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Bohmnian Trajectories for the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation

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Abstract: In this paper we study the Bohmnian Trajectories for the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation, linearized along a classical trajectory, by using the quantum mechanical formalism of the de Broglie-Bohm.

Keywords: De Broglie-Bohm Quantum Mechanics; Bohmnian Trajectories of the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation

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1. Introduction: The Bohmnian Trajectories

In this article, we calculated the *Bohmnian Trajectories* for the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation. To obtain these trajectories we adopted the quantum mechanical formalism of de Broglie-Bohm. This was done because this formalism permits to perform essential linear approximations along the classical trajectories that are the basic ingredients of the of the Feynman's principle of minimum action of quantum mechanics. [1]

2. The Bohmnian Trajectories for the Süssmann-Hasse-Albrechr-Kostin-Nassar Equation

Now, let us calculate the Bohmnian trajectories for the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation, linearized along a classical trajectory, by using the quantum mechanical formalism of the de Broglie-Bohm. [2]

2.1. The Süssmann-Hasse-Albrecht-Kostin-Nassar Equation

In 1973, D. Süssmann [3] and, in 1975, R. W. Hasse, [4] K. Albrecht, [5] and M. D.

Kostin [6] proposed a non-linear Schrödinger equation, that was generalized by A. B. Nassar, in 1986, [7] to represent time dependent physical systems, given by:

$$i\,\hbar\,\frac{\partial\Psi(x,t)}{\partial t} = -\,\frac{\hbar^2}{2\,m}\,\frac{\partial^2\,\Psi(x,t)}{\partial x^2} +$$

$$+\{\,V(x,t) + \nu\,(\,[x - q(t)]\times[c\;\hat{p} + (1-c) < \hat{p} >] - \frac{i\,\hbar\,c}{2}\,)\,\}\times\Psi(x,t)\ , \quad (2.1.1)$$

where $\hat{p}$ is the operator of linear momentum:

$$\hat{p} = -\,i\,\hbar\,\frac{\partial}{\partial x}, \quad (2.1.2)$$

and c is a constant, where: $c = 1$, for Süssmann; $c = 1/2$, for Hasse; and $c = 0$, for Albrecht and Kostin. Besides $\Psi(x,t)$ and $V(x,t)$ are, respectively, the wavefunction and the time dependent potential of the physical system in study, $q(t) = < x >$, and ν is a constant.

2.1.1. The Wave Function of the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation

Initially, let us write the wave function $\Psi(x,t)$ in the polar form defined by the Madelung-Bohm transformation [8, 9]:

$$\Psi(x,t) = \phi(x,t)\times exp\,[i\,S(x,t)]\ , \quad (2.1.1.1)$$

where $\phi(x,t)$ will be defined in what follows.

Calculating the derivatives, temporal and spatial, of (2.1.1.1), we get [remembering that $exp\,[i\,S]$ is common factor]: [2]

$$\frac{\partial\Psi}{\partial t} = exp\,(i\,S)\,(\,\frac{\partial\phi}{\partial t} + i\,\phi\frac{\partial S}{\partial t}\,)\ , \quad (2.1.1.2a)$$

$$\frac{\partial^2\Psi}{\partial x^2} = exp\,(i\,S)[\,\frac{\partial^2\phi}{\partial x^2} + 2i\,\frac{\partial S}{\partial x}\,\frac{\partial\phi}{\partial x} + i\,\phi\,\frac{\partial^2 S}{\partial x^2} - \phi\,(\,\frac{\partial S}{\partial x}\,)^2\,]\ . \quad (2.1.1.2b)$$

Putting the eqs. (2.1.1.1) and (2.1.1.2a,b) in to eq. (2.1.1) and considering the eq. (2.1.2), we have: [2]

$$i\,\hbar\,(\,\frac{\partial\phi}{\partial t} + i\,\phi\,\frac{\partial S}{\partial t}\,) = -\,\frac{\hbar^2}{2\,m}\,[\,\frac{\partial^2\phi}{\partial x^2} + 2\,i\,\frac{\partial S}{\partial x}\,\frac{\partial\phi}{\partial x} + i\,\phi\,\frac{\partial^2 S}{\partial x^2} - \phi\,(\,\frac{\partial S}{\partial x}\,)^2\,] +$$

$$+\{\,V(x,t) + \nu\,(\,[x - q(t)]\times[\,c\,\hbar\,(\,\frac{\partial S}{\partial x} - \frac{i}{\phi}\,\frac{\partial\phi}{\partial x}\,) +$$

$$+ (1-c) < \hat{p} >\,] - \frac{1}{2}\,i\,\hbar\,c\,)\,\}\times\phi(x,t)\ , \quad (2.1.1.3)$$

