On the Confinement of Gentileons

M. Cattani

Instituto de Física da Universidade de São Paulo.C.P. 66318, CEP 05315−970
São Paulo, SP, Brasil. E-mail < mcattani@if.usp.br>

Abstract

Usually, in undergraduate physics courses we learn that in nature there exist only two kinds of particles: *bosons* and *fermions*. However, it is known that within the framework of the Quantum Mechanics are predicted particles that are different from *bosons* and *fermions.* These particles are, for instance, paraparticles, anyons and gentileons. Due to peculiar symmetry properties of the gentileonic states selection rules have been deduced showing that they must be confined entities and that gentileonic systems are non-coalescent. In this paper, using these symmetry properties and assuming that gentileons are real particles a simple dynamical model is proposed using the color gauge theory and the Dirac equation to explain the physical confinement of the gentileons.

(1)Introduction.

Many papers have been published[1] on statistics that are different from the traditional bosonic and fermionic. *Bosons* are particles with integer spins s = 0,1,2,.. and *fermions* with half−integer spins s = 1/2,3/2, …that obey different statistics: *bosonic* and *fermionic*.[2] In the quantum limit bosons and fermions have a completely different behavior, but, in the classical limit they have identical behavior. In this limit they are named *maxwellons* and obey the Maxwell−Boltzmann Statistics.[2−4] Inside the framework of the Quantum Mechanics are predicted particles that are different from *bosons* and *fermions* that obey, for instance: Intermediate Statistics, Paraestatistics, Fractionary Statistics and Gentileonic Statistics.[1] Since in these studies are taken into account systems composed by "particles" we need at the outset to make clear what this word means. So, use the name "particles" to refer generically to "real particles" (which can be observed freely) and “quasi-particles”("collective excitations "). Particles that can have internal structures like, for example, helium atoms formed by a nucleus and two electrons, hadrons composed of quarks and anyons are called "composite particles". Particles that have no internal structures such as, for example, electrons and photons, will be called “elementary". If in a given system the particles have internal structures but the effects of these structures can be neglected they will be treated as “elementary”. According to large number of theoretical and experimental works,[1] there is a tendency to believe that *real particles* can only be *bosons, fermions* and *maxwellons* and that all other particles would be *quasi−particles*. Up to now days, in 3−dim systems only *bosons*, *fermions* e *maxwellons* have been detected. In 2−dim systems have been detected the *quasi−particles* called *anyons* that have fractionary charges and fractionary spins which obey the Fractionary Statistics.[1]

In Section 1 we present a brief review of the Gentileonic Statistics showing how to obtain the fermionic, bosonic and gentileonic quantum states of systems formed by three identical particles. In Section 2 is proposed a simple dynamical model to explain the physical confinement of gentileons. In this model we use the symmetry properties of the smallest gentileonic system formed by three gentileons, the color gauge theory and the Dirac equation. In Section 3 we present the conclusions and discussions.

(1) Gentilionic Statistics.

We have developed for 3−dim systems, according to the postulates of quantum mechanics and the Principle of Indistinguishability, a *general statistics* that was named Gentileonic Statistics.[1,5−11] According to this statistics three kinds of particles could exist in nature: bosons, fermions and gentileons. Bosons and fermions are represented by horizontal and vertical Young shapes, respectively, and gentileons would be represented by intermediate Young shapes. Bosonic and fermionic systems are described by one−dimensional totally symmetric (Ψ_s) and totally anti−symmetric (Ψ_a) wavefunctions, respectively. Gentileonic systems would be described by wavefunctions (Y) with mixed symmetries. In preceding papers,[1,5-11] based on very peculiar symmetry properties of the intermediate gentileonic states Y, selection rules have been deduced predicting that gentileons are confined entities and gentileonic systems are non−coalescent. The simplest physical explanation for these properties is that gentileons are *quasi−particles* (*collective excitations)*. On the other hand, if they are real particles it would be necessary to develop a confinement mechanism to explain the confinement. In this paper assuming gentileons as real particles a very simple dynamical model will be constructed taking into account symmetry properties of the $Y_+ = Y(123)$ states, the color gauge theory [12−14] and the Dirac equation.[15,16] We will see that at least for three gentileons systems our model gives a fair explanation for the physical confinement of gentileons.

