Stochastic Phenomenon:Autocorrelation Function Approach

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Abstract. It is shown how the autocorrelation function theory based on the Wiener-Khintchine Theorem (WKT) is used to analyze stochastically fluctuating phenomena. This theory might be called "time-dependent statistical mechanics" since it permits to describe fluctuations that are outside the scope of the equilibrium statistical mechanics. It is of prime importance in the investigations of noise problems. Are analyzed here the Drag and Brownian motion and Electric noises. It will be also briefly shown how these phenomena can be understood taking into account the Fluctuation-Dissipation Theorem (FDT).
Key words: stochastically fluctuating phenomena; noise.

(I) Introduction.

According to the Wiener-Khintchin Theorem (WKT) the autocorrelation function of a wide-sense-stationary random process has a spectral decomposition given by the power spectrum of that process.[1-3]

Here we show the main aspects of this autocorrelation function approach[4] which is used to analyze stochastically fluctuating phenomena. To do this, let us consider a random function of time y(t) of some stochastically fluctuating system under observation. In **Figure 1** is seen y(t), measured in a time interval τ, as a function of the time t

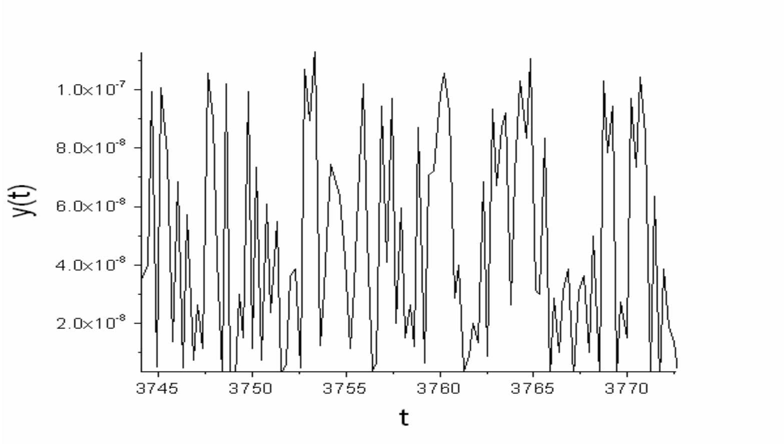

Figure 1. Stochastic y(t) as function of the time t.

Assuming that y(t) can be expanded using a Fourier integral[4] we get:

$$y(t) = \int_o^\infty a(f)\cos(ft)df + \int_o^\infty b(f)\sin(ft)df \qquad (I.1),$$

where a(f) and b(f) are given by

$$a(f) = \int_o^\infty I(t)\cos(ft)\,dt \quad \text{and} \quad b(f) = \int_o^\infty I(t)\sin(ft)dt .$$

In the time interval τ, during which y(t) is measured, is defined the **correlation function**:[4]

$$\psi_y(\tau) = < y(t)\, y(t+\tau) > \qquad (I.2),$$

where the average value < ... > is estimated taking into account a large number of systems in thermodynamic equilibrium for different instants of time t. Here we are assuming the validity of the "ergodic theorem", that is, that the time average for a single system in statistical equilibrium may be replaced by an average over an equilibrium ensemble.[4] In this sense, t is the "ergodic time". Defining the **"noise power spectrum"** $\omega_y(f)$ by

$$\omega_y(f) = 4\int_o^\infty \psi_y(\tau)\cos(2\pi f\tau)\,d\tau \qquad (I.3),$$

we see that

$$\psi_y(\tau) = \int_o^\infty \omega_y(f)\cos(2\pi f\tau)\,df \qquad (I.4).$$

Taking into account Eq.(I.3) one can show that[4]

$$< y^2> = \int_o^\infty \omega_y(f)df \quad \text{and, consequently, that} \quad < \delta y_f^2 > = \omega_y(f)\,df . \quad (I.5),$$

where δy_f^2 gives the y^2 fluctuations due to frequencies that are in the interval between f and f+df.

