Instituto de Física
Universidade de São Paulo

FFBRST Mapping in Higher Derivative Models

Pavel Yu. Moshin,[a] Sudhaker Upadhyay,[b] Ricardo A. Castro[c]
[a]Faculty of Physics, National Research Tomsk State University, 634050, Tomsk, Russia
[b]Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, Kharagpur-721302, WB, India
[c]Institute of Physics, University of São Paulo, Rua do Matão 1371, CEP 05508-090, Cidade Universitária, São Paulo, Brazil

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Pavel Yu. Moshin,^{a} Sudhaker Upadhyay,^{b} Ricardo A. Castro^{c*}

^{a}*Faculty of Physics, National Research Tomsk State University, 634050, Tomsk, Russia*
^{b}*Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, Kharagpur-721302, WB, India*
^{c}*Institute of Physics, University of São Paulo, Rua do Matão 1371,*
CEP 05508-090, Cidade Universitária, São Paulo, Brazil

We continue the study of finite field-dependent BRST (FFBRST) symmetry in the quantum theory of gauge fields. An expression for the Jacobian of path integral measure is presented, depending on a finite field-dependent parameter, and the FFBRST symmetry is then applied to a number of well-established quantum gauge theories in a form which incudes higher-derivative terms. Specifically, we examine the corresponding versions of the Maxwell theory, non-Abelian vector field theory, and gravitation theory. We present a systematic mapping between different forms of gauge-fixing, including those with higher-derivative terms, for which these theories have better renormalization properties. In doing so, we also provide the independence of the S-matrix from a particular gauge-fixing with higher derivatives. Following this method, a higher-derivative quantum action can be constructed for any gauge theory in the FFBRST framework.

I. INTRODUCTION

Higher-derivative (HD) field theories naturally emerge, due to various reasons, as effective theories in a wide area of physics. Perhaps the best known example is gravity, in which higher-order terms in the curvature arise either from underlying string dynamics, or from quantizing matter fields. Quite often, HD terms are added to a given standard theory as corrections. In gravity theories, HD terms ensure renormalizability [1]. Besides the renormalization properties, the known facts about the theory structure include the particle contents, given by the linear decomposition of the HD propagator into the parts containing second-order poles. Some issues related to the equations of motion have also been discussed [2]. Unitarity in renormalizable HD quantum gravity has been examined, and the presence of a massive spin-2 ghost in the bare propagator is found to be inconclusive [3].

On the other hand, in the case of a massive relativistic particle, the action is extended by the curvature term, being higher-derivative by its nature. This particle model, introduced quite a long time ago [4], is still under active consideration [5, 6, 7, 8, 9, 10, 11, 12]. The introduction of HD fields is not limited to this particular area. Instead, it has been considered in diverse theoretical models, such as electrodynamics [13, 14], supersymmetry [15, 16], noncommutative theory [17, 18], cosmology [19, 20], extended Maxwell–Chern–Simon theory [21, 22], theory of anyons [23, 24, 25], relativistic particles with torsion [26], membrane description for the electron [27, 28], etc. There are many more gravity models in which HD corrections are added to the Einstein–Hilbert action [29, 30, 31, 32]. HD terms acquire relevance also in the context of string theory [33, 34]. Thus, the importance of HD terms cannot be overestimated.

In quantizing gauge field theories, the Becchi–Rouet–Stora–Tyutin (BRST) formalism [35, 36, 37, 38] provides a comparatively rigorous mathematical scheme. Even though the BRST formulation is a powerful approach to quantize gauge theories, which simplifies the study of renormalizability and unitarity of gauge theories, the implementation of this approach in HD theories is quite nontrivial and poses problems. Despite this fact, in usual gauge field theories the standard BRST symmetry has been generalized by allowing the transformation parameter to be finite and field-dependent [39].