Thus, according to preceding papers[1,5–11] the wavefunction $Y_+ = Y(1,2,3) = Y(uvw)$ that represent three identical weakly interacting gentileons that occupy three different states | u >, | v > and | w > is given by

$$Y_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \qquad (1.1), \text{ where}$$

$$Y_1 = [u(1)v(2)w(3) + u(2)v(1)w(3) - u(2)v(3)w(1) - u(3)v(2)w(1)] /\sqrt{4},$$

$$Y_2 = [u(1)v(2)w(3) + 2u(1)v(3)w(2) - u(2)v(1)w(3) + u(2)v(3)w(1) - 2u(3)v(1)w(2) - u(3)v(2)w(1)] /\sqrt{12},$$

As seen elsewhere[1,5–11] the particles permutations operations P_i on the state Y(1,2,3) can be interpreted as rotations of an equilateral triangle in the Euclidean space E_3. To show this we have assumed that in the E_3 the states u,v and w occupy the vertices of an equilateral triangle taken in the (x,z) plane, as seen in Fig.1. The unit vectors along the x, y and z axes are indicated by **i**, **j** and **k**. In Fig.1 the unit vectors $\mathbf{m}_4$, $\mathbf{m}_5$ and $\mathbf{m}_6$ are given by $\mathbf{m}_4 = -\mathbf{k}$, $\mathbf{m}_5 = -(\sqrt{3}/2)\,\mathbf{i} + (1/2)\,\mathbf{k}$ and $\mathbf{m}_6 = (\sqrt{3}/2)\,\mathbf{i} + (1/2)\,\mathbf{k}$, respectively.

We represent by Y(123) the initial state whose particles 1, 2 and 3 occupy the vertices u, v and w, respectively. The permutations $P_iY = UY$ and $P_iY = VY$ can be represented by unitary operators $U = \exp[i\,\mathbf{j}.\boldsymbol{\sigma}(\theta/2)]$ and $V = i\exp[i\,\mathbf{m}_i.\boldsymbol{\sigma}(\varphi/2)]$; $\theta = \pm 2\pi/3$ are rotations angles around the unit vector **j**, $\varphi = \pm\pi$ are rotations angles around the unit vectors $\mathbf{m}_4$, $\mathbf{m}_5$ and $\mathbf{m}_6$ and $\boldsymbol{\sigma}$ are the Pauli matrices.[15,16]

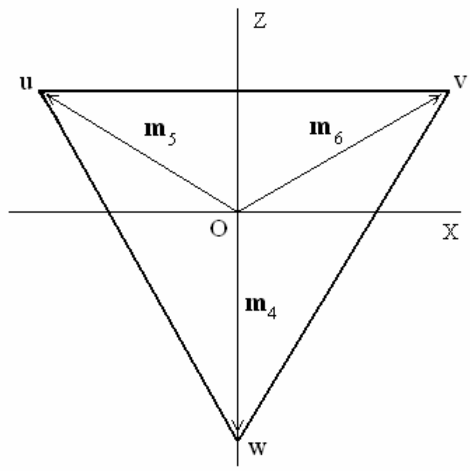

Figure 1.The equilateral triangle in the Euclidean space (x,y,z) with vertices occupied by the states u, v and w.

We have called AS_3 the algebra[1,5–11] of the symmetric group S_3 .This algebra is spanned by 6 vectors, the irreducible matrices $\{D^{(2)}(P_i)\}_{i= 1,2,..,6}$ that are indicated by $\{\eta_i\}_{i= 1,2,..,6}$. It was shown that associated to this algebra there is an algebraic invariant $K_{inv} = \eta_4 + \eta_5 + \eta_6 = (\mathbf{m}_4 + \mathbf{m}_5 + \mathbf{m}_6)\, \boldsymbol{\sigma} = 0$. From this equality results that $K_{inv} = 0$ can be represented geometrically in the (x,z) plane by the vector $\mathbf{M}$ identically equal to zero $\mathbf{M} = \mathbf{m}_4 + \mathbf{m}_5 + \mathbf{m}_6 = 0$. Usually, for continuous groups, we define the *Casimir invariants* which commute with all of the generators (in our case the generators are η_4 and η_6) and are, therefore, invariants under all group transformations. These simultaneously diagonalized invariants are the conserved quantum operators associated with the symmetry group. In our discrete case we use the same idea. So, the operator $K_{inv} = 0$ which corresponds to the genuine gentilionic representation of the AS_3 is identified with a quantum operator which gives a new conserved quantum number related to the S_3.

