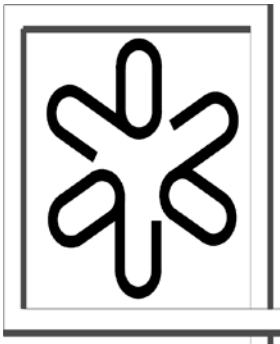

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Comments on Lorentz Transformation and Minkowski Space-Time Geometry

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Comments on Lorentz Transformation and Minkowski Space-Time Geometry

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Abstract. Graduate students of Special Relativity (**SR**) are lead generally to believe that Lorentz Transformation (**LT**) and the Space-Time Minkowski Geometry (**MG**) are responsible by the amazing effects predicted by **SR**. Presenting reasonable arguments we try to show that "geometry does not act on matter". Thus, in our understanding **LT** and **MG** constitute a geometrical approach or a useful mathematical tool that only displays the predicted effects. Are also pointed out mathematical details of the **LT** that could suggest create "nonlocal" or "action at distance" effects.
Key words: *special relativity; geometry; nonlocal effects.*

(I) Introduction.

As well known, all amazing Special Relativity (**SR**) predictions [1-4] are experimentally confirmed.[5] Myself and many students were induced to believe that Lorentz Transformation (**LT**) and the Minkowski Space-Time Geometry (**MG**) are responsible for these effects. However, we do not think that geometry "acts" on matter. Geometric approach would be understood as useful mathematical tool that only describes the measured effects. We are aware that we are venturing into controversial areas where it is difficult to proceed entirely free from bias. As this article is written to graduate students of Physics we avoid a very rigorous mathematical approach, philosophical discussions and subtle problems inherent to geometries, geometry and physical reality, time and reality and non-reality in Physics. Broad and thorough discussion on these topics can be found elsewhere.[6-8] In spite of this our mathematical approach will be as rigorous as possible. In Section 1 we consider the space-time and physics before the 20.th century. In Section 2 is seen the constancy of the light speed, the Minkowski space-time and the new theory named **SR** governed by the **Special Relative Principle (SRP)** . In Section 3 is shown how time

and length are measured according to the International System of Units. In Section 4 are mentioned many experimental works that have confirmed the theoretical predictions of the **SR**. In Classical Physics it is supposed that the only way to produce a measurable effect is by means of transmission of energy and moment densities through a *physical field*. These *fields* are *potentials* (gravitational or electromagnetic) or propagating mechanical or electromagnetic waves. Here "entities" that produce measurable effects will be simply called **physical actions.** Probably there is a primordial physical action responsible for the constancy of the light speed (**CC**) but we do not know how it is. As **CC** is a direct consequence of this **physical action** it will be taken as responsible for the **SR** predictions. So, we have following causal sequence **CC → LT → MG → SRP → measured effects.** In this way, **LT** and **MG** would be only a geometric approach or useful tool which displays mathematically the **SR** effects of a **physical action** responsible for **CC**. In Section 5 are seen comments on physical actions and conjectured the existence of nonlocal effects in **LT**. In Section 6 are presented Conclusions and Discussions.

(1) Space-Time and Physics before the 20.th Century.

Long time ago, men were constrained to move on the Earth surface. As men dislocations on the Earth were usually very small compared with the earth radius R they believe that the Earth was flat. The spatial properties, for instance, distances, areas and volume were described, for instance, with the 3-dim Euclidean geometry. Much time later since Earth is spherical they saw that it was necessary to use a curved Riemannian geometry to describe geometrical features of its surface. However, for small regions where Earth surface curvatures can be neglected a 3-dim flat Euclidean geometry can be used. The pioneer works on Physics have been performed in these "local regions" were Euclidean geometry are valid. In this way, the physical world laws have been developed using a 3-dim Euclidean geometry with coordinates (x,y,z). Besides coordinates was introduced a new independent variable, the time t. In this (3 +1) "arena" [(x,y,z) &t] that is, in this Euclidean geometry with an "outsider" parameter time t physical laws were written. More or less, at the middle of the 16.th century and beginning of 17.th centuries, due mainly to Galileo and Newton, was formulated a theory known as Newtonian Mechanics. The Italian physicist Galileo Galilei is credited with being the first to measure speed by considering the distance covered and the time interval Δt it takes. Galileo defined speed as the distance covered per unit of time. In equation form $v = \ell/\Delta t$, where ℓ is length or distance of the trajectory and Δt the time interval spent to cover ℓ. Note that the time interval Δt measurement is