Thus generalized BRST symmetry transformations, or so-called finite field-dependent BRST (FF-

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BRST) symmetry, lead to a non-trivial Jacobian of functional measure and find applications in a wide area of gauge theories, including gravity [39, 40, 41]. For instance, the celebrated Gribov problem [42, 43, 44] has been addressed in the framework of FFBRST formulation (see Ref. [45] and references therein). In this article, we present an elegant approach to derive the Jacobian of functional measure, as compared to the original study of [39]. The advantage of the present approach is that one has no need to provide an ansatz for a local functional subjected to some boundary conditions. On top of that, one has no need to solve differential equations satisfying certain initial boundary conditions to obtain a precise expression for the Jacobian. Here, the evaluation of a Jacobian only requires that one provide a suitable infinitesimal field-dependent parameter.

FFBRST transformations have been given an emphasis in higher-form gauge theories [46]. Further, in supersymmetric M-theories [47, 48, 49] such developments have also been studied [50, 51, 52]. Recently, gravity models have been explored in the context of FFBRST transformations [53]. Such generalizations are established at the quantum level [54, 55], using the BV technique [56]. Recently, the FFBRST formulation has acquired relevance in topological gauge theories [57]. Moshin and Reshetnyak, for the first time [58], systematically incorporated BRST-antiBRST symmetry into Yang–Mills theories within the context of finite transformations that deals with the case of a quadratic dependence on the transformation parameters. Further, the concept of finite BRST-antiBRST symmetry in general gauge theories has been used in Refs. [59, 60], whereas Ref. [61] by the same authors generalizes the corresponding parameters to the case of arbitrary Grassmann-odd field-dependent parameters, as compared to the so-called "potential" form of parameters [58, 59, 60].

A natural question arises concerning the application of the FFBRST formalism to HD theories. Indeed, it is not surprising, despite a considerable amount of research on HD models, that this issue so far remains unstudied. The basic motivation for this paper is to express FFBRST transformations in a more transparent way and to explore the possible applications of this formalism to HD gauge theories. In this context, we make a simplified way to FFBRST transformations by following Ref. [39] up to some good extent. As originally, we make all the fields parameter-dependent by a continuous interpolation such that, at one limit, it corresponds to the original field and, at another limit, to a transformed field. Further, we define an infinitesimal field-dependent transformation by making the constant parameter infinitesimally field-dependent. Now, we integrate such an infinitesimal field-dependent transformation to obtain an FFBRST transformation. Then, we evaluate the Jacobian of functional measure under FFBRST with an arbitrary field-dependent parameter. Further, we apply the resulting FFBRST transformation to various HD models, which leads to some interesting observations. First, we examine the FFBRST transformation in Maxwell theory and find that for a particular choice of the field-dependent parameter it maps gauge-fixing to an HD version of this theory, which also preserves the independence of the S-matrix from any particular gauge-fixing. We further apply FFBRST transformations to non-Abelian and gravitational theories, so as to extend the results and validity of our treatment. Indeed, we find that this treatment works in each of the gauge theories involved. Since HD terms play an important part in achieving the renormalization of ultraviolet (UV) divergent gauge theories, the present technique could be of help in dealing with UV-divergent gauge theories.

The paper is organized as follows. In Section II, we present the construction of FFBRST transformations in a simplified way. We derive a manifest expression for the Jacobian with no need of boundary conditions. Further, in Section III, we illustrate various HD models and discuss their BRST quantization. To be specific, in Subsection IIIA, we discuss BRST and FFBRST transformations in Maxwell theory and its HD version. In this description, we derive a Jacobian which consists only of BRST-exact terms for the HD model. In Subsection IIIB, we use FFBRST transformations to produce an HD non-Abelian action. In Subsection IIIC, we study BRST and FFBRST transformations in HD gravity. We map HD gravity to its quantum version through FFBRST transformations. In Section IV, we summarize the results and suggest some future motivations.

II. CONSTRUCTION OF FINITE FIELD-DEPENDENT BRST TRANSFORMATIONS

In this section, we illustrate the FFBRST formulation, on general grounds, within a simplified approach following Ref. [39] up to some good extent. Let us begin by defining infinitesimal BRST transformations for a generic field $\phi(x)$ as follows:

$$\phi(x) \longrightarrow \phi'(x) = \phi(x) + s_b\phi(x)\,\Lambda, \tag{1}$$

where $s_b\phi$ is the so-called Slavnov variation, and Λ is an infinitesimal anticommuting parameter with no spacetime dependence. Under such transformations, the path integral measure remains invariant [38].

