Diffusion of spherical metallic particles in a viscous fluid submitted to electric and gravitational fields.

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Abstract. We study the motion of spherical metallic particles in a fluid submitted to viscous, random, electric and gravitational forces. Are determined conditions for the average quadratic displacement" $\delta(t)^2$ between particles be given by $\delta(t)^2 = < x^2> - < x >^2 = 2Dt$, where D is the diffusion coefficient.

Key words: diffusion of metallic particles; viscous fluids; electric and gravitational fields.

(I)Introduction.

This is a didactical paper written to postgraduate students of Physics and Engineering**.** Taking into account the Langevin equation within the stochastic context[1-3] we analyze the motion of spherical metallic particles in a viscous medium. These particles are submitted to viscous $F_{visc}(t)$, stochastic $F_a(t)$, gravitational and electric forces. Are determined necessary conditions for the "average" quadratic displacement" $\delta(t)^2$ between particles be given by $\delta(t)^2 = < x^2> - < x >^2 = 2Dt$, where D is the diffusion coefficient. In **Section 1** is written the Langevin equation[1-3] for a particle with mass m submitted simultaneously to a viscous $F_{visc}(t)$, a stochastic $F_a(t)$ and a generic static force F. We first study the case when $F_{visc}(t) \neq 0$, $F_a(t) \neq 0$ and $F = 0$; after when $F_{visc}(t) \neq 0$, $F_a(t) = 0$ and $F \neq 0$. In **Section 2** is seen the case when the fluid is in gravitational field and $F= F_b$ is the buoyancy force. In **Section 3** is studied the case when the viscous force $F_{visc}(t) = \gamma(dx/dt)$, where $\gamma = (\alpha/m)$, is very large, the mass is very small and $F/\gamma = f(x) \neq 0$. In **Section 4** is analyzed the case when the medium is submitted to an uniform electric field.

(1) Diffusion of particles in a viscous medium.

According to the stochastic theory,[1-3] the motion of a particle with mass m in a viscous medium is governed, by the Langevin equation,

$$md^2x/dt^2 = - \alpha(dx/dt) + F + F_a(t) \qquad (1.1),$$

where α is the viscosity coefficient, $F_{visc}(t) = \alpha(dx/dt)$, $F_a(t)$ the stochastic force and F an static applied force. This equation can also be written as,

$$dv/dt = - \gamma v + f + \zeta(t) \qquad (1.2),$$

where $\gamma = \alpha/m$, $f = F/m$ and the noise function $\zeta(t) = F_a(t)/m$ obeys the following properties

$$< \zeta(t) > = 0, \qquad (1.3)$$

and

$$< \zeta(t)\, \zeta(t') > = \Gamma\delta(t - t') \qquad (1.4).$$

(1.a)Diffusion coefficient D when F = 0 (Brownian diffusion).

When f = 0 Eq.(1.2) becomes,

$$dv/dt = - \gamma v + \zeta(t) \qquad (1.a.1).$$

Solving (1.a.1), in the stationary regime, we get ,[1,2] $< v^2 > = \Gamma/2\gamma$. Since $m<v^2>/2 = k_BT/2$ it is shown that $\Gamma = 2\gamma k_BT/m$. We verify that the "average quadratic displacement" $\delta^2 = < x^2> - < x >^2$ is given by[1,2]

$$\delta^2 = < x^2> - <x>^2 = 2Dt \qquad (1.a.2),$$

If the viscous coefficient $\alpha = 6\pi\eta a$ the "diffusion coefficient" D becomes,

$$D = k_BT/\alpha = k_BT/6\pi\eta a \qquad (1.a.3).$$

Note that to obtain these results it is assumed that in the stationary regime the particles are in thermal equilibrium with the fluid, that is, $m<v^2>/2 = k_BT/2$. So, the average viscous force $< F_{viscous} >$ on the particles is given by $< F_{viscous} > = \alpha < v > = 6\pi\eta a(k_BT/m)^{1/2}$.

(1.b) $F \neq 0$ and noise force $F_a(t) = 0$.

In these conditions Eq.(1.1) is given by

$$mdv/dt = - \alpha v + F \qquad (1.b.1),$$

which is studied in basic text books.[4] When the initial velocity is zero Eq.(1.b.1) gives,

$$v(t) = (F/\alpha)\,(1 - e^{-\alpha t/m}) \qquad (1.b.2).$$

In the stationary regime, that is, for $t \to \infty$, we see that $v_{stat} = F/\alpha$ and that there is no Brownian diffusion.

(1.c) $F_{visc} \neq 0$, $F \neq 0$ and $F_a(t) \neq 0$.

In this case Eq.(1.2) is written as,

$$dv/dt = - \gamma v + f + \zeta(t) \qquad (1.c.1).$$

So, if $f \neq 0$ the particle would be always accelerated. Thus, taking into account Section (1.a), to have Brownian diffusion described by Eqs.(1.a.1) and (1.a.2) it would be necessary that $\gamma\, v >> f$, that is, $\alpha v >> F$. Since $< v > = (k_B T/m)^{1/2}$ a good estimation to get this condition is that

$$6\pi\eta a(k_B T/m)^{1/2} >> F \qquad (1.c.2),$$

that is, when $< F_{viscous} > >> F$.

(2)Particles in a gravitational field.

