Instituto de Física
Universidade de São Paulo

Linear Sigma Model at finite baryonic density: symmetry breakings

Braghin, Fábio L.

Instituto de Física, Universidade de São Paulo, CP 66.318
05315-970, São Paulo, SP, Brasil

Publicação IF – 1597/2004

UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

Linear Sigma Model at finite baryonic density: symmetry breakings

Fábio L. Braghin *

Instituto de Física, Universidade de São Paulo; C.P. 66.318; CEP 05315-970; São Paulo, SP, Brazil.

Abstract

The linear sigma model at finite baryonic density with a massive vector meson is investigated considering that all the bosonic fields develop non zero expected classical values corresponding to dynamical symmetry breakings. The ground state stability condition is analyzed with particular prescriptions. A modified equation for the classical field of the vector meson is proposed with its respective solution. General properties of nuclear matter are reproduced.

Key-words: Spontaneous symmetry breaking, finite density, gauge symmetry, chiral symmetry, isospin, pion, sigma, baryon, condensates, QCD.

IF- USP - 2001-2002, 2004

*e-mail: braghin@if.usp.br

1 Introduction

Quantum Cromodynamics (QCD) has intrincated color non abelian and flavor structures and strong coupling constants at low energies being therefore very difficult to obtain exact solutions. Besides lattice calculations effective models are developed such that the main properties and symmetries are respected for the energy range of interest [1, 2]. In the vacuum, the lightest strong interacting particles are known to respect, approximately at least, chiral symmetry $SU_L(2) \times SU_R(2)$ which is spontaneously broken down to $SU(2)$ [1, 2, 3]. Pions, whose masses are small in the hadronic scale, are viewed as the (quasi) Goldstone bosons of such spontaneous symmetry breaking (SSB) [2, 4]. The vacuum is expected to acquire a non trivial structure due to the formation of scalar quark-anti quark condensate $< \bar{q}q >$, the order parameter. The emergence of condensates in the ground state from a SSB must appear in an exact calculation, they rearrange the properties of the theory [2, 5]. These features can be taken into account via sigma models which, in the linear realization, implement chiral symmetry with two spin zero fields: the (pseudo-scalars) pions and the (scalar) sigma [3]. At finite density, QCD is known, and expected, to have a very complex phase diagram with the appearance of other condensates at very high densities (color superconductive phase). With different approaches finite density effective models at finite density have been extensively studied including for nuclear models for infinite matter and nuclei, at the normal and high densities [6, 7, 9, 10]. Although some authors have claimed that the non linear realization is the one Nature has chosen for finite density systems [6] there are actually several indications that the linear realization can be good also due to the recent investigations on the light scalar mesons [10, 11, 12, 13, 7] and those arguments may not be completely correct. In this work the O(N) Linear Sigma Model (LSM) at finite baryonic density is investigated with a massive vector meson. Particular prescriptions for the stability condition considered which solve the field equations. All the mesons in the model are considered to develop classical counterparts breaking spontaneously the symmetries. The pion field whose expected value is called pion condensate although it is not the same investigated previously [14]. The numerical investigation of the results will be presented elsewhere [7].

2 Linear sigma model at finite density calculations

The Lagrangian density of the Linear Sigma Model for nucleons $N(\mathbf{x})$, sigma and pions $(\sigma, \bar{\pi})$ coupled to a massive vector meson V_μ, is given by:

$$\begin{aligned}\mathcal{L} = \bar{N}(\mathbf{x})\left(i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma + i\gamma_5 \boldsymbol{\tau}.\bar{\pi})\right) N(\mathbf{x}) + \frac{1}{2}\left(\partial_\mu\sigma.\partial^\mu\sigma + \partial_\mu\bar{\pi}.\partial^\mu\bar{\pi}\right) + \\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{4}\left((\sigma)^2 + (\bar{\pi})^2 - v^2\right)^2 + \frac{1}{2}m_V^2 V_\mu V^\mu + c\sigma,\end{aligned} \tag{1}$$

