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Buchbinder , I. L.; Krykhtin, V. A.; Reshetnyak, A. A.

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Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

Hamiltonian BRST approach to Lagrangian construction for fermionic higher-spin fields in (A)dS space

I.L. BUCHBINDER[a]*, V.A. KRYKHTIN[b]†, A.A. RESHETNYAK[c]‡

[a]*Department of Theoretical Physics,*
Tomsk State Pedagogical University,
Tomsk 634041, Russia

[b]*Laboratory of Mathematical Physics,*
Tomsk Polytechnic University,
Tomsk 634034, Russia

[c]*Laboratory of Non-equilibrium State Theory,*
Institute of Strength Physics and Materials Science,
Tomsk 634021, Russia

Abstract

A general gauge-invariant Lagrangian formulation for massive and massless half-integer higher-spin fields in the (A)dS space of any dimension is derived on the basis of a Hamiltonian BRST approach. The theory is proposed in terms of an auxiliary Fock space. Closed nonlinear symmetry algebras of higher-spin fermionic fields in the (A)dS space are found and a method of constructing the BRST operator for such algebras is suggested. A universal procedure of constructing Lagrangians describing the dynamics of fermionic fields of any spin is given on the basis of the BRST operator. No off-shell constraints for the fields and gauge parameters are used from the very beginning. It is shown that all the constraints determining the irreducible representation of the (A)dS group arise automatically as a consequence of the equations of motion and gauge transformations. As an example of the general procedure, we obtain the Lagrangians for massive fermionic fields of spin 1/2 and 3/2 containing the total set of auxiliary fields and gauge symmetries.

1 Introduction

The study of various aspects of Classical Higher Spin Field Theory is in the frontline of modern high energy physics (for reviews, see, e.g., [1]). Among the fundamental problems in this area, one can mention the development of methods for constructing Lagrangian formulations, the investigation of possible field-theoretic structures in such theories, the finding of a correspondence

*joseph@tspu.edu.ru †krykhtin@mph.phtd.tpu.edu.ru ‡ reshet@tspu.edu.ru

with superstrings and M theory and the derivation of interacting HS fields recently elaborated, e.g., in [2]- [8] for massive and in [9]- [17] for massless HS theories.

At present, the study of HS field theory is concentrated on constant curvature backgrounds, the (anti-)de Sitter ((A)dS) and Minkowski spaces of any dimension. This choice of spaces provides, firstly, a consistent propagation of HS gauge fields (see, e.g., [18]). Secondly, the AdS space radius guarantees the presence of a natural dimensional parameter for the accommodation of compatible self-interactions for massless HS fields [19, 20]. Finally, the conjecture [21] of AdS/CFT correspondence of conformal $\mathcal{N} = 4$ SYM theory and superstring theory in the $AdS_5 \times S_5$ Ramond–Ramond background validates the intensive study of field (string) dynamics in the AdS space including finding its classical Lagrangian formulation, thus preparing an application to the resultant theory of the conventional Lagrangian, e.g., BV [22], and Hamiltonian, e.g., BFV–BRST [23], quantization schemes.

Among the procedures applied to deriving a gauge-invariant Lagrangian formulation for bosonic and fermionic HS fields on (A)dS space, originally described in [24]- [27], one traditionally considers the Vasiliev approach [28] in the "frame-like formulation", the Hamiltonian BRST [29]- [33] approach, as well as the ambient space [34, 35] and Zinoviev [36] approaches in the "metric-like formulation". Each of these methods has its advantages. Thus, the BRST approach presents a universal generating mechanism of constructing Lagrangian formulations for HS fields, which depends neither on off-shell constraints nor on the fields and gauge parameters, as well as provides an explicit relation with first-quantized string theory [37]- [39] and permits to find Lagrangian actions with a sufficient set of auxiliary fields of lower spins for HS fields with a mixed symmetry (see, e.g., [40] for bosonic HS fields on an arbitrary flat space) and for interacting massless bosonic HS fields on the flat and AdS background [41].

In this paper, we generalize the gauge-invariant approach of Refs. [42, 43] for the fields of half-integer arbitrary fixed spin given in a d-dimensional Minkowski space to the case of totally-symmetric fermionic HS fields on the AdS and dS spaces of any dimension. In this connection, note that a Lagrangian formulation for massive fermionic HS fields of fixed spin in the (A)dS space was recently suggested by Metsaev in [44] on the basis of the approach of Ref. [36]. In order to solve our problem, we reformulate the conditions determining the irreducible (from the viewpoint of Dirac-like tensor-spinors) representation of the (A)dS group with a given spin in terms of operator constraints in an auxiliary Fock space which form, due to the requirement of closure of the constraint algebra, a nonlinear superalgebra with a central bosonic charge. This algebra has a more involved structure as compared to its counterpart for bosonic HS fields on the AdS space [33] (being a generalization of the W_3-algebra construction [45] to the case of superalgebras), which is reflected by the fact that the obtained operator of BRST charge corresponds to a second-rank formal topological gauge theory. This approach realizes the concept of BV–BFV duality, utilized, e.g., in the papers [17,46], which permits one to construct, by means of a Hamiltonian BFV–BRST charge, the objects used in Lagrangian formalism.

The paper is organized as follows. In Section 2, we obtain a closed superalgebra of operator constraints with a central charge, which is generated by the primary constraints (with hermitian constraints containing only the kinetic term) being linearly equivalent to the original ones, in turn determining the irreducible representation of the AdS group with half-integer spin. In Section 3, we realize a representation of the superalgebra of additional parts for the constraints of the original algebra in a Fock space. In Section 4, we construct a deformed nonlinear superalgebra of the enlarged constraints without any central charge by means of an additive extension of the initial constraints by the corresponding additional parts. We find, with the help of a unitary transformation applied to the obtained BRST operator for the above deformed algebra, a BRST operator corresponding to the superalgebra of the enlarged primary constraints, which are equivalent to the conditions determining the given AdS group irrep. The derivation of the action and of a sequence of reducible gauge transformations describing the propagation of fermionic field of an arbitrary fixed spin in the (A)dS space is realized in Section 5. In Section 6, we prove that the constructed action reproduces correct conditions for the field determining the irrep of the AdS group. In Section 7, we demonstrate the procedure of constructing a Lagrangian formulation by finding gauge-invariant Lagrangians for spin-1/2 and spin-3/2 fields in an explicit manner. In Conclusion, we summarize the results of the present work. Finally, in Appendix 9 we present useful formulas of differential calculus in the case of a special gravitational background.

In addition to the conventions of Refs. [33, 43], we use the notations $\varepsilon(A)$, $gh(A)$ for the respective values of Grassmann parity and ghost number of a quantity A, and denote by $[A, B\}$ the supercommutator for quantities A, B, which in the case of definite Grassmann parity values is given by $[A\,, B\} = AB - (-1)^{\varepsilon(A)\varepsilon(B)}BA$.

This paper is devoted to the memory of a sister of A.A.R., a mathematician and remarkable person, Olga Pavlicheva, nee Reshetnyak, who untimely passed away last year.

2 Auxiliary Fock space for higher-spin fields in (A)dS space-time.

Massive and massless, for $m = 0$, half-integer spin $s = n + \frac{1}{2}$ representations of the (A)dS group are realized in the space of totally symmetric tensor-spinor fields $\Phi_{\mu_1 \ldots \mu_n}(x)$, the Dirac index being suppressed, given in a d-dimensional (A)dS space and subject to the conditions (see, e.g., [47])

$$\left[i\gamma^{\mu}\nabla_{\mu} - r^{\frac{1}{2}}\sqrt{-\omega}(n + \tfrac{d}{2} - 2) - m\right]\Phi_{\mu_1\mu_2\ldots\mu_n}(x) = 0, \tag{1}$$

$$\gamma^{\mu}\Phi_{\mu\mu_2\ldots\mu_n}(x) = 0. \tag{2}$$

where $\omega = -1, 0, 1$, respectively, for the AdS, flat, and dS space, $r = \frac{R}{d(d-1)}$, with R being the scalar curvature of space-time. We use the metric of minus signature (see below) and Dirac's matrices obey the relation

$$\{\gamma_{\mu}, \gamma_{\nu}\} = 2g_{\mu\nu}. \tag{3}$$

In order to avoid an explicit manipulation with some of the indices, it is convenient, following the technics developed for the first-quantized string theory [37], to introduce an auxiliary Fock space[1] generated by creation and annihilation operators with tangent space indices $(a, b = 0, 1, \ldots, d-1)$

$$[a_a, a_b^+] = -\eta_{ab}, \qquad \eta_{ab} = \mathrm{diag}(+, -, \ldots, -). \tag{4}$$

An arbitrary vector in this Fock space has the form

$$|\Phi\rangle \;=\; \sum_{n=0}^{\infty} \Phi_{a_1 \ldots a_n}(x)\, a^{+a_1} \ldots\, a^{+a_n}|0\rangle = \sum_{n=0}^{\infty} \Phi_{\mu_1 \ldots \mu_n}(x)\, a^{+\mu_1} \ldots\, a^{+\mu_n}|0\rangle, \tag{5}$$

where $a^{+\mu}(x) = e^{\mu}_{a}(x)a^{+a}$, $a^{\mu}(x) = e^{\mu}_{a}(x)a^{a}$, with $e^{\mu}_{a}(x)$ being the vielbein. It is evident that

$$[a_{\mu}, a_{\nu}^+] = -g_{\mu\nu}. \tag{6}$$

[1] See [33] for more details.

We refer to the vector (5) as the basic vector. The fields $\Phi_{\mu_1...\mu_n}(x)$ are the coefficient functions of the vector $|\Phi\rangle$ and its symmetry properties are stipulated by the properties of the product of the creation operators. We also assume the standard relation

$$\nabla_\mu e^a_\nu = \partial_\mu e^a_\nu - \Gamma^\lambda_{\mu\nu} e^a_\lambda + \omega_\mu{}^a{}_b e^b_\nu = 0. \tag{7}$$

We wish to realize the relations (1), (2) as certain constraints on the vectors $|\Phi\rangle$ (5). To this end, we define an operator D_μ acting on the vectors $|\Phi\rangle$,

$$D_\mu = \partial_\mu - \omega^{ab}_\mu (a^+_a a_b - \tfrac{1}{4}\gamma_{ab}), \qquad \partial_\mu|0\rangle = 0, \qquad \gamma_{ab} = \tfrac{1}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a) \tag{8}$$

and the operators

$$\tilde{t}_0 = -i\gamma^\mu D_\mu + m + r^{\frac{1}{2}}(g_0 - 2), \tag{9}$$

$$\tilde{t}_1 = \gamma^\mu a_\mu, \tag{10}$$

$$g_0 = -a^{+\mu}a_\mu + \frac{d}{2}. \tag{11}$$

One can see that the constraints

$$\tilde{t}_0|\Phi\rangle = \tilde{t}_1|\Phi\rangle = 0 \tag{12}$$

for the basic vector (5) are equivalent to equations (1), (2), with each component $\Phi_{\mu_1...\mu_n}(x)$ of (5) obeying (1), (2), thus describing spin $n + 1/2$ field.

Because of the fermionic nature of equations (1), (2) with respect to the standard Grassmann parity and the bosonic nature of any operators from the set $\{\tilde{t}_0, \tilde{t}_1\}$: $\varepsilon(a) = 0$, $a \in \{t'_0, t'_1, g_0\}$, we need to introduce a set of $d+1$ Grassmann-odd "gamma-matrix-like objects" $\tilde{\gamma}^\mu$, $\tilde{\gamma}$ obeying[2]

$$\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2g^{\mu\nu}, \qquad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \qquad \tilde{\gamma}^2 = -1 \tag{13}$$

and related to the "true" gamma-matrices as follows:

$$\gamma^\mu = \tilde{\gamma}^\mu\tilde{\gamma}. \tag{14}$$

Therefore, a noncontradictory set of initial constraints includes the following fermionic operators:

$$\tilde{t}'_0 = -i\tilde{\gamma}^\mu D_\mu + \tilde{\gamma}\left(m + r^{\frac{1}{2}}(g_0 - 2)\right), \qquad t_1 = \tilde{\gamma}^\mu a_\mu. \tag{15}$$

A realization of the set of enlarged first-class constraints O_i constructed from the initial constraints o_i ($\tilde{t}'_0, t_1 \in \{o_i\}$) by means of the additive composition of the quantities o_i with additional parts o'_i given in the auxiliary operator space, $O_i = o_i + o'_i$, which supercommute with each other, $[o_i, o'_i\} = 0$, as was realized in the case of bosonic fields in the AdS space [33], requires the conversion of o_i into an equivalent set of constraints $\tilde{o}_i$, which do not contain the $\tilde{\gamma}$-matrix because the set o'_i contains this object by construction. Thus, in order to simplify the subsequent calculations, we choose the following operators $\tilde{o}_i$ connected by means of a nondegenerate linear transformation, $o_i = U^i_j \tilde{o}_j$, to the initial operators reproducing the relations (1), (2)

$$t_0 = -i\tilde{\gamma}^\mu D_\mu\,, \tag{16}$$

$$t_1 = \tilde{\gamma}^\mu a_\mu \qquad\qquad t^+_1 = \tilde{\gamma}^\mu a^+_\mu\,, \tag{17}$$

$$l_1 = -ia^\mu D_\mu\,, \qquad\qquad l^+_1 = -ia^{+\mu} D_\mu\,, \tag{18}$$

$$l_2 = \tfrac{1}{2}\,a^\mu a_\mu \qquad\qquad l^+_2 = \tfrac{1}{2}\,a^{+\mu} a_\mu\,, \tag{19}$$

$$\tilde{m} = m - 2r^{\frac{1}{2}}, \qquad\qquad g_0 = -a^+_\mu a^\mu + \tfrac{d}{2}, \tag{20}$$