Now, the field $\phi(x)$ turns into a continuous parameter $(\kappa; 0 \leq \kappa \leq 1)$ such that $\phi(x, \kappa = 0) = \phi(x)$ is the original field, and $\phi(x, \kappa = 1) = \phi'(x) = \phi(x) + s_b\phi(x)\Theta[\phi]$ is an FFBRST-transformed field characterized by a finite field-dependent parameter $\Theta[\phi]$. To justify FFBRST transformations, we construct the following infinitesimal field-dependent BRST transformations [39]:

$$\frac{d\phi(x, \kappa)}{d\kappa} = s_b\phi(x, \kappa)\Theta'[\phi(\kappa)], \tag{2}$$

where $\Theta'[\phi(\kappa)]$ is an infinitesimal field-dependent parameter.

Further, we proceed by making integration over κ and arrive at the following field-dependent transformation [39]:

$$\phi(x, \kappa) = \phi(x, 0) + s_b\phi(x, 0)\Theta[\phi(\kappa)].$$

Here, $\Theta[\phi(\kappa)]$ is related to $\Theta'[\phi(\kappa)]$ through

$$\Theta[\phi(k)] = \int_0^\kappa d\kappa\ \Theta'[\phi(\kappa)] = \Theta'[\phi(0)]\frac{\exp\left(\kappa f[\phi(0)]\right) - 1}{f[\phi(0)},$$

with $f[\phi(k)] = \frac{\delta\Theta'}{\delta\phi}s_b\phi$. For the boundary value of κ (i.e., $\kappa = 1$), this yields the FFBRST transformation

$$\delta_b\phi(x) = \phi'(x) - \phi(x) = s_b\phi(x)\Theta[\phi(1)]. \tag{3}$$

It is easy to verify that the resulting FFBRST transformations with a field-dependent parameter also provide a symmetry of the quantum action, but the price to pay is that these are no longer nilpotent and do not leave the functional measure invariant. Incidentally, the path integral measure also changes non-trivially under these transformations, leading to a non-trivial Jacobian within functional integration. So it is worthwhile to compute an explicit Jacobian of functional measure under such transformations and follow the pertaining consequences.

A. Jacobian for finite field-dependent BRST transformations

In this subsection, we compute the Jacobian for path integral measure under FFBRST transformations with arbitrary and specific parameters. Let us start by defining the vacuum functional in Maxwell theory, described by a quantum action $S_{FP}[\phi]$,

$$Z[0] = \int \mathcal{D}\phi\ e^{iS_{FP}[\phi]}, \tag{4}$$

where $\mathcal{D}\phi$ stands for the complete functional measure. Furthermore, in order to compute the Jacobian of functional measure under FFBRST transformations, we observe [39]

$$\mathcal{D}\phi(\kappa) = J(\kappa)\mathcal{D}\phi(\kappa) = J(\kappa + d\kappa)\mathcal{D}\phi(\kappa + d\kappa).$$

Because of its infinitesimal nature, the transformation from $\phi(\kappa)$ to $\phi(\kappa + d\kappa)$ can be presented as [39]

$$\frac{J(\kappa)}{J(\kappa + d\kappa)} = \sum_\phi \pm \frac{\delta\phi(\kappa + d\kappa)}{\delta\phi(\kappa)},$$

where the $+$ sign is used for bosonic fields, and $-$ is used for fermionic fields. Now, upon making the Taylor expansion, we obtain [39]

$$1 - \frac{1}{J}\frac{dJ}{d\kappa}d\kappa = 1 + d\kappa \int d^4x \sum_\phi \pm s_b\phi(x,\kappa)\frac{\delta\Theta'[\phi(x,\kappa)]}{\delta\phi(x,\kappa)},$$

which simplifies to

$$\frac{d\ln J[\phi]}{d\kappa} = -\int d^4x \sum_\phi \pm s_b\phi(x,\kappa)\frac{\delta\Theta'[\phi(x,\kappa)]}{\delta\phi(x,\kappa)}. \tag{5}$$