Separating the real and imaginary parts of the relation (2.1.1.3), results (remember that $< \hat{p} > = m < \hat{v}_{qu} > = m < v_{qu} > =$ real):

a) imaginary part

$$\frac{\hbar}{\phi}\frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{2}{\phi}\frac{\partial S}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial^2 S}{\partial x^2}\right) - \nu\,[x - q(t)] \times c\,\frac{\hbar}{\phi}\frac{\partial \phi}{\partial x} - \frac{\nu}{2}\hbar c\ , \quad (2.1.1.4)$$

b) real part

$$-\hbar\frac{\partial S}{\partial t} = -\frac{\hbar^2}{2\,m}\left[\frac{1}{\phi}\frac{\partial^2 \phi}{\partial x^2} - \phi\left(\frac{\partial \phi}{\partial x}\right)^2\right] +$$

$$+ \nu\,[x - q(t)] \times c\,\hbar\frac{\partial S}{\partial x} + V(x,t) + \nu\,[x - q(t)] \times (1 - c)\, m < v_{qu} > . \quad (2.1.1.5)$$

2.1.2. Dynamics of the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation

Now, let us see the correlation between the expressions (2.1.1.4-5) and the traditional equations of the Ideal Fluid Dynamics: [10] a) *Continuity Equation*, b) *Euler's equation* (for conservative systems) or b') *Navier-Stokes equation* (for non-conservative systems). To do this let us perform the following correspondences:

Quantum density probability: $|\Psi(x,t)|^2 = \Psi^*(x,t)\,\Psi(x,t) \quad \leftrightarrow$

Quantum mass density: $\rho(x,t) = \phi^2(x,t) \ \leftrightarrow\ \sqrt{\rho} = \phi\ ,$ (2.1.2.1a,b)

Gradient of the wave function: $\frac{\hbar}{m}\frac{\partial S(x,t)}{\partial x} \quad \leftrightarrow$

Quantum velocity: $v_{qu}(x,t) \equiv v_{qu}\ ,$ (2.1.2.1c,d)

Bohm quantum potential:

$$V_{qu}(x,t) \equiv V_{qu} = -\left(\frac{\hbar^2}{2m}\right)\left(\frac{1}{\phi}\right)\frac{\partial^2 \phi}{\partial x^2} = -\frac{\hbar^2}{2\,m}\frac{1}{\sqrt{\rho}}\frac{\partial^2 \sqrt{\rho}}{\partial x^2}\ , \quad (2.1.2.1e,f)$$

Thus, putting the eqs. (2.1.2.1a-f) into the eq. (2.1.1.2), we obtain: [2, 11]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_{qu})}{\partial x} = -\nu\, c\{\rho + [x - q(t)]\frac{\partial \rho}{\partial x}\}, \quad (2.1.2.2)$$

expression that indicates decoherence of the considered physical system represented by the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation ($SHAKN - E$) [eq. (2.1.1)]; then the *Continuity Equation* it is not preserved. To preserved them, let us use the definition: [12]

$$v_{qum} = v_{qu} + \nu\, c\,[x - q(t)], \quad (2.1.2.3)$$

the *quantum velocity non-conservative*.

Now, let us considerer that: [13]

$$\rho(x,t) = [2\,\pi\,a^2(t)]^{-1/2} \times exp\{-\frac{[x-q(t)]^2}{2\,a^2(t)}\} \qquad (2.1.2.4a)$$

or [use eq. (2.1.2.1b)]:

$$\phi(x,t) = [2\,\pi\,a^2(t)]^{-1/4} \times exp\{-\frac{[x-q(t)]^2}{4\,a^2(t)}\} \qquad (2.1.2.4b)$$

where $a(t)$ and $q(t) = < x >$ are auxiliary functions of time, to will be determined in what follows, representing the *width* and the *center of mass of wave packet*, respectively.