$[\downarrow,\rightarrow\}$	t_0	t_1	t_1^+	l_0	l_1	l_1^+	l_2	l_2^+	g_0	$\tilde{m}$
t_0	$-2l_0$	$2l_1$	$2l_1^+$	0	(22)	$-(25)$	0	0	0	0
t_1	$2l_1$	$4l_2$	$-2g_0$	(23)	0	$-t_0$	0	$-t_1^+$	t_1	0
t_1^+	$2l_1^+$	$-2g_0$	$4l_2^+$	$-(26)$	t_0	0	t_1	0	$-t_1^+$	0
l_0	0	$-(23)$	(26)	0	$-(24)$	(27)	0	0	0	0
l_1	$-(22)$	0	$-t_0$	(24)	0	(28)	0	$-l_1^+$	l_1	0
l_1^+	(25)	t_0	0	$-(27)$	$-(28)$	0	l_1	0	$-l_1^+$	0
l_2	0	0	$-t_1$	0	0	$-l_1$	0	g_0	$2l_2$	0
l_2^+	0	t_1^+	0	0	l_1^+	0	$-g_0$	0	$-2l_2^+$	0
g_0	0	$-t_1$	t_1^+	0	$-l_1$	l_1^+	$-2l_2$	$2l_2^+$	0	0
$\tilde{m}$	0	0	0	0	0	0	0	0	0	0

Table 1: The algebra of the initial operators

$$l_0 = g^{\mu\nu}(D_\nu D_\mu - \Gamma^\sigma_{\mu\nu} D_\sigma) - r\left(g_0 + t_1^+ t_1 + \tfrac{d(d-3)}{4}\right), \tag{21}$$

which form the algebra given in Table 1 with a bosonic central charge $\tilde{m}$, where

$$[t_0\,, l_1\} = r(2t_1^+ l_2 + g_0 t_1 - \tfrac{1}{2}t_1), \tag{22}$$
$$[t_1\,, l_0\} = r(4t_1^+ l_2 + 2g_0 t_1 - t_1), \tag{23}$$
$$[l_1\,, l_0\} = r(4l_1^+ l_2 + 2g_0 l_1 - l_1), \tag{24}$$
$$[l_1^+\,, t_0\} = r(2l_2^+ t_1 + t_1^+ g_0 - \tfrac{1}{2}t_1^+), \tag{25}$$
$$[l_0\,, t_1^+\} = r(4l_2^+ t_1 + 2t_1^+ g_0 - t_1^+), \tag{26}$$
$$[l_0\,, l_1^+\} = r(4l_2^+ l_1 + 2l_1^+ g_0 - l_1^+), \tag{27}$$
$$[l_1\,, l_1^+\} = l_0 + r(g_0^2 - \tfrac{1}{2}\, g_0 - 4l_2^+ l_2 + \tfrac{3}{2}\, t_1^+ t_1). \tag{28}$$

We also note that this set of operators (16)–(21) forms a nonlinear quadratic superalgebra which is generalization of the bosonic higher-spin algebra in AdS [33] and which is invariant under Hermitian conjugation with respect to the scalar product

$$\langle\tilde{\Psi}|\Phi\rangle = \int d^d x \sqrt{|g|} \sum_{n,\,k=0}^{\infty} \langle 0|a^{\nu_1}\ldots\, a^{\nu_k}\Psi^+_{\nu_1\ldots\nu_k}(x)\tilde{\gamma}_0 \Phi_{\mu_1\ldots\mu_n}(x)\, a^{+\mu_1}\ldots\, a^{+\mu_n}|0\rangle, \tag{29}$$

where the pairing in the right-hand side is a x-local scalar product.

[2]See [43] for more details.

In terms of the operators (16)–(21), we can present equations equivalent to (12), namely,

$$\big[t_0 + \tilde{\gamma}(\tilde{m} + r^{\frac{1}{2}} g_0)\big]|\Phi\rangle = 0, \qquad t_1|\Phi\rangle = 0. \tag{30}$$

In what follows, we will show how to construct Lagrangians on the basis of the BRST construction developed, partially, in [33] for the AdS space, which reproduces equations (12) or equivalently (30). According to the above approach, we must enlarge the operators of the superalgebra given in Table 1, $\tilde{o}_i \to \tilde{O}_i = \tilde{o}_i + o'_i$, $\tilde{o}_i$ being all the operators of the algebra, so that: 1) $\tilde{O}_i$ are in involution $[\tilde{O}_i, \tilde{O}_j\} \sim \tilde{O}_k$ and 2) each Hermitian operator must contain at least one arbitrary parameter whose values will be determined later (see [33,42,43] for details).

The next step in this procedure is finding the additional parts o'_i for the operators (16)–(21).

3 Additional parts of operators

Following the procedure described in the bosonic case in [33], we should first of all find the algebraic relations for the superalgebra of the additional parts and then construct its representation in terms of new (additional) creation and annihilation operators. Extending the method given in [33] to operators including fermionic constraints, we obtain the superalgebra of the additional parts o'_i in the form

$$[\,o'_i, o'_j\}_s \;=\; f^k_{ij} o'_k - (-1)^{\varepsilon(o_k)\varepsilon(o_m)} f^{km}_{ij} o'_m o'_k, \tag{31}$$

which, in its turn, is defined so as to preserve the form of the involution relations for the initial operators $\tilde{o}_i$,

$$[\,\tilde{o}_i, \tilde{o}_j\}_s \;=\; f^k_{ij}\tilde{o}_k + f^{km}_{ij}\tilde{o}_k\tilde{o}_m, \tag{32}$$

under its extension to the algebraic relations for the enlarged constraints $\tilde{O}_i$,

$$[\,\tilde{O}_i, \tilde{O}_j\}_s \;=\; f^k_{ij}\tilde{O}_k - \left(f^{mk}_{ij} + (-1)^{\varepsilon(O_k)\varepsilon(O_m)} f^{km}_{ij}\right) o'_m \tilde{O}_k + f^{km}_{ij}\tilde{O}_k\tilde{O}_m, \tag{33}$$

where the quantities f^k_{ij}, f^{km}_{ij} obey the properties $(f^k_{ij}, f^{km}_{ij}) = -(-1)^{\varepsilon(O_i)\varepsilon(O_j)}(f^k_{ji}, f^{km}_{ji})$. As a result, the required superalgebra is presented in Table 2, where

$$[l'_1, t'_0\} = r(2t'^+_1 l'_2 + g'_0 t'_1 - \tfrac{1}{2}t'_1), \tag{34}$$
$$[t'_0, l'^+_1\} = r(2l'^+_2 t'_1 + t'^+_1 g'_0 - \tfrac{1}{2}t'^+_1), \tag{35}$$
$$[l'_0, t'_1\} = r(4t'^+_1 l'_2 + 2g'_0 t'_1 - t'_1), \tag{36}$$
$$[t'^+_1, l'_0\} = r(4l'^+_2 t'_1 + 2t'^+_1 g'_0 - t'^+_1), \tag{37}$$
$$[l'_0, l'_1\} = r(4l'^+_1 l'_2 + 2g'_0 l'_1 - l'_1), \tag{38}$$
$$[l'^+_1, l'_0\} = r(4l'^+_2 l'_1 + 2l'^+_1 g'_0 - l'^+_1), \tag{39}$$
$$[l'_1, l'^+_1\} = l'_0 - r(g'^2_0 - \tfrac{1}{2}g'_0 - 4l'^+_2 l'_2 + \tfrac{3}{2}t'^+_1 t'_1). \tag{40}$$

In accordance with our method, we, firs of all, set $\tilde{m}' = -\tilde{m}$, so that the enlarged central charge $\tilde{M}$ vanishes, secondly, the additional parts corresponding to Hermitian operators t'_0, l'_0, g'_0 must contain arbitrary parameters whose values will be defined later from the condition of reproducing the proper equations of motion (30). Explicit expressions for the additional parts can be found following the method described in papers [42,48], which is adapted to the case

$[\downarrow,\rightarrow\}$	t'_0	t'_1	t'^{+}_1	l'_0	l'_1	l'^{+}_1	l'_2	l'^{+}_2	g'_0	$\tilde{m}'$
t'_0	$-2l'_0$	$2l'_1$	$2l'^{+}_1$	0	$-(34)$	(35)	0	0	0	0
t'_1	$2l'_1$	$4l'_2$	$-2g'_0$	$-(36)$	0	$-t'_0$	0	$-t'^{+}_1$	t'_1	0
t'^{+}_1	$2l'^{+}_1$	$-2g'_0$	$4l'^{+}_2$	(37)	t'_0	0	t'_1	0	$-t'^{+}_1$	0
l'_0	0	(36)	$-(37)$	0	(38)	$-(39)$	0	0	0	0
l'_1	(34)	0	$-t'_0$	$-(38)$	0	(40)	0	$-l'^{+}_1$	l'_1	0
l'^{+}_1	$-(35)$	t'_0	0	(39)	$-(40)$	0	l'_1	0	$-l'^{+}_1$	0
l'_2	0	0	$-t'_1$	0	0	$-l'_1$	0	g'_0	$2l'_2$	0
l'^{+}_2	0	t'^{+}_1	0	0	l'^{+}_1	0	$-g'_0$	0	$-2l'^{+}_2$	0
g'_0	0	$-t'_1$	t'^{+}_1	0	$-l'_1$	l'^{+}_1	$-2l'_2$	$2l'^{+}_2$	0	0
$\tilde{m}'$	0	0	0	0	0	0	0	0	0	0

Table 2: The algebra of the additional parts for the operators

of the Verma module construction for a supersymmetric nonlinear algebra defined by Table 2. Having omitted the tedious calculations in constructing the Verma module, we have as a result

$$t_1'^{+} = f^{+} + 2b_2^{+} f, \qquad l_1'^{+} = m_1 b_1^{+}, \tag{41}$$

$$g_0' = b_1^{+} b_1 + 2b_2^{+} b_2 + f^{+} f + h, \qquad l_2'^{+} = b_2^{+}, \tag{42}$$

$$\begin{aligned} t_0' &= 2m_1 b_1^{+} f - \frac{m_1}{2}(f^{+} - 2b_2^{+} f)\, b_1^{+} \sum_{k=1}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k-1} b_1^{2k}}{(2k)!} + \tilde{\gamma} m_0 \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k} b_1^{2k}}{(2k)!} \\ &+ \frac{r(h-\frac{1}{2})}{m_1}(f^{+} - 2b_2^{+} f) \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+1}}{(2k+1)!}, \end{aligned} \tag{43}$$

$$\begin{aligned} t_1' &= -2g_0' f - (f^{+} - 2b_2^{+} f) b_2 + \frac{1}{2}(h - \tfrac{1}{2})(f^{+} - 2b_2^{+} f) \sum_{k=1}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k-1} b_1^{2k}}{(2k)!} \\ &+ \frac{1}{2}(f^{+} - 2b_2^{+} f)\, b_1^{+} \sum_{k=1}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k-1}\, b_1^{2k+1}}{(2k+1)!} \\ &- \frac{\tilde{\gamma} m_0}{m_1} \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+1}}{(2k+1)!}, \end{aligned} \tag{44}$$

$$\begin{aligned} l_0' &= m_0^2 - r\frac{\tilde{\gamma} m_0}{m_1}\, (f^{+} - 2b_2^{+} f) \sum_{k=1}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+1}}{(2k+1)!}\, (1 - 4^{-k}) \\ &- r b_1^{+} \sum_{k=0}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+1}}{(2k+1)!}\, (2h - 4^{-k}) + 4r\, \frac{\tilde{\gamma} m_0}{m_1}\, f \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k+1}\, b_1^{2k+1}}{(2k+1)!} \\ &+ r\left(h - \frac{1}{2}\right) \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k+1} \frac{(b_2^{+})^{k+1}\, b_1^{2k+2}}{(2k+2)!} - 2r\, (b_1^{+})^2 \sum_{k=0}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+2}}{(2k+2)!} \\ &- 2r f^{+} f \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \left\{\frac{(h-\frac{1}{2})}{(2k)!} + \frac{b_1^{+} b_1}{(2k+1)!}\right\} (b_2^{+})^{k} b_1^{2k} \\ &+ \frac{m_0^2 - r(h^2 - \frac{1}{4})}{2} \sum_{k=0}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k+1} \frac{(b_2^{+})^{k+1}\, b_1^{2k+2}}{(2k+2)!}, \end{aligned} \tag{45}$$