(2) Dynamical Confinement Model.

Let us consider now a gentileonic system that is formed by three identical gentileons with spin ½. We will assume[1,5–11] that this system is represented by the wavefunction $\Psi = \varphi\, Y(123)$ where φ is symmetric function that obeys Dirac´s equation $H\varphi = E\varphi$ where H is the hamiltonian operator of the stationary state with energy E. It will be also assumed[1,5–11]that the states u, v and w are the three $SU(3)_{color}$ states blue (b), red (r) and green (g) given by $|b\rangle = (\sqrt{3}/2)|+\rangle - (1/2)|-\rangle$, $|r\rangle = (\sqrt{3}/2)|+\rangle + (1/2)|-\rangle$, and $|g\rangle = |-\rangle$, where $|+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$, that are defined in the color plane (I_3,Y). The axes I_3 and Y correspond to color isospin and color hypercharge, respectively In this way the permutations of the particles around the unit vectors $\mathbf{m}_4$, $\mathbf{m}_5$ and $\mathbf{m}_6$ correspond to *discrete* rotations in the $SU(3)_{color}$ plane (I_3,Y) by angles $2\pi/3$.. These rotations can now be interpreted as being produced by color exchange between gentileons. If we were in a quantum field theory context we would say that the color exchange between gentileons is due to *gluons*.[11,12] Following the Yang–Mills gauge theory[12–14] we will write the gauge field $A_\mu(x)$ that acts on the internal color space represented by Y(brg) as

$$A_\mu(x) = \Sigma_k\, [\partial\theta^k(x)/\partial x_\mu]\, F_k \qquad (2.1),$$

where $\theta^k(x)$ are the rotation angles in (I_3,Y) and F_k the generators of the internal symmetry of the $SU(3)_{color}$ group.

In this context[1,5–11] the constant of motion, that is, $K_{inv} = 0$ would be an invariant named *color Casimir.* This invariant has now a beautiful and simple interpretation in the color space: "the color charge is an equal to zero constant of motion". This color charge conservation would be a selection rule for gentileons confinement. In this scheme, color charge conservation and gentileons confinement rules are dictated by the symmetry of the gentileonic state Y(brg). Taking into account these properties we will propose a simple phenomenological dynamical model to explain the gentileon confinement. This model will elaborated within the framework of the Dirac equation assuming that the gentileons are submitted to an external gauge field A_μ (x) given by (2.1). So, putting in a first approximation F_k = constant = C the statefunction $\psi(x)$ of the gentileon will be given by[12,13,15,16]

$$[\gamma^\mu (p_\mu - g\, a_\mu(x)) - im]\, \psi(x)= 0 \qquad (2.2),$$

where $a_\mu(x) = \partial\theta(x)/\partial x_\mu$, g is the coupling constant for the color exchange between gentileons and m the rest mass of the gentileon. It will be also assumed that $a_\mu(x)$ is a vector field, that is, $a_\mu(x) = (0,\mathbf{a}(x))= (0, \mathrm{grad}\theta(x))$. Now we suppose that the gentileon moves freely in the region $r < r_o$, where r_o is the radius of the 3−gentileon system and that it interacts with the field $\mathbf{a}(x)$ only when it reaches the frontier $r = r_o$. In this interaction the gentileon color is abruptly changed. Analyzing this interaction in terms of rotations in the (I_3,Y) plane we see that one color state is effectively transformed into another when a rotation by an angle $2\pi/3$ is accomplished. Thus, we could imagine $\theta(x)$ as a step function that at the point $r = r_o$ varies from zero up to $2\pi/3$. This would imply that

$$\mathbf{a}(r) = \mathrm{grad}\theta(x) = \delta(r - r_o)\, \boldsymbol{r} \qquad (2.3),$$

where $\boldsymbol{r}$ is the unit vector along the radial direction. In these conditions (2.2) becomes

$$[\, i\, \gamma^o + i\, \boldsymbol{\gamma}\cdot\mathrm{grad} - i\, g\, \boldsymbol{\gamma}\cdot\boldsymbol{r}\, \delta(r - r_o) - m]\, \psi(x)= 0 \qquad (2.4).$$

In order to solve (2.4) we use the polar co−ordinates and write[16]

$$\psi(x) = \exp(-iEt)\begin{pmatrix} f(r)\Omega_1 \\ g(r)\Omega_2 \end{pmatrix} \qquad (2.5),$$

where $\Omega_1 = \Omega_{J\,\ell\,m}$ and $\Omega_2 = (-1)^{(1+\ell-\ell')/2}\ \Omega_{J\,\ell' m}$ are the spinor spherical harmonics, $\ell = J \pm 1/2$ and $\ell' = 2J - \ell$.