connected with the measurement of the space interval $\Delta x = \ell$. That is, measured intervals of time Δt are associated with measured displacements Δx in space. In this classical "arena" [(x,y,z) &t] displacement and time were used to define the dynamical entities like velocity, momentum, force, acceleration, etc. In this context [9,10] it can be stated: "**dynamics becomes an aspect of geometry**". That is, dynamical properties are intimately related with geometry. To exemplify dynamics & geometry let us consider a body moving on the spherical Earth surface. Besides tangential forces there are also radial forces that are dictated by the spherical geometry because bodies are constrained to move on a curved surface. Geometry is a useful mathematical tool to describe the **physical actions** generated by interactions between Earth and bodies. Geometry does not act on matter…

Up to the revolutionary works of Michelson-Morley[1-4] the Universe was described by a continuum Euclidean space-time geometry [(x,y,z) &t] and filled by an ideal static medium, named *aether*, perfectly transparent with a very large elasticity coefficient where light propagates with speed $c \approx 3\ 10^8$ m/s. In this *aether* was anchored a clock that would measure an *absolute time* τ and that in absence of applied forces a body would move with a constant speed **v.** References frames moving with constant speeds are called *inertial frames*. All frames moving with constant relative speeds are also inertial frames.[11] Mechanical laws were governed by the *Principle of Galilean Relativity*: "**The laws of mechanics have the same mathematical form in all inertial frames**". In other words these laws were *covariant* by *Galilean Transformation* (**GT**) between two inertial frames. Let us take two inertial frames S(x,y,z) and S´(x´,y´,z´) that move relatively with a constant speed V along the x-axis and with the axes y and z parallel to y´and z´, respectively. The Galilean coordinates transformation (**GT**) between S and S´ is given by

$$x' = x - Vt, \quad y' = y, \quad z' = z \quad \text{and} \quad t = t' \qquad (1.1).$$

Finally, let us remember that speed has a relative meaning; on the other hand, according to Newton´s point of view, acceleration has an absolute meaning [See accelerated frames elsewhere.[1,2,12,13].].

Galilean Relativity was consistent only with Newtonian mechanics. The laws of electromagnetism were not covariant by **GT.** Lorentz has shown that electromagnetic laws are covariant by another kind of transformations called Lorentz Transformation (**LT**)(see Section 2) which is one of the major landmarks in the development of physics.

(2)Constancy of Light Speed, Lorentz Transformation, Minkowski Space-time and the Special Relativity Principle.

The marvelous interferometric optical experiments performed by Michelson and Morley [1-4] at the last decade of 19.th century have shown that *aether* does exist and that the light speed (c) in vacuum is constant independently of the speeds of the source or of the observer (**CC**). This result was the cornerstone of the new theory that was developed at the end of the 19.th century, mainly due memorable works of Lorentz, Minkowski and Poincaré.[14] The importance of this theory named Special Relativity (**SR**) was established by Einstein[15] in 1905. According to **SR** to describe the light propagation and all physical phenomena involving speeds v ~ c it is necessary to use, instead of the earlier 3-dim Euclidean space [(x,y,z)&t], a new 4-dim pseudo-Euclidean continuum space-time with coordinates (x, y, z, ict), called *Minkowski space-time geometry* (**MG**). Time became the fourth component of the new space-time. This new space-time geometry was not assumed ad hoc but *determined* taking into account the **CC**. The old space-time substratum or "**arena**"[(x,y,z)&t] was substituted by the new **MG** substratum where light propagates with constant speed c and where all Universe are immersed. In this new **arena** [x, y, z, ict] were constructed the relativistic dynamical entities like speed, momentum, energy, inertial mass and so on. So, relativistic dynamics became an "**aspect of the Minkowski space-time geometry**". It has been claimed [9] the most important consequence of the **SR** is that space and time are not concepts which can be considered independently of each other, but they must be combined in such a fashion as to give a 4-dim description of the physical phenomena. The classical vacuum ("absence of matter") is extended everywhere in this new arena filling the empty spaces between the material bodies.

The coordinate transformation (x, y, z, ict) → (x´,y´,z´, ict´) between two inertial frames S and S´ that move with relative speed V along the x-axis is given by the *Lorentz Transformation* (**LT**): [1-4,13]

$$x' = \gamma(V)(x - Vt) , \quad y' = y , \quad z' = z \quad \text{and} \quad t' = \gamma(V)(t - Vx/c^2) \quad (2.1),$$

where $\gamma(V) = 1/\sqrt{1-\beta^2}$ and $\beta = V/c$. The LT (2.1) are represented in Fig. 1 by a (*hyperbolic*) **rotation** in a pseudo-Euclidean Minkowski space, where $V/c = \beta = i \tan(i\alpha)$. Note that **LT** does not depend of the distance between the inertial frames.

