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Coherence and Decoherence of Schrödinger Quantum States: Basic Aspects

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Abstract. Basic ideas on coherence and decoherence of quantum states within the framework of Schrödinger's equation is presented to graduate and postgraduate students of Physics. With simple calculations and plausible arguments we see that is possible to understand how coherent states of quantum systems become decoherent when they interact with a thermal bath at a constant temperature T . Are also analyzed: connections between decoherence and measurement, density matrix, wavefunction collapse, quantum states of macroscopic bodies and the Schrödinger's cat proposition.
Key words: *Schrödinger equation; coherent and incoherent quantum states.*

(I) Introduction

We present to graduate and postgraduate students of Physics the basic aspects about coherence and decoherence of quantum states^[1,2] within the framework of Schrödinger equation. These aspects of the quantum theory that are subjects of active research since the 1980s are not usually analyzed or mentioned in textbooks adopted in basic Physics course.^[3-6] Detailed and extensive studies can be found, for instance, in references 1 and 2. In Section 1 are defined *coherent* and *incoherent* quantum states within the Schrödinger's equation formalism. In Section 2 we explain using simple quantum calculations how *decoherence* of *coherent quantum states* of molecules occurs, for instance, when they interact with a thermal bath at a constant temperature T . In Section (2.a) are done few comments on decoherence effect, quantum dissipation and *qubit*. Finally, in Section 3 are analyzed relations of the decoherence effect with quantum states of macroscopic bodies, quantum measurement, wavefunction collapse and the Schrödinger's cat proposition. Note that the concept of *coherent states* (microscopic and macroscopic quantum coherence) has been considerably generalized becoming a major topic in applied mathematics, mathematical physics and physics (quantum optics, Bose-Einstein condensates, quantum field theory, string theory, quantum gravity,...).^[7]

(1) Schrödinger Equation, Coherent and Incoherent Quantum States.

(1.a) Coherent State

According to Schrödinger formalism^[3,8-11] an isolated system S composed by N particles is described in the *coordinate representation* by a wave function

$$|\psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)\rangle = |\psi(\mathbf{x}_N)\rangle \quad (1.1),$$

where $\mathbf{x}_i$ ($i=1,2,\dots,N$) $\equiv (\mathbf{x}_N)$ are the particles coordinates. If H is the Hamiltonian operator of the system let us indicate by $|\varphi_n(\mathbf{x}_N)\rangle$ (with $n=1,2,\dots$) the eigenstates of the H, that is, $H|\varphi_n\rangle = \varepsilon_n |\varphi_n\rangle$, where ε_n are the energies of the eigenstates $|\varphi_n\rangle$. Taking into account that the eigenfunctions $\{|\varphi_n\rangle\}$ of the Hermitean operator H form a complete orthonormal set of functions which is a basis in the Hilbert space the function $|\psi(\mathbf{x}_N)\rangle$ can be written as a “*coherent state*” or “*pure state*”,^[10,11]

$$|\psi(\mathbf{x}_N)\rangle = \sum_n a_n |\varphi_n(\mathbf{x}_N)\rangle \quad (1.2),$$

where a_n are arbitrary constants and $|\varphi_n\rangle$ the basis vectors in the Hilbert space which obey the orthonormality condition

$$\langle \varphi_m | \varphi_n \rangle = \int \varphi_m^*(\mathbf{x}_N) \varphi_n(\mathbf{x}_N) d^N \mathbf{x} = \delta_{mn} \quad (1.3),$$

$\mathbf{x}_N = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ and $d^N \mathbf{x} = d^3 \mathbf{x}_1 d^3 \mathbf{x}_2 \dots d^3 \mathbf{x}_N$ is the element of volume in the particles coordinate space. Note that in many papers and text books^[2,11] a more compact Dirac notation is used putting $|\varphi_n\rangle = |n\rangle$ and $a_n = \langle n | \psi \rangle$. As $\langle \psi | \psi \rangle = 1$ we see, taking into account (1.3), that the constants a_n obey the normalization condition

$$\langle \psi | \psi \rangle = (\sum_n a_n^* \langle \varphi_n |) (\sum_m a_m |\varphi_m \rangle) = (\sum_n \sum_m a_n^* a_m \langle \varphi_n | \varphi_m \rangle) = \sum_n |a_n|^2 = 1 \quad (1.4).$$