Equations (I.3) and (I.4) together comprise what is known as the Wiener-Khintchine Theorem[1-4] which is of prime importance in the analysis of noise problems. In **Figure 2**[4] are shown sketches of two different stochastic noises y(t) and the respective functions $\omega_y(f)$ and $\psi_y(f)$.

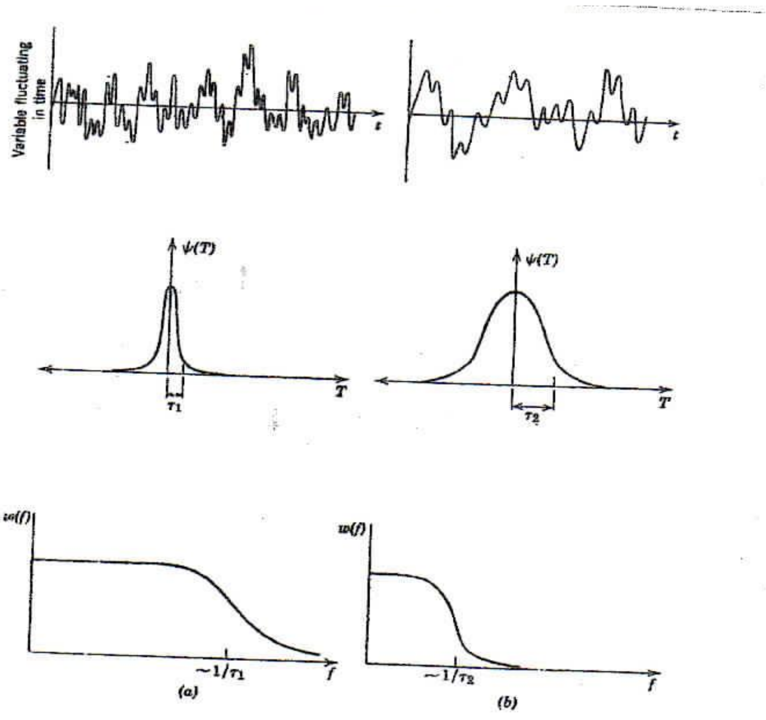

Figure 2. Sketches of noise, autocorrelation function $\psi_y(f)$ and power spectrum $\omega_y(f)$ for two types (a and b) of statistically stationary fluctuations.

Defining
$$F_y(\tau) = \int_o^{t+\tau} y(t)\, dt \qquad (I.6)$$
we can show that[4]
$$< F_y^{\,2}(\tau)> = (1/2\pi^2) \int_o^{\infty} [\omega_y(f)/f^2]\{1- \cos(2\pi f\tau)\}df \qquad (I.7)$$
and that
$$\partial< F_y^2(\tau)>/\partial\tau = (1/\pi) \int_o^{\infty} [\omega_y(f)/f] \sin(2\pi f\tau)]\, df \qquad (I.8),$$

and hence by inversion,
$$\omega_y(f) = 4\pi f \int_o^{\infty} \partial< F_y^2(\tau)>/\partial\tau \sin(2\pi f\tau)\, d\tau \qquad (I.9).$$

In the sequence the above approach is used to analyze many stochastic phenomena . In **Section 1**, the Brownian motion. In **Section 2,** the electric RL circuit. In **Section 3,** the electric RC circuit. In **Section 4,** the two resistors Johnson-Nyquist electric circuit. In **Section 5**, complex electrical circuits with impedance Z(f). In **Section 6**, a circuit with R and a frequency amplifier. In **Appendix A** is shown the estimation of $< v^2(\tau) >$ and $< x^2(\tau) >$ for Brownian motion and, finally, in **Appendix B** is commented how these phenomena can be interpreted taking into account the Fluctuation-Dissipation Theorem.

(1) Brownian motion.