a covariant derivative is used: $\mathcal{D}^\mu = \partial^\mu - i g_V V^\mu$ and the kinetic tensor is: $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. g_V, g_S and λ are the coupling constants and the chiral radius $v = f_\pi$ in the vacuum. The coupling of the temporal component of the vector meson to the baryons is equivalent to the redefinition of a chemical potential. The pion mass breaks the chiral symmetry explicitly by the term $\mathcal{L}_{sb} = c\sigma$, where $c \propto m_\pi^2$ by imposing low energy theorems [2]. This does not have great relevance for our original results. Classical components (condensates) for all the mesonic fields are considered and solutions for the respective equations are found. In this work the whole baryon masses come from the the coupling to the scalar mesonic field by the Higgs mechanism ($M^* = g_S\bar{\sigma}$). Considering an explicit mass term for the baryons in the Lagrangian due to the gluonic content does not change the original results of this work. This would lead to an *in medium* mass: $M^* = M \pm g_S\bar{\sigma}$.

The baryon (nucleon) field is quantized in terms of creation and annihilation operators and the Dirac equation with the coupling to the bsonic classical fields is solved. Its wave function can be written as superposition of spinor (χ), isospinor (η) and coordinate components $u(\mathbf{p})$, $v(\mathbf{p})$. The baryonic degrees of freedom remain in the densities: baryonic, scalar and pseudo-scalar densities (ρ_B, ρ_s and ρ_{PS}). We will not explicitly evaluate here all these quantities. The fermionic density can be approximatedly written in terms of the nucleon momentum at the Fermi surface, k_F, as usually considered [6]:

$$\rho_f = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + (M^*)^2}, \qquad \rho_B = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}. \tag{2}$$

These expressions correspond to an approximation since the contribution of the classical vector fields were not considered. This will be shown elsewhere [7].

The Euler-Lagrange equations for the static and homogeneous system define the minimum of the potential for the (classical) fields, "condensates". The following set of equations is obtained:

$$\bar{\sigma}\lambda\left(\bar{\sigma}^2 + \bar{\pi}^2 - v^2\right) + \frac{d\rho_f}{d\bar{\sigma}} = 0, \quad \lambda\bar{\pi}_a\left(\bar{\pi}^2 + \bar{\sigma}^2 - v^2\right) + \frac{d\rho_f}{d\bar{\pi}_a} = 0. \tag{3}$$

We have therefore that:

$$\frac{d\rho_f}{d\bar{\pi}_a} \propto \rho_{PS} \neq 0,$$
$$\frac{|\bar{\pi}|}{\bar{\sigma}} \propto \frac{\rho_{PS}}{\rho_S}. \tag{4}$$

The ratio of the scalar and pseudo scalar densities settle the ratio of the respective condensates.

The sigma and pion masses are given by:

$$\mu_P^2 = \lambda\left(3\bar{\pi}^2 + \bar{\sigma}^2 - v^2\right) + \frac{c}{v}, \qquad \mu_S^2 = \lambda\left(3\bar{\sigma}^2 + \bar{\pi}^2 - v^2\right). \tag{5}$$

For the vacuum $\bar{\pi} = 0$ and $\bar{\sigma} = v$ leading to a zero pion if $c = 0$ in agreement with the Goldstone theorem. However at finite density $\bar{\pi} \neq 0$ yielding a strong density dependence of the masses at finite density due to the dependence of the condensates on the densities.

The total averaged energy density can be written as:

$$\mathcal{H} = \rho_f + g_V V_0 \rho_B - \frac{1}{2} m_V^2 V_0^2 + \frac{\lambda}{4}(\bar{\sigma}^2 + \bar{\pi}^2 - v^2)^2. \tag{6}$$