In this context $|a_n|^2$ is the probability to find a system in the state $|\varphi_n\rangle$ with energy ε_n and, consequently, their values depend how the system is created, that is, depend on the *preparation* of the system.^[10,11] According to the QM postulates all properties of the system in the $\{\mathbf{x}_N\}$ space are completely determined by the wave function $|\psi(\mathbf{x}_N)\rangle$. The statefunction $|\psi(\mathbf{x}_N)\rangle$ is named *pure state* or *coherent state*. The mean value $\langle f \rangle$ of an observable $f(\mathbf{x}_N)$ of the system in the *coherent state* $|\psi(\mathbf{x}_N)\rangle$ is given by, putting $f_{nm} = \langle \varphi_n | f | \varphi_m \rangle$,

$$\langle f \rangle = (\sum_n a_n^* \langle \varphi_n |) f (\sum_m a_m |\varphi_m \rangle) = \sum_n |a_n|^2 f_{nn} + \sum'_{nm} a_n^* a_m f_{nm} \quad (1.5),$$

where $\sum'_{nm}$ is a sum over n and m, with $n \neq m$. This last term of (1.5) suggests “interference” between the different basis elements or quantum alternatives. This is a purely quantum effect and represents the non-additivity of the probabilities of quantum alternatives.

Taking into account the time variable t the wavefunction $|\Psi(\mathbf{x}_N, t)\rangle$ is given by^[9]

$$|\Psi(\mathbf{x}_N, t)\rangle = \sum_n a_n \exp(-i\varepsilon_n t/\hbar) |\varphi_n(\mathbf{x}_N)\rangle \quad (1.6)$$

that evolves with the time according to the equation

$$|\Psi(\mathbf{x}_N, t)\rangle = T(t, t_0) |\Psi(\mathbf{x}_N, 0)\rangle = \exp[-iH(t-t_0)/\hbar] |\Psi(\mathbf{x}_N, 0)\rangle \quad (1.7),$$

where $T(t, t_0) = \exp[-iH(t-t_0)/\hbar]$ is the unitary time evolution operator and $t = t_0$ is the initial time. From (1.7) we see that the normalization $|\langle \Psi(\mathbf{x}_N, t) | \Psi(\mathbf{x}_N, t) \rangle|^2 = 1$ is a constant of motion; it does not depend on the time t . This unitarity time-evolution demands that any state basis of $|\Psi(\mathbf{x}_N, t)\rangle$ must remain orthonormal.

One example of the unitary wavefunction evolution is the *Young diffraction* of particles (photons, electrons, ...). Governed by the Schrödinger equation an incident particle in a coherent state (a plane wave) passes by two orifices originating a *coherent final state* $|\Psi\rangle$ composed by two waves $|\varphi_1\rangle$ and $|\varphi_2\rangle$, that is, $|\Psi\rangle = (|\varphi_1\rangle + |\varphi_2\rangle)/\sqrt{2}$. Similar example is that observed in *Michelson interferometer*. A photon incident upon a semitransparent glass lame that is divided into two equal parts, 50% reflected ($|\varphi_1\rangle$) and 50% transmitted ($|\varphi_2\rangle$), form a coherent state $|\Psi\rangle = (|\varphi_1\rangle + |\varphi_2\rangle)/\sqrt{2}$.

1.b) Incoherent State.

Now, let us see what happens when the system S initially in the state $|\psi_0(\mathbf{x}_N)\rangle$ given by (1.2) interacts with the environment, often modeled as a thermal bath B maintained at a constant temperature T . Let us indicate by $|\chi(\xi)\rangle$ the state function of the bath B , where ξ (a very large number) are the “degrees of freedom” of B . We suppose that the (classic) macroscopic thermal bath is represented by quasi-classic wavefunctions provided by the “correspondence principle”.^[8,11] We also assume that from the initial instant $t = 0$, when S and B begin to interact, the system $S\&B$ is a closed system. Let us indicate by $H = H_S + H_B + V_{int}$ the total Hamiltonian of $S\&B$, where V_{int} is the interaction potential between S and B . From $t \geq 0$ the evolution of system $S\&B$ will be represented by $|\Theta_{SB}(\mathbf{x}_N, \xi, t)\rangle$ which obeys the equation

$$|\Theta_{SB}(\mathbf{x}_N, \xi, t)\rangle = T_{SB}(t, 0) |\Theta_{SB}(\mathbf{x}_N, \xi, 0)\rangle_1 = \exp[-iHt/\hbar] T_{SB}(t, 0) |\Theta_{SB}(\mathbf{x}_N, \xi, 0)\rangle_1 \quad (1.10),$$

where, for $t = 0$,

$$|\Theta_{SB}(\mathbf{x}_N, \xi, 0)\rangle_1 = |\Theta_{oSB}\rangle_1 = |\psi_0(\mathbf{x}_N)\rangle |\chi(\xi)\rangle = \sum_n a_n |\chi(\xi)\rangle |\varphi_n(\mathbf{x}_N)\rangle.$$