As well known, the motion of a "***Brownian particle***" with mass M is governed by the Langevin´s equation[4, 5]

$$M dv/dt = - Bv(t) + F(t) \qquad (1.1),$$

where B is the dissipative parameter and F(t) the stochastic force. This equation can be also written as

$$dv/dt + v/\tau_1 = A(t) \qquad (1.2),$$

where τ_1 is the "relaxation time" given by $\tau_1 = M/B$ and $A(t) = F(t)/M$.

Average velocity fluctuations $< v^2(\tau) >$.

Solving Langevin´s equation (1.1) we have v(t) given by[4]

$$v(t) = v_o\, e^{-t/\tau 1} + e^{-t/\tau 1} \int_o^t e^{u/\tau 1}\, A(u)\, du \qquad (1.3).$$

Following MacDonald[4] one see that Eq.(I.2) is given by

$$\psi_v(\tau) = <v(t)v(t+\tau)> = <v_o^2>\, e^{-(2t+\tau)/\tau 1} + e^{-(2t+\tau)/\tau 1} \int_o^t\int_o^{t+\tau} e^{(u+\omega)} <A(u)A(\omega)>\, du\, d\omega$$

$$= <v_o^2>\, e^{-(2t+\tau)/\tau 1} + (kT/M)\, e^{-\tau/\tau 1}(1 - e^{-2t/\tau 1}) \qquad (1.4).$$

Note that the time t corresponds to the *observation time in the ensemble.* As $\psi_v(\tau)$, defined by (I.2), which is evaluated for an equilibrium ensemble, we can assume that $< v_o^2> = kT/M$. Thus, for a sufficiently long time-average $t\rightarrow\infty$ we have

$$\psi_v(\tau) = (kT/M)\, e^{-\tau/\tau 1} \qquad (1.5).$$

So, according to WKT, $\omega_v(f)$ defined by (I.3) is given by[4]

$$\omega_v(f) = (4kT/M) \int_o^\infty e^{-\tau/\tau 1} \cos(2\pi f\tau)\, d\tau ,$$

that is,

$$\omega_v(f) = (4kT\tau_1/M)\ \{1/[1+(2\pi f\tau_1)^2]\} \qquad (1.6),$$

showing clearly how the **velocity fluctuations** δv_f^2 of the particle would be distributed in frequencies f. As $M \rightarrow 0$, that is, $\tau_1 \rightarrow 0$ the "power" spectrum of the velocity fluctuations becomes uniform, that is,

$$\omega_v(f) \approx 4kTB \qquad (1.7),$$

which corresponds to the "perfect random motion".

According to Eqs.(I.6) and (1.6) it can be verified that[4]

$$< \delta v_f^2 > = \omega_v(f)\, df = (4kTB)\ \{1/[1+(2\pi f\tau_1)^2]\}df \qquad (1.8).$$

For time intervals $\tau >> \tau_1$, that is, for frequencies $f\tau_1 <<1$, Eq.(1.8) gives,

$$< \delta v_f^2 > \approx 4kBT\, df \qquad (1.9).$$

The total average squared velocity **fluctuations** $< v^2 >$, that is, due to all frequencies f, is given by

$$< v^2 > = \int_o^\infty < \delta v_f^2 > df = \int_o^\infty \omega_v(f)\, df$$

$$= 4kTB \int_o^\infty df/[1+ (2\pi f\tau_1)^2]$$

$$= 4KT(\tau_1/M)[1/4\tau_1] = kT/M \qquad (1.10).$$

which is the equilibrium average squared velocity, according to the ("equilibrium") Maxwell-Boltzmann statistical mechanics. If we have a measuring instrument with a limiting response time $\tau << \tau_1$, then the observed fluctuations would be

$$< v^2 > \approx 4kTB \int_o^{1/\tau} df/[1+ (2\pi f\tau_1)^2] \approx (4kT/M)(\tau_1/\tau) \qquad (1.11).$$

In other words, for times $\tau << \tau_1$ the observed velocity (1.11) would be increased roughly by ratio $(\tau_1/\tau)^{1/2}$ as compared with value $[\sim(kT/M)^{1/2}]$, predicted by the "equilibrium" statistical mechanics.[4,5]

Average displacement fluctuations $< x^2(\tau) >$.