Now, considering the eq. (2.1.2.4b), let us write V_{qu} given by eq. (2.1.2.1e) in terms of potencies of [x - q(t)]. Before, we calculate the following derivations [remembering that $\frac{d}{dz}\,exp\,(z) = exp\,(z)$]:

$$\frac{\partial\phi(x,t)}{\partial x} = \frac{\partial}{\partial x}\,(\,[2\,\pi a^2(t)]^{-1/4} \times exp\,\{-\frac{[x-q(t)]^2}{4\,a^2(t)}\,\}\,) =$$

$$= [2\,\pi\,a^2(t)]^{-1/4} \times exp\,\{-\frac{[x-q(t)]^2}{4\,a^2(t)}\,\} \times \frac{\partial}{\partial x}\{-\frac{[x-q(t)]^2}{4\,a^2(t)}\,\} \;\; \rightarrow$$

$$\frac{\partial\phi(x,t)}{\partial x} = -\,[2\,\pi a^2(t)]^{-1/4} \times exp\,\{-\frac{[x-q(t)]^2}{4\,a^2(t)}\,\} \times \{\,\frac{[x-q(t)]}{2\,a^2(t)}\,\} = -\,\phi \times \{\,\frac{[x-q(t)]}{2\,a^2(t)}\,\},$$

$$\frac{\partial^2\phi(x,t)}{\partial x^2} = \frac{\partial}{\partial x}\,(\,-\,\phi \times \{\,\frac{[x-q(t)]}{2\,a^2(t)}\}\,) =$$

$$= -\,\phi\,\frac{\partial}{\partial x}\,\{\frac{[x-q(t)]}{2\,a^2(t)}\,\} - \frac{[x-q(t)]}{2\,a^2(t)}\,\frac{\partial\phi}{\partial x} =$$

$$= -\,\phi \times \frac{1}{2\,a^2(t)} + \frac{[x-q(t)]^2}{4\,a^2(t)} \times \phi \;\; \rightarrow$$

$$\frac{1}{\phi(x,t)}\,\frac{\partial^2\,\phi(x,t)}{\partial x^2} = \frac{[x-q(t)]^2}{4\,a^4(t)} - \frac{1}{2\,a^2(t)}. \qquad (2.1.2.5)$$

Substituting the eq. (2.1.2.5) in the eq. (2.1.2.1e), results:

$$V_{qu}(x,t) = -\,\frac{\hbar^2}{2\,m}\,\{\frac{[x-q(t)]^2}{4\,a^4(t)} - \frac{1}{2a^2(t)}\,\}. \qquad (2.1.2.6)$$

Differentiating the expression (2.1.2.4a) in the variable t, and remembering that

x and t are independent variables, results:

$$\frac{\partial\rho}{\partial t} = -\frac{1}{2}[2\,\pi\,a^2(t)]^{-3/2} \times [4\,\pi\,a(t)\,\dot{a}(t)] \times exp\,\{-\frac{[x-q(t)]^2}{2a^2(t)}\} +$$

$$+[2\,\pi\,a^2(t)]^{-1/2} \times exp\,\{-\frac{[x-q(t)]^2}{2\,a^2(t)}\} \times \frac{\partial}{\partial t}\{-\frac{[x-q(t)]^2}{2\,a^2(t)}\} =$$

$$= -\rho\,\{[2\,\pi\,\dot{a}(t)] \times [2\pi\,a^2(t)]^{-1} + \frac{4\,a^2(t)[x-q(t)] \times [-\dot{q}(t)] - 4a(t)\dot{a}(t) \times [x-q(t)]^2}{4a^4(t)}\} \rightarrow$$

$$\frac{\partial\rho}{\partial t} = \rho\,\{-\frac{\dot{a}(t)}{a(t)} + \frac{\dot{q}(t)}{a^2(t)}[x-q(t)] + \frac{\dot{a}(t)}{a^3(t)}[x-q(t)]^2\}\,. \quad (2.1.2.7)$$

Then multiplying the eq. (2.1.2.2) by $\rho(x,t)$ [eq. (2.1.2.4a)], differenting the result in the variable x and remembering that $\partial q(t)/\partial x = 0$, we have: [11]

$$-\nu\,c\,\{\,\rho\,[x-q(t)]\,\frac{\partial\rho}{\partial x}\,\} = \frac{\partial(\rho\,v_{qu})}{\partial x} - \frac{\partial(\rho\,v_{qum})}{\partial x}\,. \quad (2.1.2.8)$$

Putting the eq. (2.1.2.3) into eq. (2.1.2.2), we obtain:

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho\,v_{qum})}{\partial x} = 0\,, \quad (2.1.2.9)$$

expression that indicates coherence of the $SHAKN-E$.