Let us assume that the state basis $|\varphi_n(\mathbf{x}_N)\rangle$ of S is not modified by the bath. Due to the entanglement between the S and B the complete function of the $S\&B$ system for $t > 0$ will now be written as^[2,10]

$$|\Theta_{SB}(\mathbf{x}_N, \xi, t)\rangle_1 = |\Theta_{SB}\rangle_1 = \sum_n A_n |\chi_n(\xi)\rangle |\varphi_n(\mathbf{x}_N)\rangle \quad (1.11),$$

where $|\chi_n(\xi)\rangle$ is the basis vector of B due to the entanglement effect. Note that the unitary time-evolution (1.10) demands that the basis vectors remains orthonormal, that is, $\langle \varphi_m(\mathbf{x}_N) | \varphi_n(\mathbf{x}_N) \rangle = \langle \chi_m(\xi) | \chi_n(\xi) \rangle = \delta_{mn}$.

In these conditions the average value $\langle f \rangle$ of the quantity $f(\mathbf{x}_N)$ of the system S will be given by, taking into account that $\langle \chi_m | \chi_n \rangle = \langle \varphi_n | \varphi_m \rangle = \delta_{mn}$:

$$\langle f \rangle = (\sum_n a_n^* \langle \chi_n(\xi) | \langle \varphi_n(\mathbf{x}_N) |) f(\mathbf{x}_N) (\sum_m a_m |\chi_m(\xi)\rangle |\varphi_m(\mathbf{x}_N)\rangle) = \sum_n |A_n|^2 f_{nn} \quad (1.12),$$

where $f_{nn} = \langle \varphi_n(\mathbf{x}_N) | f(\mathbf{x}_N) | \varphi_n(\mathbf{x}_N) \rangle$.

Comparing (1.5) with (1.12) we verify that the interference terms $\sum'_{nm} a_n^* a_m f_{nm}$ present in (1.5) is not present in (1.12). Only the diagonal terms n contribute to $\langle f \rangle$. It shows that due to the interaction with the thermal bath the system S is not represented by the *coherent state* $|\psi\rangle$ given by (1.2). Thus, we say that the environment created a *decoherence effect*. The bath interaction has irreversibly converted the quantum

behavior characterized by the *additive probability amplitudes* to classical behavior with *additive probabilities*. Decoherence can be modelled as a non-unitary process by which the system S couples with the environment (although the combined system S&B evolves in a unitary fashion).^[1,2] Thus, the dynamics of S, treated in isolation, are non-unitary and, as such, are represented by irreversible transformations acting on the S Hilbert space. In the Appendix we show how S can be described using the *density matrix* approach.

(2) Decoherence Produced by Collisions in a Thermal Bath.

Using a non-rigorous approach we show that it is possible to explain the *decoherence effect* of *coherent quantum states* of systems (molecules, for instance) when they are immersed in a thermal bath (macroscopic system) maintained at a constant temperature T. Let us consider the bath as a gas with $N_o \sim 10^{23}$ rigid particles, each one with mass m, obeying the classical Maxwell-Boltzmann distribution. So, the number of particles $N(v)dv$ with velocities between v and v + dv is given by^[12] $N(v)dv = 4\pi N_o (m/k_B T)^{3/2} v^2 \exp(-mv^2/k_B T) dv$, where k_B is the Boltzmann constant. According to the kinetic theory of gases^[12,13] the gas particles are in continuous motion colliding between them. The mean free path between two collisions is given by $\lambda \approx 1/n\pi d^2$, where n is the number of particles per unit of volume and is d is particle diameter. The estimated mean free time t^* between two successive collisions is $t^* \approx \lambda/\langle v \rangle$, where $\langle v \rangle$ is the mean velocity of the particles. For the air, for instance, at T = 273 K (0° C) and pressure P = 1 atm we have^[12] $\langle v \rangle \approx 10^5$ cm/s, $\lambda \approx 2 \cdot 10^{-5}$ cm and $t^* \approx 2 \cdot 10^{-10}$ s. We see that each particle has in average 5 billion of collisions by second. The effective collision time T_c between two particles is determined by^[11] $T_c = D/v$, where D is the closest approach distance. Putting $D \sim 10^{-8}$ cm and $\langle v \rangle = 10^5$ cm/s results $T_c \sim 10^{-13}$ s.