If we put y(t) = v(t) we verify from Eq.(I.6) that $F_v(\tau)$ is the mean square displacement x(t) of the particle in interval τ, that is, $< x^2(\tau) >$. So, from Eq.(I.7) we have,

$$< x^2(\tau) > = (1/2\pi^2) \int_o^\infty [\omega_v(f)/f^2]\{1- \cos(2\pi f\tau)\}df$$

$$= 2kTB\tau - (2kT\tau_1^2/M)\ (1 - e^{-\tau/\tau 1}) \qquad (1.12),$$

For long time intervals of observation $\tau >> \tau_1$ we verify that

$$< x^2(\tau) > \;\rightarrow 2kTB\tau = 2D\tau \qquad (1.13),$$

where D = kTB is the "diffusion coefficient", which is the familiar Einstein equation characteristic of the random Brownian movement.[4,5]. On the other hand for $\tau \leq \tau_1$ we obtain

$$< x^2(\tau) > \;\approx v_o^2\tau^2 , \quad \text{where} \quad v_o = (kT/M)^{1/2} \qquad (1.14),$$

which describes the *motion of a free molecule*.[4,5]

Thus, for **observation times** τ comparable with the relaxation time τ_1 we have two different results for particle displacements (see **Appendix A).**

(2) Electrical Circuit RL.

The RL circuit obeys the Langevin´s equation LdI/dt + RI = E(t), that is,[4,5]

$$dI/dt + I/\tau_2 = A(t), \qquad (2.1),$$

where E(t) is a fluctuating voltage, A(t) = E(t)/L and τ_2 = L/R the relaxation time, characteristic of the RL circuit.

The charge current I(t) obeys an equation (2.1) that is formally equivalent to Eq.(1.2). In this case we have, instead of Eq.(1.5), $\psi_I(\tau) = (kT/L)\, e^{-\tau/\tau 2}$, where is taken into account an average over an equilibrium ensemble of RL systems. Thus, following the same procedure adopted in **Section 1** we verify that the "**current noise**" in the frequency interval df is given by

$$< \delta I_f^2>_{RL} = (4kT/R)df/[1+ (2\pi f\tau_2)^2] \qquad (2.2).$$

So, the **current noise** in the RL circuit $< I^2 >_{RL}$ would be given by

$$< I^2 >_{RL} = \int_o^\infty < \delta i_f^2> df = (4kT\tau_2/L) \int_o^\infty df/[1+ (2\pi f\tau_2)^2] = (4kT\tau_2/L)[1/4\tau_2]$$

that is,

$$< I^2 >_{RL} = kT/L \qquad (2.3).$$

(3) Electrical Circuit RC.

The RC circuit obeys the Langevin´s equation C dV/dt + V/R = E(t), that is,[6]

$$dV/dt + V/\tau_3 = F(t), \qquad (3.1),$$

where E(t) is a fluctuating voltage, F(t) = E(t)/C and τ_3 = RC the relaxation time characteristic of the RC circuit. Following the same procedure used in **Section 1** and **2** we see that $\psi_V(\tau) = (kT/C)\, e^{-\tau/\tau_3}$ and that

$$< \delta V^2>_{RC} = (4kT\tau_3/C)df/[1+ (2\pi f\tau_3)^2] \qquad (3.2),$$

which is the voltage noise in the frequency interval df.

