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IN MEDIUM HADRON PROPERTIES AND SPONTANEOUS SYMMETRY BREAKINGS IN MODIFIED LINEAR SIGMA MODEL AT FINITE BARYONIC DENSITY

BRAGHIN, FL

Instituto de Física, Universidade de São Paulo,
C. P. 66.318; CEP:05315-970; São Paulo – SP, Brazil

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UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

In medium hadron properties and spontaneous symmetry breakings in a modified linear sigma model at finite baryonic density

Fábio L. Braghin *

Instituto de Física, Universidade de São Paulo,

C.P. 66.318; CEP 05315-970; São Paulo - SP, Brazil.

Abstract

The two-flavor linear sigma model coupled to a vector meson is investigated at finite baryonic density. The sigma and pion fields develop classical counterparts ("condensates") spontaneously breaking the internal symmetries. Their quantum fluctuations are computed with the Gaussian approximation with truncations. Analytical solutions, which satisfy ground state stability condition with a particular prescription, are proposed. The incompressibility modulus K is obtained as a boundary condition of a differential equation. Hadron properties in the medium are investigated with particular attention. The sigma mass is found either to increase or decrease with density whereas the pion mass tends to increase in most of the solutions. A finite density symmetry radius is defined. The baryon (effective) masses become different due to the isospin symmetry breaking. A modified (variational) equation is proposed for the vector meson. Its solution may eventually be associated to another dynamical symmetry breaking at finite density.

Key-words: Spontaneous symmetry breaking, sigma, finite density, gauge symmetry, chiral symmetry, condensates, isospin, superconductivity, superfluidity, hadron masses, mass splitting, state oscillations, quantum fluctuations, effective mass, nuclear matter, incompressibility, Fermi liquid, photon, QCD, symmetry restoration.

*e-mail: braghin@if.usp.br

1 Introduction

When a Lagrangian theory is invariant under a transformation group and the ground state of this theory is not, this symmetry is spontaneously (or dynamically) broken (SSB) [1]. There may appear non zero expected value(s) of field(s) (composite or not) in the ground state, the so-called condensate(s). The energy is lowered and the system is re-arranged [2, 3]. This is the Nambu picture in contrast to the Wigner-Weyl one for which, in the case of hadronic models invariant under chiral symmetry, there would be degenerated states in multiplets with opposite parity that are not observed in the low energy hadronic phenomenology. The symmetries and their SSB are deeply related to the phase diagram of the theory [3].

Quantum Chromodynamics has intrincated flavor and non abelian color structures, strong coupling constants for processes mainly at low and intermediary energies. At not high energies the degrees of freedom of quarks and gluons are confined such that no colored states are observed. Due to these theoretical and experimental aspects it is very difficult, if not impossible, to obtain exact solutions for the theory. Such solutions are more expected to be accomplished in finite lattices where space-time is discretized. An alternative procedure is to develop effective models which respect the main properties and symmetries of the QCD for the range of energy densities (temperature, density) for the process(es) of interest and/or to integrate out some degrees of freedom to derive simplified models. In the vacuum, the lightest hadrons are known to respect, at least approximatedly, chiral symmetry $SU_L(2) \times SU_R(2)$ which is spontaneously broken down to $SU(2)$. Vacuum should acquire a non trivial structure due to the formation of scalar quark–anti-quark condensate $< \bar{q}q >$, which would be the order parameter of the Chiral SSB [3]. QCD vacuum is expected to have a preferencial direction in the chiral/flavor space.

These features can be taken into account in sigma models (invariant under $O(4)$ or $SU(2) \times SU(2)$ with two-flavor for the lightest hadrons) In the linear realization with mesons in the Nambu realization pseudo scalar pions have small masses in the hadronic scale playing a special role in hadronic and nuclear physics. They are associated to (quasi) Goldstone modes of the chiral symmetry breakdown [4, 3] since their masses are small in the hadronic spectrum. The (ground state) expected value of sigma is considered to be the order parameter of chiral SSB. One of the main links between fundamental and hadronic degrees of freedom is settled by the Gell-Mann Oaks Renner (GOR) relation:

$$m_q < \bar{q}q >= -f_\pi^2 m_\pi^2. \tag{1}$$

The order parameter $< \bar{q}q >$ can be associated to $\bar{\sigma} \propto f_\pi$ keeping fixed quark and pion masses. Time ago S. Weinberg pointed out difficulties in the linear sigma model for describing hadronic processes and proposed the non linear realization of chiral symmetry in which the sigma field is eliminated [5]. This is rediscussed nowadays. There are nowadays experimental and theoretical evidences for light scalar mesons [6, 7, 8, 9, 10, 11, 12, 13] although some results indicate they (all) may not be a quark-anti-quark meson [14]. These (experimental and theoretical) developments demand new investigations for the linear realization of chiral symmetry not only in the vacuum but also at finite energy densities. Both realizations of chiral symmetry have been extensively developped since the earliest works and they are expected to be nearly equivalent [1, 3, 5].

The linear realization of chiral symmetry exhibits several advantadges over the non linear realization for example for the description of finite density hadronic matter. QCD is known/expected to have a complex phase diagram with several different phases at non zero chemical potential. The restoration of the chiral symmetry is expected to occur $< \bar{q}q >$ would be equal (or very close) to zero at high energy densities. While the sigma acquires a classical value the pion field (ground state) expected value is zero in the vacuum, $\bar{\pi} =< vac|\pi|vac >= 0$. The properties of the hadrons as functions of the density below and close to the chiral symmetry restoration phase transition is also intensively studied, to quote few examples [15, 16, 17, 18, 19, 20, 21, 22]. Some analysis of the *in medium* linear sigma model show that the sigma becomes less broad than in the vacuum, feature associated to the trend of restoration of chiral symmetry, among other related effects [23, 15, 24]. Extended linear sigma models including a gluball degree of freedom, for considering trace anomaly, and different effects have also been investigated [25, 26, 27]. It becomes therefore relevant to establish appropriate relations between high energy density systems and the description of normal nuclear systems (at the saturation density) by reanalysing the issue differently than it has been done so far.

There have been different attempts to describe general properties of normal nuclear matter and finite nuclei within the framework of the linear realization of chiral symmetry. The linear sigma model with a vector meson, with and without vacuum polarization of nucleons has been applied to nuclear matter and, in some cases, partially appropriated description of properties have been obtained besides the normal solution, abnormal bound states has also been found [28, 29]. One has also considered quantum fluctuations for mesons but unaccetable behaviors have appeared in some works [30, 31]. However some of these results may be expected in approximations used to calculate loop corrections in effective models. It is worth to remind that usual perturbation theory for strong coupling constants is not a suitable framework. Other attempts considering the linear sigma model with vector meson have been discussed in

[32, 33]. In these works, however, the fermionic densities (and the full self consistency of the equations) were truncated as it is shown below. These results have been interpreted as indications of a failure of the linear realization for the description of nuclear systems. It is argued in this paper that these conclusions may not be correct. The linear and non linear realizations of chiral symmetry in the sigma models can be expected to be equivalent. In the vacuum this is directly seen from ealier Weinberg's works on this subject [5]. Investigations on the light scalar mesons which have been observed still allow for the possibility of identifying the σ to the chiral partner of the pion, is spite of controversial results as discussed in several works [7, 9, 10, 11, 12, 14]. Furthermore, there are indications that the sigma becomes less broad at finite density [17] being eventually associated to a relevant degree of freedom for the radial (in medium) nucleon-nucleon interaction [34, 35, 36]. The strong constraint between the couplings from chiral symmetry in the linear sigma model with the scalar condensate has been pointed as the main difficulty [32]. This can be relaxed or changed in some ways, some of which are discussed in the present paper. Hadron properties, such as their masses, are expected to vary in medium. The spontaneous breakdown of chiral symmetry provides a reasonable mechanism for generating masses for many hadrons as well as their behavior in medium by means of the scalar condensate [15, 19]. In the papers quoted above the pion was not considered to develop a non zero expected value at finite density, like a "pion condensate". This condensate may not be the same of that investigated during the 70's [38].

In this paper it is argued that previous limitations for using the linear realization of chiral symmetry may be eliminated. The present investigation also aims to provide partial links between the descriptions of nuclear and hadronic systems at higher energy densities. This work is an extension of [39] and it is organized as follows. In the next section the linear sigma model (LSM) with nucleons and a vector meson is described. Although massive vector field theories are not renormalizable they are relevant for several branches of Physics [3]. This particular issue will not be discussed here. In section 3 the Gaussian variational approximation for the sigma and pions is used and truncations of the corresponding effective potential are performed. The averaged values of the pion and sigma fields in the ground state will be referred to as pion and sigma "condensates". In the following, from a variation of the total energy density a modified equation for the vector meson is proposed. With this it is expected to take into account more non linearities than usually done for a free Fermi gas picture. This is completely consistent with a dependence of the nucleonic densities on the vector field from the full solution of the Dirac equation with a vector field. The vector meson is not quantized however and this procedure corresponds to a variational one. In section 4 the ground state stability condition for a bound homogeneous system is investigated by means of differential equations. Analytical solutions are found within a particular ansatz which satisfy the variational equations of each field. General properties of nuclear matter are well reproduced for many sets of values of the parameters of the model. We mostly find solutions for which the scalar condensate (associated with the chiral order parameter) decreases with density although there may exist others for which it may increase whose meaning is not clear. Aspects of interest for higher densities are discussed. The vector meson mass varies with density. This mass may decrease with density or it may be reduced to zero in the vacuum depending on the boundary conditions for its differential equation. Numerical results for some relevant variables are shown in section 5. In final part there is a summary.

