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PULSARS LIGHTHOUSES SIMPLE MODEL

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Abstract. This paper was written to graduate and undergraduate students of physics and engineering. Using only basic concepts of undergraduate physics course we propose a simple model to roughly estimate the very intense radiation beams like lighthouses (LHR) emitted by pulsars along their magnetic axis. We adopt two different approaches to estimated the LHR: cyclotron radiation and Faraday´s disk effect.
Key words *:pulsar electromagnetic radiation; lighthouse radiation.*

(I)Introduction.

Pulsars are neutron stars (NS)[1-3] that have a radius of 10 Km and masses of about 1.4 $M_{sun.}$ They result from explosions ("supernova") of a massive stars, combined with gravitational collapse, that compresses their cores which arrive to have a nuclear density.[1-3] Neutron stars that can be observed are very hot and typically have a surface temperature of around 600 000 K.[2,3]Their magnetic fields are between 10^8 and 10^{15} Gauss. The gravitational field at the neutron´s star surface is ~2 x10^{11} times the Earth´s gravitational field. As the star´s core collapses, its rotation rate increase as a result of the angular momentum conservation, and the newly formed NS hence rotates at up a several hundred times per second. Let us call this final angular momentum by **S**. They become very high density stars with a magnetic dipole $\boldsymbol{\mu}$ along **S** assuming that it is spinning rapidly around an external fixed axis with angular velocity $\boldsymbol{\Omega}$ with periods between 1ms and a few seconds. When their magnetic dipole $\boldsymbol{\mu}$ spinning axis are not aligned with the external rotation axis[2]they become pulsars (see **Figure 1**). By peculiar mechanisms, intense coherent beams of radiation are produced along the magnetic axis and these beams swing round like lighthouses as the star rotates. The electromagnetic radiation spectrum is composed, in general, by frequencies that go from radio waves up to X-rays and γ-rays. A beam from a favorably located pulsar can illuminate the earth once per cycle and its behavior is detected, usually with radio-telescopes, as pulses of radiation at regular intervals. In reference [2] one can see an illustration of the lighthouse effect. Below, in Figure 1[2] is shown the schematic vision of a pulsar.

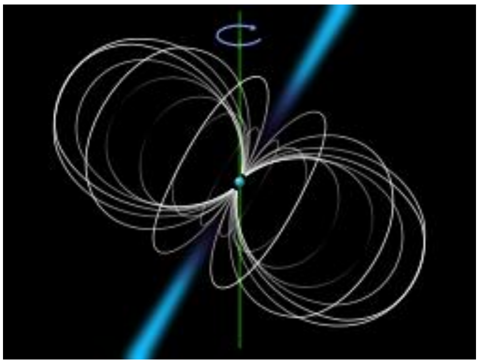

Figure 1. Schematic vision of a pulsar.[2] The sphere in the center represents the NS, the curved lines indicate the lines of the magnetic field, the blue cones indicate the radiation emission zones and the red line represents the rotation axe of the NS.

Note that the pulsar magnetic axis that determines the direction of the electromagnetic emission is not necessarily equal to the rotation axis.

In Figure 2 is shown the image of the Crab Pulsar obtained by the Hubble and Chandra space telescopes.[3] From this figure we verify that around the core of the NS there is a very large region which may extend to r_{max} > 2000 Km from the surface, named *magnetosphere*. There are found the residues of the supernova explosion composed by plasmas of ions, electrons and macro particles. Pulsars emit not only *lighthouse radiation* but also *surface radiation*. The electromagnetic radiation spectrum is composed, in general, by frequencies that go from radio waves up to X-rays. Description of the complete radiation process is extremely complicated. Many models have been proposed, but no satisfactory theory.[4]

Figure 2. Image of the Crab Pulsar obtained by the Hubble (red region) and Chandra (X-ray, blue region) space telescopes.[2]