Now, let us consider that a very large number N of molecules is immersed in the bath gas of rigid particles at a constant temperature T. To simplify let us take $N_o \gg N$ so that we can neglect molecule-molecule collisions: only collisions between rigid particles and molecules will be relevant.

Assuming that the immersing molecules are initially in a coherent state $|\psi(x_N)\rangle$, given by (1.2), they will be submitted to an enormously large number of collisions with the rigid particles. In order to estimate the effect of the time dependent interaction potential V(t) between the molecules and the rigid particles the molecular state function $|\Psi(x_N, t)\rangle$ will be written as^[9]

$$|\Psi(x_N, t)\rangle = \sum_n a_n(t) \exp(-i\varepsilon_n t/\hbar) |\varphi_n(x_N)\rangle \quad (2.1).$$

Let us show that the collisions transform the molecular *coherent state* $|\Psi(x_N, t)\rangle$ given by (2.1) in an *incoherent sum* of states $|\varphi_n(x_N)\rangle = |n\rangle$. To do this we analyze the transitions produced by collisions between the initial coherent state $|I\rangle = |\Psi(x_N, t)\rangle$ to generic states $|m\rangle = |\varphi_m(x_N)\rangle$. According to the time-dependent perturbation theory^[8] the transition probability W_{Im} between these states, estimated in the first order Born approximation, is given by

$$W_{Im} \approx (2\pi/\hbar) \{ | \langle \mathbf{q} | \langle m | V(t) | I \rangle |^2 \rho(E) \}_{Av} \quad (2.2),$$

where $|\mathbf{q}\rangle$ and $|\mathbf{q}'\rangle$ are, respectively, the initial and final momentum of the colliding rigid particle, $\rho(E)$ is the density of the final states of these particles with energy E given by $E = \varepsilon_I + \mathbf{q}^2/2m = \varepsilon_m + \mathbf{q}'^2/2m$ and the bracket $\{\dots\}_{Av}$ means an average over all

velocities (according to the Maxwell-Boltzmann distribution) of the colliding particles. Since the initial state $|I\rangle$ is given by a sum over the states $|n\rangle$ we will estimate the transition probabilities W_{nm} between generic states $|n\rangle \rightarrow |m\rangle$. Analyzing these probabilities one can evaluate (2.2). Assuming that the trajectory of the colliding rigid particle is a straight line,^[11] that is, putting $\mathbf{q} \approx \mathbf{q}'$ one can easily see that W_{nm} is proportional to the square modulus of the matrix element

$$\langle \mathbf{q} | \langle m | V(t) | n \rangle | \mathbf{q}' \rangle = \int \langle m | V(t) | n \rangle \exp(i\omega_{mn}t) dt \quad (2.3),$$

where $\omega_{mn} = (\varepsilon_m - \varepsilon_n)/\hbar$ and the time integration is extended from $-\infty$ to ∞ . From (2.3) we see that if the effective collision time $T_c = D/v$ is appreciably larger than the period ω_{mn}^{-1} , that is, when

$$\omega_{mn}(D/v) \gg 1 \quad (2.4),$$

the function $\exp(i\omega_{mn}t)$ in (2.3) oscillates many times during the effective collision time and the integral is practically equal to zero, that is, $W_{nm} \approx 0$. That is, these collisions are not able to induce transition between the quantum states. These collisions are named *adiabatic*.

On the hand, if during the effective collision time is satisfied the inequality

$$\omega_{mn}(D/v) \leq 1 \quad (2.5),$$

we have $\exp(i\omega_{mn}t) \sim 1$ and (2.3) can be different from zero, resulting $W_{nm} \neq 0$. Since v can assume large values it is possible to have $W_{nm} \neq 0$. In other words, these collisions would be able to induce transitions between the molecular quantum states. Due to the very high frequency of collisions there will be millions of transitions per second between the molecular quantum states.