2 Modified Linear sigma model at finite baryonic density

The Lagrangian density of the two-flavor Linear Sigma Model with baryons, $N_i(\mathbf{x})$, sigma and pions, $(\sigma, \boldsymbol{\pi})$, covariantly coupled to a vector meson, V_μ, is considered to be given by:

$$\begin{array}{l}\mathcal{L} = \bar{N}_i(\mathbf{x})\left(i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma + i\gamma_5 \boldsymbol{\tau}.\boldsymbol{\pi})\right) N_i(\mathbf{x}) + \frac{1}{2}\left(\partial_\mu \sigma.\partial^\mu \sigma + \partial_\mu \boldsymbol{\pi}.\partial^\mu \boldsymbol{\pi}\right) + c\sigma \\ -\frac{1}{4}F^v_{\mu\nu}F^{\mu\nu}_v - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{4}\left(\sigma^2 + \boldsymbol{\pi}^2 - v^2\right)^2 + \frac{1}{2}m_V^2 V_\mu V^\mu,\end{array} \tag{2}$$

where the covariant derivative including the vector meson with minimal coupling is: $\mathcal{D}^\mu = \partial^\mu - i g_V V^\mu$. The kinetic tensor is $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. Although there is a term for the vector meson mass, m_V, which breaks explicitely symmetries, the scalar condensate can also contribute as it will be discussed below. An explicit mass term for the baryons, M_0 which would also break chiral symmetry, is usually considered due to quark and gluon degrees of freedom which do not correspond to the scalar classical field. It is not included in this formulation. For instance gluon condensates are expected to contribute for some hadron masses [40]. The resulting effective nucleon mass is given by: $M^* = \pm g_S \bar{\sigma}$. The inclusion of the explicit term for the baryon mass does not change the conclusions of this paper. The coupling constants are: g_V, g_S, g and λ; and $v = F_\pi \simeq 88$ MeV is the bare chiral radius. This is equal to the pion decay constant in the vacuum in the so called chiral limit [41]. The term proportional to $c\sigma$ provides a mass to pions. It breaks slightly the (chiral) symmetry explicitely, $c \propto m_\pi^2$. It will not be considered along all this paper. With a non zero classical pion field ($\bar{\boldsymbol{\pi}}$) the pion mass also becomes different from zero since there is another broken symmetry [3]. Besides that, the inclusion of $< \pi >$ leads to a natural description of the behavior of the pion mass with density. The pion mass induces a shift of the chiral condensate from F_π to $f_\pi \simeq 92$ MeV, among shifts in other variables [41]. There is also a rearrangement of the variables due to quantum fluctuations [42, 3]. The introduction of a chemical potential, with an extra term $\delta\mathcal{L} = -\bar{N}\gamma_0\mu_{chem}N$, is equivalent to a shift of the classical temporal component of the vector field V_0 coupled to the nucleons. The same reasoning would apply for a classical counterpart of the photon which couples directly to protons and charged pions.

Non zero ("classical") expected values (condensates) for all the bosonic fields will be considered and their equations will be solved basically within variational approximations. Since the condensates (such as $\bar{\sigma}$) depend (strongly) on the density so the hadronic masses do. There are several ways of coupling vector mesons to the sigma and pions. In the Appendix A the chiral partners rho and A_1 mesons are coupled within a local symmetric way. This procedure however does not yield the full $\vec{\rho}$ mass from the chiral SSB.

The mass of the vector field can also be generated by the scalar condensate. For this the derivative ∂_μ is substituted by a covariant derivative in the kinetic terms. This can be written as:

$$\mathcal{L}_{V-m} = \frac{1}{2}\left(\partial_\mu + ig_{ef}V_\mu\right)\sigma.\left(\partial^\mu - ig_{ef}V^\mu\right)\sigma + \frac{1}{2}\left(\partial_\mu + ig_{ef}V_\mu\right)\boldsymbol{\pi}.\left(\partial^\mu - ig_{ef}V^\mu\right)\boldsymbol{\pi} + \frac{1}{2}m_V^2 V_\mu V^\mu. \quad (3)$$

Where the mass term from the Lagrangian was also included. In most part of this work we will consider only the component V_0 to be non zero. The non renormalizability issue which arises will not be addressed since this field will not be quantized. Two non excludent possibilities arise for the vector meson (effective) mass. The expression for the (in medium) mass $\tilde{m}_V^2$ will be given by:

$$\tilde{m}_V^2 = m_V^2 + 2g_{ef}^2(\bar{\sigma}^2 + \bar{\boldsymbol{\pi}}^2). \quad (4)$$

However, the sigma and pion masses can be modified due to a classical component of V_μ. They appear as coefficients of σ^2 and $\boldsymbol{\pi}^2$, as mass terms. It yields (for $i = \sigma, \boldsymbol{\pi}$):

$$\tilde{m}_i^2 = m_i^2 + g_{ef}^2(\bar{V}_0^2 - \bar{V}_i^2). \quad (5)$$

The spatial components of (massive and coupled) V_μ will not be considered in most part of this paper. Exactly the same reasoning may be considered for the electromagnetic field if it develops a classical counterpart (in a "condensation") $\bar{A}_\mu$ which couples directly to charged pions. The charged pion masses would be modified to:

$$\tilde{m}_{\pi\pm}^2 = m_\pi^2 \pm g^2(\bar{A}_0^2 - \bar{A}_i^2). \quad (6)$$

This may also be considered for the other hadrons which couple directly to photons. This would mean a non degenerescence of the charged hadron masses inside the same flavor-multiplet. This kind of degenerescence is not observed in the vacuum although it may be present somewhere in the phase diagram.

2.1 Densities

The nucleon field is written in terms of creation and annihilation operators. It generates non zero baryonic density as well as scalar and pseudo-scalar densities (ρ_B, ρ_s and ρ_{PS}). These will not be explicitely evaluated here. For a non interacting Fermi gas, the following expressions are obtained:

$$\rho_B = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}, \qquad \rho_f = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + (M^*)^2}. \tag{7}$$

In these expressions k_F is the baryon momentum at the Fermi surface. This picture is not exact when nucleons are coupled to (classical or not) vector fields. These quantities are to be functions of the classical vector field. This means that in the energy density (or binding energy) these densities are to depend on the bosonic parameters. The full expression of the densities, as indicated in the Appendix B, will not be computed explicitely. Instead of this, a variational procedure will be adopted below to extract a dependence of the densities on the classical component of V_μ. With this, higher order non linear and interacting effects with relation to the usual picture are considered. The nucleon effective mass, $M^* = \pm g_S\bar{\sigma}$, and the nucleon momentum at the Fermi surface are the parameters for the determination of baryon densities. Below it will be shown that M^* may be defined as a matrix which depends on the isospin (and spin) of the baryons when $\bar{\pi} \neq 0$.

3 Gaussian Approximation for the sigma and pions

The variational approach with Gaussian trial wave-functionals in the Schroedinger picture [43, 44] is considered for pions and sigma. Instead of doing the complete calculation, the GAP and field equations at finite density are calculated and the effective potential is truncated such that the main features of the approximation are preserved.

An estimate for the energy density of the ground state can be obtained by calculating the averaged energy with trial wave-functionals $|\Psi[\sigma,\pi]>$ which is normalized to unit: $<\Psi[\sigma,\pi] \mid \Psi[\sigma,\pi]>= 1$. This averaged is written, for the complete hamiltonian, as:

$$\mathcal{H}_{tot} \equiv <\Psi_{s.z.}[\sigma,\pi]\,|H[\sigma,\pi,V_\mu,N]\,|\Psi_{s.z.}[\sigma,\pi]>$$

where the subscript $(s.z.)$ stands for spin zero components. Only the terms containing the sigma and pion fields however yield non trivial contributions, the remaining ones are simply factorized with the normalization of the Gaussian The action of pion and sigma field operators ($\hat{\phi}_i = \sigma, \pi$) and their respective canonical conjugated momenta ($\hat{\xi}_i$) when applied to the wavefunctional are respectively given by:

$$\hat{\phi}_i|\Psi[\phi_i]>=\phi_i|\Psi[\phi_i]>, \quad \hat{\xi}_i|\Psi[\phi_i]>=-i\frac{\delta}{\delta\phi_i}|\Psi[\phi_i]>. \tag{8}$$

Two Gaussians are considered: $|\Psi[\sigma,\pi]>=|\Psi_S[\sigma]>.|\Psi_P[\pi]>$. For the sigma component it is given by:

$$\Psi_S[\sigma(\mathbf{x})]=\mathcal{N}_S exp\left\{-\frac{1}{4}\int d^3\mathbf{x}d^3\mathbf{y}\delta\sigma(\mathbf{x})G_S^{-1}(\mathbf{x},\mathbf{y})\delta\sigma(\mathbf{y})\right\}, \tag{9}$$

Where the shift due to the non zero expected value in the ground state is done: $\delta\sigma(\mathbf{x})=\sigma(\mathbf{x})-\bar{\sigma}$; the normalization is $\mathcal{N}_S$ such that $\int\mathcal{D}[\sigma]\Psi_S^*\Psi_S=1$ (omitting space coordinates), the two variational parameters are: the condensate $\bar{\sigma}=<\Psi_S|\sigma|\Psi_S>$ and the the width of the Gaussian, which is the two-point function,

$$G_S(\mathbf{x},\mathbf{y})=<\Psi_S|\delta\sigma(\mathbf{x})\delta\sigma(\mathbf{y})|\Psi_S>. \tag{10}$$

This function is the same as the Feynman Green's function of an explicitely covariant formalism with time integrated and with the sign of the imaginary part changed and the mass replaced by the bare (Lagrangian) mass [2]. The Gaussian approach corresponds to a summation of *cactus* type Feynman diagrams in loops [45] and it is equivalent to the large N leading order approximation [43]. An analogous expression for the pions is considered with variational parameters given by: $\bar{\pi}=<\Psi_P|\pi|\Psi_P>$ and $G_P^{a,b}(\mathbf{x},\mathbf{y})$, which is a matrix in isospin space. This two point function can be considered to be diagonal as a particular case which is developped along this work ($G_P^{a,a}=G_P$). This reduces the corresponding functional space and it guarantees the explicit "chiral and isospin" invariances.