The above expression is nothing else but the expression for an infinitesimal change in the Jacobian of functional measure. To reach the expression for a finite Jacobian, it is straightforward to integrate (5) over κ within the limits from 0 to 1. This leads to the series

$$\ln J[\phi] = -\int_0^1 d\kappa \int d^4x \sum_\phi \pm s_b\phi(x,\kappa)\frac{\delta\Theta'[\phi(x,\kappa)]}{\delta\phi(x,\kappa)}.$$

Upon making the Taylor expansion of $\sum_\phi \pm s_b\phi(x,\kappa)\frac{\delta\Theta'[\phi(x,\kappa)]}{\delta\phi(x,\kappa)}$ in κ and then integrating over κ, we find

$$\ln J[\phi] = -\left(\int d^4x \sum_\phi \pm s_b\phi(x)\frac{\delta\Theta'[\phi(x)]}{\delta\phi(x)}\right).$$

Further simplifications give us a precise expression for the Jacobian of functional measure under FFBRST transformations:

$$J[\phi] = \exp\left(-\int d^4x \sum_\phi \pm s_b\phi(x)\frac{\delta\Theta'[\phi(x)]}{\delta\phi(x)}\right). \tag{6}$$

Here, we notice that, in order to calculate the Jacobian, we have no need of a local functional $S_1[\phi]$ replacing the Jacobian as e^{iS_1} and satisfying, together with Θ', certain conditions presented in Ref. [39]. In the FFBRST formulation [39], one first presents an ansatz for S_1 in terms of an arbitrary κ-dependent parameter; then a physicality condition leads to certain differential equations with respect to an arbitrary parameter. By satisfying the boundary conditions, one solves these differential equations to obtain a precise expression for S_1.

Jacobian (6) therefore extrapolates the quantum action (within functional integration) of the theory in (4) as follows:

$$\int \mathcal{D}\phi' e^{iS_{FP}[\phi']} = \int \mathcal{D}\phi \, J[\phi]e^{iS_{FP}[\phi]} = \int \mathcal{D}\phi \, e^{i\left\{S_{FP}[\phi] - \int d^4x \left(\sum_\phi \pm s_b\phi\frac{\delta\Theta'[\phi]}{\delta\phi}\right)\right\}},$$

which is nothing else but the vacuum functional of the same theory, since this extra piece does not change the theory on physical grounds, but rather simplifies various issues in a dramatic way.

III. HIGHER-DERIVATIVE MODELS AND FFBRST TRANSFORMATIONS

In this section, we present some HD models in the context of FFBRST description.

A. Higher-derivative Maxwell theory

The presence of a local gauge symmetry in Maxwell theory requires, as usual, the introduction of a gauge-fixing term and a compensating Faddeev–Popov (FP) ghost term to the classical action, resulting in the Faddeev–Popov quantum action

$$S_{FP} = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \zeta^2 (\partial_\mu A^\mu)^2 - \bar{c} \Box c \right], \tag{7}$$

where ζ is a dimensionless gauge parameter. In the auxiliary field formulation, the action becomes

$$S_{FP} = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + B \partial_\mu A^\mu + \frac{1}{2\zeta^2} B^2 - \bar{c} \Box c \right],$$

where B is the Nakanishi–Lautrup field, and $\Box = \partial_\mu \partial^\mu$. The Faddeev–Popov action breaks the local gauge invariance. However, the action S_{FP} remains invariant under a rigid BRST transformation with a fermionic parameter. The infinitesimal BRST transformations are

$$\begin{aligned} \delta_b A_\mu &= -\partial_\mu c \Lambda, \quad \delta_b c = 0, \\ \delta_b \bar{c} &= B \Lambda, \quad \delta_b B = 0, \end{aligned} \tag{8}$$

where Λ is the transformation parameter. There exists a conserved charge corresponding to the above transformation, which plays an important role in constructing the physical state space.