$$\begin{aligned} l_1' &= -m_1 b_1^{+} b_2 + \frac{m_1}{4}\, b_1^{+} \sum_{k=1}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k} \left\{\frac{2h - 4^{-k}}{(2k)!} + \frac{2b_1^{+} b_1}{(2k+1)!}\right\} (b_2^{+})^{k-1} b_1^{2k} + \\ &+ \frac{\tilde{\gamma} m_0}{4}\, (f^{+} - 2b_2^{+} f) \sum_{k=1}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k-1}\, b_1^{2k}}{(2k)!}\, (1 - 4^{-k}) \\ &+ \frac{r(h-\frac{1}{2})}{2m_1} \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+1}}{(2k+1)!} + \frac{m_1}{2}\, b_1^{+} f^{+} f \sum_{k=1}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k-1} b_1^{2k}}{(2k)!} \\ &- \frac{r(h-\frac{1}{2})}{m_1} f^{+} f \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k} b_1^{2k+1}}{(2k+1)!} - \tilde{\gamma} m_0 f \sum_{k=0}^{\infty} \left(\frac{-2r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k} b_1^{2k}}{(2k)!} \\ &+ \frac{m_0^2 - r(h^2 - \frac{1}{4})}{m_1} \sum_{k=0}^{\infty} \left(\frac{-8r}{m_1^2}\right)^{k} \frac{(b_2^{+})^{k}\, b_1^{2k+1}}{(2k+1)!}, \end{aligned} \tag{46}$$

$$
\begin{aligned}
l'_2 \;=\;& g'_0 b_2 - b_2^+ b_2^2 - \frac{m_0^2 - r(h^2-\frac{1}{4})}{m_1^2}\sum_{k=0}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+2}}{(2k+2)!}\\
&- \frac{r(h-\frac{1}{2})}{2m_1^2}\sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+2}}{(2k+2)!} + \frac{\tilde{\gamma} m_0}{m_1}\, f \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k b_1^{2k+1}}{(2k+1)!}\\
&- \frac{\tilde{\gamma} m_0}{4m_1}\,(f^+ - 2b_2^+ f)\sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!}\,(1-4^{-k})\\
&- \frac{1}{4}\, b_1^+ \sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \left\{\frac{2h-4^{-k}}{(2k+1)!} + \frac{2b_1^+ b_1}{(2k+2)!}\right\}(b_2^+)^{k-1} b_1^{2k+1}\\
&- \frac{1}{2}\, f^+ f \sum_{k=1}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \left\{\frac{h-\frac{1}{2}}{(2k)!} + \frac{b_1^+ b_1}{(2k+1)!}\right\}(b_2^+)^{k-1} b_1^{2k},
\end{aligned}
\tag{47}
$$

In the above expressions, h is a dimensionless arbitrary constant, while m_0 and m_1 are arbitrary constants with the dimension of mass. To obtain explicit expressions for the additional parts (41)–(47) in the Fock space, we have introduced two new pairs of bosonic and a new pair of fermionic creation and annihilation operators satisfying the standard commutation relations

$$
[b_1, b_1^+] = 1, \qquad [b_2, b_2^+] = 1, \qquad \{f, f^+\} = 1. \tag{48}
$$

The found additional parts of the operators possess all the necessary properties, i.e., the additional parts which correspond to hermitian operators contain arbitrary parameters. In particular, the operator t'_0 contains an arbitrary parameter m_0; the operator g'_0 contains an arbitrary parameter h, but the operator l'_0 cannot contain any more independent arbitrary parameters since $l'_0 = -(t'_0)^2$. The values of the arbitrary parameters h and m_0 will be determined later by the condition of reproducing the correct equations of motion (12).

The massive parameter m_1 remains arbitrary, and it can be expressed through the other parameters of the theory,

$$
m_1 = f(m, r) \neq 0. \tag{49}
$$

This arbitrariness does not affect the equations for the basic vector (5).

We note that the additional parts do not obey the usual properties

$$
(l'_0)^+ \neq l'_0, \qquad (l'_1)^+ \neq l'^+_1, \qquad (l'_2)^+ \neq l'^+_2, \tag{50}
$$

$$
(t'_0)^+ \neq t'_0 \qquad (t'_1)^+ \neq t'^+_1, \tag{51}
$$

if we use the standard rules of Hermitian conjugation for the new creation and annihilation operators,

$$
(b_1)^+ = b_1^+, \qquad (b_2)^+ = b_2^+, \qquad (f)^+ = f^+. \tag{52}
$$

To restore the correct Hermitian conjugation properties for the additional parts, we change the scalar product in the Fock space generated by the new creation and annihilation operators as follows:

$$
\langle \tilde{\Psi}_1 | \Psi_2 \rangle_{\text{new}} = \langle \tilde{\Psi}_1 | K' | \Psi_2 \rangle \tag{53}
$$

for any vectors $|\Psi_1\rangle, |\Psi_2\rangle$ with some yet unknown operator K'. This operator is determined by the condition that all the operators of the algebra must have the correct Hermitian properties with respect to the new scalar product,

$$\langle\tilde{\Psi}_1|K'l'_0|\Psi_2\rangle = \langle\tilde{\Psi}_2|K'l'_0|\Psi_1\rangle^*, \qquad \langle\tilde{\Psi}_1|K't'_0|\Psi_2\rangle = \langle\tilde{\Psi}_2|K't'_0|\Psi_1\rangle^*, \tag{54}$$

$$\langle\tilde{\Psi}_1|K'l'_1|\Psi_2\rangle = \langle\tilde{\Psi}_2|K'l'^+_1|\Psi_1\rangle^*, \qquad \langle\tilde{\Psi}_1|K't'_1|\Psi_2\rangle = \langle\tilde{\Psi}_2|K't'^+_1|\Psi_1\rangle^*, \tag{55}$$

$$\langle\tilde{\Psi}_1|K'l'_2|\Psi_2\rangle = \langle\tilde{\Psi}_2|K'l'^+_2|\Psi_1\rangle^*, \qquad \langle\tilde{\Psi}_1|K'g'_0|\Psi_2\rangle = \langle\tilde{\Psi}_2|K'g'_0|\Psi_1\rangle^* \tag{56}$$

These relations permit one to define a Hermitian (in the usual sense) operator K' as follows:

$$K' = Z^+Z, \qquad Z = \sum_{n_1,n_2=0}^{\infty}\sum_{s=0}^{1}\left(\frac{l'^+_1}{m_1}\right)^{n_1}(t'^+_1)^s(l'^+_2)^{n_2}|0\rangle_V\frac{1}{n_1!n_2!}\langle 0|b_1^{n_1}f^s b_2^{n_2} \tag{57}$$

where the auxiliary vector $|0\rangle_V$ satisfies the relations

$$t'_1|0\rangle_V = l'_1|0\rangle_V = l'_2|0\rangle_V = 0, \tag{58}$$

$$t'_0|0\rangle_V = \tilde{\gamma}m_0|0\rangle_V, \qquad g'_0|0\rangle_V = h|0\rangle_V, \qquad {}_V\langle 0|0\rangle_V = 1. \tag{59}$$

Since the problem of Hermitian conjugation for the operators is related to the b_i, b_i^+, f, f^+ sector of the Fock space, we assume that the operator K' acts as a unit operator in the remaining part of the Fock space. For low numbers n_1+n_f, where n_i are the numbers of "particles" associated with b_i^+, and n_f is the number of "particles" associated with f^+, the operator K' reads

$$\begin{aligned} K' &= |0\rangle\langle 0| - \frac{m_0^2 + rh(h-\frac{1}{2})}{m_1^2}\, b_1^+|0\rangle\langle 0|b_1 - 2hf^+|0\rangle\langle 0|f \\ &\quad + \frac{m_0}{m_1}\Big(\tilde{\gamma}f^+|0\rangle\langle 0|b_1 + b_1^+|0\rangle\langle 0|f\tilde{\gamma}\Big) + \ldots \end{aligned} \tag{60}$$

This expression for the operator K' will be used later in the construction of examples in section 7.

4 The deformed algebra and the BFV–BRST operator

Let us turn to the algebra of the enlarged operators $\tilde{O}_i = \tilde{o}_i + o'_i$. Since the algebra is quadratic, then there exist different possibilities of operator ordering in the right-hand side of the commutation relations. This situation is similar to that in the bosonic case [33]. Each form of ordering can, in principle, lead to a different BRST operator. We will not investigate all the possibilities of ordering as this was done in [33] and choose only one of them, which corresponds to the standard physical choice of supersymmetric, i.e., Weyl ordering for the constraints[3]. These choice of ordering leads to the shortest expression for the corresponding BRST operator with nonvanishing terms of the third degree in powers of ghosts. Such a realization of the algebra of the enlarged operators is presented in Table 3 ,

[3]Within this ordering, we (instead of the general product $\tilde{O}_i\tilde{O}_j$ in the r.h.s. of the algebraic relations for enlarged operators) substitute its Weyl-ordered expression, as follows: $\tilde{O}_i\tilde{O}_j = \frac{1}{2}(\tilde{O}_i\tilde{O}_j + (-1)^{\varepsilon(O_i)\varepsilon(O_j)}\tilde{O}_j\tilde{O}_i) + \frac{1}{2}[\tilde{O}_i, \tilde{O}_j\}$

$[\downarrow,\rightarrow\}$	T_0	T_1	T_1^+	L_0	L_1	L_1^+	L_2	L_2^+	G_0
T_0	$-2L_0$	$2L_1$	$2L_1^+$	0	(61)	$-(62)$	0	0	0
T_1	$2L_1$	$4L_2$	$-2G_0$	(63)	0	$-T_0$	0	$-T_1^+$	T_1
T_1^+	$2L_1^+$	$-2G_0$	$4L_2^+$	$-(64)$	T_0	0	T_1	0	$-T_1^+$
L_0	0	$-(63)$	(64)	0	$-(65)$	(66)	0	0	0
L_1	$-(61)$	0	$-T_0$	(65)	0	(67)	0	$-L_1^+$	L_1
L_1^+	(62)	T_0	0	$-(66)$	$-(67)$	0	L_1	0	$-L_1^+$
L_2	0	0	$-T_1$	0	0	$-L_1$	0	G_0	$2L_2$
L_2^+	0	T_1^+	0	0	L_1^+	0	$-G_0$	0	$-2L_2^+$
G_0	0	$-T_1$	T_1^+	0	$-L_1$	L_1^+	$-2L_2$	$2L_2^+$	0

Table 3: The algebra of the enlarged operators

where

$$[T_0, L_1\} = r\Big[\tfrac{1}{2}G_0T_1 + \tfrac{1}{2}T_1G_0 + T_1^+L_2 + L_2T_1^+ - g_0'T_1 - t_1'G_0 - 2(t_1'^+L_2 + l_2'T_1^+)\Big], \quad (61)$$

$$[L_1^+, T_0\} = r\Big[\tfrac{1}{2}T_1^+G_0 + \tfrac{1}{2}G_0T_1^+ + L_2^+T_1 + T_1L_2^+ - t_1'^+G_0 - g_0'T_1^+ - 2(l_2'^+T_1 + t_1'L_2^+)\Big], \quad (62)$$

$$[T_1, L_0\} = r\Big[G_0T_1 + T_1G_0 + 2T_1^+L_2 + 2L_2T_1^+ - 2\,(g_0'T_1 + t_1'G_0) - 4(t_1'^+L_2 + l_2'T_1^+)\Big], \quad (63)$$

$$[L_0, T_1^+\} = r\Big[T_1^+G_0 + G_0T_1^+ + 2L_2^+T_1 + 2T_1L_2^+ - 2(t_1'^+G_0 + g_0'T_1^+) - 4(l_2'^+T_1 + t_1'L_2^+)\Big], \quad (64)$$

$$[L_1, L_0\} = r\Big[G_0L_1 + L_1G_0 + 2L_1^+L_2 + 2L_2L_1^+ - 2\,(l_1'G_0 + g_0'L_1) - 4\,(l_1'^+L_2 + l_2'L_1^+)\Big], \quad (65)$$

$$[L_0, L_1^+\} = r\Big[L_1^+G_0 + G_0L_1^+ + 2L_2^+L_1 + 2L_1L_2^+ - 2\big(l_1'^+G_0 + g_0'L_1^+\big) - 4\,\big(l_1'L_2^+ + l_2'^+L_1\big)\Big] \quad (66)$$

$$[L_1, L_1^+\} = L_0 + r\Big\{G_0^2 - 2L_2^+L_2 - 2L_2L_2^+ + \tfrac{3}{4}T_1^+T_1 - \tfrac{3}{4}T_1T_1^+\Big\} + r\Big\{-2g_0'G_0 + 4\,\big(l_2'^+L_2 + l_2'L_2^+\big) - \tfrac{3}{2}\,\big(t_1'^+T_1 - t_1'T_1^+\big)\Big\}. \quad (67)$$