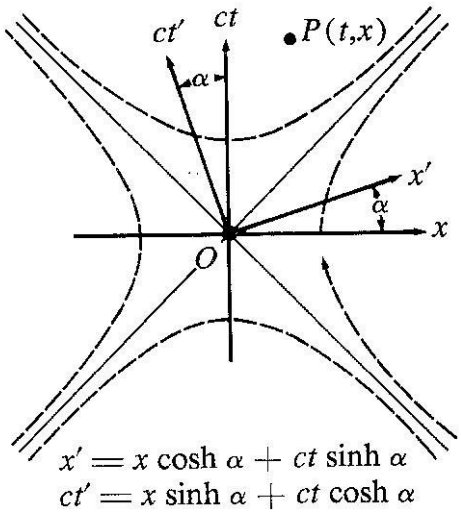

Figure 1. LT represented by a hyperbolic rotation in Minkowski space, where V/c = β = I tan(iα).

With the advent of **SR** not only the mechanical laws but all physical laws begin to obey the **Special Relativity Principle** (**SRP***)*:
"**The laws of physics must be covariant by Lorentz Transformations in all inertial frames**". In other words, the physical laws must be written in a new form which is invariant by **LT**. So, relativistic dynamical variables are defined and written as 4-vectors and 4-tensors.[1-4,11] Let us recall the causal sequence: **CC → LT → MG → SRP**.

Note that the **LT** considered above are only *rotations* in space-time, that is, they are "*proper transformations*" of the Lorentz group [16] which are valid only to kinematics and dynamics processes. Not all equations of physical laws are invariant under the complete transformations of the Lorentz group that involve rotations and inversions **(r,t) → (-r,-t).** Space and time inversions belong to the *improper Lorentz group* transformations. It was verified [4,16] that processes associated with β-decay, for instance, violate space inversion invariance. Internal physical properties of some *material* systems like, for instance, elementary particles obey many different symmetry groups.[10,16,17] To give a complete description of these systems it is necessary to take into account all these symmetries.

Until 1913 physical laws were written using the Minkowski arena where evolve physical phenomenon. In 1913 when Einstein proposed his Gravitation Theory [2,4,13,18,19] (**EGT**) the flat Minkowski geometry was substituted by the *curved* 4-dim Riemannian geometry (**RG**). In this theory the Minkowskian line-element $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$ are modified by the gravitational effect becoming $ds^2 = g_{ij}\, dx^i\, dx^j$ where g_{ij} (metric tensor) that depends of the gravitation effects are functions of general curvilinear coordinates $(x_i)_{i=1,2,3,4}$. The tensor g_{ij} takes into account the distortions of the **MG** caused by the gravitation field. Far from gravitational effects ds^2 is given by the Minkowskian line-element and the time coordinate $x_4 = ct$ is

interpreted as a "reference time". **SR** plays a fundamental role in the description of all physical phenomena, as long as gravitation is not significant. Thus, in **EGT** the Riemannian arena is determined by **CC** and also by the gravitational interaction. The gravitational interaction transforms the flat **MG** into a curved **RG**. Note that in **EGT** the light speed is only constant locally.

(3)Time, Length and International System of Units.

Let us recall how are measured intervals of *time* and *space* according to the International System of Units (SI).

Second is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium 133 atom. Note that for physicists "time" is not a metaphysical abstraction, it is a reading on a clock.[1]

Meter is defined as the distance travelled by light in (1/c) x *second*, where c = 299 792 458 m/s.

This shows that measurements of space and time intervals are closely connected. According to quantum mechanics hyperfine energy levels of Cesium depend of interaction *distances* between electrons and nucleus. Thus, if these distances change (according to **LT**) the duration of the periods also change and, consequently, the *second* changes. This naturally would imply that measurements of time intervals change.

(4) Experimental Confirmations of the Special Relativity.

