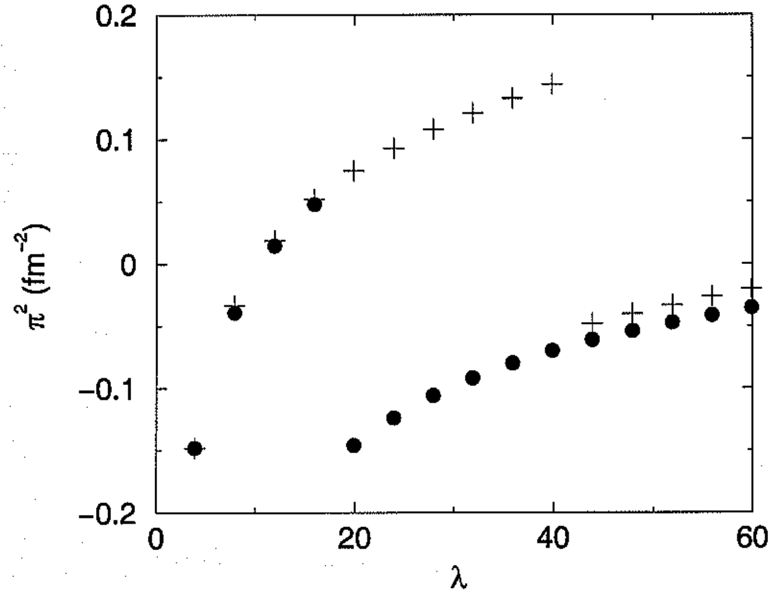

Figure 2

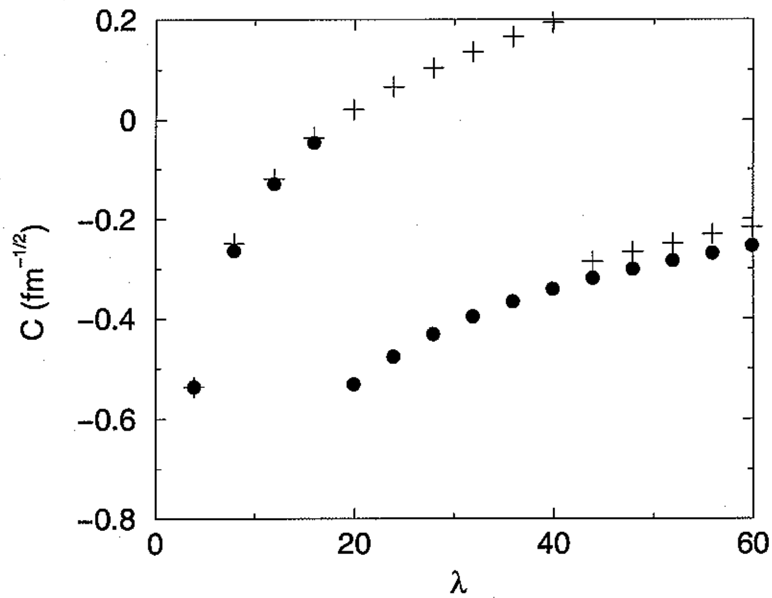

Figure 3

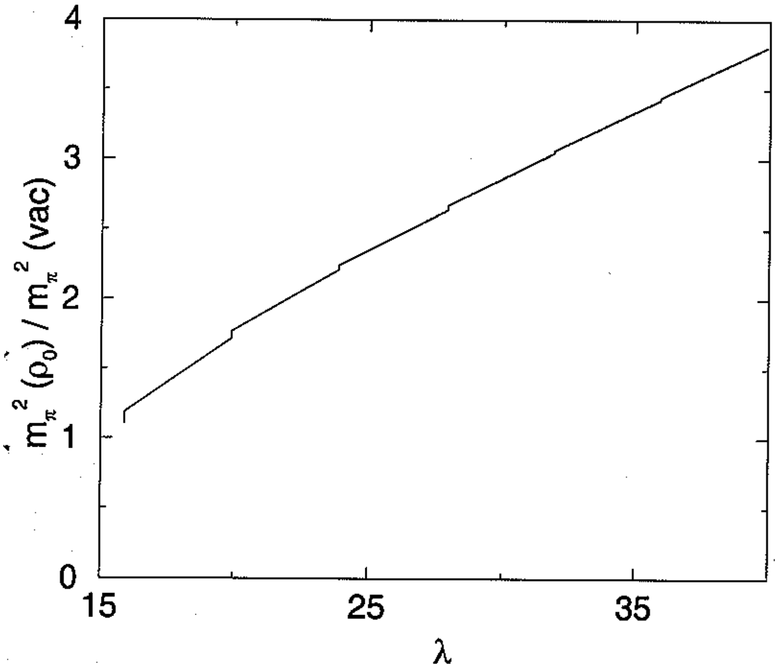

Figure 4