Pulsars are catalogued with the initials PSR;[4] many of them are mentioned, for instance, in reference [4].The first one, PSR 1913+16, was discovered in ~1967 by J.B - Burnell and A. Hewish. The stability of isolated pulsars rivals that of atomic clocks. The PSR 1937+21, for example, has a period of 1.5578 ms with a rate change of only 10^{-19}s/s. Over 2000 pulsars have been detected in total. Most of them rotate on the order of once per second (these are sometimes called "slow pulsars"), while more than 200 pulsars that rotate hundreds of times per second (called "millisecond pulsars") have been found. The discovering of pulsar became possible to astronomers to study an object never observed before: the NS. This is the only kind of object where the matter behavior with a nuclear density can be indirectly observed . In addition, millisecond pulsars became possible to study the General Theory of Relativity in the presence of intense gravitational field.[1,5] In **Section 1** are seen the structure, compositions and properties of the magnetosphere and core. In **Section 2** are analyzed the surface radiation and the LHR. We assumed that the magnetosphere can be divided into two regions. In the first one that goes from R up to r* the free electrons move attached to the NS surface just as the air in the Earth's atmosphere moves attached to the Earth. In the second region, that goes from r* up to r_{max}, the magnetosphere is considered to be at rest, that is, not attached to the NS. In **Section 3** the LHR is estimated assuming that, in the region that goes from R up to r*, it is generated by the *cyclotron mechanism.* In **Section 4,** assuming that in the region that goes from r* up to r_{max}, is calculated the LHR energy spectrum using the *Faraday´s disk effect*. In **Section 5** are presented the **Conclusions** about the estimated frequencies obtained in Sections 3 and 4 that would be observed in the LHR.

(1) Pulsar Magnetosphere and Core.

To explain the PSR radiation we need to know the composition, structure, internal properties of its magnetosphere and core.[1-4] Around the NS core there is very large region with a radius $r_{max} >> R$ where R is the NS radius. Current models indicate[5] that matter at the surface of a neutron star is essentially composed of ordinary iron nuclei crushed into a solid lattice with a sea of electrons flowing through the gaps between them and by a very dense electron plasma. The density of this region is ~10^6 g/cm^3.[5] This "outer crust" should be solid with T $\leq 10^6$ K and fluid with T > 10^6 K only for younger NS. Below this crust, with at most several micrometers thick, one encounters a solid crust composed by different structures (see **Figure (3**) described in reference [3]. Temperatures in the internal cores, in newly formed NS, are very high, going from 10^{11} up to 10^{12} K with densities comparable with that of atomic nucleus.

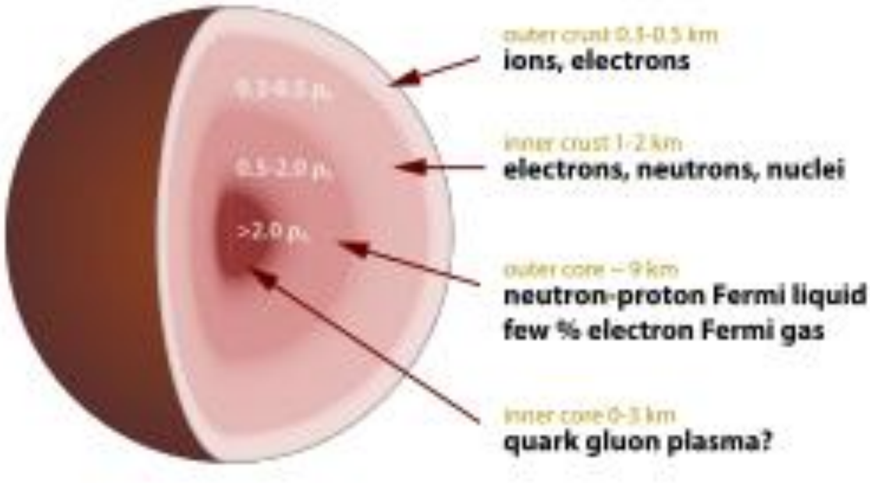

Figure 3.Cross section of the NS core.[3]

According to some models[6–11]the NS cores are not in equilibrium. Are submitted to perturbations of the self-gravitating neutron matter. These would be governed by an equation of state very close to that of an elastic solid. Perturbations of the equilibrium state would create pulsar glitches[12] Pulsating frequencies ω^* of the star, when its equilibrium state is disturbed, can be estimated taking into account that the compressibility χ of neutron matter. It is given by $\chi = -(\Delta\rho/\Delta p)/\rho$ where p is its pressure and ρ its density. Recalling that the speed v_s propagation of sound waves is given by $v_s = 1/\sqrt{\chi\rho} = \sqrt{\partial p/\partial\rho}$[13,14] it can be written as $v_s \sim \sqrt{p/\rho}$. As typical pressure p in a self-gravitating sphere [15] of radius R and mass M is $p = GM^2/R^4$ we obtain $v_s \sim (G/\rho)^{1/2}(M/R^2) \sim (GM/R)^{1/2}$. Assuming that the wavelengths λ of the lowest mode of vibration oscillations are given by $\lambda \sim R$ we obtain $\omega^* = (2\pi/\lambda)v_s \sim 2\pi\,(G\rho)^{1/2}$. For $\rho \sim 10^{14}$ g/cm^3 we have frequencies $f \sim 10^3$ Hz.