The construction of a nilpotent fermionic BRST operator for the nonlinear supersymmetric algebra generated by the constraints $\tilde{O}_i$ is based on the same principles as those developed in [29, 33]. (For a general consideration of operator BFV quantization, see the reviews [50]). The BRST operator constructed on the basis of the algebra presented in Table 3 can be found with the use of $(\mathcal{CP})$-ordering for the ghost coordinates $\mathcal{C}^i$ and the momenta $\mathcal{P}_i$ operators in the form

$$
\begin{aligned}
\tilde{Q}' &= q_0T_0 + q_1^+T_1 + q_1T_1^+ + \eta_0L_0 + \eta_1^+L_1 + \eta_1L_1^+ + \eta_2^+L_2 + \eta_2L_2^+ + \eta_GG_0 \\
&\quad + i(\eta_1^+q_1 - \eta_1q_1^+)p_0 - i(\eta_Gq_1 + \eta_2q_1^+)p_1^+ + i(\eta_Gq_1^+ + \eta_2^+q_1)p_1 \\
&\quad + (q_0^2 - \eta_1^+\eta_1)\mathcal{P}_0 + (2q_1q_1^+ - \eta_2^+\eta_2)\mathcal{P}_G + (\eta_G\eta_1^+ + \eta_2^+\eta_1 - 2q_0q_1^+)\mathcal{P}_1 \\
&\quad + (\eta_1\eta_G + \eta_1^+\eta_2 - 2q_0q_1)\mathcal{P}_1^+ + 2(\eta_G\eta_2^+ - q_1^{+2})\mathcal{P}_2 + 2(\eta_2\eta_G - q_1^2)\mathcal{P}_2^+ \\
&+ r(q_0\eta_1^+ + 2q_1^+\eta_0)\Big[\tfrac{i}{2}G_0p_1 + \tfrac{1}{2}T_1\mathcal{P}_G + T_1^+\mathcal{P}_2 + iL_2p_1^+ - t_1'\mathcal{P}_G - ig_0'p_1 - 2(il_2'p_1^+ + t_1'^+\mathcal{P}_2)\Big] \\
&+ r(q_0\eta_1 + 2q_1\eta_0)\Big[-\tfrac{i}{2}G_0p_1^+ - \tfrac{1}{2}T_1^+\mathcal{P}_G - T_1\mathcal{P}_2^+ - iL_2^+p_1 + t_1'^+\mathcal{P}_G + ig_0'p_1^+ + 2(il_2'^+p_1 + t_1'\mathcal{P}_2^+)\Big] \\
&\quad + 2r\eta_0\eta_1^+\Big[\tfrac{1}{2}G_0\mathcal{P}_1 + \tfrac{1}{2}L_1\mathcal{P}_G + L_1^+\mathcal{P}_2 + L_2\mathcal{P}_1^+ - l_1'\mathcal{P}_G - g_0'\mathcal{P}_1 - 2(l_1'^+\mathcal{P}_2 + l_2'\mathcal{P}_1^+)\Big] \\
&\quad - 2r\eta_0\eta_1\Big[\tfrac{1}{2}G_0\mathcal{P}_1^+ + \tfrac{1}{2}L_1^+\mathcal{P}_G + L_1\mathcal{P}_2^+ + L_2^+\mathcal{P}_1 - l_1'^+\mathcal{P}_G - g_0'\mathcal{P}_1^+ - 2(l_2'^+\mathcal{P}_1 + l_1'\mathcal{P}_2^+)\Big] \\
&\quad - r\eta_1\eta_1^+\Big[2L_2^+\mathcal{P}_2 + 2L_2\mathcal{P}_2^+ - G_0\mathcal{P}_G - \tfrac{3i}{4}T_1^+p_1 + \tfrac{3i}{4}T_1p_1^+\Big] \\
&\quad + r\eta_1\eta_1^+\Big[4(l_2'^+\mathcal{P}_2 + l_2'\mathcal{P}_2^+) - 2g_0'\mathcal{P}_G - \tfrac{3i}{2}(t_1'^+p_1 - t_1'p_1^+)\Big] \\
&+ r^2\eta_0\eta_1\eta_1^+\Bigg[G_0\left(p_1^+p_1 + 2\mathcal{P}_2^+\mathcal{P}_2\right) + L_2^+p_1^2 + L_2(p_1^+)^2 + \frac{i}{2}\left(T_1^+p_1^+\mathcal{P}_2 + T_1p_1\mathcal{P}_2^+\right) \\
&\qquad + \frac{i}{4}\left(T_1^+p_1 + T_1p_1^+\right)\mathcal{P}_G - 2\left(L_2\mathcal{P}_2^+ - L_2^+\mathcal{P}_2\right)\mathcal{P}_G\Bigg]. \qquad (68)
\end{aligned}
$$

Here, q_0, q_1, q_1^+ and η_0, η_1^+, η_1, η_2^+, η_2, η_G are, respectively, the bosonic and fermionic ghost "coordinates" corresponding to their canonically conjugate ghost "momenta" p_0, p_1^+, p_1, $\mathcal{P}_0$, $\mathcal{P}_1$, $\mathcal{P}_1^+$, $\mathcal{P}_2$, $\mathcal{P}_2^+$, $\mathcal{P}_G$. They obey the (anti)commutation relations

$$
\begin{aligned}
\{\eta_1, \mathcal{P}_1^+\} = \{\mathcal{P}_1, \eta_1^+\} = \{\eta_2, \mathcal{P}_2^+\} = \{\mathcal{P}_2, \eta_2^+\} = \{\eta_0, \mathcal{P}_0\} = \{\eta_G, \mathcal{P}_G\} = 1, \\
[q_0, p_0] = [q_1, p_1^+] = [q_1^+, p_1] = i \qquad (69)
\end{aligned}
$$

and possess the standard ghost number distribution, $gh(\mathcal{C}^i) = -gh(\mathcal{P}_i) = 1$, providing the property $gh(\tilde{Q}') = 1$.

As a next step, we need to realize a transition from the operator $\tilde{Q}'$ (68) to a BRST operator Q' encoding the algebra of the enlarged constraints O_i which differ from $\tilde{O}_i$ only in the definition of the operators T_0, L_0, as well as contain the initial constraint (15) for the vanishing of all the additional annihilation and creation operators b_i, b_i^+, f, f^+, and correspond to the correct equations of motion (30) as follows:

$$
T_0^t\big|_{\{b_i=b_i^+=f=f^+=0\}} = \left(t_0 + t_0' + \tilde{\gamma}r^{\frac{1}{2}}G_0\right)\Big|_{\{b_i=b_i^+=f=f^+=0\}} = \tilde{t}_0', \qquad (70)
$$

for a special choice of the constants m_0, h. The above-mentioned linear transformation from the set of operators o_i to $\tilde{o}_i$, $o_i = U_i^j\tilde{o}_j$ can be, firstly, extended to the enlarged operators O_i, $O_i = \tilde{U}_i^j\tilde{O}_j$, with an operator nondegenerate supermatrix $\tilde{U}$ being derived from U by the change $(\tilde{m}, g_0) \to (0, G_0)$. Secondly, the obtained transformation permits one to obtain the BRST operator Q' by means of a unitary transformation (constructed through the matrix $\tilde{U}$) of the operator $\tilde{Q}'$ (68) in the space of all the operators, in the form

$$
\begin{aligned}
Q' &= \mathcal{U}Q'\mathcal{U}^{-1}, \qquad \mathcal{U}K\mathcal{U}^+ = \mathcal{U}^+K\mathcal{U} = K, \qquad \mathcal{U}Q'\mathcal{U}^{-1}\big|_{\mathcal{P}=0} = O_iC^i, \qquad (71) \\
\mathcal{U} &= \exp\Big\{r^{\frac{1}{2}}\mathcal{A}\mathcal{P}_G\Big\}, \quad \mathcal{A} = \left(q_0\tilde{\gamma} + \eta_0\Big[r^{\frac{1}{2}}G_0 + 2m_0\sum_{m\geq 0}\left(\frac{-2\lambda^2}{m_4^2}\right)^m\frac{1}{2m!}(b_2^+)^m b_1^{2m}\Big]\right) \qquad (72)
\end{aligned}
$$

with the operator K being the enlargement of K' into space of all the operators

$$
\begin{aligned}
K \;=\;& \sum_{(l_0,l_1,l_2,l_9)\geq 0}^{\infty} \; \sum_{(l_3,\ldots,l_8)\geq 0}^{(1,\ldots,1)} \frac{i^{l_0+l_1+l_2}(-1)^{l_9+l_2}}{l_0!l_1!l_2!l_9!} \Bigg[\prod_{(i_0,i_1,i_2)\geq 1}^{(l_0,l_1,l_2)} (q_0)^{i_0}(q_1^+)^{i_1}(p_1^+)^{i_2} \prod_{(i_3,\ldots,i_8)\geq 1}^{(l_3,\ldots,l_8)} (\eta_1^+)^{i_3}\times \\
& \times (\mathcal{P}_1^+)^{i_4}(\eta_2^+)^{i_5}(\mathcal{P}_2^+)^{i_6}(\eta_0)^{i_7}(\eta_G)^{i_8} \prod_{i_9=1}^{l_9} a^{+\mu_{i_9}} \Bigg] K' \Bigg[\prod_{i_9=1}^{l_9} a_{\mu_{i_9}} \prod_{(i_8,\ldots,i_3)\geq 1}^{(l_8,\ldots l_3)} (\mathcal{P}_G)^{i_8}(\mathcal{P}_0)^{i_7}(\eta_2)^{i_6}\times \\
& \times (\mathcal{P}_2)^{i_5}(\eta_1)^{i_4}(\mathcal{P}_1)^{i_3} \prod_{(i_2,i_1,i_0)\geq 1}^{(l_2,l_1,l_0)} (q_1)^{i_2}(p_1)^{i_1}(p_0)^{i_0} \Bigg], \quad \text{where } \prod_{i=1}^{0}(G)^i \equiv 1 \text{ for any } G. \qquad (73)
\end{aligned}
$$

As a consequence of Eqs. (71), (72), the BRST operator Q' is nilpotent, has the same value of ghost number as $\tilde{Q}'$ and takes the final form[4]

$$
\begin{aligned}
Q' \;&=\; \tilde{Q}' + r^{\frac{1}{2}}\mathcal{A}(\sigma+h) - r^{\frac{1}{2}}\Big([\mathcal{A},\tilde{Q}'] - r^{\frac{1}{2}}q_0^2(\sigma+h)\Big)\mathcal{P}_G, && (74)\\
[\mathcal{P}_G, Q'\} \;&=\; \sigma + h \equiv G_0 + iq_1^+p_1 - iq_1p_1^+ + \eta_1^+\mathcal{P}_1 - \eta_1\mathcal{P}_1^+ + 2\eta_2^+\mathcal{P}_2 - 2\eta_2\mathcal{P}_2^+. && (75)
\end{aligned}
$$

The property of the BRST operator $\tilde{Q}'$ as well as of Q' to be Hermitian is determined by the rule

$$
Q'^{+}K = KQ'. \qquad (76)
$$

Now we turn to the construction of the Lagrangians for fermionic higher-spin fields in AdS space.

5 Construction of Lagrangians

In order to construct the Lagrangians, we extract, according to the procedure developed in Refs. [42,43] the terms proportional to the ghosts $\eta_G, \mathcal{P}_G$ from the BRST operator Q' as follows:

$$
Q' \;=\; Q + (\eta_G + r^{\frac{1}{2}}\mathcal{A})(\sigma+h) + \Big[\mathcal{B} - r^{\frac{1}{2}}[\mathcal{A},\tilde{Q}'] + rq_0^2(\sigma+h)\Big]\mathcal{P}_G, \qquad (77)
$$