The minimization of the averaged energy calculated with the Gaussian wave functional with respect to the (Gaussian) variational parameters yield the GAP and condensate equations which define the ground state. The following set of expressions is obtained:

$$\begin{array}{ll}(i) & \frac{\delta\mathcal{H}^{tot}}{\delta\bar{\sigma}}=0\to\bar{\sigma}\lambda\left(\bar{\sigma}^2+3G_S+\bar{\pi}^2+G_P-v^2\right)+\frac{\partial\rho_f}{\partial\bar{\sigma}}+2g_{ef}^2(V_0^2-V_i^2)\bar{\sigma}+c=0;\\ (ii) & \frac{\delta\mathcal{H}^{tot}}{\delta G_S}=0\to\frac{\partial\rho_f}{\partial G_S}-\frac{G_S^{-2}}{8}-\frac{\Delta}{2}+\frac{\lambda}{4}\left(6\bar{\sigma}^2+2\bar{\pi}^2+6G_S+2G_P-2v^2\right)+g_{ef}^2(V_0^2-V_i^2)=0,\\ (iii) & \frac{\delta\mathcal{H}^{tot}}{\delta\bar{\pi}_a}=0\to\bar{\pi}_a\lambda\left(\bar{\sigma}^2+3G_S+\bar{\pi}^2+G_P-v^2\right)+\frac{\partial\rho_f}{\partial\bar{\pi}_a}+2g_{ef}^2(V_0^2-V_i^2)\bar{\pi}_a=0;\\ (iv) & \frac{\delta\mathcal{H}^{tot}}{\delta G_P}=0\to\frac{\partial\rho_f}{\partial G_P}-\frac{G_P^{-2}}{8}-\frac{\Delta}{2}+\frac{\lambda}{4}\left(6\bar{\sigma}^2+2\bar{\pi}^2+6G_S+2G_P-2v^2\right)+g_{ef}^2(V_0^2-V_i^2)=0,\end{array} \tag{11}$$

where V_0, V_i stand for the expected values ("classical" fields) and derivatives of the fermionic density with respect to the variational parameters were included. The second and fourth of these expressions can be re-written as the functional forms for G_i:

$$G_S=G_S(\mathbf{x},\mathbf{x})=<\mathbf{x}|\frac{1}{\sqrt{\Delta+\mu_S^2}}|\mathbf{x}>, \qquad G_P=G_P(\mathbf{x},\mathbf{x})=<\mathbf{x}|\frac{1}{\sqrt{\Delta+\mu_P^2}}|\mathbf{x}>, \tag{12}$$

where Δ is the Laplacian and μ_i the physical masses. It will be done a truncation in the effective potential such that the calculation of the two point functions, G_i, is not needed. Therefore ultraviolet divergences will be avoided. It will be considered the following rearrangement of the variables due to quantum fluctuations ($G_i = G_i(\mathbf{x},\mathbf{x})$):

$$\tilde{\sigma}^2 = \bar{\sigma}^2 + G_S, \qquad \tilde{\pi}^2 = \bar{\pi}^2 + G_P. \tag{13}$$

Considering the contribution of the vector field, discussed in section 2, the masses are given by:

$$\begin{aligned} \mu_s^2 &= \lambda(3\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2) + 2g_{ef}^2(V_0^2 - V_i^2), \\ \mu_P^2 &= \lambda\left(3\tilde{\pi}^2 + \tilde{\sigma}^2 - v^2\right) + 2g_{ef}^2(V_0^2 - V_i^2). \end{aligned} \tag{14}$$

Where the limit $\mu_P^2 \to 0$ follows from the Goldstone theorem in the vacuum for $c = 0$ and $V_\mu = 0$ using expression (15) written below. The equations for the averaged fields - condensates -(the first and third of the set (11)) can be written in the forms:

$$\begin{aligned} \lambda\bar{\sigma}\left(\bar{\pi}^2 + 3G_S + \bar{\sigma}^2 + G_P - v^2\right) + \frac{\partial\rho_f}{\partial\bar{\sigma}} + g_{ef}^2(V_0^2 - V_i^2)\bar{\sigma} + c = 0; \\ \lambda\bar{\pi}_a\left(\bar{\pi}^2 + 3G_P + \bar{\sigma}^2 + G_S - v^2\right) + \frac{\partial\rho_f}{\partial\bar{\pi}_a} + g_{ef}^2(V_0^2 - V_i^2)\bar{\pi}_a = 0. \end{aligned} \tag{15}$$

For the sake of clearness usual approximations will be done, as discussed below. Equations (15) can be written with the scalar density $\rho_s \propto \partial\rho_f/\partial\bar{\sigma}$ and the pseudoscalar density $\rho_{ps} \propto \partial\rho_f/\partial|\bar{\pi}|$, explicitly. The baryonic and fermionic densities are considered to depend on $\bar{\pi}$ which is consistent with the effective nucleon mass dependence on $\bar{\pi}$.

As it is shown in the Appendix B the fermionic density ρ_f has an intrincated expression for the case in which baryons are coupled to the classical vector meson. Analogous procedures are valid for scalar and pseudo scalar ones. Therefore, the dependence of the fermionic densities on $\bar{\sigma}$ and $\bar{\pi}$, in equations (15), will be considered to be determined from these differential equations. The condensates have very different values already in the vacuum. These equations, with redefinition (13), can be rewritten as:

$$\frac{\partial\rho_f}{\partial\tilde{\pi}^2} = \frac{\partial\rho_f}{\partial\tilde{\sigma}^2} \simeq \frac{\lambda}{2}(\tilde{\pi}^2 + \tilde{\sigma}^2 - \tilde{v}^2), \tag{16}$$

where $\tilde{v}^2 = v^2 - 2G$.

It will be assumed that the quantum fluctuations of spin zero bosons - through the two two-point Green's functions G_S and G_P - have two effects only:

(I) they change the resulting sigma mass whereas the pion mass is kept zero in the vacuum for $c = 0$, due to the Goldstone theorem, as well as

(II) they cause shifts of the respective condensates with respect to their values at the tree level - expressions (13). Quantum fluctuations (loop corrections) always re-arrange the parameters of the model. Due to these two hypothesis it is not needed to evaluate the G_i functions explicitely. These two approximations (I and II) correspond to a truncation of the full (self-consistent) Gaussian effective potential of spin zero fields, neglecting non linear terms in G of the effective potential. The model is an effective model and this would correspond nearly to fix an energy scale from the renormalized theory. The full "self consistency", typical of the variational approximation, is not always needed. Exact calculations will be compared elsewhere.

3.1 Total energy density and vector meson equation

The total energy density is written in terms of the four variational parameters for σ, π, plus nucleonic densities and vector meson variables. With the truncation of the effective potential of sigma and pion discussed above, $\mathcal{H}^{tot}$ can be written as:

$$\mathcal{H}^{tot} = \rho_f + g_V V_0 \rho_B - \frac{1}{2}\tilde{m}_V^2 V_0^2 + \frac{\lambda}{4}(\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2)^2, \tag{17}$$

Where $\tilde{m}_V$ is an effective mass for the vector meson (expression (4)). The densities ρ_B and ρ_f depend on $\tilde{\sigma}$, $\tilde{\pi}$ and V_0. Although the full explicit dependence of the baryonic density on V_0 is not calculated it is considered an implicit undetermined dependence on it. The exact expression is obtained from the exact solution of the Dirac equation coupled to the vector meson field. This is indicated in the Appendix B. It is considered that the energy density is varied with respect to the classical vector field, which is not quantized. The modified equation for the vector meson will be given by:

$$\frac{\partial \mathcal{H}}{\partial V_0} = 0 \quad \to \quad g_V \left(\rho_B + V_0 \frac{\partial \rho_B}{\partial V_0} \right) - \tilde{m}_V^2 V_0 = 0. \tag{18}$$

Where V_0 is now a sort of variational parameter for a more exact density $\rho_B = \rho_B[V_0]$. For this derivation $\tilde{m}_V$ was kept constant. This is, however, a first analysis as discussed above and the full equations and solutions will be investigated in a forthcoming work.

4 Ground state stability and solutions for the field equations

In this section solutions for the above equations (11,18) are searched with a stable ground state. However before investigating those equations the condition of stable ground state from the expression of the energy density (17) is analysed. The stability condition for the ground state, with binding energy $E_0/A = \mathcal{H}/\rho_B < 0$, can be written as:

$$\frac{\partial \frac{\mathcal{H}}{\rho_B}}{\partial \rho_B} = 0 \quad \rightarrow \quad \frac{\partial \mathcal{H}}{\partial \rho_B} = \left.\frac{\mathcal{H}}{\rho_B}\right|_{\rho_B=\rho_0} < 0, \qquad \left.\frac{\partial^2 \frac{\mathcal{H}}{\rho_B}}{\partial \rho^2}\right|_{\rho_B=\rho_0} > 0, \tag{19}$$

where ρ_0 is the stability density. The first derivative equation can be written as:

$$\frac{\partial \rho_f}{\partial \rho_B} + \frac{1}{2}\frac{\partial(2g_V V_0 \rho_B - \tilde{m}_V^2 V_0^2)}{\partial \rho_B} + \frac{\lambda}{2}(\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2)\frac{\partial(\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2)}{\partial \rho_B} = \frac{\mathcal{H}^{tot}}{\rho_B}, \tag{20}$$

where $\mathcal{H}^{tot}$ is given by expression (17). This equation is faced as a differential equation for the dynamical variables. The expression for the energy density (17) is separated into three parts such that each component of the hadronic matter satisfy equation (19 (i)) separatedly. The equations (prescriptions) are the following:

$$\begin{array}{ll} (i) & \dfrac{\partial \rho_f}{\partial \rho_B} = \dfrac{\rho_f}{\rho_B}, \\ (ii) & \dfrac{\partial(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2)}{\partial \rho_B} = \dfrac{(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2)}{2\rho_B}, \\ (iii) & \dfrac{\partial \mathcal{H}_V}{\partial \rho_B} = \dfrac{\mathcal{H}_V}{\rho_B}. \end{array} \tag{21}$$

In this last expression the energy density terms with contributions of the vector field are:

$$\mathcal{H}_V = g_V V_0 \rho_B - \frac{1}{2}\tilde{m}_V^2 V_0^2.$$

Equation (21 - (iii)) is the same equation as (18). A detailed comparison of these prescriptions with the exact calculation will not be shown here but results indicating they are reasonable will be given below. Furthermore the solutions for (stability) equations (ii) and (iii) satisfy the equations of the corresponding fields.

4.1 Fermionic density

From the first of the differential equations (21 - (i)) we find a solution for the dependence of ρ_f on the baryonic density ($\rho_f = \rho_f(\rho_B)$). This is numerically nearly in agreement with that resulting from the integration of expression (7) in the range of densities not far from ρ_0. A solution for the above prescription (21)-(i) is given by:

$$\rho_f = K\frac{\rho_B}{9}Ln\left(\frac{\rho_B}{\rho_0}\right) + B\rho_B - K\frac{\rho_B^2}{9\rho_0}, \tag{22}$$

where B is a constant fixed to reproduce ρ_f according to expression (7) ($B \simeq 3.8 fm^{-1}$ for the values adopted in section 4) and K is the usual incompressibility modulus:

$$K = 9\rho_0^2 \left.\frac{\partial^2(E/A)}{\partial \rho_B^2}\right|_{\rho_0} > 0. \tag{23}$$

Expression (22) for $\rho_f(\rho_B)$ agrees with the one written in (7) close to the stability density (although they have different slopes with k_F).

To compare with the resulting expression from expression (7) its derivative is calculated, $\partial\rho_f/\partial\rho_B$. This expression can be written as a function of the momentum at the Fermi surface k_F as:

$$\rho_f = \frac{1}{16\pi^2}\left(2k_F E_f^3 - k_F(M^*)^2 E_f - (M^*)^4 Ln\left(\frac{k_F + E_f}{M^*}\right)\right), \tag{24}$$

where $\rho_B = 2k_F^3/(3\pi^2)$. Its derivative is given by:

$$\frac{\partial\rho_f}{\partial\rho_B} = \frac{\rho_f}{\rho_B} + C,, \tag{25}$$

where C is a constant of the order of 1fm^{-1} for the values typical of the saturation density $\rho_0 \simeq 0.15$ fm^{-3}. For this it was considered that:

$$\frac{\partial M^*}{\partial\rho_B} = \left.\frac{M^*}{4\rho_B}\right|_{\rho_0}.$$

This last relation for the effective mass is to be compatible to the equation for the scalar condensate yielding eventually $\bar{\sigma}(\rho_0)$. These last two expressions will not be considered.