Now, let us obtained another dynamic equation of the $SHAKN-E$. First, considering that:

$$<\,f(x,t)\,> = \int_{-\infty}^{+\infty} \rho(x,t)\;f(x,t)\,dx = g(t)\,. \quad (2.1.2.10)$$

Using the eqs. (2.1.2.3,4a) and remembering that $\int_{-\infty}^{+\infty} exp(-z^2)\,dz = \sqrt{\pi}$ and $<x> = q(t)$, results:

$$<q(t)> = \int_{-\infty}^{+\infty} \rho(x,t)q(t)\,dx = q(t)\,, \quad (2.1.2.11)$$

$$\frac{\partial <v_{qu}>}{\partial x} = \frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\rho(x,t)\,v_{qu}(x,t)\,dx = \frac{\partial g(t)}{\partial x} = 0\,. \quad (2.1.2.12)$$

By calculating the $<\ >$ of the eq. (2.1.2.3) and considering the eq. (2.1.2.11), we have:

$$<v_{qum}> = <v_{qu}>\,. \quad (2.1.2.13)$$

Now, differentiating the eq. (2.1.1.5) with respect x, and using the eqs. (2.1.2.1c-f) and (2.1.2.2,11-13), we have: [11]

$$\frac{\partial v_{qum}}{\partial t} + v_{qum}\frac{\partial v_{qum}}{\partial x} =$$

$$-\nu\,[c\,(v_{qu} - v_{qum}) + < v_{qum} >] - \frac{1}{m}\frac{\partial}{\partial x}(V + V_{qu}). \quad (2.1.2.14)$$

Considering the *substantive differentiation* (local plus convective) or *hydrodynamic differentiation*: [10]

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_{qu}\frac{\partial}{\partial x}\ , \quad (2.1.2.15a)$$

and that:

$$v_{qu}(x, t)\,|_{x=x(t)} = \frac{dx}{dt}\ , \quad (2.1.2.15b)$$

the eq. (2.1.2.14) could be written as:

$$\frac{d^2x}{dt^2} + \nu\,[c\,(v_{qu} - v_{qum}) + < v_{qum} >] = -\frac{1}{m}\frac{\partial}{\partial x}[\,V(x,t) + V_{qu}(x,t)\,], \quad (2.1.2.16)$$

what has a form of the *Dissipative Second Newton Law*, being the terms of the second member, respectively, the *classical newtonian force* and the *quantum bohmnian force*.

2.1.3 The Quantum Wave Packet of the Linearized Süssmann-Hasse-Albrecht-Kostin-Nassar Equation along a Classical Trajetory

In order to find the quantum wave packet of the linearized Süssmann-Hasse-Albrecht-Kostin-Nassar Equation ($SHAKN-E$) along a trajectory classic, we must integrated the eq. (2.1.2.9). Then, we have: [11]

$$v_{qum}(x, t) = \frac{\dot{a}(t)}{a(t)}[x - q(t)] + \dot{q}(t). \quad (2.1.3.1)$$

Considering the eqs. (2.1.2.3) and (2.1.3.1), results:

$$v_{qu}(x, t) \equiv \frac{dx(t)}{dt} = [\frac{\dot{a}(t)}{a(t)} - \nu\ c\ ] \times [x - q(t)] + \dot{q}(t). \quad (2.1.3.2)$$

We observe that the integration of the eq. (2.1.3.2) given the *bohmnian quantum trajectory* of the physical system represented by $SHAKN-E$.

To obtain the quantum wave packet [$\Psi(x\,,t)$] of the $SHAKN-E$ given by eq. (2.1.1), let us expand the functions S(x, t), V(x t), and V_{qu}(x, t) around of $q(t) = < x >$ up to *second Taylor order*. [13] In this way, we have:

$$S(x, t) = S[q(t), t] + S'[q(t), t] \times [x - q(t)] + \frac{1}{2}S''[q(t)\ t] \times [x - q(t)]^2, \quad (2.1.3.3)$$

$$V(x, t) = V[q(t), t] + V'[q(t), t] \times [x - q(t)] +$$

$$+\frac{1}{2}V''[q(t),t]\times[x-q(t)]^2, \qquad (2.1.3.4)$$

$$V_{qu}(x,t)=V_{qu}[q(t),t]+V'_{qu}[q(t),t]\times[x-q(t)]+$$

$$+\frac{1}{2}V''_{qu}[q(t),t]\times[x-q(t)]^2, \qquad (2.1.3.5)$$

where (′) and (") means, respectively: $\frac{\partial}{\partial q}$ and $\frac{\partial^2}{\partial q^2}$.