An HD version for the quantum action (7) is defined by [62]

$$\begin{aligned} S_{HD} &= \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^2} F_{\mu\nu} \Box F^{\mu\nu} - \frac{1}{2} \zeta^2 (\partial_\mu A^\mu)^2 - \frac{\zeta^2}{2M^2} (\partial_\mu A^\mu) \Box (\partial_\nu A^\nu) \right. \\ &\quad \left. - \bar{c} \left(1 + \frac{\Box}{M^2} \right) \Box c \right], \end{aligned} \tag{9}$$

where m^2 is a dimensional parameter, and M^2 is a dimensional gauge parameter. In terms of the auxiliary field B, the above expression reads

$$\begin{aligned} S_{HD} &= \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^2} F_{\mu\nu} \Box F^{\mu\nu} + B \left(1 + \frac{\Box}{M^2} \right) \partial_\mu A^\mu + \frac{1}{2\zeta^2} B \left(1 + \frac{\Box}{M^2} \right) B \right. \\ &\quad \left. - \bar{c} \left(1 + \frac{\Box}{M^2} \right) \Box c \right]. \end{aligned} \tag{10}$$

This HD quantum action is invariant under the same transformations (8).

The importance of this HD gauge theory lies in the fact that this model mimics the model of quantum gravity. For instance, the first term in (10) is reminiscent of $\sqrt{-g}R$, and the second term is similar to $\sqrt{-g}R^2$.

Next, we generalize the BRST transformation according to the above FFBRST formulation. Following (1), (2) and (3), we construct the FFBRST transformation corresponding to (8), as follows:

$$\begin{aligned} \delta_b A_\mu &= -\partial_\mu c \Theta[\phi], \quad \delta_b c = 0, \\ \delta_b \bar{c} &= B \Theta[\phi], \quad \delta_b B = 0, \end{aligned}$$

where $\Theta[\phi]$ is an arbitrary finite field-dependent parameter. An explicit choice for the parameter $\Theta[\phi]$ produces specific results. To observe the appearance of a higher-derivative quantum action, we make the following explicit choice:

$$\Theta'[\phi] = \int d^4x \left[\bar{c} \left(\frac{\Box}{M^2} \partial_\mu A^\mu + \frac{1}{2\zeta^2} \frac{\Box}{M^2} B \right) \right].$$

Using (6), we obtain the Jacobian of functional measure from the above Θ' and find

$$J[\phi] = \exp\left[\int d^4x\left(B\frac{\Box}{M^2}\partial_\mu A^\mu + \frac{1}{2\zeta^2 M^2}B\Box B - \bar{c}\frac{\Box}{M^2}\Box c\right)\right].$$

This Jacobian exhibits BRST-exact HD terms within functional integration. In other words, HD terms, essential for the quantum action, turn out to be inherent in the Jacobian for path integral measure under a change of variables. This justifies a mapping between the Maxwell theory and its HD version. By computing the Jacobian, one can calculate the HD terms in the given theory.

B.   Higher-derivative theory for non-Abelian vector field

In this subsection, we extend the above results and use FFBRST transformations in an HD non-Abelian gauge theory. The action of the theory is defined by [63]

$$S = \frac{1}{2}\int d^dx\,\mathrm{Tr}\left(-F_{\mu\nu}F^{\mu\nu} + D^\nu F_{\nu\mu}D_\rho F^{\rho\mu} + \frac{1}{4\xi^2}\partial^\mu\partial A\partial_\mu\partial A - \frac{1}{\xi}D^\nu F_{\nu\mu}\partial^\mu\partial A + \bar{\mathcal{F}}^{\mu\nu}\mathcal{F}_{\mu\nu}\right.$$
$$\left. - 2i\{\bar{c}^\mu, c^\nu\}F_{\mu\nu} + \frac{1}{\xi}\partial^\mu\bar{c}_\mu\partial^\nu c_\nu\right), \tag{11}$$

where ξ is an arbitrary gauge parameter. Here, the Yang–Mills covariant derivative is defined by $D_\mu = \partial_\mu + g[A_\mu, \bullet]$; $\mathcal{F}_{\mu\nu}$ and $\bar{\mathcal{F}}_{\mu\nu}$ are the field strengths for the fields c_μ and $\bar{c}_\mu$, respectively. The above action is invariant under the following rigid fermionic symmetry:

$$\delta_b A_\mu^a = -c_\mu^a\Lambda, \quad \delta_b c_\mu^a = 0, \quad \delta_b\bar{c}_\mu^a = \left(D^{\nu ab}F_{\nu\mu}^b + \frac{1}{2\xi}\partial_\mu\partial^\nu A_\nu^a\right)\Lambda.$$