The disagreement between the relation (25) and prescription (21 (i)) is small and could be mended in two ways:

(i) considering a modification in solution (22) which leads to a rearrangement of that expression such that (25) is satisfied. In this case, ρ_f would read:

$$\rho_f^{(b)} = K'\frac{\rho_B}{9}Ln\left(\frac{\rho_B}{\rho_0}\right) + B'\rho_B.$$

(ii) with the inclusion of a term of the form $\delta\rho_f = C\rho_B$ in the expression (24) for ρ_f (of a free Fermi gas) such that the relation (21 (i)) is satisfied.

As it is shown in Appendix B, the usual expression (24) is an approximation in the presence of the classical vector field. To illustrate this, some leading terms for the more exact fermionic density ρ_f are shown in the Appendix B. None of the two expressions for ρ_f, (22) and (24), are exact for the finite density calculation due to V_0 both modifications are nearly equivalents and will be investigated further elsewhere. It is worth emphasizing that the parameters used for these densites, M^*, K, are self consistently obtained from the solutions, such as $E/A, \rho_0$ with reasonable values.

4.2 Condensates at finite density

The solution for the equation (21 (ii)) is given by:

$$\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2 = \tilde{C}\sqrt{\rho_B}. \tag{26}$$

$\tilde{C}$ is a constant to be fixed by a boundary condition at ρ_0. This represents a symmetry radius in the medium. The above expression is, in principle, valid at zero density and at ρ_0 but it will be shown to be reasonable for different ρ_B. In the vacuum $\tilde{\pi} = 0$, $\bar{\sigma}^2 = F_\pi^2$ and $\tilde{\sigma} = f_\pi \neq \bar{\sigma}$. As the density ρ_B increases $\tilde{\sigma}$ is to decrease and the behavior of $\tilde{\pi}^2$ is thus strongly dependent on the sign of $\tilde{C}$. This is clearly seen by rewritting the above expression as: $\tilde{\pi}^2 = \tilde{v}^2 - \tilde{\sigma}^2 + \tilde{C}\sqrt{\rho_B}$ For chiral symmetry to be restored at high baryonic densities $\tilde{\sigma} \to 0$ and $\tilde{\pi} \to 0$. For a negative value, ($\tilde{C} < 0$) which will be found below to be more plausible, the parameter $\tilde{\pi}^2$ decreases with increasing densities from its value at the stable density up to the point in which $\tilde{\sigma}^2 = \tilde{\pi}^2 = 0$. For a positive $\tilde{C} > 0$ the negative sign solutions for $\tilde{\pi}^2$ make the absolute value to decrease with the increase of the baryonic density. There is another possibility, namely that $\tilde{\pi}^2 > 0$ and $\tilde{C} > 0$ which does not yield reasonable behavior.

The Gell-Mann Oakes Renner relation (1) can be substituted into the equation of the scalar $\tilde{\sigma}$ condensate (15) yielding:

$$(< \bar{q}q >)^{\frac{3}{2}} \left(\frac{m_q}{m_\pi^2}\right)^{\frac{3}{2}} + < \bar{q}q >^{\frac{1}{2}} \left(\tilde{\pi}^2 - \tilde{v}^2\right) \left(\frac{m_q}{m_\pi^2}\right)^{\frac{1}{2}} \simeq \frac{\rho_s}{\lambda} g_s M^*, \qquad (27)$$

where $\rho_s \propto d\rho_f/d\tilde{\sigma}$ is the scalar density. With this substitution it was assumed that modifications of the medium in the form of the G.O.R. relation are not large, expression (1).

An expression with a similar structure can be roughly and qualitatively obtained by considering that the nucleon wavefunction is composed by three quarks. Considering a sort of heuristic factorization of the scalar and pseudoscalar densities into (colorless) scalar two point functions of quarks, which can be shifted with the formation of the scalar condensates,the baryon structure has to be taken into account by means of parameters associated to the "bag constant". It can be written that:

$$\begin{aligned}
&\rho_S = \bar{N}N \;\to\; B < \bar{q}\bar{q}\bar{q}qqq > + F \;\to \\
&\to \rho_S \simeq\; 6\tilde{B}(< \bar{q}q >)^3 + 9\tilde{B}_2 < \bar{q}q > + \; o.c.t.(i), \\
&\rho_{PS}^{i,j} = (\bar{N}\gamma_5 N)_{i,j} \to B^p < \bar{q}\bar{q}\bar{q}\gamma_5(-\gamma_5^2)qqq >_{i,j} - F^p \;\to \\
&\to \rho_{PS} \simeq -6\tilde{B}^p(< \bar{q}\gamma_5 q >)^3_{i,j} + 9\tilde{B}_2^p < \bar{q}\gamma_5 q >_{i,j} + \; o.c.t.(ii),
\end{aligned} \qquad (28)$$

where i, j are for the isospin, the functions $\tilde{B}^{(p)}_{(i)}, \tilde{F}^{(p)}$ stand for the effect of having confined quarks inside each nucleon of the nucleonic matter, a factor $-\gamma_5^2$ has been inserted in the pseudoscalar factorization, $o.c.t.(i)$ (as well as $\tilde{B}^{(p)}_{(2)}$) stand for other colorless terms. The term $\tilde{B}^{(p)}_{(2)}$, for example, can include four quark/antiquark densities ($< \bar{q}\bar{q}qq >$ and $< \bar{q}\bar{q}\tau_a^2\gamma_5^2 qq >$ with isospin and gamma matrices, with pseudoscalar and/or scalar condensate squared).

Expressions (28) have terms analogous to those of the expression (27), with an analogous expression for the pseudoscalar condensate. The scalar condensate appear with squared-root in (27). When the scalar density is zero $\rho_S = 0$ there are three solutions for the scalar condensate $\tilde{\sigma} \propto < \bar{q}q >$ in the vacuum. The first solution is $\tilde{\sigma} = 0$. The other two solutions are given by: $\tilde{\sigma}^2 = \tilde{v}^2$. From this limit it yields: $< \bar{q}q >^2 = -9\tilde{B}_2/6\tilde{B}$. An analogous reasoning holds for the pseudoscalar condensate $\tilde{\pi}$ in terms of the pseudoscalar density, when $\rho_{PS} \neq 0$.

The scalar and pseudoscalar densities can also be expanded in terms of the scalar and pseudoscalar condensates for example as:

$$\begin{aligned} \rho_S &= \rho_S^{(0)} + \frac{\tilde{\sigma}}{\tilde{\sigma}_{vac}}\rho_S^{(1)} + \frac{\tilde{\sigma}^2}{\tilde{\sigma}_{vac}^2}\rho_S^{(2)} + \frac{\tilde{\sigma}^3}{\tilde{\sigma}_{vac}^3}\rho_S^{(3)}, \\ \rho_{PS} &= |\tilde{\pi}|\rho_{PS}^{(1)} + |\tilde{\pi}^2|\rho_{PS}^{(2)} + |\tilde{\pi}^3|\rho_{PS}^{(3)}, \end{aligned} \tag{29}$$

where the coefficients $\rho_i^{(j)}$ are such that $\rho_S = \rho_{ps} = 0$ when $\tilde{\sigma} = \tilde{\sigma}_{vac}$ and $\tilde{\pi} = 0$. Comparing to expression (28) it follows that: $\rho_S^2 = \rho_{PS}^2 = 0$. By substituting these expressions into the condensate expressions (15) it is seen that the terms proportional to $\tilde{\sigma}$ and $\tilde{\pi}$ yield contributions to the *in medium* masses of sigma and pions from the finite fermionic density in expressions (14). The contributions are given by $\rho_S^{(1)}$ and $\rho_{PS}^{(1)}$. These terms appears numerically through the derivatives of the fermionic density, $\partial\rho_f/\partial\tilde{\sigma} \propto \rho_S$ and $\partial\rho_f/\partial|\tilde{\pi}_a| \propto \rho_{PS}$. These factors together with the density dependence of $\tilde{\sigma}$ and $\tilde{\pi}$ determine the density dependence of the masses. The terms proportional to $\tilde{\sigma}^3$ and $\tilde{\pi}^3$ yield (effective) contributions for the coupling constant, i.e., $\lambda \to \lambda \pm \rho_S^{(3)}/(\tilde{\sigma}_{vac}^3) = \lambda \pm \rho_{PS}^{(3)}$. These two corrections for the in medium effective coupling constant may also be different from each other, leading to different interactions of the pion and sigma in the baryonic medium breaking chiral symmetry further.

4.3 Equations of $\tilde{\sigma}$ and $\tilde{\pi}^2$

The "condensate" equations (16), for $\tilde{\sigma}$ and $\tilde{\pi}$, can be faced as differential equations for ρ_f:

$$\begin{aligned} (\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2) &= \frac{2}{\lambda}\frac{\partial\rho_f}{\partial\tilde{\sigma}^2}, \quad (i) \\ (\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2) &= \frac{2}{\lambda}\frac{\partial\rho_f}{\partial\tilde{\pi}^2}, \quad (ii). \end{aligned} \tag{30}$$

These equations are isomorfic.