(2)Pulsar Electromagnetic Radiation.

(2.a)Surface Radiation.

The surface radiation is due essentially to the core surface temperature. The young pulsar has surface temperatures $T \sim 10^9$ K. The whole surface is so high that it emits X-rays. There are also charged particles in the pulsar´s magnetic surroundings that also emit synchrotron and X radiation as they move, accelerated outwards, along the magnetic field lines.[4]Detailed calculations can be seen, for instance, using the Magnetic Hydrodynamics theory[16-18](MHD). This radiation from ions and electrons is due to the star rotation and by the intense magnetic field. As soon NS are born they begin to cool down. After several million years, they have cooled from billions of degrees to much less than 500 000 K. The surface radiation intensity decreases very much. Their surface-wide X-ray emission, for instance, has faded from the view.

(2.b)Lighthouse radiation.

The *lighthouse radiation* (**LHR**) is also very difficult to be estimated. As occurs with the surface radiation, many models using the Magnetic Hydrodynamics (MHD)[16-18] have been proposed but no satisfactory theory.[4] Probably because there are no precise information about the magnetosphere compositions that are essential to perform reliable MHD calculations.

In this paper we intend to obtain only *orders of magnitude* of the LHR. This will be done in a *non relativistic limit*, avoiding the use of the MHD theory[19] and assuming that the magnetosphere can be divide into two regions (see **Introduction**). In **Section 3** is shown that the region from $r = R$ up to r^*, attached to the NS, would be responsible mainly by the emission of very high energy photons like X-Rays and γ -rays. In **Section 4** is shown that the region going from r^* up to r_{max}, will be responsible for low the energy radiation like radio waves and infrared. Of course, this sharp frontier is a "poetic license" assumed to simplify the calculations.

(3) LHR cyclotron radiation model.

To perform the calculation we will assume that:

(i)free electrons in the magnetosphere **move attached** to the NS surface just as the air in the Earth's atmosphere moves attached to the Earth.

(ii) These electrons are submitted to a static magnetic field **B**(r), parallel to the rotation (spin) axis **k** of the pulsar (**Figure 1**), given by

$$\mathbf{B}(r) \approx (2\mu/r^3)\,\mathbf{k} \qquad (3.1),$$

where r is the distance measured from the origin of the NS.

(iii) Free electrons move along **perpendicular planes** to **B(r)** inside a long cylinder with height r^* and basis with area πR^2,where R is the NS radius.

With hypothesis **(1)** it is unnecessary to take into account the electric induction field $\mathbf{E} = -(\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{B}$ where $\mathbf{\Omega} = \Omega\,\mathbf{k}$ is the rotational angular velocity of the NS around the axis **k.**[20,21]

In our calculations we are at the North Pole of the NS. Similar results will be obtained to the South Pole.

So, electrons moving along a perpendicular plane to **k,** passing by r , are submitted to a force $\mathbf{F}(r) = q\mathbf{v} \times \mathbf{B}(r)$. Thus, all them would describe circular motions with radius ρ with the same angular velocity $\omega_{cr}(\rho)$[20]

$$\omega_{cr}(\rho) = \omega_{cr}(r) = qB(r)/m = (qB_o/m)(R/r)^3 \qquad (3.2).$$

As $B(r) = 2\mu/r^3$ and $\mu = B_oR^3/2$,[19,21,22] the maximum emitted frequency, for a given B_o, would be $\omega_{sr\ max}(\rho) = qB_o/m$.