Using the auxiliary field b_μ^a, we present the action (11) in the form

$$S = \frac{1}{2}\int d^dx\,\mathrm{Tr}\left[-F_{\mu\nu}F^{\mu\nu} + b^\mu\left(D^\nu F_{\nu\mu} + \frac{1}{2\xi}\partial_\mu\partial^\nu A_\nu - \frac{1}{2}b_\mu\right) + \bar{\mathcal{F}}^{\mu\nu}\mathcal{F}_{\mu\nu}\right.$$
$$\left. - 2i\{\bar{c}^\mu, c^\nu\}F_{\mu\nu} + \frac{1}{\xi}\partial^\mu\bar{c}_\mu\partial^\nu c_\nu\right],$$

which is invariant under the following off-shell nilpotent BRST transformations:

$$\delta_b A_\mu^a = -c_\mu^a\Lambda, \quad \delta_b c_\mu^a = 0, \quad \delta_b\bar{c}_\mu^a = b_\mu^a\Lambda, \quad \delta_b b_\mu^a = 0.$$

This structure has been discussed in topological quantum field theories [64]. These transformations are generalized by making the transformation parameter finite and field-dependent:

$$\delta_b A_\mu^a = -c_\mu^a\Theta[\phi], \quad \delta_b c_\mu^a = 0, \quad \delta_b\bar{c}_\mu^a = b_\mu^a\Theta[\phi], \quad \delta_b b_\mu^a = 0,$$

where the finite parameter is constructed explicitly from the infinitesimal field-dependent parameter

$$\Theta'[\phi] = \frac{1}{2}\int d^dx\,\mathrm{Tr}\left[\bar{c}^\mu\left(D^\nu F_{\nu\mu} + \frac{1}{2\xi}\partial_\mu\partial^\nu A_\nu - \frac{1}{2}b_\mu\right)\right]. \tag{12}$$

The Jacobian of functional measure under the FFBRST transformation with a parameter constructed by (12) reads as follows:

$$J[\phi] = \exp\left\{\frac{1}{2}\int d^dx\,\mathrm{Tr}\left[b^\mu\left(D^\nu F_{\nu\mu} + \frac{1}{2\xi}\partial_\mu\partial^\nu A_\nu - \frac{1}{2}b_\mu\right) + \bar{\mathcal{F}}^{\mu\nu}\mathcal{F}_{\mu\nu}\right.\right.$$
$$\left.\left. - 2i\{\bar{c}^\mu, c^\nu\}F_{\mu\nu} + \frac{1}{\xi}\partial^\mu\bar{c}_\mu\partial^\nu c_\nu\right]\right\}. \tag{13}$$

Now, we can see that under FFBRST transformations with a specific parameter one can produce an HD action for the non-Abelian theory in question. This also justifies the validity of our approach in non-Abelian gauge theories. Consequently, using FFBRST transformations, one can generate appropriate HD terms which allow one to get rid of UV divergencies. Since the HD theory is BRST-invariant, the unitarity problem associated with HD theories can be overcome.

C. Higher-derivative gravity

In this subsection, we examine FFBRST transformations in HD gravity. To this end, we start with a general fourth-order gravity action in curved spacetime [3],

$$S_g = \int d^4x \sqrt{-g} \left[-\frac{1}{\alpha^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) + \beta R^2 + \frac{\gamma}{\zeta^2} R \right]. \tag{14}$$