Since the beginnings of the 20.th century up to now a large number of experimental works[5] have confirmed all predictions of the SR. These experiments played (and still play) an important role in its development and justification of the **SR**. The strength of the theory lies in its unique ability to correctly predict to high precision the outcome of an extremely diverse range of experiments. Repeats of many of those experiments are still being conducted with steadily increased precision, with modern experiments focusing on effects such as at the Planck scale and in the neutrino sector. These innumerous experimental works are seen in a detailed review given in reference [5]. They have confirmed interval time dilation, length contraction, relativistic mass $m(v) = m_o\gamma(v)$, relativistic momentum $\mathbf{p} = m_o\gamma(v)\mathbf{v}$, rest mass energy m_oc^2, total energy of a particle $m_oc^2\gamma(v)$ and many other correlated effects. Since the first days of **SR** these predicted effects have been questioned. This happened due erroneous impressions usually created if one insists on understanding the **LT** purely in geometrical terms. Due to superficial application of the **LT** formulas many paradoxes have been proposed. However, these have been solved with accurate

mathematical analysis, "thought experiments" and taking into account a large number of precise experiments.

Let us briefly analyze only two effects that are mentioned in all basic physics textbooks:[20,21] (a) interval time dilation and (b) length contraction.

(a)*Interval Time Dilation* or simply ***Time Dilation.*** According to basic textbooks time intervals Δt measured by a clock C fixed in a inertial system S are longer than the respectively time intervals $\Delta t'$ measured by a clock C´ fixed in a moving inertial system S´, that is, $\Delta t = \gamma \Delta t'$. Since systems S and S´ are equivalent due to the symmetry of the relative motion, we would have $\Delta t' = \gamma \Delta t$.

Now let us assume now that C is fixed at the origin O in the laboratory and C´ which was initially at O moves on a closed path and comes back to its initial place O. One may think that C´ will be found delayed compared with C. One may also think that with respect to S´ the system S has also performed a closed loop motion that upon arrival there will be a confusion as to which is retarded. However, since S is an **inertial** system S´ cannot be one. This impasse was called the "**clock paradox**" or "**twin paradox**". According to Mϕller[2] it would be desirable an extension of the **SR** theory to a **General Relativity** theory (**GRT**) which allows the use of systems of coordinates in arbitrary motion. To *solve* this problem inside the context of the **SR** it was assumed [1,2,12,13] that in an infinitesimal time interval dt the accelerated system S´ has an instantaneous speed V(t) relative to S. In this time interval dt the S´ is *considered* as *instantaneously inertial* with an instantaneous speed V(t). In this way the time interval dt´ of the accelerated clock would be given by $dt' = dt \{1- [V(t)/c]^2\}^{1/2}$, that is, the clocks in motion are retarded. The inverse **LT** from S to S´ is not valid because S´ is not an inertial system. [2] This "extended" **LT** [1,2,12,13] is experimentally confirmed measuring, for instance, the "**transverse Doppler effect**" where a radioactive source emitting γ-rays (with frequency f_o) describes a circle around a fixed center O at the laboratory where there is a γ detector.[22]The γ-rays emitted radially by the source are detected at O with frequency $f = f_o\sqrt{1-\beta^2}$, with $\beta=V(t)/c$, where V(t) is the *instantaneous tangential speed* of the source. Other confirmation of the validity of the "extended" **LT** was given, for instance, by Hafele-Keating[23] experiment: two cesium atomic clocks flew east and west around the Earth in commercial airlines and compared the elapsed time against that of a clock that remained fixed at the U.S. Naval Observatory. They verified that the flying clocks are retarded with respect of the fixed clock. The final solution of the "clock paradox" using **GRT** is seen, for instance, in Mϕller´s book.[2]

Note that "**time dilation"** is a very real phenomenon verified by many experiments. It is an intrinsic property of the "coordinate" time. Not only the clocks in motion are retarded, but all physical processes become

retarded. Chemical reactions, for instance, occur more slowly when are in motion. Since life consists in a complex of chemical reactions, life also would pass more slowly by the same factor. The space traveler gets actually younger than his twin ("**twin paradox**") to return to Earth. The clock of the spacecraft and the heartbeat of the astronaut would be diminished in relation to a stationary clock. It is more adequate to regard the retardation phenomenon as an elementary phenomenon which is a direct consequence of the **SRP**.

Time dilation is also verified measuring the increase of the **lifetime of unstable elementary particles** produced by the cosmic radiation in the high atmosphere and in particle accelerators.