Taking into account (2.4) and using the property[16]

$$\Omega_{J\,\ell' m} = (-1)^{\ell-\ell'}\ \Omega_{J\,\ell\,m}$$

we get from (2.4):[17]

$$\begin{aligned} &df(r)/dr + (1 + K)f(r)/r - i\, g\, \delta(r - r_o)f(r)/2 - (E-m)g(r) = 0, \\ &dg(r)/dr + (1- K)g(r)/r - i\, g\, \delta(r - r_o)g(r)/2 + (E-m)f(r) = 0 \end{aligned} \qquad (2.6),$$

where $K = -(\ell + 1)$ when $J = \ell + 1/2$ and $K = \ell$ when $J = \ell - 1/2$.

From (2.6) we verify that for $r \approx r_o$ we obtain, considering f(r) and g(r) as exact solutions of (2.6),

$$\begin{aligned} &df(r)/dr \approx i\, g\, \delta(r - r_o)f(r)/2 \\ \text{and} \qquad & \\ &dg(r)/dr \approx i\, g\, \delta(r - r_o)g(r)/2 \end{aligned} \qquad (2.7)$$

At the neighborhood of r_o an obvious solution of (2.7) is that f(r) = g(r). Consequently, at $r = r_o$ we must have

$$f(r_o) = g(r_o) \qquad (2.8),$$

This result, as will be shown below, is a necessary condition to explain the confinement of the gentileons. Indeed, let us calculate the radial flux $J_r(r)$[15,16] of the gentileons through the surface of a sphere with radius r that is given by

$$J_r(r) = \Psi_a (\boldsymbol{r}\,\boldsymbol{\gamma})\Psi = \Psi_a \gamma_r \Psi \qquad (2.9).$$

Taking into account that $\Psi_a = \Psi^+\gamma_4 = \Psi^+\beta$, $\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and that

$\gamma_r = \begin{pmatrix} 0 & -\sigma_r \\ \sigma_r & 0 \end{pmatrix}$ we see that (2.9) is written as

$$J_r = \Psi^+\beta\gamma_r\Psi = -i(f^*\,\Omega_1^* - i\, g^*\,\Omega_2^*) \begin{pmatrix} ig(\sigma_r\Omega_2) \\ f(\sigma_r\Omega_1) \end{pmatrix}$$

$$= [f^*g\,(\sigma_r\Omega_2)\,\Omega_1^* - f\, g^*(\sigma_r\Omega_1)\Omega_2^*] \qquad (2.10),$$

where $\Omega_1 = \Omega_{J\,\ell\,m}$ and $\Omega_2 = \Omega_{J\,\ell' m} = (-1)^{\ell-\ell'}\ \Omega_{J\,\ell\,m} = (-1)^{\ell-\ell'}\ \Omega_1$. Since[16]

$\Omega_{J\,\ell' m} = (-1)^{\ell-\ell'}(\sigma_r\Omega_{J\,\ell\,m})$ we verify that (2.10) becomes

$$J_r = \; = (f^*g\,|\Omega_1|^2 - f\,g^*|\Omega_2|^2) \qquad (2.11).$$

As $\Omega_{J\,\ell' m} = (-1)^{\ell'-\ell}(\boldsymbol{\sigma}\,\mathbf{n})\Omega_{J\,\ell\,m}$ we see that $|\Omega_2|^2 = |(\boldsymbol{\sigma}\,\mathbf{n})|^2\,|\Omega_1|^2$. Since for a generic unit vector $\mathbf{n}$ we have[15,16] $|(\boldsymbol{\sigma}\,\mathbf{n})|^2 = (\boldsymbol{\sigma}\,\mathbf{n})\,(\boldsymbol{\sigma}\,\mathbf{n}) = \mathbf{n}\,\mathbf{n} + i\,\sigma\,(\mathbf{n}\times\mathbf{n})$ we verify that $|(\boldsymbol{\sigma}\,\mathbf{n})|^2 = 1$ which implies that $|\Omega_2|^2 = |(\boldsymbol{\sigma}\,\mathbf{n})|^2|\Omega_1|^2 = |\Omega_1|^2$. This means that the radial flux $J_r(r)$ given by (2.11) becomes