The solutions were found by eliminating $\tilde{\sigma}^2$ from the equation of $\tilde{\pi}^2$ and, conversely, eliminating $\tilde{\pi}^2$ from the equation of $\tilde{\sigma}^2$. This was done as follows: $\rho_f(\tilde{\sigma}^2)$ is obtained with $\tilde{\pi}^2$ kept constant from the first equation. Inverting this solution, written like $\tilde{\sigma}^2(\rho_f, \tilde{\pi}^2)$, it is introduced in the second equation, for $\rho_f(\tilde{\pi}^2)$. The variables are changed to: $t^2 = z^2 - 4\rho_f/\lambda - v^2\tilde{v}^2$, and $z = \tilde{v}^2 - \tilde{\pi}^2$ and the following differential equation is obtained:

$$\frac{dt}{dz} = \frac{z}{t} \mp 1. \tag{31}$$

A solution for this differential equation is given by:

$$z\sqrt{t^2 - zt - z^2} = C_\pi exp\left(-\frac{\sqrt{5}}{5}arctanh\left(\frac{\sqrt{5}(z-2t)}{5z}\right)\right), \tag{32}$$

Where C_π is a constant determined by the boundary condition: $\tilde{\pi}(\rho_f = 0) = 0$. There are two values for this constant, they are given by:

$$\begin{aligned} C_\pi^+ &= \sqrt{v^4 - \tilde{v}^4 - \tilde{v}^2 v^2} exp\left(\frac{\sqrt{5}}{5}arctanh\left(\frac{2v^2 - \tilde{v}^2}{\tilde{v}^2}\right)\right), \\ C_\pi^- &= \sqrt{v^4 + \tilde{v}^4 - 3\tilde{v}^2 v^2} exp\left(\frac{\sqrt{5}}{5}arctanh\left(\frac{-2v^2 + 3\tilde{v}^2}{\tilde{v}^2}\right)\right),. \end{aligned} \tag{33}$$

Expression (32) for $\tilde{\pi}(\rho_f)$ is transcendental and it yields several values for a given fermionic density. Numerical values are showed in the next section. The same reasoning was applied for the $\tilde{\sigma}^2$ and an analogous expression is obtained with a different boundary condition, $\tilde{\sigma}(\rho_f = 0) = \tilde{v}$. For this solution $\tilde{\pi}$ in (32) is replaced by $\tilde{\sigma}$ and the constant C_π of exprèssion (32) is replaced by $C_\sigma^\pm$ which is given by:

$$C_\sigma^\pm = \left(\frac{3}{2} \pm \frac{1}{2}\right) Gexp\left(\frac{\sqrt{5}}{5}arctanh\left((1 \pm 2)\frac{\sqrt{5}}{5}\right)\right). \tag{34}$$

Nevertheless in the present work $\tilde{\sigma}(\rho_0)$ was fixed by a reasonable value of the baryonic effective mass as discussed above. Different estimates for the $\tilde{\pi}^2$ are shown in Appendix C.

4.4 Higher and variable densities and $\tilde{C}$

To investigate the consistency of the expression for the symmetry radius (26) for densities different from ρ_0 it is imposed that the equations (16) are satisfied simultaneously with expression (26) for any density $\rho_f(\rho_B)$. Equations (16) are rewritten with:

$$\frac{\partial \rho_f}{\partial \tilde{\sigma}^2} = \frac{\partial \rho_f}{\partial \rho_B}\frac{\partial \rho_B}{\partial \tilde{\sigma}^2}, \qquad \frac{\partial \rho_f}{\partial \tilde{\pi}^2} = \frac{\partial \rho_f}{\partial \rho_B}\frac{\partial \rho_B}{\partial \tilde{\pi}^2}.$$

With the prescription for the fermionic density (21 (i)) at $\rho_B = \rho_0$ it results that:

$$\tilde{C} \simeq \pm\sqrt{\frac{8\rho_f}{\lambda\rho_0}}. \tag{35}$$

Any other fermionic density which satisfy stability condition (21 (i)) will produce the same result. For $\lambda = 20$ and $\rho_0 = 0.15\text{fm}^{-3}$ it results:

$$\tilde{C} \simeq \pm 0.41\text{fm}^{-\frac{1}{2}}. \tag{36}$$

This will be discussed below. For a different relation from prescription (21 (i)), for $\partial\rho_f/\partial\rho_B$, there appears modification(s) in expression (26) and (36).

In the following it will be assumed that the symmetry radius, expression (26), is nearly valid for a large range of baryonic densities including those close to that of the restoration of chiral symmetry (ρ_c). This is a crude approximation because, for example, heavier hadrons are expected to be relevant for such energetic situation. At this (critical) density $\tilde{\sigma} \to 0$. Three values of ρ_B and $\tilde{\sigma}^2 + \tilde{\pi}^2$ are considered to find the $\tilde{C}$, which is a "free parameter", $\rho = 0$, ρ_0 and ρ_c. This critical density can be reparametrized as

$$\rho_c = u\rho_0, \tag{37}$$

i.e., the restoration of chiral symmetry occurs u times the saturation density of nuclear matter. In this point it is expected that $\tilde{\sigma} = \tilde{\pi} = 0$, and thus:

$$\tilde{C} = \mp\tilde{v}^2\sqrt{\frac{1}{u.\rho_0}}. \tag{38}$$

At the saturation density the expression (26) can be written as:

$$\tilde{\sigma}^2 + \tilde{\pi}^2 \simeq \tilde{v}^2\left(1 \pm \sqrt{\frac{1}{u}}\right). \tag{39}$$

Four values are considered as examples: (i) $u = 2$, (ii) $u = 3$, (iii) $u = 3.5$ and (iv) $u = 4$ yielding the following values for $\tilde{\sigma}^2 + \tilde{\pi}^2 < \tilde{v}^2$ at $\rho_B = \rho_0$:

$$\begin{aligned} &\sqrt{\tilde{\sigma}^2 + \tilde{\pi}^2}\Big|_{\rho_0} \simeq 0.54\tilde{v} \ \ (i), \qquad &&\sqrt{\tilde{\sigma}^2 + \tilde{\pi}^2}\Big|_{\rho_0} \simeq 0.65\tilde{v} \ \ (ii), \\ &\sqrt{\tilde{\sigma}^2 + \tilde{\pi}^2}\Big|_{\rho_0} \simeq 0.68\tilde{v} \ \ (iii), \qquad &&\sqrt{\tilde{\sigma}^2 + \tilde{\pi}^2}\Big|_{\rho_0} \simeq 0.71\tilde{v} \ \ (iv). \end{aligned} \tag{40}$$

The numerical solutions for $\tilde{\sigma}^2 + \tilde{\pi}^2 > \tilde{v}^2$ are not presented because they are seemingly non realistic. Considering that $\tilde{\pi}$ should be expected to be small the larger values are in good agreement with other theoretical analysis and experimental observation [15, 46]. It may be that $f_\pi^* \simeq \sqrt{\tilde{\sigma}^2 + \tilde{\pi}^2}$. The constraint (26) may be therefore useful for relating descriptions of different ranges of the nuclear matter phase diagram. Expression (26) can be rewritten in terms of the pion decay constant in the vacuum (f_π) and of the density:

$$\tilde{\sigma}^2 + \tilde{\pi}^2 = (f_\pi^0)^2\left(1 \pm \sqrt{\frac{\rho_B}{\rho_c}}\right). \tag{41}$$

The values obtained for $\tilde{C}$ from estimates (40) are respectively given by:

$$\begin{aligned} &\tilde{C} \simeq \pm 0.41\text{fm}^{-\frac{1}{2}} \ \ (i), \qquad &&\tilde{C} \simeq \pm 0.33\text{fm}^{-\frac{1}{2}} \ \ (ii), \\ &\tilde{C} \simeq \pm 0.30\text{fm}^{-\frac{1}{2}} \ \ (iii), \qquad &&\tilde{C} \simeq \pm 0.28\text{fm}^{-\frac{1}{2}} \ \ (iv). \end{aligned} \tag{42}$$

These values, in particular (i) for $u = 2$, are in fair agreement with (36).

Another way of estimating $\tilde{C}$ is shown by considering the GAP equations (14). With them the symmetry radius can be written as:

$$\tilde{C}\sqrt{\rho_B} = \frac{1}{4\lambda}\mu_T^2 - \frac{v^2}{2} = \frac{1}{4\lambda}((\mu_T^*)^2 - (\mu_T^{vac})^2), \tag{43}$$

where $\mu_T^2 = \mu_S^2 + \mu_P^2$ at a given density ρ_B with other contributions, from the classical vector meson, set to zero. The sum of the pion and sigma masses is therefore given by:

$$\mu_T^2 = 2\lambda v^2 \pm 4\lambda\tilde{C}\sqrt{\rho_B}. \tag{44}$$

Two possible behaviors are obtained in this picture: the sum of these masses may decrease or increase depending on the sign of $\tilde{C}\sqrt{\rho_B}$. For $\rho_0 = 0.15\text{fm}^{-3}$ and $\tilde{C} \simeq -0.15$ this expression yields approximated values $\mu_T^2(\rho_0) \simeq (1 \pm 0.53)\ \mu_T^2(\rho_B = 0)$. The sigma and pion masses (and the sum of them) may increase or decrease at finite density.

These are the main remarks concerning high density behavior of the model analysed in this paper. Therefore three ways of calculating the parameter $\tilde{C}$ were shown, expressions (35,38,43). They provide crude (but interesting) estimations which do not consider other degrees of freedom (quarks, gluons, heavier flavors, higher order interactions and approximations).

4.5 Vector meson field extended solutions

Finally, considering the extended equation for V_0 - expression (18) - as a differential equation of the baryonic density ρ_B as a function of V_0 we obtain the following solution:

$$V_0(\rho_B) = \frac{-g_V\rho_B \pm \sqrt{g_V^2\rho_B^2 - 2C_V\rho_B\tilde{m}_V^2}}{\tilde{m}_V^2}, \tag{45}$$

where C_V is a (negative) constant to garantee real solutions. This expression also satisfies the prescription equation (21 (iii)). These two solutions for V_0 yield the same contribution to the energy density. It is given by: $\mathcal{H}_V = C_V\rho_B$. The expression above can be inverted and written as:

$$\tilde{m}_V^2 = \frac{-2V_0 g_V\rho_B - C_V\rho_B}{V_0^2}. \tag{46}$$

In this form there are two possible behaviors. The first one corresponds to a decrease of $\tilde{m}_V^0$ as density increases, at least for a finite range of ρ_B, in agreement with usual expectations [19, 15, 47]. However there is a different possibility, namely that ρ_B and V_0 go to zero such that the mass of the vector meson disappears in the vacuum for $\rho_B/V_0 \to 0$. In fact, for the limit $\rho_B \to 0$ it is not possible to keep such mass non zero depending on how $V_0 \to 0$. If $V_0^2 \to 0$, exactly like to the denominator of the expression above of $\tilde{m}_V^2$, it is obtained that $m_V^2(\rho_B \to 0) \neq 0$, otherwise $\tilde{m}_V \to 0$ or $m_V \to \infty$ in this framework. In this case V_0 cannot be completely identified to the vector meson ω. The parameter m_V is a Lagrangian parameter which however can be associated to the finite density formulation of the model as well as the expected value V_0. This is consistent with the assumption of equivalence of the redefinition of V_0 and the introduction of the chemical potential. It is seen that the baryonic density generates a non zero value of V_0 - which may be viewed as a "condensate", eventually from another SSB. A massive vector field may be characteristic from a superconductive state. The issue of (non) renormalizability of the model couple to massive gauge theory (moreover forming a finite density medium) will not be addressed here.