[4]Note that Q' contains a nonvanishing term of the fourth degree in power of ghosts, $r^{\frac{5}{2}}q_0\eta_0\eta_1\eta_1^+\times$ $\times\left([\tilde{\gamma},l_2'\}\,(p_1^+)^2 - \frac{i}{2}\,\{\tilde{\gamma},t_1'\}\,p_1\mathcal{P}_2^+\right)\mathcal{P}_G$

where the enlarged spin operator σ is defined by (75) whereas the quantities Q and $\mathcal{B}$ have the form

$$
\begin{aligned}
Q &= q_0T_0 + q_1^+T_1 + q_1T_1^+ + \eta_0L_0 + \eta_1^+L_1 + \eta_1L_1^+ + \eta_2^+L_2 + \eta_2L_2^+ \\
&\quad + i(\eta_1^+q_1 - \eta_1q_1^+)p_0 - i\eta_2q_1^+p_1^+ + i\eta_2^+q_1p_1 + (q_0^2 - \eta_1^+\eta_1)\mathcal{P}_0 + (\eta_2^+\eta_1 - 2q_0q_1^+)\mathcal{P}_1 \\
&\quad + (\eta_1^+\eta_2 - 2q_0q_1)\mathcal{P}_1^+ - 2q_1^{+2}\mathcal{P}_2 - 2q_1^2\mathcal{P}_2^+ \\
&+ r(q_0\eta_1^+ + 2q_1^+\eta_0)\Big[\tfrac{i}{2}G_0p_1 + T_1^+\mathcal{P}_2 + iL_2p_1^+ - ig_0'p_1 - 2(il_2'p_1^+ + t_1'^+\mathcal{P}_2)\Big] \\
&+ r(q_0\eta_1 + 2q_1\eta_0)\Big[-\tfrac{i}{2}G_0p_1^+ - T_1\mathcal{P}_2^+ - iL_2^+p_1 + ig_0'p_1^+ + 2(il_2'^+p_1 + t_1'\mathcal{P}_2^+)\Big] \\
&\quad + 2r\eta_0\eta_1^+\Big[\tfrac{1}{2}G_0\mathcal{P}_1 + L_1^+\mathcal{P}_2 + L_2\mathcal{P}_1^+ - g_0'\mathcal{P}_1 - 2(l_1'^+\mathcal{P}_2 + l_2'\mathcal{P}_1^+)\Big] \\
&\quad - 2r\eta_0\eta_1\Big[\tfrac{1}{2}G_0\mathcal{P}_1^+ + L_1\mathcal{P}_2^+ + L_2^+\mathcal{P}_1 - g_0'\mathcal{P}_1^+ - 2(l_2'^+\mathcal{P}_1 + l_1'\mathcal{P}_2^+)\Big] \\
&\quad - r\eta_1\eta_1^+\Big[2L_2^+\mathcal{P}_2 + 2L_2\mathcal{P}_2^+ - \tfrac{3i}{4}T_1^+p_1 + \tfrac{3i}{4}T_1p_1^+\Big] \\
&\quad + r\eta_1\eta_1^+\Big[4(l_2'^+\mathcal{P}_2 + l_2'\mathcal{P}_2^+) - \tfrac{3i}{2}(t_1'^+p_1 - t_1'p_1^+)\Big] \\
&+ r^2\eta_0\eta_1\eta_1^+\left[G_0\left(p_1^+p_1 + 2\mathcal{P}_2^+\mathcal{P}_2\right) + L_2^+p_1^2 + L_2(p_1^+)^2 + \frac{i}{2}\left(T_1^+p_1^+\mathcal{P}_2 + T_1p_1\mathcal{P}_2^+\right)\right], \quad (78)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{B} &= 2q_1q_1^+ - \eta_2^+\eta_2 + r(\tfrac{1}{2}q_0\eta_1^+ + q_1^+\eta_0)\Big[T_1 - 2t_1'\Big] - r(\tfrac{1}{2}q_0\eta_1 + q_1\eta_0)\Big[T_1^+ - 2t_1'^+\Big] \\
&\quad + r\eta_0\eta_1^+\Big[L_1 - 2l_1'\Big] - r\eta_0\eta_1\Big[L_1^+ - 2l_1'^+\Big] + r\eta_1\eta_1^+\Big[G_0 - 2g_0'\Big] \\
&+ r^2\eta_0\eta_1\eta_1^+\left[\frac{i}{4}\left(T_1^+p_1 + T_1p_1^+\right) - 2\left(L_2\mathcal{P}_2^+ - L_2^+\mathcal{P}_2\right)\right]. \quad (79)
\end{aligned}
$$

As a next step, we should extract the zero ghost modes from the operator Q (78). This operator has the structure

$$
Q = q_0\tilde{T}_0 + \eta_0\tilde{L}_0 + i(\eta_1^+q_1 - \eta_1q_1^+)p_0 + (q_0^2 - \eta_1^+\eta_1)\mathcal{P}_0 + \Delta Q, \quad (80)
$$

where the enlarged (by means of the ghosts) constraints $\tilde{T}_0$ and $\tilde{L}_0$ are defined by the relations

$$
\begin{aligned}
\tilde{T}_0 &= T_0 - 2q_1^+\mathcal{P}_1 - 2q_1\mathcal{P}_1^+ \\
&\quad + r\eta_1^+\Big[\tfrac{i}{2}G_0p_1 + T_1^+\mathcal{P}_2 + iL_2p_1^+ - ig_0'p_1 - 2(il_2'p_1^+ + t_1'^+\mathcal{P}_2)\Big] \\
&\quad + r\eta_1\Big[-\tfrac{i}{2}G_0p_1^+ - T_1\mathcal{P}_2^+ - iL_2^+p_1 + ig_0'p_1^+ + 2(il_2'^+p_1 + t_1'\mathcal{P}_2^+)\Big], \quad (81) \\
\tilde{L}_0 &= L_0 + 2rq_1^+\Big[\tfrac{i}{2}G_0p_1 + T_1^+\mathcal{P}_2 + iL_2p_1^+ - ig_0'p_1 - 2(il_2'p_1^+ + t_1'^+\mathcal{P}_2)\Big] \\
&\quad + 2rq_1\Big[-\tfrac{i}{2}G_0p_1^+ - T_1\mathcal{P}_2^+ - iL_2^+p_1 + ig_0'p_1^+ + 2(il_2'^+p_1 + t_1'\mathcal{P}_2^+)\Big] \\
&\quad + 2r\eta_1^+\Big[\tfrac{1}{2}G_0\mathcal{P}_1 + L_1^+\mathcal{P}_2 + L_2\mathcal{P}_1^+ - g_0'\mathcal{P}_1 - 2(l_1'^+\mathcal{P}_2 + l_2'\mathcal{P}_1^+)\Big] \\
&\quad - 2r\eta_1\Big[\tfrac{1}{2}G_0\mathcal{P}_1^+ + L_1\mathcal{P}_2^+ + L_2^+\mathcal{P}_1 - g_0'\mathcal{P}_1^+ - 2(l_2'^+\mathcal{P}_1 + l_1'\mathcal{P}_2^+)\Big] \\
&\quad + r^2\eta_1\eta_1^+\left[G_0\left(p_1^+p_1 + 2\mathcal{P}_2^+\mathcal{P}_2\right) + L_2^+p_1^2 + L_2(p_1^+)^2 + \frac{i}{2}\left(T_1^+p_1^+\mathcal{P}_2 + T_1p_1\mathcal{P}_2^+\right)\right], \quad (82)
\end{aligned}
$$

which preserve the relation, $[\tilde{T}_0, \tilde{T}_0\} = -2\tilde{L}_0$, whereas the independent (from the zero ghost modes part) of the operator Q has the form

$$\begin{aligned}\Delta Q &= q_1^+T_1 + q_1T_1^+ + \eta_1^+L_1 + \eta_1L_1^+ + \eta_2^+L_2 + \eta_2L_2^+ - i\eta_2 q_1^+p_1^+ + i\eta_2^+q_1p_1 \\ &+ \eta_2^+\eta_1\mathcal{P}_1 + \eta_1^+\eta_2\mathcal{P}_1^+ - 2q_1^{+2}\mathcal{P}_2 - 2q_1^2\mathcal{P}_2^+ \\ &- r\eta_1\eta_1^+\left[2L_2^+\mathcal{P}_2 + 2L_2\mathcal{P}_2^+ - \tfrac{3i}{4}T_1^+p_1 + \tfrac{3i}{4}T_1p_1^+\right] \\ &+ r\eta_1\eta_1^+\left[4(l_2'^+\mathcal{P}_2 + l_2'\mathcal{P}_2^+) - \tfrac{3i}{2}(t_1'^+p_1 - t_1'p_1^+)\right]. \end{aligned} \tag{83}$$

Following the procedure of Ref. [42,43], we choose the representation of the Hilbert space given by the relations

$$(p_0, q_1, p_1, \mathcal{P}_0, \mathcal{P}_G, \eta_1, \mathcal{P}_1, \eta_2, \mathcal{P}_2)\,|0\rangle = \vec{0} \tag{84}$$

and suppose (as it was shown in [42] in order to avoid the doubling of the physical component states) that vectors, as well as the gauge parameters, do not depend on η_G

$$\begin{aligned}|\chi\rangle = \sum_{k_i}(q_0)^{k_1}(q_1^+)^{k_2}(p_1^+)^{k_3}(\eta_0)^{k_4}(f^+)^{k_5}(\eta_1^+)^{k_6}(\mathcal{P}_1^+)^{k_7}(\eta_2^+)^{k_8}(\mathcal{P}_2^+)^{k_9}(b_1^+)^{k_{10}}(b_2^+)^{k_{11}}\times \\ \times a^{+\mu_1}\cdots a^{+\mu_{k_0}}\chi^{k_1\cdots k_{11}}_{\mu_1\cdots\mu_{k_0}}(x)|0\rangle.\end{aligned} \tag{85}$$

The sum in (85) is taken over over $k_0, k_1, k_2, k_3, k_{10}, k_{11}$ running from 0 to infinity, and over $k_4, k_5, k_6, k_7, k_8, k_9$ running from 0 to 1 with a vanishing ghost number for $|\chi\rangle$. As a consequence, we derive from the equations determining the physical vector, $Q'|\chi\rangle = 0$, and the reducible gauge transformations, $\delta|\chi\rangle = Q'|\Lambda\rangle$, a sequence of relations:

$$Q|\chi\rangle = 0, \qquad (\sigma+h)|\chi\rangle = 0, \qquad (\varepsilon, gh)\,(|\chi\rangle) = (1,0), \tag{86}$$

$$\delta|\chi\rangle = Q|\Lambda\rangle, \qquad (\sigma+h)|\Lambda\rangle = 0, \qquad (\varepsilon, gh)\,(|\Lambda\rangle) = (0,-1), \tag{87}$$

$$\delta|\Lambda\rangle = Q|\Lambda^{(1)}\rangle, \qquad (\sigma+h)|\Lambda^{(1)}\rangle = 0, \qquad (\varepsilon, gh)\,(|\Lambda^{(1)}\rangle) = (1,-2), \tag{88}$$

$$\delta|\Lambda^{(i-1)}\rangle = Q|\Lambda^{(i)}\rangle, \qquad (\sigma+h)|\Lambda^{(i)}\rangle = 0, \qquad (\varepsilon, gh)\,(|\Lambda^{(i)}\rangle) = (i+1,-i-1). \tag{89}$$

These relations guarantee both the extraction of the vectors with the required value of spin and the nilpotency of Q on this Hilbert subspace.

We then expand the state vector and the gauge parameters in powers of the zero mode ghosts as follows:

$$|\chi\rangle = \sum_{k=0}^{\infty} q_0^k(|\chi_0^k\rangle + \eta_0|\chi_1^k\rangle), \qquad gh(|\chi_m^k\rangle) = -(m+k), \tag{90}$$

$$|\Lambda^{(i)}\rangle = \sum_{k=0}^{\infty} q_0^k(|\Lambda^{(i)k}_{\ \ 0}\rangle + \eta_0|\Lambda^{(i)k}_{\ \ 1}\rangle), \qquad gh(|\Lambda^{(i)k}_{\ \ m}\rangle) = -(i+k+m+1). \tag{91}$$

Following the procedure described in [31,42], we get rid of all the fields except two $|\chi_0^0\rangle$, $|\chi_0^1\rangle$, and, hence, the relations (80), (86) yield two independent equations for these fields

$$\Delta Q|\chi_0^0\rangle + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\chi_0^1\rangle = 0, \tag{92}$$

$$\tilde{T}_0|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle = 0, \tag{93}$$

where $\{A, B\} = AB + BA$ for any quantities A, B.

Next, due to the fact that the operators Q, $\tilde{T}_0$, $\eta_1^+\eta_1$ commute with σ, we derive from equations (92), (93) the equations of motion for the fields with a fixed value of spin:

$$\Delta Q|\chi_0^0\rangle_n + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\chi_0^1\rangle_n = 0, \tag{94}$$

$$\tilde{T}_0|\chi_0^0\rangle_n + \Delta Q|\chi_0^1\rangle_n = 0. \tag{95}$$

These field equations are Lagrangian and can therefore be deduced from the following Lagrangian action:[5]

$$\begin{aligned} \mathcal{S}_n &= {}_n\langle\tilde{\chi}_0^0|K_n\tilde{T}_0|\chi_0^0\rangle_n + \frac{1}{2}{}_n\langle\tilde{\chi}_0^1|K_n\{\tilde{T}_0, \eta_1^+\eta_1\}|\chi_0^1\rangle_n \\ &\quad + {}_n\langle\tilde{\chi}_0^0|K_n\Delta Q|\chi_0^1\rangle_n + {}_n\langle\tilde{\chi}_0^1|K_n\Delta Q|\chi_0^0\rangle_n, \end{aligned} \tag{96}$$

where we assume under the pairing $\langle\ |\ \rangle$ the standard definition for a scalar product as was given in (29). The above action is expressed for the fields with a given spin, which are defined by the number n chosen according to (75), (86) as follows:

$$\sigma|\chi_0^0\rangle_n = \big(n + (d-4)/2\big)|\chi_0^0\rangle_n, \qquad \sigma|\chi_0^1\rangle_n = \big(n + (d-4)/2\big)|\chi_0^1\rangle_n \tag{97}$$

and the operator K_n is the operator K (73) where the following substitution is assumed: $h \to -\big(n + (d-4)/2\big)$.