Thus, the **voltage noise** in RC circuit $< V^2 >_{RC}$ would be given by

$$< V^2 >_{RC} = \int_o^\infty < \delta V_f^2> df = (4kT\tau_3/C) \int_o^\infty df/[1+ (2\pi f\tau_2)^2] = (4kT\tau_3/C)[1/4\tau_3]$$

That is,

$$< V^2 >_{RC} = kT/C \qquad (3.3).$$

Note that, according to the **Equipartition Theorem of Energy** (ETE) we must have

$$C < V^2 >/2 = L < I^2 >/2 = kT/2, \quad \text{that is,}$$

$$< I^2 >_{RL} = kT/L \quad \text{and} \quad < V^2 >_{RC} = kT/C \qquad (3.4).$$

Eqs.(3.4) are used by many authors to estimate voltage and current noises in RL and RC circuits,[6-9] respectively.

(4) Two resistors Johnson-Nyquist Circuit.

In his original paper,[6] Nyquist analyzed a circuit formed *only* by two ideal resistors in parallel, with resistance R each one, at a temperature T, connected by a long non-dissipative transmission line. He has shown that the resistance R generates a **voltage noise** $< V^2 >_R$, given by

$$< V^2 >_R = 4kTR \qquad (4.1),$$

and a **current noise**

$$< I^2 >_R = < V^2 >_R /R^2 = 4kT/R \qquad (4.2).$$

Nyquist´s formula[6] is essentially the same as that derived by Planck in 1901 for the electromagnetic radiation of a black body in one dimension i.e., it is the one-dimension version of the blackbody Planck´s law.[8] In other words, a hot resistor will create electric energy on a transmission line just like a hot object in free space. The electric current in the circuit, are generated by the thermal energy stored in a body at temperature T obeying by Planck distribution. So, indicating by hf df the electric power, with frequencies in the interval f and f + df, transferred to the circuit by each one of the resistors it would be given by

$$hfdf = (I_{circuit})^2 Rdf = [(\delta V_f{}^2)_R/4R^2]R \qquad (4.3),$$

where $(\delta V_f)_R$ is the resistance noise voltage in the interval f and f + df. In this way we see that

$$(\delta V_f{}^2)_R = 4R\ hf\ df \qquad (4.4).$$

Taking into account the radiation Planck´s law the **"voltage noise"** $< V^2 >_R$, due to all degrees of freedom f, would be given by

$$< V^2 >_R = \int_o^\infty (\delta V_f{}^2)_R P(f)\ df = 4R \int_o^\infty hfdf /(e^{hf/kT} - 1) = 4RkT,$$

according to Eq.(4.1). For **high temperatures**, that is, **kT >> hf** we see that

$$(\delta V_f{}^2)_R \approx 4RkT\ df \qquad (4.5),$$

that is, the "voltage noise" $(\delta V_f)_R{}^2$ does not depend of the frequency f. It is known as a **"white noise".**

(5) Complex Electrical Circuits with Impedance Z(f).

Nyquist´s original paper[6] also provided the generalized noise for components having partly reactive response, e.g., sources that contain capacitors or inductors. Such component can be described by a frequency-dependent impedance Z(f). He has shown that, over a span of frequencies f_1 and f_2 the **voltage noise** $< V^2 >$ will be given by [4,7]

$$< V^2 > = \int_{f_1}^{f_2} S_v(f)\ df \qquad \text{where} \qquad S_v(f) = 4kT\ \eta(f)\ Re[Z(f)] \qquad (5.1),$$

and $\eta(f) = (hf/kT) /(e^{hf/kT} - 1)$.

In what follows it will be analyzed only circuits at high temperatures, that is, when kT >> hf and in which electric currents are generated by an external voltage. Thus, putting $\eta(f) \approx 1$ in Eq.(5.1) we have

$$(\delta V_f^2)_z = 4kT\ Re[Z(f)]. \qquad (5.2).$$

Alternatively, the noise current $< I^2 >$ could be obtained by[4,7-10]

$$S_I(f) = 4kT\ Re[Y(f)], \quad \text{where the } admittance\ Y(f) = 1/Z(f),$$

that is,

$$(\delta I_f^2)_z = S_I(f)\,df \qquad \text{and} \qquad < I^2 >_z = 4kT \int_{f_1}^{f_2} Re[Y(f)]\,df \qquad (5.3).$$

(5.1)Circuit RC.