Differenting the eq. (2.1.3.3) in the variable x, multiplying the result by $\hbar/m$ and using the eqs. (2.1.2.1c,d) and (2.1.3.2), results:

$$\frac{\hbar}{m}\frac{\partial S(x,t)}{\partial x}=\frac{\hbar}{m}\{\, S'[q(t),t]+S''[q(t),t]\times[x-q(t)]\,\}=v_{qu}(x,t)=$$

$$=[\,\frac{\dot{a}(t)}{a(t)}-\nu\, c\,]\times[x-q(t)]+\dot{q}(t) \quad\to$$

$$S'[q(t),t]=\frac{m\,\dot{q}(t)}{\hbar}\,,\;\; S''[q(t),t]=\frac{m}{\hbar}[\,\frac{\dot{a}(t)}{a(t)}-\nu\, c\,]\;. \qquad (2.1.3.6a,b)$$

Substituting the eqs. (2.1.3.6a,b) into eq. (2.1.3.3), we have:

$$S(x,t)=S_0(t)+\frac{m\,\dot{q}(t)}{\hbar}\times[x-q(t)]+\frac{m}{2\,\hbar}[\frac{\dot{a}(t)}{a(t)}-\nu\, c]\times[x-q(t)]^2\,, \qquad (2.1.3.7a)$$

where:

$$S_o(t)\equiv S[q(t),t] \qquad (2.1.3.7b)$$

are the *quantum action*.

Differenting the eq. (2.1.3.7a) in relation to the time t and using the eq. (2.1.3.2), results (remember that $\partial x/\partial t=0$): [11]

$$\frac{\partial S(x,t)}{\partial t}=\dot{S}_o(t)-\frac{m\,\dot{q}^2(t)}{\hbar}+\frac{m}{\hbar}\{\,\ddot{q}(t)-\dot{q}(t)\,[\,\frac{\dot{a}(t)}{a(t)}-\nu\, c\,]\}\times[x-q(t)]+$$

$$+\frac{m}{2\,\hbar}[\,\frac{\ddot{a}(t)}{a(t)}-\frac{\dot{a}^2(t)}{a^2(t)}\,]\times[x-q(t)]^2\,. \qquad (2.1.3.8)$$

Using the eqs. (2.1.2.1c-e) and (2.1.1.3), we obtain:

$$-\hbar\frac{\partial S}{\partial t}=-\frac{\hbar^2}{2\,m}[\,\frac{1}{\phi}\frac{\partial^2\phi}{\partial x^2}-\phi\,(\frac{\partial S}{\partial x})^2\,]+V(x\,,t)+\nu\,[x-q(t)]\times c\,\hbar\,\frac{\partial S}{\partial x}+$$

$$+\nu\,[x-q(t)]\,(1-c)\,m<v_{qu}> \quad\to$$

$$\hbar \frac{\partial S}{\partial t} + \frac{1}{2} m\, v_{qu}^2 + V(x,t) + V_{qu}(x,t) +$$

$$+ \nu\, m\, [x - q(t)] \times [c\, v_{qu} + (1-c) < v_{qu} >] = 0 . \quad (2.1.3.9)$$

Considering the eqs. (2.1.2.6,13), (2.1.3.2,4,5,6a,b) and inserting into eq. (2.1.3.9), ordering the result in potencies of $[x - q(t)]$, and considering that $[x - q(t)]^0 = 1$, we have: [11]

$$\{\hbar\, \dot{S}_o(t) - \frac{m}{2}\, \dot{q}^2(t) + \nu\, m\, (1-c)\, \dot{q}(t) + V[q(t),t] + \frac{\hbar^2}{4\, m\, a^2(t)}\} \times [x - q(t)]^0 +$$

$$+ \{m\, \ddot{q}(t) + m\, \nu\, c\, \dot{q}(t) + V'[q(t),t]\} \times [x - q(t)] +$$

$$+ \{\frac{m}{2} \frac{\ddot{a}(t)}{a(t)} - \frac{m\, \nu^2\, c^2}{2} + \frac{1}{2} V''[q(t),t] - \frac{\hbar^2}{8\, m\, a^4(t)}\} \times [x - (q)t)]^2 = 0 . \quad (2.1.3.10)$$