The equations of motion (94), (95) and the action (96) are invariant with respect to the gauge transformations

$$\delta|\chi_0^0\rangle_n = \Delta Q|\Lambda_0^0\rangle_n + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\Lambda_0^1\rangle_n, \tag{98}$$

$$\delta|\chi_0^1\rangle_n = \tilde{T}_0|\Lambda_0^0\rangle_n + \Delta Q|\Lambda_0^1\rangle_n. \tag{99}$$

which are reducible with gauge parameters $|\Lambda^{(i)}{}_0^k\rangle_n$, $k = 0, 1$ subject to the same conditions as for $|\chi_0^k\rangle_n$ in (97),

$$\delta|\Lambda^{(i)}{}_0^0\rangle_n = \Delta Q|\Lambda^{(i+1)}{}_0^0\rangle_n + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\Lambda^{(i+1)}{}_0^1\rangle_n, \qquad |\Lambda^{(0)}{}_0^0\rangle_n = |\Lambda_0^0\rangle_n, \tag{100}$$

$$\delta|\Lambda^{(i)}{}_0^1\rangle_n = \tilde{T}_0|\Lambda^{(i+1)}{}_0^0\rangle_n + \Delta Q|\Lambda^{(i+1)}{}_0^1\rangle_n, \qquad |\Lambda^{(0)}{}_0^1\rangle_n = |\Lambda_0^1\rangle_n, \tag{101}$$

with a finite number of reducibility stages at $i_{max} = n - 1$ for spin $s = n + 1/2$.

Now we can find the specific value of the arbitrary parameter m_0. It is defined from the condition of reproducing the equations (30) for the basic vector $|\Phi\rangle$ (5). To this end, it is necessary that the conditions (30) be reproduced from Eqs. (94), (95). Note that the general vector $|\chi_0^0\rangle_n$ includes the basic vector $|\Phi\rangle$ (5) as follows:

$$|\chi_0^0\rangle_n = |\Phi\rangle_n + |\Phi_A\rangle_n, \qquad |\Phi_A\rangle_n\Big|_{\mathcal{C}=\mathcal{P}=b_1^+=b_2^+=f^+=0} = 0 \tag{102}$$

with the vector $|\Phi_A\rangle$ essentially depending on all of the auxiliary fields in its components. In the following section we shall demonstrate, by using the gauge transformations and the equations of motion, that the vector $|\Phi_A\rangle_n$ can be completely removed from the decomposition (102) so that the resulting equations of motion have the form

$$T_0\big|_{\{b_i=b_i^+=f=f^+=0\}}|\Phi\rangle_n = (t_0 + \tilde{\gamma}m_0)|\Phi\rangle_n = 0, \qquad t_1|\Phi\rangle_n = 0. \tag{103}$$

[5]The action is defined, as usual, up to an overall factor.

The above relations permit one to determine the parameter m_0 in a unique way as follows:

$$m_0 = m - r^{\frac{1}{2}}\sqrt{-\omega}h = m + r^{\frac{1}{2}}\sqrt{-\omega}\big(n + (d-4)/2\big). \tag{104}$$

In the next section, we show that the action, indeed, reproduces the correct equations of motion (12).

Thus, we construct a Lagrangian for fermionic fields of any fixed spin using the BRST approach. Note that the Lagrangian formulation for massless higher-spin theory on (A)dS is the partial case of the corresponding massive case for $m = 0$ in opposite to the analogous construction in the flat space [42, 43][6].

6 Reduction to the Initial Irreducible Relations

Let us show that the action $\mathcal{S}_n$ (96) reproduces the correct equations (12) on the physical field. As a first step, following the analysis of Ref. [42] we can express the dependence of the fields and gauge parameters on the ghost fields explicitly. For the gauge functions of the $(n-1)$-stage, we have, due to the ghost number and spin distribution, that

$$|\Lambda_0^{(n-1)0}\rangle_n = (p_1^+)^{n-1}\left\{\mathcal{P}_1^+|\omega\rangle_0 + p_1^+|\omega_1\rangle_0\right\}, \tag{105}$$

$$|\Lambda_0^{(n-1)1}\rangle_n \equiv 0, \tag{106}$$

where we recall that the subscripts of the state vectors are associated with the eigenvalues of the corresponding state vectors (97). With the help of these functions, we can eliminate the dependence on the ghost $\mathcal{P}_2^+$ in the gauge functions of the $(n-2)$-stage

$$|\Lambda_0^{(n-2)0}\rangle_n = (p_1^+)^{n-3}\left\{\mathcal{P}_1^+\mathcal{P}_2^+|\omega_2\rangle_0 + p_1^+\mathcal{P}_2^+|\omega_3\rangle_0 + p_1^+\mathcal{P}_1^+|\omega_4\rangle_1 + (p_1^+)^2|\omega_5\rangle_1\right\} \tag{107}$$

adding to $|\Lambda_0^{(n-2)0}\rangle_n$ the term, $\delta|\Lambda_0^{(n-2)0}\rangle_n = \Delta Q|\Lambda_0^{(n-1)0}\rangle_n$ with $(|\omega\rangle_0, |\omega_1\rangle_0) = \frac{1}{2(n-1)}\big(\frac{1}{n-2}|\omega_2\rangle_0,$ $-\frac{1}{n}|\omega_3\rangle_0\big)$, whereas the vector $|\Lambda_0^{(n-2)1}\rangle_n$ has the structure analogous to $|\Lambda_0^{(n-1)0}\rangle_n$. In doing so, we can get rid of any dependence on the ghost $\mathcal{P}_2^+$ in all the fields and the gauge functions, which corresponds to the gauge fixing conditions

$$\eta_2|\Lambda_0^{(i)k}\rangle_n = \eta_2|\chi_0^k\rangle_n = 0, \qquad i = 0, ..., n-1, k = 0, 1, \tag{108}$$

and means that all the fields and gauge parameters are annihilated by the operators $\tilde{q}_1^2 = q_1^2 - r\eta_1\eta_1^+(L_2 - 2l_2')$ and $(T_1 - 2t_1')$.

Secondly, we restrict the whole set of gauge fields only to be equal or to be less than of the first degree in powers of p_1^+, which corresponds to the gauge theory of first stage reducibility. Indeed, the only vectors $|\Lambda_0^{(1)0}\rangle_n, |\Lambda_0^{(0)k}\rangle_n$ survive due to the lowest ghost number value given by $p_1^+, \mathcal{P}_1^+$. This step corresponds to the partial gauge-fixing of the complete theory, as follows:

$$q_1^2|\Lambda_0^{(i)k}\rangle_n = q_1^2|\chi_0^k\rangle_n = 0, \qquad i = 0, ..., n-1, k = 0, 1 \tag{109}$$

and therefore, due to the restriction (108), the constraint $\tilde{q}_1^2$ is simplified to the form: $(L_2 - 2l_2')$.

[6]The construction of the Lagrangian action describing the propagation of all fermionic fields in the (A)dS space simultaneously is analogous to that in the flat space [42], and we do not consider it here. We merely note that the necessary condition for solving this problem is to exchange, in Q', Q, the parameter $-h_n$ by the operator σ.

Thirdly, we can get rid of the dependence on the ghost η_2^+ in all of the remaining physical and gauge fields. For instance, for the gauge functions

$$
\begin{aligned}
&|\Lambda_0^{(1)0}\rangle_n = p_1^+\mathcal{P}_1^+|\xi^{(1)}\rangle_{n-2}, \qquad |\Lambda_0^1\rangle_n = p_1^+\mathcal{P}_1^+|\xi^1\rangle_{n-2},\\
&|\Lambda_0^0\rangle_n = \mathcal{P}_1^+|\xi\rangle_{n-1} + p_1^+|\xi_1\rangle_{n-1} + p_1^+\mathcal{P}_1^+\left\{q_1^+|\xi_2\rangle_{n-3} - \eta_1^+|\xi_3\rangle_{n-3} - \eta_2^+|\xi_4\rangle_{n-4}\right\}, \quad (110)
\end{aligned}
$$

we obtain the required result after adding to $|\Lambda_0^0\rangle_n$ the summand $\delta|\Lambda_0^0\rangle_n = \Delta Q|\Lambda_0^{(1)0}\rangle_n$ given by Eq.(100) with the choice $|\xi^{(1)}\rangle_{n-2} = \left((L_2L_2^+ - 2)^{-1}L_2^+|\xi_4\rangle_{n-4}\right)$, for a nondegenerate operator $(L_2L_2^+ - 2)$. Similar transformation can be made to eliminate η_2^+ in $|\chi_0^k\rangle_n$, so that as a result we impose the gauge fixing

$$
\mathcal{P}_2|\Lambda_0^{(i)k}\rangle_n = \mathcal{P}_2|\chi_0^k\rangle_n = 0, \qquad i,k = 0,1, \qquad (111)
$$

equal to the statement that all the fields and gauge parameters are annihilated by the operator $L_2' = L_2 + iq_1p_1 + \eta_1\mathcal{P}_1$.

Fourth, we get rid of the gauge vector $|\Lambda_0^1\rangle_n$ with the help of $|\Lambda_0^{(1)0}\rangle_n$, by means of the gauge transformation (101) $\delta|\Lambda_0^1\rangle_n = \tilde{T}_0|\Lambda_0^{(1)0}\rangle_n$, where we choose an arbitrary vector $|\xi^{(1)}\rangle_{n-2}$ subject to the conditions

$$
|\xi^{(1)}\rangle_{n-2} = T_0L_0^{-1}|\xi^1\rangle_{n-2}, \ (T_1 - 2t_1', l_2, l_{2,}')\,|\xi^{(1)}\rangle_{n-2} = (0,0,0) \qquad (112)
$$

with the operator $\tilde{L}_0$ being nondegenerate nearly everywhere on a space of gauge vectors, where the last equations provide the conservation of the gauge-fixing conditions (108), (109), (111) for the transformed vectors $|\Lambda_0^k\rangle_n$.

Next we write down the remaining fields as follows:

$$
\begin{aligned}
|\chi_0^0\rangle_n &= |\Psi\rangle_n + \eta_1^+\mathcal{P}_1^+|\Psi_1\rangle_{n-2} + q_1^+p_1^+|\Psi_2\rangle_{n-2} + p_1^+\eta_1^+|\Psi_3\rangle_{n-2} + q_1^+\mathcal{P}_1^+|\Psi_4\rangle_{n-2}\\
&\quad + q_1^+p_1^+\eta_1^+\mathcal{P}_1^+|\Psi_5\rangle_{n-4} + (q_1^+)^2p_1^+\mathcal{P}_1^+|\Psi_6\rangle_{n-4}, \qquad (113)\\
|\chi_0^1\rangle_n &= \mathcal{P}_1^+|\chi\rangle_{n-1} + p_1^+|\chi_1\rangle_{n-1} + p_1^+\mathcal{P}_1^+\left(q_1^+|\chi_2\rangle_{n-3} - \eta_1^+|\chi_3\rangle_{n-3}\right), \qquad (114)
\end{aligned}
$$

with the vectors $|\Psi_{j_0}\rangle_{k_0}, |\chi_{j_1}\rangle_{k_1}$ not depending on the ghost operators as well as on the gauge vectors $|\xi_{j_2}\rangle_{k_2}$ in (110) for $|\xi_4\rangle_{n-4} = 0$.

As a fifth transformation, we eliminate, with the use of the gauge transformations for $|\chi_0^1\rangle_n$ in the form $\delta|\chi_0^1\rangle_n = \tilde{T}_0|\Lambda_0^0\rangle_n$, the vectors $|\chi_2\rangle_{n-3}$, $|\chi_2\rangle_{n-3}$ by means of fixing the parameters $|\xi_2\rangle_{n-3}$, $|\xi_3\rangle_{n-3}$ simultaneously with new conditions on the remain functions $|\xi\rangle_{n-1}$, $|\xi_1\rangle_{n-1}$,

$$
(T_1 - 2t_1', L_2, l_2')\,|\xi\rangle_{n-1} = (T_1 - 2t_1', L_2, l_{2,}')\,|\xi_1\rangle_{n-1} = (0,0,0). \qquad (115)
$$

Then, from the equations of motion (94), (95), it follows that

$$
|\Psi_2\rangle_{n-2} = i(|\Psi_1\rangle_{n-2} + L_2|\Psi\rangle_n), \qquad |\Psi_5\rangle_{n-4} = 0. \qquad (116)
$$