As, for the **RC circuit** $1/Z = 1/R + i2\pi fC$, that is,

$Re[Z] = |Z| = R/(1 + 4\pi^2 f^2 C^2 R^2)^{1/2}$ we obtain,

$$(\delta V_f^2)_{RC} = 4kTR/(1 + 4\pi^2 f^2 C^2 R^2)^{1/2} \qquad (5.4).$$

According to Eq.(5.1) we must have

$$< V^2 >_{RC} = 4kT \int_o^\infty Re[Z(f)]\,df = 4kTR \int_o^\infty df/(1 + 4\pi^2 f^2 C^2 R^2)^{1/2}$$

$$= (4kTR/a) \int_o^\infty dx/(1 + x^2)^{1/2}, \text{ where } a = 2\pi RC.$$

In this way we obtain, in agreement with Eq.(3.4):

$$< V^2 >_{RC} = kT/C ,$$

So, we have

$$< I^2 >_{RC} = < V^2 >_{RC}/R^2 = (kT/R^2C) \qquad (5.5).$$

(5.2)Circuit RL.

In this case $Z(f) = R + i2\pi f L$, that is, $Re[Z(f)] = [R^2 + 4\pi^2 f^2 L^2]^{1/2}$. In this way we obtain, using Eq.(4.8),

$$(\delta V_f^2)_{RL} = 4kT\ Re[1/Z(f)]\,df = 4kTR\,df /(1 + 4\pi^2 f\tau_2^2)^{1/2} \qquad (5.6),$$

where $\tau_2 = L/R$.

When $L/R << 1$, that is, $f\tau_2 << 1$, Eq.(5.6) becomes,

$$(\delta V_f^2)_{RL} \approx 4kTRdf \qquad (5.7),$$

showing that when the resistive effects are much larger than the inductive ones results:

$$(\delta V_f^2)_{RL} = (\delta V_f^{\ 2})_R \approx 4kTR\,df \quad \text{and} \quad (\delta I_f^{\ 2})_{RL} = (\delta V_f^{\ 2})_R \approx (4kT/R)\,df \qquad (5.8),$$

that is, the noise effects are essentially the resistive ones.

The total RL noise, due to all frequencies, is given by

$$< I^2 >_{RL} = 4kT \int_o^\infty Re[1/Z(f)]\,df = 4kT \int_o^\infty df /(R^2 + 4\pi^2 f^2 L^2)^{1/2} \qquad (5.9),$$

that is, $$< I^2 >_{RL} = kT/L \qquad (5.10),$$

in agreement with Eq.(2.3). Consequently,

$$< V^2 >_{RL} = < I^2 >_{RL} R^2 = kTR^2/L. \qquad (5.11).$$

Finally, let us note that for the **RC circuit** we have

$$< V^2 >_{RC} = kT/C \quad \text{and} \quad < I^2 >_{RC} = < V^2 >_{RC}/R^2 = (kT/R^2C),$$

and for the **RL circuit**,

$$< V^2 >_{RL} = kT/L \quad \text{and} \quad < I^2 >_{RL} = < V^2 >_{RL}/R^2 = (kT/R^2L).$$

From these equations result:

$$< V^2 >_{RC} / < V^2 >_{RL} = L/C \quad \text{and} \quad < I^2 >_{RC} / < I^2 >_{RL} = L/C .$$

(6) Circuit with R and a Frequency Amplifier A(f).[4]

Let us consider a circuit (see **Figure 3**) with R in parallel with an amplifier network A(f) with a very high input impedance (|Z| >> R).