From Eq.(3.2) all electrons at a distance r from the NS center would move with the same angular frequency. Half of them moving in the horary sense and half in the anti-horary. If their average velocity is $v^{@}$, the average radius $\rho^{@}(r)$ of motion would be given by,

$$\rho^{@}(r) = mv^{@}/qB(r) \qquad (3.3),$$

and their average centrifugal acceleration $a_{sr}{}^{@}(r)$ by,

$$a_{cr}{}^{@}(r) = \omega_{cr}{}^2(\rho)\rho^{@}(r) = (qB(r)/m)v^{@}\,. \qquad (3.4).$$

Eqs.(3.2)-(3.4) show that $\omega(r)$, $\rho^{@}(r)$ and $a_\rho{}^{@}(r)$ are constants in a plane at a distance r from the NS center. Note that $B(r) = 2\mu/r^3$ and $\mu = B_oR^3/2$.[21,22]

The radiation intensity I = ℓ(luminosity) emitted by a charge in a circular motion is given by[21] $\ell(r) = \zeta(2/3)q^2\mathbf{a}^2/c^3$, where $\zeta = 1/2\pi\varepsilon_o$ and **a** is the centripetal acceleration. In a rough approximation, it will be assumed that the ℓ(r) emitted from NS pole by an electron located in a plane at distance r is given by,

$$\ell(r) \approx \zeta q^2 a_{cr}{}^{@}(r)^2/c^3 \qquad (3.5).$$

(3.1)Cyclotron luminosity $L_{cr}(r)$ as function of r.

Free electrons are inside a cylinder with a basis with area πR^2 and height that goes from r = R up to r = r*. If $\rho_e(r)$ is the electrons density at r, the cyclotron luminosity $dL_{cr}(r)$ due the electrons that are inside a volume $dV = \pi R^2 dr$ is given by, using Eqs.(3.5) - (3.4), where $\mu = B_o R^3/2$,

$$dL_{cr}(r) = \ell(r)\rho_e(r)\,\pi R^2 dr = \zeta\{q^2 a_\rho^{@}(r)^2/c^3\}\,\rho_e(r)\,\pi R^2 dr =$$

$$= \zeta\{4\mu^2 q^4 v^{@2}/m^2c^3r^6\}\,\rho_e(r)\,\pi R^2 dr \text{ , that is,}$$

$$dL_{cr}(r) = \zeta\,\pi\,\rho_e(r)\,\{B_o^2 R^8 q^4 v^{@2}/m^2c^3\}\,dr/r^6 \qquad (3.6).$$

Assuming that $\rho_e(r) \approx$ constant $= \rho_e$ and that they are spread in the magnetosphere, from R up to r = r*, the **total** cyclotron lighthouse luminosity L_{cr} would be given by

$$L_{cr}(r^*) = \zeta\,\pi\,\rho_e\{B_o^2 R^8 q^4 v^{@2}/5m^2c^3\}(1/R^5 - 1/r^{*5}) \qquad (3.7).$$

If $r^* > R$, Eq.(3.7) becomes,

$$L_{cr} \approx \zeta\,\pi\,\rho_e\,v^{@2}\{B_o^2R^3\}\{q^4/5m^2c^3\} \qquad (3.8).$$

All frequencies $\omega_{cr}(r)$, defined by Eq.(3.2), from qB_o/m up to $(qB_o/m)(R/r^*)^3$, contribute to the luminosity. The maximum and minimum emitted frequencies $\omega(r)$ are given by

$$\omega_{max} = qB_o/m \qquad (3.9)$$

and $$\omega_{min} = (qB_o/m)(R/r^*)^3.$$

(3.2) Cyclotron Luminosity $L_{cr}(\omega)$ as a function of ω.

In **Appendix A** the luminosity is calculated as a function of the frequencies ω, that is, $L_{cr} = L_{cr}(\omega)$. According to Eq.(A.6)

$$dL_{cr}(\omega)/d\omega = -\zeta\,(\pi/3)\,\rho_e\{B_o^2R^3v^{@2}\}\{q^4/m^2c^3\}\,\omega^{2/3}/\omega_{max}^{5/3} \qquad (3.10),$$

and the luminosity δL_{12} due to frequencies in the frequency interval $\omega_2 - \omega_1$ where $\omega_2 \le \omega_{max}$, is given by Eq.(A.9),

$$(\delta L_{cr})_{12} = -\,\zeta\,(\pi/5)\rho_e\{B_o^2R^3v^{@2}\}\{q^4/m^2c^3\}\{\omega_2^{5/3} - \omega_1^{5/3}\}/\omega_{max}^{5/3} \qquad (3.11).$$

(3.3)Numerical estimates.