The dynamical equation for the vector meson was calculated considering the fact that the baryonic densities do depend on V_μ. Although this is obtained from the solution of the Dirac equation this full solution will not be calculated here. Instead, It will be assumed that $\rho_B = \rho_B[V_0]$ without any previously determined form. Only the component spatial component V_0 is considered. If we consider the other components $V_i \neq 0$ the equations are slightly changed, but the conclusions remain valid with another coupled equation. The modified vector meson equation is therefore obtained by the variation of the energy density with respect to its classical field. It is given by:

$$g_V\left(\rho_B + V_0 \frac{d\rho_B}{dV_0}\right) - m_V^2 V_0 = 0. \tag{7}$$

3 Ground state stability and remarks

The stability equation is faced, in the following, as a dynamical equation whose solutions are found within a particular prescription such that the main properties of a (bound) finite density system are consistently described. The stability condition of the system can be written as: $\frac{d\mathcal{H}}{d\rho_B} = \frac{\mathcal{H}}{\rho_B} < 0$ at $\rho_B = \rho_0$, where ρ_0 is the stable density and $\mathcal{H}$ is given by expression (6). To guarantee that this expression is satisfied we consider some prescriptions for the dependence of its dynamical variables on the baryonic density. This equation is considered to be equivalent to the following ones:

$$\begin{aligned}
\frac{d\rho_f}{d\rho_B} &= \frac{\rho_f}{\rho_B},\\
\frac{d(\bar{\sigma}^2+\bar{\pi}^2-v^2)}{d\rho_B} &= \frac{(\bar{\sigma}^2+\bar{\pi}^2-v^2)}{\rho_B},\\
\frac{d\mathcal{H}_V}{d\rho_B} &= \frac{\mathcal{H}_V}{\rho_B}.
\end{aligned} \tag{8}$$

In this last expression $\mathcal{H}_V = g_V V_0 \rho_B - \frac{1}{2} m_V^2 V_0^2$ is the energy density with contributions of the vector meson. This last expression is equivalent to the modified equation (7) which is solved below.

From the first of the differential equations (8) it is found an expression for ρ_f as a function of the baryonic density ($\rho_f = \rho_f(\rho_B)$) which is in excellent agreement with that resulting from the integration of the usual expression (2) for densities close to ρ_0. It is given by:

$$\rho_f = \frac{K\rho_B}{9} Ln\left(\frac{\rho_B}{\rho_0}\right) - \frac{K}{9\rho_0}\left[\left(\frac{B}{2} - \frac{\rho_B}{\sqrt{\rho_B}}\right)^2 + \frac{B^2}{4}\right], \tag{9}$$

where B is a constant to be adjusted numerically. None of these expressions , (2) or (9), are exact because the coupling to V_0 introduces relevant modifications [7].

From the second expression in (8) there appears a solution which can be defining a symmetry radius in the medium:

$$(\bar{\sigma}^2+\bar{\pi}^2-v^2) = \tilde{C}\sqrt{\rho_B}. \tag{10}$$

In this expression $\tilde{C}$ is a constant to be determined from the parameters of the model [7]. Therefore in the vacuum: $\bar{\sigma}^2 = v^2 = f_\pi^2$ as discussed above.

The condensate equations (3), for $\bar{\sigma}$ and $\bar{\pi}$ can be written as:

$$(\bar{\sigma}^2+\bar{\pi}^2-v^2) + \frac{2}{\lambda}\frac{d\rho_f}{d\bar{\sigma}^2} \simeq 0, \quad (\bar{\pi}^2+\bar{\sigma}^2-v^2) + \frac{2}{\lambda}\frac{d\rho_f}{d\bar{\pi}^2} \simeq 0. \tag{11}$$

These equations have an isomorfism. A solution for the two condensate equations can be found considering that ρ_f is a function of these fields independently. This yields $\rho_f = \rho_f^{(1)}(\bar{\sigma})$ and $\rho_f = \rho_f^{(2)}(\bar{\pi})$ and by inverting these expressions $\bar{\sigma}(\rho_f)$ and $\bar{\pi}(\rho_f)$. Eliminating ρ_f from one solution into the other the following approximated value for the pion condensate is found (if $|\bar{\pi}^2| << v^2$):

$$\bar{\pi}^2 \simeq \frac{\bar{\sigma}^2(\bar{\sigma}^2-\tilde{v}^2)}{4(-\frac{\bar{\sigma}^2}{2} \pm \tilde{v}^2)}, \tag{12}$$

In these solutions, as well as in others more exact, $\bar{\pi}^2$ may be either positive or negative.