Assuming that at some point in the phase diagram the temporal component of the photon also develops a classical couterpart, at sufficiently large densities, the binding energy modification, according to the solution (45), will be: $E/A \simeq g\bar{A}_0\rho_B/2 - \frac{1}{2}\tilde{m}_A^2\bar{A}_0^2$. The contribution of this field is evaluated in the same variational way as the vector meson V_0, leading to a mass of the form:

$$m_A^2 = \frac{-2A_0 g\rho_B - C_A\rho_B}{A_0^2}, \tag{47}$$

where C_A is a (negative) constant. There is an additional requirement for this solution: that $A_0 \to 0$ slower than $\rho_0 \to 0$, because $A_0(\rho_B) = 0$ necessarily.

The resulting binding energy per nucleon with all the contributions calculated so far at a density ρ_B can be written as:

$$\frac{E}{A} = -\frac{\mathcal{H}}{\rho_B} = g_V V_0 \rho_B - \frac{1}{2}\tilde{m}_V^2 V_0^2 + \frac{\rho_f}{\rho_B} + \frac{\lambda}{4}\frac{\bar{\sigma}^2 + \bar{\pi}^2 - \bar{v}^2}{\sqrt{\rho_B}} + M, \tag{48}$$

where $\tilde{m}_V$ is the modified vector meson mass due to $\bar{\sigma}, \bar{\pi}$.

5 Some Numerical Results

The following values were considered in this section: $-E/A = 16.0$ MeV, $\rho_0 = 0.16\text{fm}^{-3}$, $g_S = 9$, $M^* = 0.8M$, $m_V \simeq 780$ MeV, $g_V \simeq -5$ and $K \simeq 1$ fm^{-1} [48]. They were used in expressions (48,22,26,32,45). The coupling constant λ is the less known parameter and therefore a large range of values was considered. The scalar condensate at the saturation density is given by $\bar{\sigma} = M^*/g_S$ for which $\bar{\sigma}(\rho_B) \simeq 0.78 f_\pi$, is in agreement with the usual idea of symmetry restoration as discussed by Brown and Rho [15] and with experimental results [46]. It is worth to emphasize that the effective mass in nuclear matter is usually considered to be higher than in (part of the) finite nuclei, in which it depends on the radial spatial coordinate r. However considering smaller or larger effective mass these calculations would still exhibit the numerical solutions presented below. The values of $\bar{\sigma}$ and $\tilde{\sigma}$ will be given respectively by $F_\pi = 88$ MeV and $f_\pi = 92$ MeV.

In Figure 1 the solutions of expression (32) for the squared pion "condensate" are shown as a function of the coupling λ. There are several solutions for only one value of λ and ρ_f at the stable density ρ_0. The dots (crosses) correspond to the minimum (maximum) values which the squared pion "condensate" may acquire for each value of λ, i.e., the classical pion field, $\tilde{\pi}^2$, may have several values between the dots and crosses. The relatively smaller values with relation to $f_\pi^2 \simeq .22$ fm^{-2} seem more acceptable. There are discontinuities in two points due to the form of the expression (32): when $\lambda \simeq 16$ and $\lambda \simeq 43$.

With the values found for $\tilde{\pi}^2$ shown in Figure 1 the corresponding values for the symmetry radius $\tilde{C}$ are obtained from expression (26) for fixed $\tilde{\sigma}$. The maximum and minimum values for each λ are shown in Figure 2 also as a function of λ at the stabitily density. These values for $\tilde{C}$, obtained from the numerical values of $\tilde{\pi}^2$, are in good agreement with those estimations of expressions (35,38,43). They have the same order of magnitude and very similar values. The same investigation for $\tilde{\sigma}$ was done, however the corresponding solutions (using expressions (32),(34)), which yield reasonable results, are not shown. Although the expected behavior of a continuous decrease of $\tilde{\sigma}$ with increasing density is obtained there are also solutions of further chiral symmetry breaking at some finite density, i.e. $\tilde{\sigma} > v$, although not realistic [42, 49]. The restoration of chiral symmetry also seems to occur inside the nucleon as a skyrmion [13]. It is interesting to note that the variables usually identified to the *in medium* chiral order parameter may be in fact including the squared pion condensate. In other words, it seems to be fair to ask: is the usual parameter associated to the scalar condensate at variable densities only $\tilde{\sigma}$ or it takes into account somehow $\tilde{\pi}^2$?

For some solutions of Figures 1 and 2 we show results for C_V, the constant in expression (45), which yields a correct value for the binding energy in expression (17). Two particular solutions were chosen:

$$
\begin{aligned}
\lambda \simeq 16.0 \rightarrow \; & \sqrt{-\tilde{C}\sqrt{\rho_0}} \simeq 25\text{MeV} \;\; \rightarrow \;\; C_V \simeq -3.6\text{fm}^{-1} + M, \\
\lambda \simeq 40.0 \rightarrow \; & \sqrt{+\tilde{C}\sqrt{\rho_0}} \simeq 55\text{MeV} \;\; \rightarrow \;\; C_V \simeq -3.9\text{fm}^{-1} + M,
\end{aligned}
\tag{49}
$$

where M is the nucleon (effective) mass. Although the resulting values of C_V are not too different there are some important differences between these two solutions. In the first one the value of $|\tilde{\pi}|$ is small ($\tilde{\pi}^2 \simeq -0.05 fm^{-2}$), the constant $\tilde{C}$ is small and negative indicating that $\tilde{\sigma}^2 + \tilde{\pi}^2 < v^2$ at the saturation density and making $\tilde{\sigma}$ smaller and not very different from f_π. In the second solution $\tilde{C}$ is large and positive due to the large value of $\tilde{\pi}^2$, $\tilde{\pi}^2 \simeq 0.14 fm^{-2}$ and consequently $\tilde{\sigma}^2 + \tilde{\pi}^2 > v^2$.

From the GAP equations for pion and sigma we can write the ratio of their *in medium* masses as:

$$\frac{\mu_P^2}{\mu_S^2} = \frac{2\tilde{\pi}^2 + \tilde{C}\sqrt{\rho_B} + c/(\lambda v)}{2\tilde{\sigma}^2 + \tilde{C}\sqrt{\rho_B}}. \tag{50}$$

This expression reduces to zero in the vacuum if the parameter c is set to zero. As discussed above, the scalar and pseudo-scalar densities contribute to the sigma and pion in medium masses (expressions (29)) and those contributions appear in this expression.

In Figure 3 it is shown the behavior of the pion mass in the medium divided by its value in the vacuum for some of the solutions shown in Figures 1 and 2 as a function of the coupling λ - keeping $\tilde{\sigma}$ fixed. For the lowest values of the coupling, $\lambda < 15$, the pion mass is smaller than in the vacuum. For larger values of λ the pion mass becomes larger than $m = 140$MeV at the saturation density. A complete analysis of the behavior with varying density is more involved and it will be shown elsewhere. The increase of the value of μ_P may be associated to the tendence of the restoration of chiral SSB at finite density with the decrease of the sigma mass. In this case the sigma and pion masses would become degenerated at the time chiral symmetry is restored. This is approximatedly present in the expression (50) above for large $\sqrt{\rho_B}$.

In Figure 4 we show values of the ratio of the *in medium* sigma mass ($\mu_S^2 = \mu_S^2(\tilde{\sigma}, \tilde{\pi}^2)$) to its value in the vacuum as a function of λ also for the solutions of the previous Figures. The sigma mass, for lower values of the λ, is lower than its value in the vacuum ($\mu_S^0 \simeq 482MeV$, smaller than usually considered) whereas for larger λ it becomes higher than μ_S^0. This is a similar behavior of that obtained for the pion mass shown in Figure 3.

Some values obtained for the *in medium* sigma mass (at ρ_0) divided by its value in the vacuum for the case when the pion mass is zero (chiral limit) are shown in Figure 5. There is a strong difference with relation to the results of Figure 4 for massive pions. It is seen that the sigma mass is found to be lower at the saturation density than in the vacuum for a larger range of values of the coupling λ. For a meaningful range of values of λ there are no solutions.

The behavior of the sigma and pion masses with the variation of density is controversial. The following ansatz for the in medium sigma mass was considered in several works [50]

$$m_\sigma(\rho_B) = m_\sigma \left(1 - \alpha \frac{\rho_B}{\rho_0}\right).$$

This is qualitatively consistent with the above results for zero pion mass (from Figure 5) if $\alpha > 0$. However this relation is linear in the density. With the inclusion of quantum fluctuations there are results from other calculations for the linear sigma model [51] and quark-meson coupling model [49] which are in agreement with results from figure 4: m_σ^2 increases at finite baryonic density (with $\alpha < 0$ in the expression above).

The Brown-Rho scaling can be summarized, in what concerns the hadronic masses, in the following relations:

$$\frac{f_\pi^*}{(f_\pi^0)} = \frac{M^*}{M} = \frac{m_m^*}{m_m} = \left(\frac{< \bar{q}q(\rho) >}{< \bar{q}q(0) >} \right)^\alpha \tag{51}$$

where M^* are *in medium* light baryon masses and m^* *in medium* (non Goldstone) light meson masses. Usually $\alpha = 1/3$ when the property of chiral transformation of L_{sb} is considered (consistent with the usual G.O.R.) whereas in earlier lattice calculations for the for the chiral transition $\alpha = 1$ [15]. In this picture the chiral SSB is responsible for the masses as assumed along the present work. However the ratios obtained here (not completely analytical) are slightly different from those of the Brown-Rho scaling. Several other investigations concerning the behavior of masses and their relation to the the scalar condensate have been done for finite densities and temperatures, for example [17, 52, 53, 54, 55]. They present similar trends and they will not be discussed.

With the quantity $\bar{\pi} \neq 0$ the nucleon effective mass M^* can be considered to be a matrix written as:

$$M^*_{a,b;s} = \pm g_S < \Psi[\sigma, \pi]|. < N_{a,s}|(\sigma + i\gamma_5 \tau.\pi)|N_{b,s} > .|\Psi[\sigma, \pi] >= \pm g_S(\bar{\sigma} + i\tilde{M}^d_{(a,b),s}\bar{\pi}_d). \tag{52}$$

In this expression a, b, d stand for the isospin index (neutrons/protons), $\tilde{M}$ is a non diagonal isospin matrix and the final spin (s) structure is not written. The nucleonic effective mass is to be therefore obtained by the averaged value $\bar{\sigma}$ plus a contribution from the averaged value of the pion field. This leads a splitting between neutron and proton masses, but it also means possible *in medium* oscillations between proton and neutron, which can be expected to be related to electroweak processes. This matrix includes a dependence on the baryon spins. The baryonic/fermionic densities become non degenerated [56]. However, the densities (ρ_f,ρ_B) along this paper were the usual ones, i.e., with a constant diagonal effective mass due to $\bar{\sigma}$, i.e. $M^*_{a,b} \simeq \pm g_S\bar{\sigma}$, which is the leading term. These effective masses do not take into account contributions of other isovector mesons such as the $\vec{\delta}$ [57, 58] and they should not be regarded as the full effective masses for baryons in an neutron-proton asymmetric nuclear medium. This non trivial solution corresponds to a non invariant ground state under an isospin transformation, although the Lagrangian is symmetric. This is a breaking of isospin symmetry.