In the SI system of units, $\zeta = 1/2\pi\varepsilon_o \sim 10^{10}$, $B_o = 10^8 - 10^{15}$ Gauss, R = 10 Km = 10^4 m, q = e =1.6 10^{-19} C, m = 9,11 10^{-31} Kg and c = 3 10^8 m/s. At the temperature T we have $v^{@2} = 8k_BT/\pi m \approx 3.8\ 10^7$ T. In this way Eq.(3.2) and Eq.(3.6) give, respectively,

$$\omega_{cr}(r) = (qB_o/m)(R/r)^3 \approx 1.76\ 10^{11}\ B_o\ (R/r)^3 \text{ rad/s} \qquad (3.12),$$

and

$$dL_{cr}(r) \approx \zeta\ \pi\ v^{@2}\{B_o^2R^8\}\{q^4/m^2c^3\}\ \rho_e(r)\ dr/r^6$$

$$\approx \pi\ \{T\ B_o^2R^8\}10^{-30}\ \rho_e(r)\ dr/r^6 \qquad (3.13).$$

Taking into account that $\{q^4/m^2c^3\} \approx 2.30\ 10^{-47}$, $\rho_e(r)$ = constant = ρ_e and integrating Eq.(3.11) from r = R up to r* we have *total Luminosity*,

$$L_{cr}(r^*) \approx \pi\ \rho_e\ \{T\ B_o^2R^8\}\ 10^{-29}\ (1/R^5 - 1/r^{*5}) \qquad (3.14),$$

that, for r* >> R = 10 Km = 10^4 m gives,

$$L_{cr} \approx \pi\ \rho_e\ \{T\ B_o^2R^3\}10^{-29} \quad \text{W} \qquad (3.15),$$

where are present all frequencies $\omega(r)$, with r = R up to r* >> R. Note that

$$\omega_{max} = \omega(R) = 1.76\ 10^{11}\ B_o \text{ rad/s} \qquad \text{and}$$

$$\omega_{min}(r^*) = 1.76\ 10^{11}\ B_o\ (R/r^*)^3 \text{ rad/s} \qquad (3.16).$$

Using Eqs.(3.10) -(3.11) -(3.15) and (3.16) we obtain,

$$dL_{cr}(\omega)/d\omega \approx -\pi\rho_e\{T\ B_o^2R^3\}10^{-29}\ \omega^{2/3}/\omega_{max}^{5/3} \qquad (3.17).$$

and $$(\delta L_{cr})_{12} \approx \pi\rho_e\{T\ B_o^2R^8\}10^{-29}\{\omega_2^{5/3} - \omega_1^{5/3}\}/\omega_{max}^{5/3} \qquad (3.18).$$

Let us analyze some cases, using Eqs.(3.12)-(3.18), taking $\rho_e = 10^{12}/m^3$, R =10 Km and T~10^6 K, for B_o=10^{15},10^{12},10^{10} and 10^8 Gauss.

(A)B_o = 10^{15} Gauss.
$L_{cr} \approx 10^{31}$ W
$(\delta L_{cr})_{12}$ = contributions of emitted frequencies in **rad/s** in the intervals 1-2:
(a) $10^{25} - 10^{21}$ (γ-rays) → $\delta L_{12} \sim 10^{29}$ W
(b) $10^{19} - 10^{17}$ (x-rays) → $\delta L_{12} \sim 10^{19}$ W
(c) $10^{15} - 10^{12}$ (infrared-ultraviolet) → $\delta L_{12} \sim 10^{12}$ W

(d) 10^{11}-10^{8} (microwave -TV) → $\delta L_{12} \sim 10^{6}$ W
(e) 10^{9} -10^{3} (radio) → $\delta L_{12} \sim 10^{2}$ W

(B)$B_o = 10^{12}$ Gauss
$L_{cr} \approx 10^{25}$ W
$(\delta L_{cr})_{12}$
(a) 10^{22} -10^{19} (γ rays) → $\delta L_{12} \sim 10^{22}$ W
(a) 10^{18} -10^{15} (x-rays - ultraviolet) → $\delta L_{12} \sim 10^{16}$ W
(b) 10^{14} -10^{9} (infrared-microwave) → $\delta L_{12} \sim 10^{9}$W
(c) 10^{8} - 10^{4} (radio) → $\delta L_{12} \sim 10$ W

(C)$B_o = 10^{8}$ Gauss,
$L_{cr} \approx 10^{17}$ W
$(\delta L_{cr})_{12}$
(a) 10^{18} -10^{15} (x-rays - ultraviolet) → $\delta L_{12} \sim 10^{15}$ W
(b) 10^{14} - 10^{9} (infrared-microwave) → $\delta L_{12} \sim 10^{8}$ W
(c) 10^{8} - 10^{4} (radio) → $\delta L_{12} \sim 10^{-2}$ W

(3.4)Conclusions.