In these inelastic $n \rightarrow m$ collisions there is an energy exchange between the $\sim 10^{23}$ translational degrees of freedom of the particles and the internal and translational degrees of freedom of the molecules. The molecular gas once immersed in the thermal bath after a very short time interval $\tau \sim 10^{-9}$ s (that is, after a few consecutive molecule-atoms collisions) will be in a state of thermal equilibrium^[13] at the temperature T . That is, the occupation numbers of the translational and *internal energy levels* of the atoms and molecules will be given by the equilibrium values predicted by the *statistical distribution function*.^[13,14] Note that the system S&B reach a stationary state, from the very definition of *thermal equilibrium* or *statistical equilibrium*.^[14] Thus, in a very short time $\tau \sim 10^{-9}$ s the probability $|a_n(\tau)|^2$ to find a molecule in the state $|n\rangle$ with energy ε_n evolves to a constant value ρ_n given by the *statistical distribution* (see Appendix).^[14]

$$\rho_n = \langle |a_n(\tau)|^2 \rangle = (1/Z) \exp(-\varepsilon_n/kT) \quad (2.5),$$

where Z is the partition function. So, for times $t > \tau$ we have $a_n = \sqrt{\rho_n}$ for a state $|\varphi_n\rangle$.

So, using (2.1), the mean value $\langle f \rangle$ of a molecular quantity $f(\mathbf{x}_N)$ is given by,

$$\begin{aligned} \langle f \rangle &= \{ \langle \Psi(\mathbf{x}_N, t) | f | \Psi(\mathbf{x}_N, t) \rangle \}_{Av} \\ &= \{ (\sum_n a_n^* \exp(i\varepsilon_n t/\hbar) \langle \varphi_n |) f (\sum_m a_m \exp(-i\varepsilon_m t/\hbar) | \varphi_m \rangle) \}_{Av} \\ &= \sum_n \rho_n f_{nn} + \{ \sum'_{nm} \sqrt{\rho_n} \sqrt{\rho_m} f_{nm} \exp[i \omega_{nm} t] \}_{Av} \end{aligned} \quad (2.6),$$

where the bracket $\{\dots\}_{Av}$ indicates a time average, the sum Σ'_{nm} is over n and m with $n \neq m$ and $\omega_{nm} = (\epsilon_n - \epsilon_m)/\hbar$. As the measurement time t is usually very large compared with the molecular periods ω_{nm}^{-1} the average value of the sum Σ'_{nm} is equal to zero. Consequently, the mean value of a quantity f of a molecule immersed in a thermal bath becomes given by....

$$\langle f \rangle = \sum_n \rho_n f_{nn} \quad (2.7).$$

Note that the mean values (2.7) and (1.5) are completely different. In (2.7) we have only the diagonal matrix elements f_{nn} and instead of $|a_n|^2$ we have the *statistical weight* ρ_n . So, due to the environment interactions the molecular state function is no more described by a coherent state (1.2) or (1.6). It is represented by an *incoherent mixture* of the states $|\varphi_n\rangle$ with a statistical weight ρ_n satisfying the relation $\sum_n \rho_n = 1$. We say that the system is in an *incoherent state*. This expresses the fact that to obtain the mean value $\langle f \rangle$ we must know the statistical probability ρ_n to find the molecule in the pure states $|\varphi_n\rangle$ and use (2.7).

(2.a) Conclusions and Addendum.

We saw that coherent quantum states of isolated molecules are destroyed when they interact with the environment and that it occurs in a very short time scale $\sim 10^{-9}$ s. In other words, the environment generate *decoherence* of *coherent* quantum states. *Quantum decoherence* ^[1,2] is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition defined by (1.2) or (1.6). The components of the coherent (or “pure”) statefunction are decoupled from the coherent system and acquire phases from their immediate surroundings. A total superposition of the global wavefunction still exists (and remains coherent at the global level). These aspects were seen in Section 1 analyzing the entanglement between the quantum states of the bath and of the molecules. The bath interaction converts irreversibly the *quantum behavior* characterized by the additive probability amplitudes to *classical behavior* with additive probabilities.

With quite similar arguments one can explain quantum decoherence effects when any kind of quantum systems initially in coherent states interact with a thermal bath (“environment”) or interact between them forming a *macroscopic aggregate* maintained at a constant temperature T .

Quantum dissipation is the branch of physics that studies the quantum analogue of the process of irreversible loss of energy observed at the classical level. Its main purpose is to derive the laws of classical dissipation from the framework of QM. It shares many features with the subjects of quantum decoherence and quantum theory of measurement.^[15] Dissipation is a decoherent process by which the populations of quantum states are changed due to entanglement with a bath.^[15]

Quantum bit or qubit. *Bit* is the classical basic unit of information always understood to be a 0 or a 1. *Quantum bit* or *qubit* is the unit of quantum information^[16] represented by the coherent state $|\Psi\rangle = \alpha |0\rangle + \beta |1\rangle$, where α and β are probability amplitudes, in general, both complex numbers that are constrained by the equation $|\alpha|^2 + |\beta|^2 = 1$. In the Bloch sphere^[16] representation $\alpha = \cos(\theta/2)$, $\beta = e^{i\phi/2} \sin(\theta/2)$ (north pole $z = |0\rangle$ and south pole $z = |1\rangle$).