As the above relation [eq. (2.1.3.10)] is an identically null polynomium, the coefficients of the potencies must be all equal to zero, that is:

$$\dot{S}_o(t) = \frac{1}{\hbar} \{ \frac{m}{2}\, \dot{q}^2(t) - V[q(t),t] - \nu\, m\, (1-c)\, \dot{q}(t) - \frac{\hbar^2}{4\, m\, a^2(t)}\} , \quad (2.1.3.11)$$

$$\ddot{q}(t) + \nu\, c\, \dot{q}(t) + \frac{1}{m} V'[q(t),t] = 0 , \quad (2.1.3.12)$$

$$\ddot{a}(t) + \{ \frac{1}{m} V''[q(t),t] - \nu\, c^2 \} \times a(t) = \frac{\hbar^2}{4\, m^2\, a^3(t)} . \quad (2.1.3.13)$$

Now, let us consider that V[q(t), t] is given by:

$$V[q(t),t] = \frac{1}{2}\, m\, \omega^2(t)\, q^2(t) , \quad (2.1.3.14)$$

which is the Time Dependent Harmonic Oscillator Potencial).
In this case, we have:

$$V'[q(t),t] = m\, \omega^2(t)\, q(t), \quad V''[q(t),t] = m\, \omega^2(t) . \quad (2.1.3.15a,b)$$

Insering the eqs. (2.1.3.15a,b) into eqs. (2.1.3.12,13), results:

$$\ddot{q}(t) + \nu\, c\, \dot{q}(t) + \omega^2(t) q(t) = 0 , \quad (2.1.3.16)$$

$$\ddot{a}(t) + \{ \omega^2(t) - \nu\, c^2\} \times a(t) = \frac{\hbar^2}{4\, m^2 a^3(t)} . \quad (2.1.3.17)$$

2.1.4. The Bohmnian Trajectories for the Süssmann-Hasse-Albrecht-Kostin-

Nassar Equation

The associated Bohmnian Trajectories, [14]-[17] for the Süssmann-Hasse-Albrecht-Kostin-Nassar Equation ($SHAKN-E$) of an evolving *ith* particle of the ensemble with an initial position x_{0i} can be calculated by considering that:

$$\frac{dx_i(t)}{dt} = v_{qu}[x_i(t), t]. \quad (2.1.4.1)$$

Then substituting the eq. (2.1.4.1) into eq. (2.1.3.2), results:

$$\frac{dx_i(t)}{dt} = [\,\frac{\dot{a}(t)}{a(t)} - \nu\, c\,] \times [x - q(t)] + \dot{q}(t) \rightarrow$$

$$x_i(t) = \int_{t_0}^{t} \{\, [\,\frac{\dot{a}(t)}{a(t)} - \nu\, c\,] \times [x - q(t)] + \dot{q}(t)\,]\,\}\, dt. \quad (2.1.4.2)$$

The eqs. (2.1.3.16,17) show that a continuous measurement of a quantum wave packet gives specific features to its evolution: the appearance of distinct classical and quantum elements, respectively. This measurement consist of monitoring the position of quantum systems and the result is the measured classical path $q(t)$ for t within a quantum uncertainty $a(t)$.

From the eq. (2.1.3.17), we note that a stationary regime can be reached and that the width $[\,a(t)\,]$ of the wave packet can be related to the resolution of measurement as follows. Then considering that $a(t) = cte$ $[\,\dot{a}(t) = 0\,;\ a_{t_0} = a_0\,]$ in the eq. (2.1.3.17) and considering the t_0 the $initial\ time$, we have:

$$\omega_0^2(t_0) = \frac{\hbar^2}{4\, m^2 a_0^4(t_0)} \quad \rightarrow \quad \omega_0 = (\frac{1}{\tau_B})\ , \quad (2.1.4.3a)$$

where [13, 18]:

$$\tau_B = (\,\frac{2\, m\, a_0^2}{\hbar}) = 6{,}8 \times 10^{-26} s\,, \quad (2.1.4.1.3b)$$

is the *Bohmtime constant* which determines the time resolution of the quantum measurement.

3. Conclusion

The eqs. (2.1.4.3a,b) means that if an initially free wave packet is kept under a certain continuous measurement, its (a_0) may not spread in time. Then, the associated *Bohmnian Trajectories* of an evolving *ith* particle of the ensemble with an initial position x_{0i} is giving by the eq. (2.1.4.2).

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