From these results we verify that the cyclotron mechanism favors high frequencies like x-rays and γ-rays. It is not adequate to estimate low frequencies emissions like, for instance, radio waves. In **Section 4**, assuming that the magnetosphere is at rest, that is, not attached to the NS, is shown that is possible to obtain a reasonable description for low frequency emissions.

(4)LHR using Faraday´s disk model.

In preceding **Section** the LHR was estimated assuming that for $r \geq R$ up to r^* free electrons in the NS magnetosphere move attached to the NS surface just as the air in the Earth's atmosphere moves attached to the Earth. They were submitted to a magnetic field **B**(r), parallel to the rotation axis **k** of the pulsar, given by Eq.(2.1), $\mathbf{B}(r) \approx (2\mu/r^3)\,\mathbf{k}$, where r is the distance measured from the origin of the NS**.** So, to estimate the LHR, it was not necessary to take into account the electric induction field $\mathbf{E} = -(\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{B}$ where $\mathbf{\Omega} = \Omega\,\mathbf{k}$ is the rotational angular velocity of the NS around the axis **k.**[20,21]

Now, it will be assumed that the magnetosphere for $r \geq r^*$ up to r_{max} is at rest, that is, not attached to the NS. In these conditions, to calculate the

LHR luminosity it will be necessary to take into account the effect of induction electric field **E** mentioned above.

Now, it will be assumed that the magnetosphere for r > r* is at rest, nor attached to the NS, and the NS is rotating with angular velocity **Ω**. In these conditions we must take into account the induction electric field **E** which will be responsible for an effect observed in "Faraday´s disk".[20]

Let us analyze for regions r > r* the effect of induced rotation of electrons in a circle with a radius R* = χR, where χ > 1, due to the separation of the magnetic field lines as expected for r* > R .This circle is located in a plane perpendicular, at point r, to the axis **k.** Its origin O is at the axis **k**. Let ρ be the distance of a point of this circle to the center O. The relative motion to the NS will create at ρ, the *emf* $\varepsilon(\rho) = (1/2)B\Omega\rho^2$.[20] This ε(ρ), due to the induced field **E(**ρ), is directed to the center of a circle. It will generate radial electronic currents in the circle. Since, on the electron at ρ is applied a central force $F(\rho) = q\varepsilon(\rho)/\rho$, its centripetal acceleration ω(ρ) would be given by,

$$F(\rho) = q\varepsilon(\rho)/\rho = m\omega(\rho)^2\rho,$$

that is,

$$q\{(1/2)B\Omega\rho^2\}/\rho = (1/2)qB\Omega\rho = m\omega(\rho)^2\rho,$$

giving, finally

$$\omega_{rot}(\rho) = \omega_{rot}(r) = \{qB(r)/2m\}^{1/2}\Omega^{1/2},$$

that is,

$$\omega_{rot}(r) = \{qB_o/2m\}^{1/2}(R/r)^{3/2}\,\Omega^{1/2} \qquad (4.1),$$

remembering that $B(r) = 2\mu/r^3$ and $\mu = B_oR^3/2$. Note that rotational frequency $\omega_{rot}(r)$ is given by the product of the resonant frequency $\omega_{rs}(r)$, defined by Eq.(3.2), and the NS angular frequency Ω.

(4.1)The rotational luminosity L_{rot} (r).

For r ≥ r*, electrons are inside a cylinder with a basis area πR*, where R*= χR and height r_{max} - r*. According to Eq.(4.1), the centripetal acceleration of each electron is, taking the average value < ρ > ~ R*/2,

$$a_c(r) = \omega_{rot}(\rho)^2\rho \approx \omega_{rot}(\rho)^2=\{qB(r)/m\}\Omega R^*/2,$$

and that its luminosity is given by $\ell(r) = \zeta\{q^2a_c(r)^2/c^3\}$.[22] Following the same procedure seen in **Section 3** the rotational luminosity $dL_{rot}(r)$ due to electrons inside a volume $dV = \pi R^{*2}dr = \pi\chi^2R^2dr$ is given by,

$$dL_{rot}(r) = \ell(r)\rho_e(r)\,\pi\chi^2R^2dr = \zeta\,\pi\chi^2R^2\{q^2a_c(r)^2/c^3\}\rho_e(r)\,dr =$$

$$= \zeta\,\pi\,\chi^4R^2\rho_e(r)\,B^2(r)R^2\Omega^2\{q^4/4m^2c^3\}\,dr$$

$$= \zeta\,\pi\,\chi^4R^2\rho_e(r)\{B_o{}^2\Omega^2R^8\}\{q^4/16m^2c^3\}\,dr/r^6 \qquad (4.2).$$