(3) Macroscopic Bodies Decoherence.

One of the most worrying difficult in the interpretation of QM is the problem of interferences between quantum states of macroscopic bodies which is predicted by linear theories and practically never observed. Let us see how this problem can be understood taking into account the decoherence quantum effect.^[1]

To describe an *isolated microscopic* system with Schrödinger's equation one uses a somewhat *small set* of *known* variables and parameters (positions of the particles, masses, spins,...) and well *known* interaction potentials between the particles. In these conditions solving Schrödinger equation the system wavefunction is given by a *coherent* superposition of states (1.2). Assuming that there are only two possible states,

$$|\psi(x)\rangle = a_1 |\phi_1(x)\rangle + a_2 |\phi_2(x)\rangle \quad (3.1),$$

where x indicates all *known* variables and parameters necessary to describe the system. So, putting $f_{ij} = \langle \phi_i(x) | f(x) | \phi_j(x) \rangle$ ($i, j=1,2$) the mean value of the observable $f(x)$ is given by

$$\langle f \rangle = \langle f(x) \rangle = |a_1|^2 f_{11} + |a_2|^2 f_{22} + a_1^* a_2 f_{12} + a_2^* a_1 f_{21} \quad (3.2),$$

showing interference between the states 1 and 2.

Now, let us suppose that we have a *macroscopic body* composed by a small system (a microscopic system) that depends on the small fraction x of *known* variables and by a large part (macroscopic) that depends on billions and billions of *unknown* degrees of freedom or variables Y . The Hamiltonian of this macroscopic body is $\mathbf{H}(x, Y)$; $|\phi_1(x, Y)\rangle$ and $|\phi_2(x, Y)\rangle$ two different eigenstates of $\mathbf{H}$, that is, $\mathbf{H} |\phi_n(x, Y)\rangle = E_n |\phi_n(x, Y)\rangle$, with $n=1,2$. Let us assume that this system could be represented by a coherent superposition of the states 1 and 2:

$$|\psi(x, Y)\rangle = A_1 |\phi_1(x, Y)\rangle + A_2 |\phi_2(x, Y)\rangle \quad (3.3).$$

One may expect that the two final functions $|\phi_1(x, Y)\rangle$ and $|\phi_2(x, Y)\rangle$ are very different in their fine Y dependence. This means particularly that for each value of Y they have very different phases, which vary strongly in each wave function with no direct relation to each other.^[1,17] If only the x -variables are observed the mean value $\langle f \rangle = \langle f(x) \rangle$ is given by

$$\langle f \rangle = \langle \psi(x, Y) | f(x) | \psi(x, Y) \rangle = \int dx f(x) \{ \sum_{ij} A_i^* A_j \int dY \phi_i^*(x, Y) \phi_j(x, Y) \} \quad (3.4).$$

Due to very different phases of the functions $|\phi_1(x, Y)\rangle$ and $|\phi_2(x, Y)\rangle$ we would have

$$\int dY \phi_i^*(x, Y) \phi_j(x, Y) = 0 \quad (3.5),$$

for any x values, showing that there is *suppression* of the macroscopic interferences. From this we can conclude that the coherent superposition of different quantum states of a macroscopic body is meaningless. The quantum states of macroscopic body are **incoherent states**; there is no quantum interference between them.

As is practically impossible to know the billions of degrees of freedom Y , decoherence can be viewed as the loss of information from the system.^[2] Decoherence can also be understood as the absence of macroscopic interferences. This helps us to

understand why the interaction with the environment is responsible for the quantum decoherence of a microscopic system.

(3.a) *Measurement, wavefunction collapse and decoherence.*

Absent measurement the state function of an isolated system evolves in a leisurely and deterministic way, according to the Schrödinger equation (see Section 1). In this way quantum mechanics looks like a rather ordinary field theory much simpler than classical electrodynamics, for example, since there is only one field (ψ), instead of two ($\mathbf{E}$ and $\mathbf{B}$).^[5] According to the Copenhagen Quantum Mechanics postulates in the *measurement process*, when a **microscopic system** interacts with an instrument (an observer) which is a *macroscopic system*, there is a *wavefunction collapse*. That is, the system wavefunction initially in a superposition of different states given by (1.2) appears to reduce to a *single one* of the states after interaction with an observer. However, it was shown in Section 2 that the interaction of a macroscopic system with microscopic one which was initially in a *coherent superposition* of states $|n\rangle$ transforms this pure state in an *incoherent mixture* of states $|n\rangle$. The weighting of each outcome in the mixture in case of measurement is exactly that which gives the probabilities of the different results of such a measurement. The measurement is a bizarre process that generates the conceptual difficulties like indeterminacy, nonlocality and the collapse of the wavefunction.^[1,2]