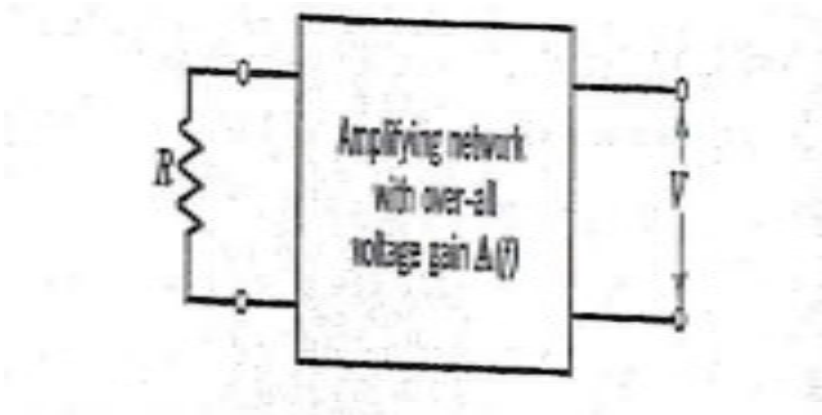

Figure 3. Sketch[4] to the illustrate amplification of electrical noise generated by a resistor R.

Assuming that the amplifier itself introduces a negligible "noise" in comparison with that of passive resistor R at the input terminals, the over-all noise at the output will be estimated by[4]

$$< \delta V^2(t) >_A = 4RkT\,|A(t,f)|^2\, df \qquad (6.1),$$

where A(t,f) is the complex voltage gain of the amplifier as a function of t and frequency f. So, the **" noises"** $< V^2(t) >_A$ and $< \delta I^2(t) >_A$, at the time t, due to the contribution of all frequencies, are given by

$$< X^2(t) >_A \sim \int_o^\infty |A(t,f)|^2\, df \qquad (6.2)$$

If the amplifier unit has a gain concentrated around a frequency f_o like $|A(f)|= \delta(f-f_o)$ the "noises" would be:

$$< V^2(t) >_A = 4RkT\,|A(t,f_o)|^2 \text{ and } < I^2(t) >_A = (4kT/R)\,|A(t,f_o)|^2 \quad (6.3).$$

APPENDIX A. Estimation of $< v^2(\tau) >$ and $< x^2(\tau) >$.

According to Eq.(13) we have[4]

$$v(\tau) = v_o\, e^{-\tau/\tau 1} + e^{-\tau/\tau 1} \int_o^\tau e^{u/\tau 1} A(u)\, du \qquad (A.1).$$

Integrating Eq.(A.1) we obtain $x(\tau)$ and taking into account "reasonable molecular chaos approximations" one can see that[4]

$$< x^2(\tau) > = <v_o^2>\, \tau_1^2(1- 2e^{-\tau/\tau 1}+2e^{-2\tau/\tau 1}) + (kT\tau_1^2/M)(-1+4e^{-\tau\tau 1}-e^{-2t/\tau 1}) + 2BkT\tau \quad (A.2).$$

For $\tau >> \tau_1$ we obtain, if $<v_o^2> = kT/M$, since the average is performed over the equilibrium ensemble,

$$< x^2(\tau) > \approx <v_o^2>\, \tau_1^2- (kT/M)\tau_1^2 + 2BkT\tau \approx 2D\tau \qquad (A.3)$$

that describes the molecular motion in a viscous medium according to Einstein.[4] On the other hand for $\tau \leq \tau_1$ we get

$$< x^2(\tau) > \approx v_o^2\tau^2, \text{ where } v_o = (kT/M)^{1/2} \qquad (A.4),$$

which gives the *motion of a free molecule*.

That is, for **observation times** $\tau > \tau_1$ or $\tau < \tau_1$, where τ_1 is the relaxation time of the system, we have two different results for the particles displacement. This shows why fluctuation theory (or "correlation function approach") might be called "time-dependent statistical" mechanics.