Hence, the only independent vectors $|\Psi_i\rangle_{n-2}$, $i = 1,3,4$, $|\Psi_6\rangle_{n-4}$ and $|\chi\rangle_{n-1}$, $|\chi_1\rangle_{n-1}$ survive in the decompositions (113), (114), so that from the representation

$$
|\Psi_j\rangle_{n-k} = \sum_{(k_1,s,k_2)\geq 0}^{(k_1+s+2k_2\leq n-k)} (b_1^+)^{k_1}(f^+)^s(b_2^+)^{k_2}|\Psi_{j0}^{k_1sk_2}\rangle_{n-k-k_1-s-2k_2}, \quad k\geq 0 \qquad (117)
$$

there follows the possibility of removing completely all the vectors $|\Psi_i\rangle_{n-2}$, $i = 1,3,4$, $|\Psi_6\rangle_{n-4}$ by means of gauge transformations. Indeed, the parameters $|\xi\rangle_{n-1}$, $|\xi_1\rangle_{n-1}$ have four, six and eight independent (on the operators b_1^+, f^+, b_2^+) vectors of spin $n-2$, $n-3$, $n-4$, respectively, which is sufficient to make the above transformation. As a result, we obtain the remaining equations of motion and the gauge transformations for the vectors $|\Psi_i\rangle_n$, $|\chi\rangle_{n-1}$, $|\chi_1\rangle_{n-1}$ in the form

$$T_0|\Psi\rangle_n + iT_1^+|\chi_1\rangle_{n-1} + L_1^+|\chi\rangle_{n-1} = 0, \qquad |\chi\rangle_{n-1} = T_1|\Psi\rangle_n, \tag{118}$$

$$L_1|\chi\rangle_{n-1} = 0, \qquad T_1|\chi\rangle_{n-1} = T_1|\chi_1\rangle_{n-1} = 0, \tag{119}$$

$$L_1|\chi_1\rangle_{n-1} + ir\left[(L_2 - 2l_2')|\Psi\rangle_n - \frac{3}{4}(T_1 - 2t_1')|\chi\rangle_{n-1}\right] = 0, \quad l_2'|\chi\rangle_{n-1} = L_2|\Psi\rangle_n = 0, \tag{120}$$

$$|\chi_1\rangle_{n-1} = i\big(T_0|\chi\rangle_{n-1} - L_1|\Psi\rangle_n\big), \tag{121}$$

$$\delta|\Psi\rangle_n = L_1^+|\xi\rangle_{n-1} + iT_1^+|\xi_1\rangle_{n-1}, \qquad \delta|\chi\rangle_{n-1} = -T_0|\xi\rangle_{n-1} - 2i|\xi_1\rangle_{n-1}, \tag{122}$$

$$\delta|\chi_1\rangle_{n-1} = T_0|\xi_1\rangle_{n-1} - \frac{ir}{2}(G_0 - 2g_0')|\xi\rangle_{n-1}. \tag{123}$$

We note that in order to check the gauge invariance of equations (118)–(121) we must use the property $G_0|\Psi\rangle_{n-k} = k|\Psi\rangle_{n-k}$, the restrictions (115), and new restrictions, $L_1|\xi\rangle_{n-1} = L_1|\xi_1\rangle_{n-1} = 0$. Having expressed the vectors $|\chi\rangle_{n-1}, |\chi_1\rangle_{n-1}$ as the components of $|\Psi\rangle_n$ from equation (121) and from the right-hand side of (118), we can present the equations of motion for $|\Psi\rangle_n$ as follows:

$$\Big[T_0 - T_1^+T_0T_1 + T_1^+L_1 + L_1^+T_1\Big]|\Psi\rangle_n = 0, \qquad L_1T_1|\Psi\rangle_n = 0, \tag{124}$$

$$\Big[L_1T_0T_1 - L_1^2 - 2r\Big(l_2' - \frac{3}{4}t_1'\Big)\Big]|\Psi\rangle_n = 0, \qquad L_2|\Psi\rangle_n = 0. \tag{125}$$

Finally, we use the decomposition (117) for $|\Psi\rangle_n$, $|\xi_1\rangle_{n-1}$, $|\xi\rangle_{n-1}$ in order to extract the only physical vector $|\Psi_0\rangle_n$ by means of gauge transformations. Indeed, there are $k+1$ vectors $|\Psi_0\rangle_{n-k}$, $k = 1,2,3$ in the expansion (117) for $|\Psi\rangle_n$, whereas there are two $|\xi_{10}\rangle_{n-1}$, $|\xi_0\rangle_{n-1}$ and one (remaining after gauge-fixing for $|\Psi_i\rangle_{n-2}$, $i = 1,3,4$) $|\xi_0\rangle_{n-2}$ independent vectors. Choosing for $|\xi_0\rangle_{n-2}$ the vector $|\xi_0^{010}\rangle_{n-2}$, we transform the vector $|\Psi\rangle_n$ to the form without any dependence on b_2^+,

$$|\Psi\rangle_n \to |\Psi\rangle_n' = |\Psi_0\rangle_n + (b_1^+)^2|\Psi_0^{200}\rangle_{n-2} + b_1^+f^+|\Psi_0^{110}\rangle_{n-2}, \tag{126}$$

so that from the relations in the right column of Eqs. (124), (125) the quantities $|\Psi_0^{200}\rangle_{n-2}$, $|\Psi_0^{110}\rangle_{n-2}$ can be expressed as linear combinations of the vector $|\Psi_0\rangle_n$ components

$$|\Psi\rangle_n = |\Psi_0\rangle_n + (b_1^+)^2 g_1\left(l_1t_1|\Psi_0\rangle_n, l_2|\Psi_0\rangle_n\right) + b_1^+f^+g_2\left(l_1t_1|\Psi_0\rangle_n, l_2|\Psi_0\rangle_n\right), \tag{127}$$

with homogeneous linear functions g_i. Thus, the equation in the left-hand side of (124) is equivalent to the set of equations in powers of $(b_1^+)^{k_1}(f^+)^s(b_2^+)^{k_2}$, $k_1, s, k_2 = 0,1$ from which at the highest degrees $(b_1^+)^3$, $f^+b_2^+$, $b_1^+b_2^+$ we obtain the equations $t_1g_i = l_1g_i = 0$, whose solutions are $t_1^3|\Psi_0\rangle_n = 0$ and $l_1|\Psi_0\rangle_n = 0$. Then from the rest equations we may find that the physical vector $|\Psi_0\rangle_n = |\Phi\rangle_n$ obeys the equations (12), or equivalently (103), and therefore the physical field $\Phi_{\mu_1\ldots\mu_n}(x)$, $|\Phi\rangle_n = \Phi_{\mu_1\ldots\mu_n}(x)\, a^{+\mu_1}\ldots a^{+\mu_n}|0\rangle$, satisfies the relations (1), (2).

7 Examples

Here, we examine a realization of the general prescriptions for Lagrangian formulation in the case of fermionic fields on (A)dS of the lowest spins.

7.1 Spin 1/2 field

For a fermionic field of spin $s = \frac{1}{2}$, we have $h = 2 - \frac{d}{2}$. Then the only nonvanishing vector $|\chi^0_0\rangle_0$ which obeys the condition (97) and has the proper ghost number (86) has the form

$$|\chi^0_0\rangle_0 = \psi(x)|0\rangle, \qquad {}_0\langle\tilde{\chi}^0_0| = {}_0\langle 0|\psi^+(x)\tilde{\gamma}^0. \tag{128}$$

Then using (57), (73), (104) for $n_i = s = n'_i = s' = 0$, we obtain that $K_0 = |0\rangle\langle 0|$, and, hence, the Lagrangian following (96) has the form

$$\mathcal{S}_0 \;=\; {}_0\langle\tilde{\chi}^0_0|K_0 T_0|\chi^0_0\rangle_0 = -\int d^dx\sqrt{|g|}\bar{\psi}\Big\{i\gamma^\mu\nabla_\mu - m - r^{\frac{1}{2}}\sqrt{-\omega}\Big(\frac{d}{2}-2\Big)\Big\}\psi. \tag{129}$$

Here, we have used the definition for the "true" gamma-matrices (14), introduced a Dirac-conjugate spinor $\bar{\psi}$, $\bar{\psi} = \psi^+\gamma^0$, and have taken into account that $\omega = -1, 0, 1$, respectively, for the AdS, flat and dS spaces. Thus, we can see that the action (129) reproduces the Lagrangian for a fermionic spin $\frac{1}{2}$ field interacting with a curved (A)dS background that coincides with the corresponding Deser–Waldron Lagrangian [49] up to an overall factor. Note, firstly, that this theory is not a gauge one and, secondly, it is easy to see that the action reproduces the equation (95) for $n = 0$.

7.2 Spin 3/2 field

In the case of spin 3/2 field the corresponding Lagrangian formulation is an irreducible gauge theory for the vectors containing the classical fields $|\chi^0_0\rangle_1, |\chi^1_0\rangle_1$ and gauge vectors containing arbitrary gauge fields $|\Lambda^0_0\rangle_1$ (for $|\Lambda^1_0\rangle_1 \equiv 0$, due to $gh(|\Lambda^1_0\rangle_1) = -2$), with the corresponding Grassmann grading and ghost number distributions,

$$(\varepsilon, gh)\left(|\chi^0_0\rangle_1\right) = (1,0), \quad (\varepsilon, gh)\left(|\chi^1_0\rangle_1\right) = (1,-1), \quad (\varepsilon, gh)\left(|\Lambda^0_0\rangle_1\right) = (0,-1), \tag{130}$$

which controls its dependence on oscillator variables in a unique form:

$$|\chi^0_0\rangle_1 = \left[-ia^{+\mu}\psi_\mu(x) + f^+\tilde{\gamma}\psi(x) + b^+_1\varphi(x)\right]|0\rangle, \qquad |\chi^1_0\rangle_1 = \left[\mathcal{P}^+_1\tilde{\gamma}\chi(x) + p^+_1\chi_1(x)\right]|0\rangle, \tag{131}$$

$${}_1\langle\tilde{\chi}^0_0| = \langle 0|\left[i\psi^+_\mu(x)a^\mu + \psi^+(x)\tilde{\gamma}f + \varphi^+(x)b_1\right]\tilde{\gamma}^0, \quad {}_1\langle\tilde{\chi}^1_0| = \langle 0|\left[\chi^+(x)\tilde{\gamma}\mathcal{P}_1 + \chi^+_1 p_1\right]\tilde{\gamma}^0, \tag{132}$$

$$|\Lambda^0_0\rangle_1 = \left[\mathcal{P}^+_1\xi_1(x) + p^+_1\tilde{\gamma}\xi_2(x)\right]|0\rangle \tag{133}$$

with spin 3/2 fields $\psi_\mu(x), \psi^+_\mu(x)$ and spin 1/2 auxiliary ones $\psi(x), \chi(x), \varphi(x), \chi_1(x)$ and their Hermitian-conjugated fields. Having substituted (131), (132) to the general definition for the action (96) at $n = 1$, we find the Lagrangian (up to an overall factor) for a spin 3/2 field interacting with the (A)dS background

$$\begin{aligned}
\mathcal{L}_1 \;=\;& \bar{\psi}^\mu\Big[(i\gamma^\nu\nabla_\nu - m_{D(1)})\psi_\mu - i\gamma_\mu\chi_1 - \nabla_\mu\chi\Big] - \bar{\varphi}\Big\{\frac{M^2_{(1)}}{m^2_1}\Big[i\gamma^\mu\nabla_\mu \\
& - m_{D(1)}\Big(1 - \frac{r(h-\frac{1}{2})}{M^2_{(1)}}\Big)\Big]\varphi + \frac{m_{D(1)}}{m_1}\Big[i\gamma^\mu\nabla_\mu + m_{D(1)}\Big(1 - \frac{2M^2_{(1)}}{m^2_{D(1)}}\Big)\Big]\psi \\
& - \frac{M^2_{(1)}}{m_1}\chi + \frac{m_{D(1)}}{m_1}\chi_1\Big\} - \bar{\psi}\Big\{2h\Big[i\gamma^\mu\nabla_\mu + m_{D(1)}\Big(1 - \frac{1}{h}\Big)\Big]\psi \\
& + \frac{m_{D(1)}}{m_1}\Big[i\gamma^\mu\nabla_\mu + m_{D(1)}\Big(1 - \frac{2M^2_{(1)}}{m^2_{D(1)}}\Big)\Big]\varphi - m_{D(1)}\chi + 2h_1\chi_1\Big\} \\
& + \bar{\chi}\Big[\Big(i\gamma^\mu\nabla_\mu + m_{D(1)}\Big)\chi + \nabla^\mu\psi_\mu + \frac{M^2_{(1)}}{m_1}\varphi + m_{D(1)}\psi - \chi_1\Big] \\
& + \bar{\chi}_1\Big[i\gamma^\mu\psi_\mu - \frac{m_{D(1)}}{m_1}\varphi - 2h\psi - \chi\Big],
\end{aligned} \tag{134}$$

where we have used the definition for h, $m_{D(n)} = m_0$, for $n = 1$, following from (104), and have also introduced a quantity $M^2_{(1)}$,

$$m_{D(1)} = m + r^{\frac{1}{2}}\sqrt{-\omega}\left(\frac{d}{2} - 1\right), \qquad h = 1 - \frac{d}{2}, \tag{135}$$

$$M^2_{(1)} = m^2_{D(1)} - rh\left(h - \frac{1}{2}\right) = m^2_{D(1)} - \frac{r}{4}(1-d)(2-d), \tag{136}$$

as well as expressions for K_1', K_1,

$$\begin{aligned} K_1' &= K_0 + \frac{M^2_{(1)}}{m_1^2} b_1^+|0\rangle\langle 0|b_1 - \tilde{\gamma}\frac{m_{D(1)}}{m_1} b_1^+|0\rangle\langle 0|f \\ &+ \tilde{\gamma}\frac{m_{D(1)}}{m_1} f^+|0\rangle\langle 0|b_1 - 2hf^+|0\rangle\langle 0|f, \end{aligned} \tag{137}$$

$$K_1 = K_1' - a^{+\mu}K_0 a_\mu + iq_1^+ K_0 p_1 - ip_1^+ K_0 q_1 + \eta_1^+ K_0 \mathcal{P}_1 + \mathcal{P}_1^+ K_0 \eta_1. \tag{138}$$