Assuming $\rho_e(r) \approx$ constant $= \rho_e$ and that electrons in the magnetosphere are spread from r* up to r_{max}, the LHR luminosity $L_{rot}(r)$, that are in region r_{max}- r*, would be:

$$L_{rot}(r) \approx \zeta\pi\chi^4R^2\rho_e\{B_o{}^2R^8\Omega^2\}\{q^4/80m^2c^3\}(1/r^{*5} - 1/r_{max}{}^5) \quad (4.3).$$

As $r_{max} >> r^*$, we obtain, with $\omega_{rot}(r^*) = qB(r^*)/m$ and $B(r^*) = B_oR^3/r^{*3}$, the **total luminosity** $L_{rot}(*)$ due to all frequencies in the range $r^*- r^{max}$:

$$L_{rot}(*) \approx \zeta\pi\chi^4\rho_e\,R^5\{B_o\Omega\}^2\,(R/r^*\}^5\{q^4/80m^2c^3\} \qquad (4.4).$$

The luminosity $L_{rot}(1,2)$ in the range r_1 and r_2 would be given by

$$L_{rot}(1,2) \approx \zeta\pi\chi^4R^2\rho_e\{B_o{}^2R^8\Omega^2\}\{q^4/80m^2c^3\}(1/r_1{}^5 - 1/r_2{}^5) \quad (4.5).$$

and the emitted frequencies $\omega_{rot}(r)$, using Eq.(4.1),

$$\omega_{rot}(r) = \{qB(r)/2m\}^{1/2}\,(\Omega)^{1/2} = (q/2m)^{1/2}\,B_o{}^{1/2}\,\Omega^{1/2}\,(R/r)^{3/2} \qquad (4.6).$$

(4.2) Rotational Luminosity $L_{rot}(\omega)$ as a function of ω.

Using Eq.(4.5) and (4.6) the contributions of frequencies in the interval $\omega_{rot}(r_1)$ - $\omega_{rot}(r_2)$, indicate by $\delta(L_{rot})_{12}$, can be obtained substituting r_1 and r_2 as functions of $\omega_{rot}(r_1)$ and $\omega_{rot}(r_2)$,respectively, in Eq.(4.6).

(4.3)Numerical estimates.

In the SI system of units, Eqs.(4.4) -(4.6)become, respectively,

$$L_{rot}(*) \approx 3\pi\chi^4\rho_e\,R^5\{B_o\Omega\}^2\,(R/r^*\}^5 10^{-39} \quad W \qquad (4.7),$$

and

$$\omega_{rot}(r) \approx 3\;10^5\;\{B_o\Omega\}^{1/2}\,(R/r)^{3/2} \quad rad/s \qquad (4.8),$$

where $B_o = 10^8 - 10^{15}$ Gauss, R = 10 Km = 10^4 m, $\rho_e \sim 10^{12}/m^3$ [23] and Ω goes from 50 milliseconds up to 1 s. Measuring the frequencies in Hertz, f = 2πω and F = 2πΩ, Eqs.(4.6)-(4.7) are given by, putting X = r/R:

$$L_{rot}(*) \approx 2.40\ \chi^4\{B_o F\}^2\{1/X^*\}^5 10^{-8} \qquad W \qquad (4.9)$$

where X = r/R and X* = r*/R,

$$f_{rot}(X) \approx 8.45\ 10^4\ \{B_o\}^{1/2}\{F\}^{1/2}(1/X)^{3/2} \quad Hz\ . \qquad (4.10).$$

Using Eqs.(4.9) and (4.10), let us estimate the emitted rotational frequency spectrum assuming, for instance, that F=1s, r*= 5R and $\chi \sim 10$. In **Figure 4** are plotted $\log\{f_{rot}(X)\}$ as functions of X = r/R and B_o= 10^{15},10^{12}, 10^{10} and 10^8 Gauss.