Finally, considering the equation of V_0 - expression (7) - as a differential equation of the baryonic density ρ_B as a function of V_0 the following solution is obtained:

$$V_0(\rho_B) = \frac{-g_V \rho_B \pm \sqrt{g_V^2 \rho_B^2 - 2C_V \rho_B m_V^2}}{m_V^2}, \tag{13}$$

where C_V is a constant. This constant will be the only contribution of the vector meson sector to the energy density $\mathcal{H}_V = C_V \rho_B$. It has negative sign to keep V_0 real. In the limit of zero density: $V_0 \to 0$. This is consistent with the assumption of equivalence of the redefinition of V_0 and the introduction of the chemical potential. It is seen that the baryonic density generates a non zero value of V_0 - which can be viewed as a condensate. This may be another dynamical symmetry breaking. From this we see that the mass of the vector meson is proportional to the density, i.e., it is an *in medium* effect. This seems to suggest the existence of other QCD condensate(s) at finite density.

From the coupling of pions to fermions in the Lagrangian, the non zero pion condensate will induce a difference between the *in medium* baryonic isospin states (which can be the neutron and proton). The neutron and proton effective masses have the following form: $M^* = g_s(\bar{\sigma} \pm \bar{\pi}_0 \tilde{M})$, where $\tilde{M}$ is dependent on their spins, being therefore non-degenerated. This non trivial solution corresponds to a non invariant ground state under an isospin transformation, although the Lagrangian is symmetric. Different baryonic masses generate different densities leading to an asymmetry [8]. Zero energy (Goldstone) collective modes are therefore expected to occur. Several collective modes identified to zero-sound like excitations have been found in nuclear matter calculations with non relativistic calculations [15]. The behavior of the properties of the model, such as masses, with density are in qualitative agreement with ideias developped in other works [10] and these aspects will be addressed elsewhere. The present model will be considered for the description of hadronic (nuclear) matter properties and eventually of nuclei elsewhere [7]. These properties depend strongly on the values and signs of the coupling constants g_S and λ.

Acknowledgement

This work was partially supported by FAPESP, Brazil.

References

[1] S. Weinberg, Physica **A 96**, 327 (1979). G.P. Lepage, in Proceedings of J.A.Swieca Summer School-Nuc. Phys., Campos do Jordão, Brazil, ed. by C. Bertulani *et al*, World Scientific, (1997).

[2] S. Weinberg, *The Quantum Theory of Fields -II*, Cambridge University Press, (1996). C. Itzykson and J.-B. Zuber, *Quantum Field Theory*, McGraw Hill Int. Editions, Singapore, (1985).

[3] Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960), Phys. Rev. **117**, 648 (1960). Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** 354 (1961), **124** 246 (1961).

[4] J. Goldstone, Nuovo Cimento **19** 154 (1961).

[5] F.L. Braghin, Doctoral thesis, unpublished.

[6] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16**, (1986) 1; Int. Jour. Mod. Phys. **E 6**, (1997) 515 and references therein. R.J. Furnstahl and B.D. Serot, Phys. Rev. **C 47** (1993), 2338. R.J. Furnstahl, B.D. Serot, H.-B. Tang, Nucl. Phys. **A 615**, 441 (1997).

[7] F.L. Braghin, pré-print IF-1570/2001, Instituto de Física da USP (2001). hep-ph/0206106.

[8] F.L. Braghin, Proceedings of IWARA 2003, Olinda, PE, Brazil, Int. Journ. of Mod. Phys. **D 13**, 1267 (2004).

[9] A. Delfino, F.S. Navarra, M. Nielsen, R.B. Prandini, M. Chiapparini, nucl-th/9806020.

[10] G.E. Brown and M. Rho, Phys. Rep. **269**, 333 (1996).

[11] K. Hagikawa *et al*, Particle Data Group, Phys. Rev. **D 66** 010001 (2002).

[12] N.A. Tornqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).

[13] E.M. Aitala *et al*, Phys. Rev. Lett. **86**, 770 (2001).

[14] A.B. Migdal, E.E. Saperstein, M.A. Troitsky, D.N. Voskresensky, Phys. Rep. **192** 179 (1990).

[15] F.L. Braghin, D. Vautherin, A. Abada, Phys. Rev. **C 52**, 2504 (1995). F.L. Braghin, Nuc. Phys. **665 A** 13 (2000).