In the weak limit, we decompose the metric into a fixed metric $\eta^{\mu\nu}$ and fluctuations $h^{\mu\nu}$, as follows:

$$\sqrt{-g} g^{\mu\nu} = \eta^{\mu\nu} + \alpha\zeta h^{\mu\nu}.$$

The action is invariant under the following general gauge transformation:

$$\delta h^{\mu\nu} = D_\rho^{\mu\nu} \omega^\rho,$$

where the manifest expression for the covariant derivative of the vector parameter ω^ρ is given by

$$D_\rho^{\mu\nu} \omega^\rho = \partial^\mu \omega^\nu + \partial^\nu \omega^\mu - \eta^{\mu\nu} \partial_\rho \omega^\rho + \alpha\zeta (\partial_\rho \omega^\mu h^{\rho\nu} + h^{\rho\mu} \partial_\rho \omega^\nu - \partial_\rho h^{\mu\nu} \omega^\rho - h^{\mu\nu} \partial_\rho \omega^\rho).$$

According to conventional quantization, one introduces gauge-fixing in order to remove the redundant degrees of freedom. Here, we choose the familiar harmonic (De Donder) gauge

$$\partial_\nu h^{\mu\nu} = 0.$$

Then the gauge-fixing term in the action is quadratic in derivatives:

$$S_{gf} = -\frac{1}{2} \int d^4x \, (\partial_\nu h^{\mu\nu})^2.$$

This implies that it is not every part of the graviton propagator that behaves as $(\text{momentum})^{-4}$ for large momenta, leading thereby to some UV divergences. This complication is easily overcome by introducing gauge-fixing terms with four or more derivatives [3]:

$$S_{gf} = -\frac{1}{2} \int d^4x \, [\hat{e}(\Box) \partial_\nu h^{\mu\nu}]^2,$$

where $\hat{e}(\Box) = b_1 \Box + b_2$, with b_1 and b_2 being constant. Using the Nakanishi–Lautrup field B_μ, one presents the linearized gauge-fixing term as

$$S_{gf} = \int d^4x \left[\frac{1}{2} (B_\mu)^2 - B_\mu b_1 \Box \partial_\nu h^{\mu\nu} - B_\mu b_2 \partial_\nu h^{\mu\nu} \right].$$

The compensating ghost term within functional integration is given by

$$\begin{aligned}
S_{gh} = \int d^4x \, \{ & \bar{c}_\mu b_1 \Box \partial_\nu \left[\partial^\mu c^\nu + \partial^\nu c^\mu - \eta^{\mu\nu} \partial_\rho c^\rho + \alpha\zeta \left(\partial_\rho c^\mu h^{\rho\nu} + h^{\rho\mu} \partial_\rho c^\nu - \partial_\rho h^{\mu\nu} c^\rho - h^{\mu\nu} \partial_\rho c^\rho \right) \right] \\
+ & \bar{c}_\mu b_2 \partial_\nu \left[\partial^\mu c^\nu + \partial^\nu c^\mu - \eta^{\mu\nu} \partial_\rho c^\rho + \alpha\zeta \left(\partial_\rho c^\mu h^{\rho\nu} + h^{\rho\mu} \partial_\rho c^\nu - \partial_\rho h^{\mu\nu} c^\rho - h^{\mu\nu} \partial_\rho c^\rho \right) \right] \}.
\end{aligned}$$

The FP quantum action $S_g + S_{gf} + S_{gh}$ admits BRST invariance under

$$\delta_b h^{\mu\nu} = D^{\mu\nu}_\rho c^\rho \Lambda, \quad \delta_b c^\mu = -\zeta \partial_\nu c^\mu c^\nu \Lambda,$$
$$\delta_b \bar{c}^\mu = -B^\mu \Lambda \quad \delta_b B^\mu = 0. \tag{15}$$

Using these symmetry transformations, one can compute a conserved (BRST) charge which annihilates the physical states in the total state space and helps one to establish unitarity in the theory.

Following Section II, we construct the FFBRST transformations corresponding to (15), namely,

$$\delta_b h^{\mu\nu} = D^{\mu\nu}_\rho c^\rho \; \Theta[\phi], \quad \delta_b c^\mu = -\zeta \partial_\nu c^\mu c^\nu \Lambda \; \Theta[\phi],$$
$$\delta_b \bar{c}^\mu = -B^\mu \Lambda \; \Theta[\phi] \quad \delta_b B^\mu = 0. \tag{16}$$

where $\Theta[\phi]$ is an arbitrary finite field-dependent parameter. This parameter admits any value; however, in the present case we assign it to the value which is derived from

$$\Theta'[\phi] = \int d^4x \; \bar{c}_\mu (b_1 \Box \partial_\nu h^{\mu\nu}). \tag{17}$$