(b) ***Length contraction*** or ***Lorentz contraction****:* direct experimental confirmations of length contraction are hard to achieve, because at the current state of technology, objects of considerable extension cannot be accelerated to relativistic speeds. The only objects traveling with the required speeds are the atomic and elementary particles, yet whose spatial dimensions are too small to allow a *direct* measurement of contraction. However, there are indirect confirmations; for example, the behavior of colliding heavy ions can only be explained if their increased density due to Lorentz contraction is considered. Contraction also leads to an increase of the intensity of the Coulomb field perpendicular to the direction of motion, whose effects already have been observed. Consequently, both time dilation and length contraction must be considered when conducting experiments in particle accelerators.

(5) Physical Action and Nonlocality in SR.

(5.a) **Physical actions and CC**.

In Classical Physics it is supposed that the only way to produce a measurable effect is by means of transmission of energy and moment densities through a *physical field*. These *fields* are *potentials* (gravitational or electromagnetic) or propagating mechanical or electromagnetic waves. Here "entities" that produce measurable effects are named **physical actions**. We have assumed that unknown physical actions are responsible for the **CC.** In this way, in **SR** the **CC** is taken as a "postulate" or a "principle" similar to "energy conservation" or "charge conservation". We hope in the future to determine these actions in order to give a physical or a mathematical justification for the **CC**. One famous example of unprovable postulate is the Newton´s second law of motion that was explained via quantum mechanics with the Ehrenfest´s theorem.[24] Dirac suggested[7] that fundamental aspects of the **SR** like, for instance, the **CC** origin could be

explained taking into account quantum mechanics and the interaction of moving particles with the quantum vacuum $| \phi >$.[25]

Thus, assuming that **CC** is created by physical actions we have supposed the following causal sequence **CC → LT → MG → SR → measured effects.** Thus, physical actions that create **CC** were considered responsible for the predicted **SR** effects. **LT** and **MG** constitute a geometric approach or useful mathematical tool that only displays the predicted effects by the **SR.**

(5.b) **Nonlocality.**

Lorentz transformation governs not only the **SR** theory but the whole of physical theory. From the continuum Minkowski space-time and **LT** are deduced all laws of the **SR** theory. If the Minkowski space-time has no limit, that is, all coordinates varies from $+\infty \rightarrow -\infty$ [6] the **SR** laws can be applied at any point and at any instant. In addition, there are mathematical details in **LT** that seem able to create "**nonlocal effects"** or "**action at a distance**" (a)**LT** does not depend of distance between the reference frames; (b)**LT** in many cases depends of instantaneous relative speeds V(t) between reference frames. This is the case, for instance, of the Doppler shift measured in the Earth of a light pulse emitted by a very far star: it depends of an "*instantaneous*" relative speed V between the star and Earth and does not depend of the distance between them that can be thousands of light years. Another cases confirmed experimentally when **LT** is performed between two systems, one at rest and another accelerated, are: the *twin paradox* and *transverse Doppler Effect* (seen in Section 4). These effects are *independent of the distance* between the twins or between detector and source and depend of a relative *instantaneou*s speeds V(t).

Thus, we could expect that theories developed in **Euclidean** arena [(x,y,z)&t] or in **MG** arena (x,y,z, ict) manifest "nonlocal" effects. This would explain nonlocal effects found in non-relativistic and relativistic Quantum Mechanics that are developed, respectively, in **Euclidean** and **MG** arenas.[25] This would happen with the **EGT** which is developed in a **MG** arena modified by gravitation. Maybe "nonlocal" effects have been circumvented in **GR** theory [2,12].

(6) Conclusions and Discussions.

Of course, it could be conjectured [7] that the constancy of the light speed (**CC)** is created by "physical actions" but it was proved that they exist. If these actions exist the amazing predicted effects of the **SR** are only mathematically displayed by the **LT** and **MG** geometric properties and according to the **SRP**. The complete understanding of the **SR** would be attained determining the physical origins of the **CC.**[7] Up to now, as can be easily verified the **CC** cannot be explained by the interaction of particles

with the Quantum Mechanics vacuum $|\phi>$[26] or with the zero point field (**ZPF**) of the Stochastic Electrodynamics[27] (**SED**).

It was also pointed out some mathematical details of the **LT** that seem able to create "nonlocal" or "action at distance" effects. If in the **SR** context these effects exist they are so small that their contributions, up to now, have not been detected**.** In **QM**, on the contrary, they are relevant measured effects.[28] They do not alter the excellent agreement between QM predictions and experimental results. Why "action at distance" is not admitted? Only because violates **SRP**? "There are more things in heaven and earth that can imagine our vain philosophy…"

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