From (A.1) we can see that ensemble average

$$< v^2(\tau) > = <v_o^2>\, e^{-2\tau/\tau 1} + 2\, e^{-2\tau/\tau 1} \int_o^\tau e^{u/\tau 1} <v_o A(u) > du$$

$$+\, e^{-2\tau/\tau 1} \int_o^\tau \int_o^\tau e^{(u+\omega)/\tau 1} <A(u)A(\omega)> du\, d\omega \qquad (A.5).$$

The second term of Eq.(A.1) vanishes for large values of τ.[4] However, the third term does not vanish because, when $\omega \approx u$ and there is a finite contribution to the double integral. So, Eq.(A.2) is well estimated by[4]

$$< v^2(\tau) > = <v_o^2>\, e^{-2\tau/\tau 1} + (kT/M)\,(1 - e^{-2\tau/\tau 1}) \qquad (A.5),$$

where, as seen above, $< v_o^2> = KT/M$.

From Eq.(A.5), for $\tau >> \tau_1$ we verify that

$$< v^2(\tau) > \approx kT/M = < v_o^2 > \quad ,$$

the average value obtained by the equilibrium Boltzmann statistics. On the other extreme, that is, for measuring instrument with limiting response time $\tau << \tau_1$, then the **observed velocity fluctuations** would be given by

$$< v^2(\tau) > \approx (4kT/M)(\tau_1/\tau) \qquad \text{(A.6)}.$$

In other words, the observed Brownian Movement velocity ($\sim <v_o^2>^{1/2}$) would be reduced roughly by the ratio $(\tau_1/\tau)^{1/2}$ as compared with value $[\sim(kT/M)^{1/2}]$, predicted directly by the "equilibrium" statistical mechanics.

APPENDIX B. The Fluctuation-Dissipation Theorem.

The Fluctuation-Dissipation Theorem(FDT)[11] was proven by H. Callen and T. Welton[12] and expanded by R. Kubo.[11] There are [11] antecedents to the general theorem, including Einstein explanations of the Brownian Motion and by H. Nyquist of the Resistance-Johnson noise.

The FDT is a powerful tool in statistical physics for predicting the behavior of systems that obey the *detailed balance*.[13]It is a general proof that thermodynamic fluctuations in a physical variable predict the response quantified by the admittance or impedance of the same physical variable (like voltage, temperature difference, etc.) and vice-versa. The FDT applies both to classical systems and quantum mechanical fluctuations.[14]

In few words, the FDT says that when there is *process that dissipates energy*, turning into heat (e.g., friction), there is a reverse process related to *thermal fluctuations*. The FDT is a general result of statistical thermodynamics that quantifies the relation between the fluctuations in a system that obeys the *detailed balance* and the response of the system to applied perturbations. This is can be best understood by considering some examples:[11]

Drag and Brownian motion.

If an object moving through fluid, it experiences ***drag*** (air resistance or fluid resistance). Drag dissipates kinetic energy, turning it into heat. The corresponding fluctuation is the ***Brownian motion.*** An object in a fluid does not sit still, but rather moves around with a small and rapidly-changing velocity, as molecules in the fluid bump into it. Brownian motion converts heat energy into kinetic energy ---the reverse of drag.

Resistance and Johnson noise.

If electric current is running through a wire loop with resistor in it, the current will rapidly go to zero because of the resistance. Resistance dissipates electrical energy turning it into heat (***Joule heating***). The corresponding fluctuation is ***Johnson noise*.** A wire loop with a resistor in it does not actually have zero current, it has a small and rapidly-fluctuating current caused by the thermal fluctuations of the electrons and atoms in the resistor. Johnson noise converts heat energy into electrical energy---the reverse of resistance.

Ligth absorption and thermal radiation. When light impinges on an object, some fraction of the light is absorbed, making the object hotter(***light heating***). In this way, light absorption turns light energy into heat. The corresponding fluctuation is **thermal radiation** (e.g.,the glow of a "red hot"object). Thermal radiation turns heat energy into light energy---the reverse of light absorption. Kirchhoff´s law of thermal radiation confirms that the more effectively an object absorbs light, the more thermal radiation it emits.

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