It is important to remark that decoherence by itself does not give a complete solution of the measurement problem since all components of the wavefunction still exist in a global superposition.^[1,2] That is, decoherence does not generate actual wave function collapse.^[1,2] It gives only an appearance of wavefunction collapse. Decoherence does not claim to provide a mechanism for the actual wavefunction collapse; rather it puts forth a reasonable mechanism for the appearance of wavefunction collapse.^[1,2]

Note that when we submit a *macroscopic system* to a measurement process it interacts with a *macroscopic instrument*. In this act there is no decoherence of the macroscopic system state since *ab initio* it was in a *decoherent state*. As seen before, macroscopic bodies are represented by an *incoherent state*: when this system is measured there is no wavefunction collapse. Wavefunction collapse happens only for systems described by coherent states.

(3.b) *Schrödinger's Cat.*

Schrödinger's cat is a thought experiment,^[18] proposed by the Austrian physicist Erwin Schrödinger in 1935, sometimes described as a *paradox*. It illustrates what he saw as the problem of the Copenhagen interpretation of QM applied to macroscopic objects. This proposition is presented and extensively analyzed in many books^[5] and articles.^[18] Shortly, “a cat, a flask of poison, and a radiative source are placed in a sealed box. If an internal monitor (Geiger detector) detects radioactivity (i.e. a single atom decaying) the flask is shattered, releasing the poison that kills the cat.”

Considering the decoherence effect let us help the reader to understand the essential aspects of this proposition answering the following questions:

(1) Is it possible before the measurement to describe the cat state $|\Psi\rangle$ inside the box with a *coherent wavefunction*

$$|\Psi\rangle = (|\Psi_{\text{alive}}\rangle + |\Psi_{\text{dead}}\rangle)/\sqrt{2} \quad (3.1),$$

where $|\Psi_{\text{alive}}\rangle$ and $|\Psi_{\text{dead}}\rangle$ describe the cat alive and dead, respectively ?

(2) The cat is neither alive nor dead, but a linear combination of both things till the observation is done, say till you observe the situation by a glass window. In this moment the observation forces the cat to “assume a position”: alive or dead. If you see that it is dead, was your observation that killed the cat?

To answer the first question let us consider Section 3: since the cat is a macroscopic system it cannot be described by a coherent state $|\Psi\rangle$ given by (3.1). That is, the cat is alive *or* dead, not both alive *and* dead. The coherent superposition of macroscopic objects in two completely different states is an absurd.

Let us consider the second question. According to the QM postulates the measurement process occurs when a microscopic system interacts with a macroscopic system. In this event there is a collapse of the microscopic wavefunction (see Section 3.a). In this context the most acceptable answer for the second question is that the Geiger counter discharge constitutes the real measurement process and not the human observation.

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APPENDIX. Density matrix approach.

Let us present two different density matrix approaches for: (1) *isolated systems* and (2) *systems immersed in a thermal bath (B)*.

(1) *Isolated Systems.*

Assuming that the composed system S&B defined in Section 1 is an isolated system let us calculate, with a *density matrix approach*,^[10,11] the mean value of a physical quantity $g(x_N)$ of the subsystem S. So, taking into account that the system S&B is represented by the *incoherent state* (see Section 1):

$$|\Theta_{SB}(x_N, \xi)\rangle = \sum_s A_s |\phi_s(\xi)\rangle |\varphi_s(x_N)\rangle \quad (A.1),$$

where $\{\varphi_s(x_N)\}$ form complete orthonormal set of eigenfunctions of some operator $S(x_N)$ acting upon the coordinates of the subsystem S, $\{\phi_s(\xi)\}$ is the set of functions which give a complete description of the bath^[10,11] and the constants A_s obey the condition $\sum_s |A_s|^2 = 1$. Thus, the mean value $\langle g(x_N) \rangle = \langle g \rangle$ is given by

$$\langle g \rangle = \langle \Theta_{SB}(x_N, \xi) | g(x_N) | \Theta_{SB}(x_N, \xi) \rangle = \iint \Theta_{SB}^*(x_N, \xi) g(x_N) \Theta_{SB}(x_N, \xi) dx_N d\xi \quad (A.2).$$