This mass splitting, is probably connected with the Nolen-Schiffer effect [59], relates the nucleon effective masses to the scalar and pseudo scalar QCD condensate(s) $< \bar{q}q >, < \bar{q}\gamma_5(\tau)q >$. It is obtained from expression (52) and it is given by:

$$\Delta M^* = M_n^* - M_p^* \simeq 2ig_S|\bar{\pi}_3|\tilde{M}. \tag{53}$$

Considering one small solution of $\bar{\pi}$ from Figure 1 with $\lambda \simeq 60$ it is found that $\Delta M^* \simeq 40 g_S \tilde{M}$ MeV which may be used to estimate or to check the value of the matrix element $\tilde{M}$ which depends on the nucleons spins. It must have very small elements. With this $g_S|\bar{\pi}|$ could also be fixed to reproduce an expected ΔM^*.

For spontaneous breakdowns of global symmetries zero energy collective modes are expected to appear [4]. In nuclear matter calculations zero sound-like collective modes are found in several channels of the nuclear interaction. These modes are zero energy-like and may be associated or manifestation of Goldstone modes. They have been associated to giant resonances in nuclei [60]. Finally, the breakdown of isospin symmetry is to be restricted by the charge conservation.

Considering that the Goldberger-Treiman relation nearly holds at the saturation density, with the rearrangement of the scalar condensate, as it was done in expression (13), it can be written as:

$$g_S\bar{\sigma} = (M^* \pm \Delta M^*)g_A^*, \tag{54}$$

Where ΔM^* is due to $\bar{\pi}$. In the vacuum ($\Delta M^* \to 0$, $\bar{\sigma} = f_\pi$ and $M^* \to M_0 + g_S\bar{\sigma}$) we obtain a small value $g_A \simeq 1.05$, which is roughly the value for the tree level linear sigma model. A realistic value should not be expected in the vacuum with these arguments although it is reasonable at the saturation density [15]. However it can be expected that the behavior at varying density may be reasonable [61]. Weak interactions should be considered for a full picture.

$\phi - \omega$ and nonlinear sigma models (cubic and quartic order scalar interacting terms) are accepted to be appropriated for the nuclear observables [32, 33]. However the linear and non linear realizations of chiral symmetry are expected to be equivalent and the recent observation of scalar mesons in several processes demand investigations on the linear realization with scalar fields (quark-anti-quark mesons or with more complicated structure). Furthermore the nonlinear models usually includes more interacting terms than the standard version of the linear sigma model in which only two free parameters (constrained) are considered. Higher order Lagrangian terms can be included as well. This relaxes constraints. A more precise calculation of fermionic densities is also of relevance. And finnally the in medium modifications of the hadron properties are expected to contribute as it was shown in this work. The next step is the calculation of finite nuclei properties with the linear realization of chiral symmetry along the lines investigated here with the full calculation. This is will be developped elsewhere. There is some results on pion condensates independently done by Toki and collaborators among others for light finite nuclei [62].

6 Summary

The two flavor linear sigma model coupled to a vector meson is investigated to describe the basic properties of a finite density bound hadronic system. This offers a suitable framework for the study of nucleonic matter properties at different densities. Non zero expected values for the meson fields ("condensates") were considered. In particular the averaged value of the pion was considered and it does not seem to be necessarily the same as investigated previously [38]. A constraint for the sigma and pion condensates at the finite density was found defining a symmetry radius $\tilde{C}$. It is given by expression (26):

$$\tilde{C}\sqrt{\rho_B} \simeq \left(\tilde{\sigma}^2 + \tilde{\pi}^2 - \tilde{v}^2\right).$$

Several different estimations for the numerical values of $\tilde{C}$ were proposed independently of each other in expressions (35,38,43). From these, some estimates for the behavior of the system at variable high densities were done. For example, the critical density for the restoration of chiral symmetry and the behavior of the sigma and pion masses with density were related qualitatively. A brief qualitative comparison of the structure of the classical (sigma and pion) fields equations in the medium considering the quark content by means of the GOR relation was done. Prescriptions for the condition of minimum energy of the bound state for each component of the system (baryons, sigma-pion, vector meson) were proposed such that the dynamical equations are also satisfied. A solution for a modified dynamical equation of the vector meson field at finite density was found. The equation of the classical vector meson field and its solution consider the dependence of the (corrected) baryonic density on V_0. This is obtained from the coupling of baryons to the vector meson and it is present in the coupled Dirac equation. The vector meson mass may either decrease with the increase of density or it may be reduced to zero in the vacuum. In the former picture it is identified to the omega mean field whereas the latter may suggest an association of V_μ to a sort of *dressed photon*. The non zero value of the pion "condensate" may lead to a splitting of the neutron and proton masses in the medium with the possibility of oscillations between these two states. With the "pion condensates" the electrical charge must be conserved. The present model will be considered for the description of finite nuclei elsewhere.

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Appendix A: Linear sigma model and local chiral symmetry

The linear sigma model invariant under local chiral transformations with vector and axial fields has a Lagrangian density given by [63]:

$$\mathcal{L} = \bar{N}_i(\mathbf{x})\left[i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma + i\gamma_5 \boldsymbol{\tau}.\boldsymbol{\pi})\right] N_i(\mathbf{x}) + \frac{1}{2}\left(\mathcal{D}_\mu\sigma.\mathcal{D}^\mu\sigma + \mathcal{D}_\mu\boldsymbol{\pi}.\mathcal{D}^\mu\boldsymbol{\pi}\right) - \frac{\lambda}{4}\left((\sigma)^2 + (\boldsymbol{\pi})^2 - v^2\right)^2 + \\ + \frac{a}{4}\left(\mathcal{F}^i_{\mu\nu\,L}.\mathcal{F}_i^{\nu\mu}{}_L + \mathcal{F}^i_{\mu\nu\,R}.\mathcal{F}_i^{\nu\mu}{}_R\right) + \frac{m_V^2}{2} V_\mu V^\mu - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}, \tag{A.1}$$

where the "left-right components" gauge kinetic tensor (with a constant coefficient a) are respectively given by:

$$\begin{aligned}
\mathcal{D}_\mu\sigma &= \partial_\mu\sigma + \gamma({A_\mu}^i_R - {A_\mu}^i_L).\pi^i, \\
\mathcal{D}_\mu\pi^i &= \partial_\mu\pi^i + \gamma\left((\vec{A}^R_\mu + \vec{A}^L_\mu)\times\pi\right)^i - \gamma\sigma\left((A^R_\mu)^i - (A^L_\mu)^i\right), \\
(\mathcal{F}^i_{\mu\nu})^{L,R} &= \partial_\mu(A^i_\nu)^{L,R} - \partial_\nu(A^i_\mu)^{L,R} + \gamma(A^i_\mu)^{L,R}\times(A^i_\nu)^{L,R},
\end{aligned} \tag{A.2}$$

for which the chiral combinations of the isovector and iso-axial vector fields expected to correspond to the mesons ρ and A_1 (with $\vec{A}^{R(L)}_\mu \propto (\vec{V}_\mu + (-)\vec{A}_\mu)/2$). In these expressions several couplings constants were introduced (λ, g) and v is the chiral radius. Scale invariance can be broken due to two effects. Due to the chiral radius v, which provides a mass to the sigma, and due the quantum fluctuations [3]. However to obtain the chiral spontaneous symmetry breaking at the tree level, with the chiral scalar condensate $\bar{\sigma}$, v shows to be necessary. In this approach the mass of the rho is not fully obtained.

For $\bar{\pi} \neq 0$ additional terms from the covariant derivatives appear, among which:

$$\mathcal{L}_{\bar{\pi},\bar{\sigma}} = \frac{\gamma^2}{2}\left((A_\mu)^i(A^\mu)^j\bar{\pi}_i\bar{\pi}_j + (V_\mu)^i(V^\mu)^j\Gamma_{ijkl}\bar{\pi}_k\bar{\pi}_l(A_\mu)^i.(A^\mu)^i\bar{\sigma}^2 + \bar{\sigma}(V_\mu)^i\bar{\pi}^j\epsilon_{ijk}(A_\mu)^k\right), \tag{A.3}$$

where the last term represents a parity non conserving mixing between mesons ρ and A_1.

The introduction of the ω field in the covariant derivatives (A.2) of the sigma and pion can generate mass term as well as a vector meson mixing term. This is given by:

$$\mathcal{L}_{\bar{\pi},\bar{\sigma}} = \frac{\gamma g_V}{2}\bar{\sigma}\bar{\pi}^j\epsilon_{ijk}\bar{\pi}_k\omega^\mu(V_\mu)^i. \tag{A.4}$$

Appendix B: Densities

For the nucleon field decomposed into creation and annihilation operators in a fixed volume V:

$$\Psi = \frac{1}{\sqrt{V}}\Sigma_{k,\alpha}\left(u(k,\alpha)A_{k,\alpha}e^{ik.x} + v(-k,\alpha)B^\dagger_{-k,\alpha}e^{-ik.x}\right), \tag{B.1}$$

where the creation operators for fermion and anti-fermion are given by $A^\dagger$ and $B^\dagger$. For the Dirac equation which considers the coupling of fermions to the vector field, the spinors:

$$u(k_\pm) = \begin{pmatrix} \left(\frac{E_\pm+M^*}{2M^*}\right)^{\frac{1}{2}}\varphi^\alpha(M,0) \\ \frac{\vec{\sigma}.\vec{k}}{\sqrt{(2M^*(M^*+E_\pm))}}\varphi_\alpha(M,0) \end{pmatrix}, \tag{B.2}$$

$$v(k_\pm) = \begin{pmatrix} \frac{\vec{\sigma}.\vec{k}}{\sqrt{(2M^*(M^*+E_\pm))}}\chi_\alpha(M,0) \\ \left(\frac{E_\pm+M^*}{2M^*}\right)^{\frac{1}{2}}\chi^\alpha(M,0) \end{pmatrix}, \tag{B.3}$$

where $\vec{\sigma}$ stands for the Pauli spin matrices, $\varphi(M,0)$ and $\chi(M,0)$ for the two dimensional components of fermion and anti-fermion spinors. The energy eigenvalues for states with four-momentum $k_\pm = k_\mu \pm g_V V_\mu$ are given by

$$E_\pm = g_V V_0 \pm \sqrt{\mathbf{k}^2 + (M^*)^2}.$$

The nucleonic densities can be calculated. In particular we write the leading terms of the expressions for the fermionic ρ_f density. The baryonic density ρ_B and the scalar density ρ_S will also contain corrections to the expressions for free Fermi gas. The leading terms for the fermionic density are given by:

$$\rho_f \simeq \frac{\gamma}{(2\pi)^3} \int d^3k \left(\frac{(M^* + E_+)^2 + \mathbf{k}^2}{2M^*(M^* + E_+)} \right) |\varphi|^2. \tag{B.4}$$

There are also terms corresponding to densities of particles and anti-particles which were not shown. Their role will be investigated elsewhere. In the limit of $V_0 = 0$ the above expression reduces to the expression (7).