Using (6) and (17), we calculate the Jacobian of functional measure:

$$J[\phi] = \exp\left\{ \int d^4x \left[-B_\mu b_1 \Box \partial_\nu h^{\mu\nu} + \bar{c}_\mu b_1 \Box \partial_\nu \left(\partial^\mu c^\nu + \partial^\nu c^\mu - \eta^{\mu\nu} \partial_\rho c^\rho \right) \right. \right.$$
$$\left. \left. + \; \alpha\zeta \left(\partial_\rho c^\mu h^{\rho\nu} + h^{\rho\mu} \partial_\rho c^\nu - \partial_\rho h^{\mu\nu} c^\rho - h^{\mu\nu} \partial_\rho c^\rho \right) \right] \right\}.$$

So, we can see that this parameter renders FFBRST transformations a source of HD terms in the quantum action of gravity. This proves our treatment to be valid also in the case of gravity. It is well known that the action (14) is renormalizable by power counting, and, in fact, this renormalizability has been demonstrated in Ref. [65]. More importantly, this theory is asymptotically free [66, 67]. The renormalizability and asymptotic freedom are entirely due to the HD terms. However, there is still redundancy in physical degrees of freedom. To remove it, one needs a higher-derivative quantum action, which can be generated using the FFBRST mechanism with suitable HD terms in the theory through the Jacobian.

IV.   CONCLUSION

HD field theories are of interest, since they play an important role in understanding the fundamental interactions of Nature. Incidentally, the theory of gravity, as we know it today, is an effective theory, and the usual Einstein–Hilbert action should be supplemented with corrections involving higher powers in the curvature tensor. This is supported by string theory or by conformal anomalies present in all quantum field theories coupled to gravity. From the practical viewpoint, HD gravity endows the effective potential and phase transitions of scalar fields with a wealth of astrophysical and cosmological properties.

In this paper, we have generalized rigid BRST transformations by allowing the transformation parameter to be finite and field-dependent. The expression for the Jacobian presented here has a more solid derivation basis. To calculate the Jacobian, we do not need any local functional satisfying some initial conditions and differential equations. Here, the Jacobian depends on an arbitrary infinitesimal field-dependent parameter. For a given value of the field-dependent parameter, one can easily compute the Jacobian of functional measure under FFBRST transformations. We have implemented such FF-BRST transformations in different HD models. For instance, we employed the FFBRST formalism first in Maxwell theory and found that, for a particular value of the field-dependent parameter, the Jacobian is the source of HD terms in the BRST-exact part of the theory. At the same time, BRST symmetry, in its finite field-dependent form, makes it possible to provide independence for the S-matrix from any particular HD gauge-fixing. That is to say, such a BRST transformation actually preserves the S-matrix and transforms a quantum theory into an equivalent one. To extend this result, we have further studied FFBRST transformations in a non-Abelian theory and in quantum gravity. Here, remarkably, we have

found the previous general results to hold true for these theories as well. Thus, we have mapped different HD theories to the BRST-exact parts of these theories. The HD terms in the quantum action have been generated in a precise form through the Jacobian of functional measure. So, we conclude that the Jacobian of functional measure plays a key role in this treatment.

Even though an HD action is renormalizable by power counting, and, in fact, this renormalizability has been established in full generality, the nature of HD terms in the quantum action requires that one remove some redundancies in gauge degrees of freedom, which are generated through BRST transformations. The present study may be of help in dealing with a theory having UV-divergent terms. It will be interesting to use the results of this paper to establish renormalizability in some models by getting rid of UV divergences.

Recently, the concept of Very Special Relativity (VSR) has been suggested [68]. It is based on the idea that the laws of physics need not be invariant under the full Lorentz group, but rather under its subgroups, which still preserves the basic SR elements, such as the constancy of the speed of light. VSR has been under active investigation by many researchers [69, 70, 71, 72, 73, 74]. It will be interesting to study FFBRST and HD theories in the VSR context.

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