Substituting (A.1) into (A.2) we have,

$$\langle g \rangle = \sum_{s's'} \rho_{ss'} g_{s's} \quad (A.3),$$

where

$$g_{s's} = \int \varphi_{s'}^*(x_N) g(x_N) \varphi_s(x_N) dx_N \quad \text{and} \quad \rho_{s's} = \int \phi_s^*(\xi) \phi_{s'}(\xi) d\xi$$

are the matrix elements of the *density matrix* ρ . Since $\langle g \rangle$ is real the matrix $\rho_{s's}$ is Hermitean, that is, $\rho_{s's} = \rho_{s's}^*$. Using the matrix operator ρ and the operator g the mean value of g will be given by the trace of matrix operator ρg :

$$\langle g \rangle = \sum_s (\rho g)_{ss} = \text{Tr}(\rho g) \quad (A.4).$$

This form (A.4) has the advantage that allows us to calculate $\langle g \rangle$ with help of any arbitrary complete, orthogonal, normalized set of wave functions: the trace of an operator is independent of the choice of the system of functions in terms of which the matrix elements are defined.^[10,14]

(2) *Systems Immersed in a Thermal Bath.*

Now, the density matrix will be calculate following the Statistical Mechanics formalism assuming that the S&B is a macroscopic system immersed in a very large thermal bath at a constant temperature T . The system S&B that is now a subsystem of the very large bath will be taken as a “closed system” (rigorously, “quasi-closed”) that is represented by a state function^[14]

$$|\Theta_{SB}(\mathbf{x}_N, \xi, t)\rangle = \sum_j c_j \exp(-iE_j t/\hbar) |\psi_j(\mathbf{x}_N, \xi)\rangle \quad (\text{A.5}),$$

where $|\psi_j(\mathbf{x}_N, \xi)\rangle$ are the basis vectors of the system S&B, $H|\psi_j(\mathbf{x}_N, \xi)\rangle = E_j|\psi_j(\mathbf{x}_N, \xi)\rangle$ and H is the total Hamiltonian of S&B. With this representation the mean value of *any* quantity $f(\mathbf{x}_N, \xi)$ of S&B is written as

$$\langle f(t) \rangle = \sum_{ij} \rho_{ij}(t) f_{ji} \quad (\text{A.6}),$$

where $\rho_{ij}(t) = c_i^* c_j \exp[-i(E_i - E_j)t/\hbar]$ and $f_{ji} = \langle \psi_j(\mathbf{x}_N, \xi) | f(\mathbf{x}_N, \xi) | \psi_i(\mathbf{x}_N, \xi) \rangle$.

In this way,

$$\partial \rho_{ij}(t) / \partial t = (i/\hbar) (E_i - E_j) \rho_{ij}(t) \quad (\text{A.7}).$$

Equations (A.7) may be rewritten in the general form^[10,14]

$$(E_i - E_j) \rho_{ij}(t) = \sum_m [\rho_{im}(t) H_{mj} - H_{im} \rho_{mj}(t)] \quad (\text{A.8}).$$

Hence we have for the operators ρ and H :^[10]

$$\partial \rho / \partial t = (i/\hbar) (\rho H - H \rho) \quad (\text{A.9}).$$

From (A.9) we verify that ρ will be a constant of motion, that is, $\partial \rho / \partial t = 0$ if the operators ρ and H commute, that is, when $[\rho, H] = 0$.^[10,14] The system would be in a stationary state. In these conditions, supposing that the bath is at a constant temperature T it implies that the system is in *thermal equilibrium* or *statistical equilibrium*.^[14] From (A.7) results that $\partial \rho_{ij}(t) / \partial t = 0$, that is, the matrix ρ_{ij} is diagonal: $\rho_{ij} = \rho_{ij} \delta_{ij}$. So, (A.6) becomes

$$\langle f \rangle = \sum_i \rho_i f_{ii} \quad (\text{A.10}).$$

Eq.(A.9) is satisfied, for instance, if $\rho = \rho(H)$: the function ρ being a function of an integral of motion H is itself an integral of motion. According to the Statistical Mechanics^{10,14}

$$\rho(H) = (1/Z) \exp(-H/kT) \quad (\text{A.11}),$$

where Z is the “partition function”. So, the function $\rho_i = \rho_{ii} = \rho(E_i)$ gives the probability to find a state with energy E_i . In quantum statistics this function, called “*distribution function*”, is given by^[14]

$$\rho(E_i) = (1/Z) \exp(-E_i/kT) \quad (\text{A.12}).$$

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