$$J_r(r) = [f^*(r)g(r) - f(r)g^*(r)]\,|\Omega_1|^2 \qquad (2.12).$$

Consequently, if at the boundary $r = r_o$ the condition $f(r_o) = g(r_o)$ is obeyed , according to (2.8), we get $J_r(r_o) = 0$, that is, there is no flux of gentileons through the boundary surface of the system. So, gentileons are confined. This can be interpreted as the manifestation in Lorentz space of the confinement selection rule predicted by the Casimir $K_{inv} = 0$ defined in the color space.

(3)Conclusions and Discussions.

In preceding papers,[1,5-11] based on peculiar symmetry properties of the intermediate gentileonic states Y we have deduced selection rules,[1,5–11] which predict that gentileons are confined entities and gentileonic systems are non−coalescent. The simplest physical explanation (probably the more plausible) for these properties is that gentileons are *quasi−particles* (*collective excitations)*. However, assuming gentileons as real particles we have proposed a simple dynamical model taking into account the symmetry properties of the Y(123) states, the color gauge theory and the Dirac equation. We have shown that (at least for three gentileons systems our model is able to explain the physical confinement of gentileons.

Finally it is interesting to remark that in the "MIT bag model" to get the confinement of quarks in hadrons it is necessary to assume the boundary condition[18]

$$\gamma_r\Psi = i\Psi. \qquad (3.1).$$

Following similar calculations performed in Section 2 and using the relation[16] $\sigma_r\,\Omega_{J\,\ell' m} = i^{(\ell-\ell')}\,\Omega_{J\,\ell m}$ it can verify that from (3.1) we have

$$f(r) = (-1)^{(1+\ell-\ell')}\,g(r),$$

that, using (2.12), also would be able to explain the gentileonic confinement.

REFERÊNCIAS

[1] M.Cattani and J.M.F.Bassalo. arXiv:0903.4773 (2009).
[2] A.Sommerfeld, "Thermodynamics and Statistical Mechanics", Academic Press (1964).
[3] L.D.Landau and E.M.Lifshitz."Statistical Physics"Addison−Wesley(1958).
[4].K. Huang, "Statistical Mechanics", John Wiley & Sons (1963).
[5] M. Cattani and N. C. Fernandes. Rev. Bras. Fís.12,585(1982).
[6] M. Cattani and N. C. Fernandes. Rev. Bras. Fís. 13, 464 (1983)
[7] M. Cattani and N.C. Fernandes. Nuovo.Cimento 79A, 107 (1984).
[8] M. Cattani and N.C. Fernandes. Nuovo.Cimento 87B, 70 (1985).
[9] M. Cattani and N.C. Fernandes. Phys. Lett. 124A, 229 (1987),
[10].M. Cattani and N.C.Fernandes. An. Acad. Bras. de Ciências 59, 283 (1987).
[11] M. Cattani. Acta Phys. Pol. B20, 983 (1989).
[12] R. Mills. Am. J. Phys .57, 493(1989).
[13] K. Moriyasu."An Elementary Primer for Gauge Theory",World Scientific (1983).
[14] J.M.F. Bassalo and M. Cattani. "Teoria de Grupos", Livraria da Física (2008).
[15] A.S.Davydov. "Quantum Mechanics". Addison−Wesley(1965).
[16] V.B.Beresteskii, E.M. Lifshitz and L.P.Pitaevskii. "Relativistic Quantum Theory", Pergamon Press (1971).
[17] M.Villani. Nuovo Cimento A72, 164(1982). *He obtained similar equations in a completely different theoretical context.*
[18] A.Chodos, R.L.Jaffe,K.Johnson,C.B.Thorn and V.F.Weisskopf. Physical Review D9, 3471 (1974).*See equation 5.11 of this paper.*