Now, let us consider particles with density ρ_p, immersed in medium with density ρ_m, submitted to a gravitational field g. In this case F would be the buoyancy force F_b:

$$F_b = (4\pi a^3/3)\{\rho_p - \rho_m\}g \qquad (2.1).$$

So, according to **Section (1.c)**, to have particles diffusion it would be necessary that $< F_{viscous} > \; >> F_b$, that is,

$$6\pi\eta a(k_B T/m)^{1/2} >> F_b \qquad (2.2).$$

That is, the viscosity coefficient η must be obey the condition;

$$\eta >> (m/k_B T)^{1/2}(F_b/6\pi a) \qquad (2.3).$$

(3)Fokker- Planck equation: diffusion and drift.

An illustrative case is when the viscous force γ (dx/dt) is very large, the mass is very small and $F/\gamma = f(x) \neq 0$. In this case Eq.(1.2) becomes,

$$dx/dt \approx f(x) + \zeta(t) \qquad (3.1),$$

remembering that v = dx/dt and f(x) = F(x)/m.

According to reference[1], associated with Eq.(3.1), we have the **Fokker-Planck equation,**

$$\partial P(x,t)/\partial t = -\partial[f(x)P(x,t)]/\partial x + (\Gamma/2)\partial^2 P(x,t)/\partial t^2 \qquad (3.2),$$

that gives the temporal evolution of the probability density P(x,t) that represents the distribution probabilities of the stochastic variable x obtained solving Eq.(3.1). In Eq.(3.1), $< \zeta(t) > = 0$ and Γ is defined by the equation $< \zeta(t)\, \zeta(t') > = \Gamma\, \delta(t - t')$.

When f(x) = constant = c, solving Eq.(3.2), we obtain[1]

$$P(x,t) = (1/2\pi\Gamma t)^{1/2} \exp\{-(x - x_o - ct)^2/2\Gamma t\} \qquad (3.3).$$

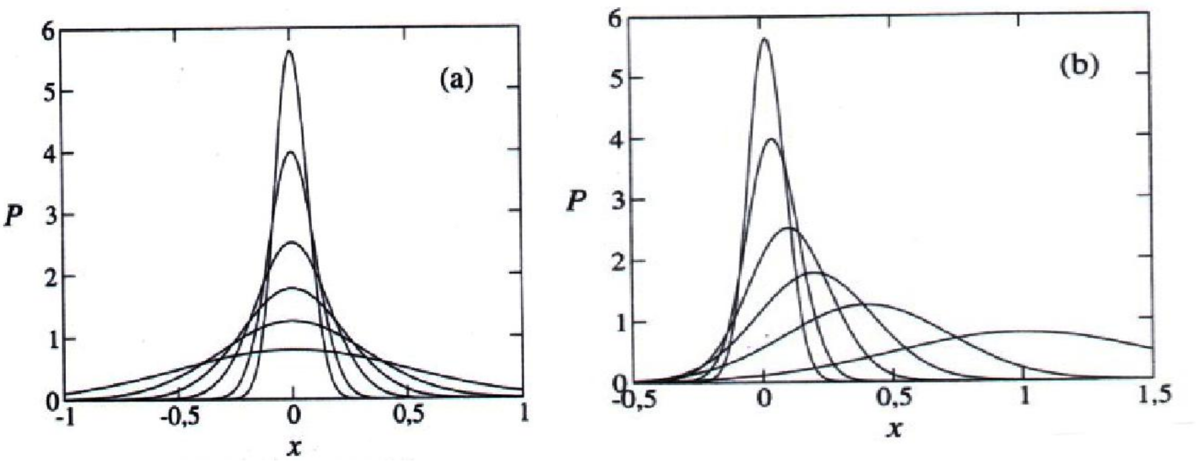

Figure 1. P(x,t) as a function of x and t to the Brownian motion. (a)Symmetric (c = 0) and (b) asymmetric with "drift" at right (c > 0). These figures shown that when c = o there is **only** diffusion described by $\delta = (2\Gamma t)^{1/2}$. On the other hand when $c \neq 0$ there is, simultaneously, diffusion and "drift" of the particles, with velocity c, induced by the external force F.

(4) Spherical metallic particles in uniform electric field E_o.

If the viscous medium, where are immersed metallic spheres, with thickness **d** is submitted to an electrical potential difference V the average electric uniform field E_o in the medium would be given by $E_o = V/d$.

This electric field E_o (**see Figure 2**) would create on the metallic spherical surfaces an induced charge density $\sigma(\theta)$ given by[5]

$$\sigma(\theta) = 3\varepsilon_o E_o \cos\theta \qquad (4.1).$$

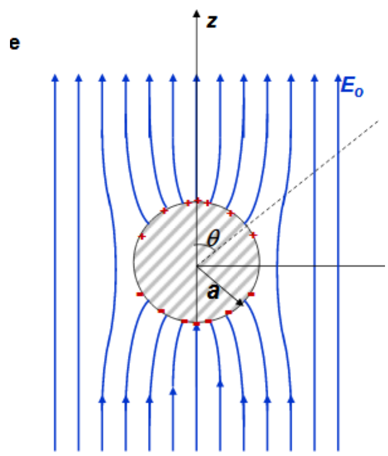

Figure 2. Spherical metallic particle in an uniform electric field E_o.

For the spherical metallic particle with radius a the induced charge Q is,

$$Q = 2\pi a^2 < \sigma(\theta) > \sim 3\pi a^2 \varepsilon_o E_o \qquad (4.2).$$

As the electron charge is 10^{-19} C and $\varepsilon_o = 8.85\ 10^{-12}$ SI, the number of induced electrons is given by

$$N \sim 8.3\ 10^7\ a^2\ E_o \qquad (4.3)$$

When N<< 1 the multipolar electric interactions between metallic particles can be neglected. Consequently, in the viscous medium there would be no electric forces beyond that created by the applied field E_o. For sufficiently small E_o values electrostriction effects can also be neglected.[6]

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