Appendix C: Other estimates for $\tilde{\pi}$

We envisage two more ways of finding approximated estimates for $\tilde{\pi}^2$ from equations (30).

(1) An approximated solution for the two condensate equations can be found as if ρ_f were a function of these fields independently. These solutions for ρ_f, labeled for each of the above equations with a and b can be inverted and written as:

$$\begin{aligned} \tilde{\sigma}^2 &= \frac{v^2}{2} - \tilde{\pi}^2 \pm \sqrt{\left(\frac{v^2}{2} - \tilde{\sigma}^2\right)^2 + \frac{4}{\lambda}\rho_f^a}, \\ \tilde{\pi}^2 &= \pm \left(-\frac{v^2}{2} \pm \sqrt{\frac{\rho_f^b}{\lambda} + \frac{v^4}{4}} \right). \end{aligned} \tag{C.1}$$

Eliminating ρ_f from the second expression ($\rho_f^a = \rho_f^b$) we find the following approximated value for the pion condensate (if $|\tilde{\pi}|^2 << v^2$):

$$\tilde{\pi}^2 \simeq \frac{\tilde{\sigma}^2(\tilde{\sigma}^2 - \tilde{v}^2)}{4(-\frac{\tilde{\sigma}^2}{2} \pm \tilde{v}^2)}. \tag{C.2}$$

With $g_S = 10$ and $M^* = 0.7M$, where $M = 940$ MeV, we find the values $\tilde{\pi}^2 \simeq 0.47 fm^{-2}$ and $\tilde{\pi}^2 \simeq -0.034 fm^{-2}$. Only the second value seems consistent with the approximation done for expression (C.2). For the sake of comparison we remind that $v^2 \simeq f_\pi^2 \simeq 0.22 fm^{-2}$. In these solutions, as well as in others more exact, $\tilde{\pi}^2$ may be either positive or negative.

(2) An alternative way can also be done by adding the two differential equations (30) which can be seen as partial differential equations. This yields $\rho_f = \rho_f^{(1)}(\tilde{\sigma})$ and $\rho_f = \rho_f^{(2)}(\tilde{\pi}^2)$ with constants fixed for the boundary $\rho_B = 0$ when $\bar{\pi} = 0$ and $\tilde{\sigma} = v$. The constant obtained in the solution of the differential equations resulting from the addition of both equations for ρ_f are fixed by requiring that in the vacuum $\bar{\pi} = 0$ and $\tilde{\sigma} = v$. We find the solutions:

$$\begin{aligned} \rho_f^{(1)} &\simeq \frac{\lambda}{2}\tilde{\sigma}^2\left(v^2 - \tilde{\sigma}^2\right) + C_f(\tilde{\sigma}^2 - v^2), \\ \rho_f^{(2)} &= \frac{\lambda}{2}\tilde{\pi}^2(v^2 - \tilde{\pi}^2) - C_f\tilde{\pi}^2, \end{aligned} \quad \text{(C.3)}$$

Where C_f is a constant.

Figure captions

Figure 1 Maximum (+) and minum (dots) values of the squared pion condensate $\tilde{\pi}^2$ (fm^{-2}) as a function of the coupling λ for $M^* = 0.7M$ and $g_S = 9$ found self consistently.

Figure 2 Maximum (+) and minum (dots) values of the symmetry radius $\tilde{C}$ $(fm^{-1/2})$ for the solutions of figure 1 as a function of λ.

Figure 3 Ratio of the squared pion mass in the medium divided by its value in the vacuum as a function of λ for a particular solution of Figure 1.

Figure 4 Ratio of the squared sigma mass in the medium divided by its value in the vacuum as a function of λ for a particular solution of figure 3.

Figure 5 Some values for the ratio of the squared sigma mass in the medium divided by its value in the vacuum as a function of λ considering zero pion mass in the vacuum.

Figure 1

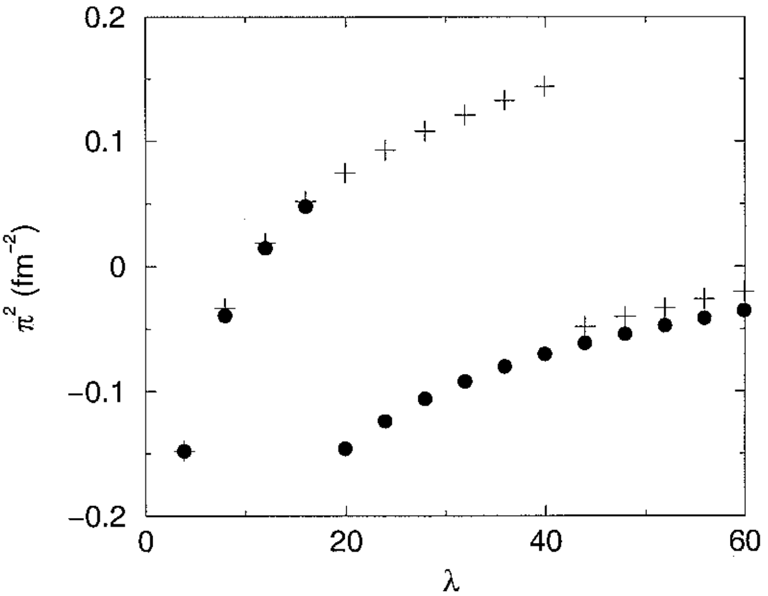

Figure 2

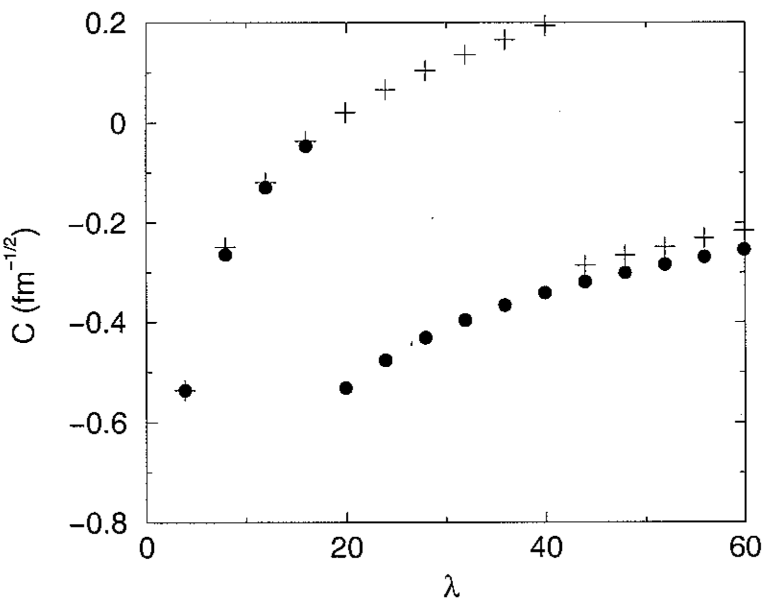

Figure 3

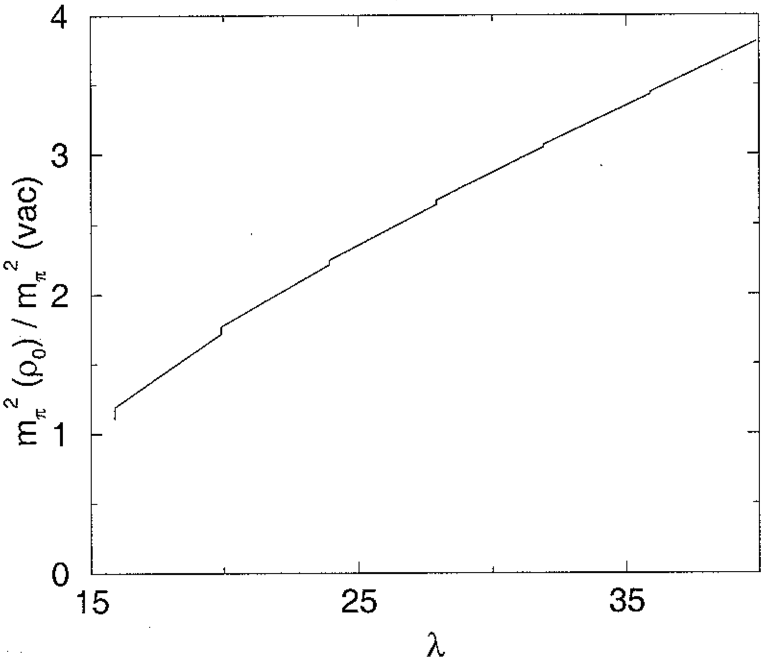

Figure 4

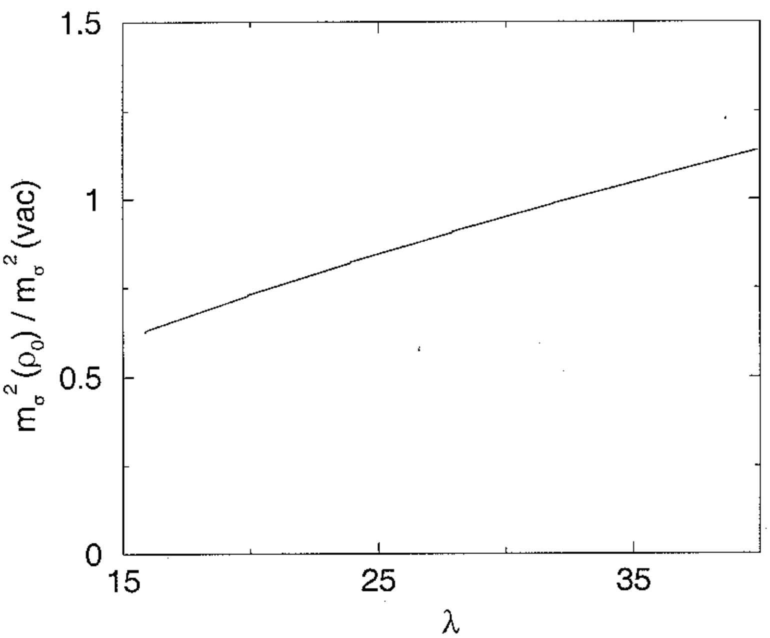

Figure 5