Substituting (131), (133) into (98), (99), we find the gauge transformations for the algebraically independent fields of the configuration space $\mathcal{M} = \{\psi_\mu, \bar{\psi}_\mu, \psi, \bar{\psi}, \varphi, \bar{\varphi}, \chi, \bar{\chi}, \chi_1, \bar{\chi}_1\}$

$$\begin{aligned} &\delta\psi_\mu = \nabla_\mu \xi_1 + i\gamma_\mu \xi_2, \qquad \delta\psi = \xi_2, \qquad \delta\varphi = m_1 \xi_1, \\ &\delta\chi = \left[i\gamma^\mu\nabla_\mu - m_{D(1)}\right]\xi_1 - 2\xi_2, \qquad \delta\chi_1 = -\left[i\gamma^\mu\nabla_\mu + m_{D(1)}\right]\xi_2 + \frac{r}{2}(d-1)\xi_1. \end{aligned} \tag{139}$$

Let us transform the Lagrangian (134) to the conventional form by rescaling the fields $\varphi, \bar{\varphi}$ as follows:

$$\frac{M_{(1)}}{m_1}(\varphi, \bar{\varphi}) = (\varphi', \bar{\varphi}') \mapsto (\varphi, \bar{\varphi}). \tag{140}$$

As a result, the Lagrangian may be written in the form

$$\begin{aligned} \mathcal{L}_1 &= \bar{\psi}^\mu\Big[(i\gamma^\nu\nabla_\nu - m_{D(1)})\psi_\mu - i\gamma_\mu\chi_1 - \nabla_\mu\chi\Big] - \bar{\varphi}\Bigg\{\Big[i\gamma^\mu\nabla_\mu - \frac{m_{D(1)}}{M^2_{(1)}}\Big(m^2_{D(1)} \\ &- \frac{r}{4}(1-d)(4-d)\Big)\Big]\varphi + \frac{m_{D(1)}}{M_{(1)}}\Big[i\gamma^\mu\nabla_\mu - m_{D(1)}\Big(1 - \frac{r}{2}\frac{(1-d)(2-d)}{m^2_{D(1)}}\Big)\Big]\psi \\ &- M_{(1)}\chi + \frac{m_{D(1)}}{M_{(1)}}\chi_1\Bigg\} - \bar{\psi}\Bigg\{(2-d)\Big[i\gamma^\mu\nabla_\mu - \frac{d}{2-d}m_{D(1)}\Big]\psi \\ &+ \frac{m_{D(1)}}{M_{(1)}}\Big[i\gamma^\mu\nabla_\mu - m_{D(1)}\Big(1 - \frac{r}{2}\frac{(1-d)(2-d)}{m^2_{D(1)}}\Big)\Big]\varphi - m_{D(1)}\chi + (2-d)\chi_1\Bigg\} \\ &+ \bar{\chi}\Big[\Big(i\gamma^\mu\nabla_\mu + m_{D(1)}\Big)\chi + \nabla^\mu\psi_\mu + M_{(1)}\varphi + m_{D(1)}\psi - \chi_1\Big] \\ &+ \bar{\chi}_1\Big[i\gamma^\mu\psi_\mu - \frac{m_{D(1)}}{M_{(1)}}\varphi + (d-2)\psi - \chi\Big], \end{aligned} \tag{141}$$

which is invariant with respect to the gauge transformations (139), where we must only make the change $\delta\varphi = M_{(1)}\xi_1$. There are no dependence on an arbitrary parameter m_1 in the final Lagrangian and gauge transformations. For $r = 0$, we obtain the Lagrangian formulation for a spin 3/2 field on a d-dimensional Minkowski space, which is different from the one given in [43] due to the ambiguity (related to gauge invariance and field redefinitions) in the definition of the additional parts of the constraints. Expressing the auxiliary fields χ, χ_1, ψ, φ with the help of the equations of motion for $\bar{\chi}$, $\bar{\chi}_1$, $\bar{\psi}$, $\bar{\varphi}$ through ψ^{μ} alone, we can obtain a Lagrangian for the physical fields ψ^{μ} alone, which generalizes the Rarita–Schwinger Lagrangian in a d-dimensional Minkowski space [43], and the Deser–Waldron Lagrangian in AdS_4 [49] with the use of formulas (I.13), (I.14) to the case of the $(A)dS_d$ space and reproduces only the equations (1), (2) for $s = 3/2$.

8 Conclusion

In this paper, we have constructed a gauge invariant Lagrangian formulation for half-integer totally-symmetric higher-spin fields in the AdS space of any dimensional in the "metric-like" formulation. The obtained results are applicable under the choice of one of the three values of the integer-valued parameter for both massive and massless fermionic higher-spin fields on the AdS, Minkowski and dS spaces. The formulation is based on the use of a Hamiltonian BRST construction for the realization of a special superalgebra of nonlinear symmetry found in the paper.

Starting from an embedding of the fermionic higher-spin fields into the vectors of an auxiliary Fock space, we treat these fields as the coordinates of Fock space vectors, reformulating the theory in terms of such vectors. We realize the conditions determining the irreducible (from the viewpoint of Dirac-like spinors) representation of the AdS group with a given mass and spin by differential operators acting in this Fock space. The mentioned conditions are interpreted as constraints for the Fock space vectors and generate a closed higher-spin nonlinear symmetry superalgebra with a bosonic central charge being a basic object of the research.

It is shown that the derivation of a correct Lagrangian formulation requires, firstly, a transition to another basis of constraints of the original symmetry algebra, algebraically equivalent to the previous one, secondly, an extension and deformation of the obtained symmetry algebra, thus providing an introduction into the Lagrangian of the Stueckelberg fields (associated with massive-like theory) and a complete set of auxiliary fields of lower spins. We have constructed a realization of additional parts for the constraints of the nonlinear superalgebra in an enlarged Fock space and have derived, on the basis of an additive extension, a deformed superalgebra of nonlinear symmetry. We have found an exact Hamiltonian operator of BRST charge, which encodes the resulting symmetry algebra with supersymmetric (Weyl) ordering of the constraints in its commutator relations and contains a nonvanishing term of third order in powers of the ghost variables, which reflects a nontrivial resolution of the Jacobi identity for the operator superalgebra under consideration. We have obtained, with respect to the above BRST operator by means of a unitary transformation, a BRST charge corresponding to the deformed set of the enlarged original constraints, equivalent to the conditions defining the irreducible representation of the AdS group. It is shown that the given BRST operator determines a consistent Lagrangian dynamics for fermionic fields of any value of spin. The corresponding Lagrangian formulation of the theory in question, is realized in an exact form by constructing the action and a sequence of the reducible gauge transformations in terms of the Fock space for any higher-spin fields propagating in the AdS space of any dimension. The resultant Lagrangian formulation represents, as in the case of the flat space, a gauge theory of finite stage of reducibility being by 3/2 less than the value of spin for the fermionic field.

We have proved, with the help of the gauge ambiguity in the choice of all the fermionic fields underlying the gauge theory, that the total Lagrangian extremals for the basic field of any fixed spin contain only the equations corresponding to the relations determining the respective irreducible representation of the AdS group. As examples, demonstrating the applicability of the general scheme, we have derived gauge-invariant Lagrangian formulations for the components of spin-1/2, spin-3/2 fields in an explicit form.

The basic results of the present work are given by the relations (96), where the Lagrangian action for a field with an arbitrary half-integer spin is constructed, and by (98)–(101), where the gauge transformations for the fields and the sequence of reducible gauge transformations for the gauge parameters are presented.

In connection with the discussed points, the following open problems appear to be of interest:

- Constructing an action for fermionic fields in the $AdS_5 \times S_5$ Ramond–Ramond background by means of incorporating, into the obtained Lagrangian formulation, of an appropriate dependence on the S_5 coordinates, as was mentioned in Ref. [44].
- Derivation of a Lagrangian formulation for fields with a tensor-spinor fields of mixed symmetry, permitting to describe simultaneously the corresponding particles of various spin on the basis of a generalization of the results obtained, for instance, in Ref. [40] in the case of bosonic massless fields with mixed symmetry in the Minkowski space.
- Obtaining, with the use of the results related to the above problem, the interacting vertices for fermionic higher-spin fields in the flat and AdS space, following, in part, the research made in Ref. [41] in the case of cubic interactions of massless bosonic fields.

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9 Definitions

The signature of the metric tensor $g_{\mu\nu}(x)$ on the d-dimensional Riemann space is $(+,-,-,\cdots,-)$. We use the standard definition for the covariant derivative ∇_μ (A, B are the spinor indices, usually omitted) with a vielbein $e^a_\mu(x)$ and a Lorentz connection $\omega_\mu{}^a{}_b$ which acts on a spinor as follows:

$$\nabla_\mu\psi_A = \partial_\mu\psi_A + \frac{1}{4}\,\omega_\mu{}^{ab}(\gamma_{ab})^B_A\psi_B, \qquad \gamma_{ab} = \frac{1}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a), \tag{I.1}$$

$$\nabla_\mu\gamma_\nu\psi = \gamma_\nu\nabla_\mu\psi, \qquad \{\gamma_a,\gamma_b\} = 2\eta_{ab}, \tag{I.2}$$

$$R_{\mu\nu\alpha\beta} = r(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}), \qquad [\nabla_\mu,\nabla_\nu]\psi = \frac{r}{2}\gamma_{\mu\nu}\psi, \tag{I.3}$$

where in the last equation we have written the components of the Riemann tensor in the $(A)dS_d$ space.

The covariant derivative D_μ on the Fock space of vectors $|\Phi\rangle$ (5) is defined by the condition that

$$D_\mu|\Phi\rangle = \sum_{n=0}^{\infty}\nabla_\mu\Phi_{\mu_1\ldots\mu_n}(x)\,a^{+\mu_1}\ldots a^{+\mu_n}|0\rangle, \tag{I.4}$$

so that, explicitly,

$$D_\mu = \partial_\mu - \omega_\mu{}^{ab}a^+_a a_b + \tfrac{1}{4}\,\omega_\mu{}^{ab}\gamma_{ab}. \tag{I.5}$$

The components of the curvature tensors $R^{ab}{}_{\mu\nu}$, $R^{\sigma}{}_{\tau\mu\nu}$ in any Riemann space with the world μ, ν, σ, τ and tangent a, b indices are given by

$$R^{ab}{}_{\mu\nu} = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} + \omega_\mu{}^{ac}\omega_{\nu c}{}^{b} - \omega_\nu{}^{ac}\omega_{\mu c}{}^{b} \tag{I.6}$$

$$R^{\sigma}{}_{\tau\mu\nu} = \partial_\mu \Gamma^\sigma_{\nu\tau} - \partial_\nu \Gamma^\sigma_{\mu\tau} + \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\nu\tau} - \Gamma^\sigma_{\nu\lambda}\Gamma^\lambda_{\mu\tau} \tag{I.7}$$

$$\left[D_\nu, D_\mu\right] = R_{\mu\nu}{}^{ab}\left(a^+_a a_b - \tfrac{1}{4}\gamma_{ab}\right), \tag{I.8}$$

where inorder to obtain the relation (I.8) we have used the identities

$$[\gamma_{ab}, \gamma^c] = 2(\gamma_a \delta^c_b - \gamma_b \delta^c_a), \tag{I.9}$$

$$[\gamma_{ab}, \gamma_{cd}] = 2(\eta_{ad}\gamma_{bc} + \eta_{bc}\gamma_{ad} - \eta_{ac}\gamma_{bd} - \eta_{bd}\gamma_{ac}). \tag{I.10}$$

For any vector $V^\mu(x)$, we have the definitions

$$V^a(x) = e^a_\mu(x)V^\mu(x), \qquad \nabla_\mu V^a = \partial_\mu V^a + \omega_\mu{}^a{}_b V^b, \tag{I.11}$$

$$\nabla_\mu e^a_\nu = \partial_\mu e^a_\nu - \Gamma^\lambda_{\mu\nu} e^a_\lambda + \omega_\mu{}^a{}_b e^b_\nu = 0, \qquad \Gamma^\lambda_{\mu\nu} = (\partial_\mu e^a_\nu)e^\lambda_a + \omega_\mu{}^{ab} e^\lambda_a e_{b\nu}. \tag{I.12}$$

We borrow from [51] for the case of the (A)dS space the fact that

$$R_{\mu\nu\rho\tau} = -\frac{2\Lambda}{(d-1)(d-2)}(g_{\mu\rho}g_{\nu\tau} - g_{\mu\tau}g_{\nu\rho}), \tag{I.13}$$

$$r = -\frac{2\Lambda}{(d-1)(d-2)}, \tag{I.14}$$

where Λ is (negative)positive for the (anti-)de Sitter space, and $r^{-\frac{1}{2}}$ is the (A)dS$_d$ radius.

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