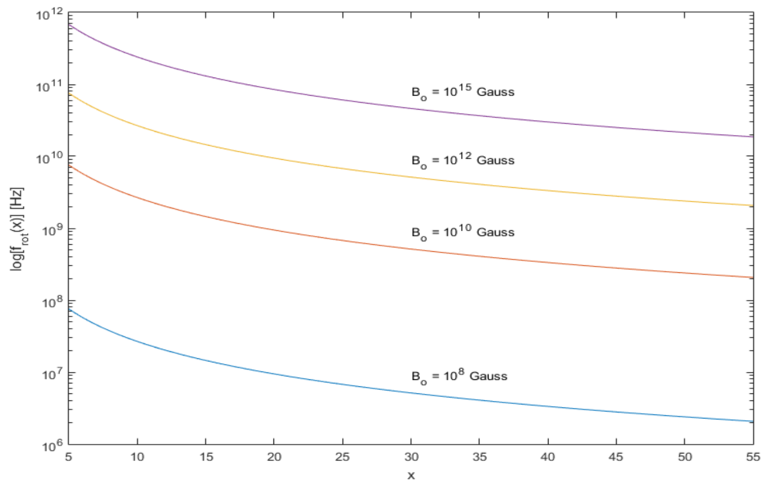

Figure 4. $\log\{f_{rot}(X)\}$measured in Hz plotted as functions of X = r/R and B_o.

The total luminosities $L_{rot}(*)$, defined by Eq.(4.9), are calculated, assuming that r* =5R, F= 1s and χ~10, for Bo = 10^{15}, 10^{12}, 10^{10} and 10^8 Gauss, respectively, are given by

$\mathbf{B_o = 10^{15}}$ **Gauss** $\rightarrow L_{rot}(X^*) \approx 7.68\ 10^{20}$ W

$\mathbf{B_o = 10^{12}}$ **Gauss** $\rightarrow L_{rot}(X^*) \approx 7.68\ 10^{14}$ W

$\mathbf{B_o = 10^{10}}$ **Gauss** $\rightarrow L_{rot}(X^*) \approx 7.68\ 10^{10}$ W

$\mathbf{B_o = 10^{8}}$ **Gauss** $\rightarrow L_{rot}(X^*) \approx 7.68\ 10^{6}$ W

From the above results and taking into account **Figure 4** we can verify that:

(a) for $B_o = 10^{15}$ Gauss → essentially infrared waves are emitted.

(b)for $B_o = 10^{12}$ Gauss → are emitted infrared and microwaves.

(c)for $B_o = 10^{10}$ Gauss → mainly microwaves and also radio waves.

(d)for $B_o = 10^8$ Gauss → only radio waves are emitted.

(5) Conclusions.

According to our model, where the **LHR** is simultaneously produced by two different mechanisms, **cyclotron radiation** and **Faraday´s disk effect**, we can conclude that:

(1) C**yclotron radiation,** created at the magnetosphere attached to the NS that goes from r = R up to $r^* \approx 5R$, would be responsible mainly by the emission of high energy photons with frequencies higher 10^{12} Hz (infrared, ultraviolet, X-rays, γ- rays).

(2) F**araday´s effect,** created in the rest magnetosphere going from r* up to r_{max}, would be responsible by emission of low energy photons with frequencies smaller than 10^{12} Hertz (infrared, microwaves, radio waves).

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APPENDIX A. The frequency spectrum $L_{cr}(\omega)$.

In **Section 3** $L_{cr}(r)$ was estimated taking into account all emitted frequencies from r = R up to r*. Here it will be obtained as a function of the frequencies ω.

Since $B(r) = 2\mu/r^3$ and $\mu = B_oR^3/2$, according to Eq.(3.2), we have

$$\omega(r) = \{qB(r)/m\} = A/r^3 \qquad (A.1),$$

where $A = (2q\mu/m) = qB_oR^3/m$. So,

$$r = \{A/\omega(r)\}^{1/3}, \qquad (A.2).$$

According to Eq.(3.6),

$$dL_{cr}(r)= \pi\rho_e(r)\{B_o{}^2R^8v^{@2}\}\{q^4/m^2c^3\}\ dr/r^6 \qquad (A.3).$$

from Eq.(A.1) we get

$$d\omega = -3Ar^{-4}\, dr = -3\, Ar^2\, (dr/r^6) \quad \rightarrow \quad dr/r^6 = -\, d\omega/3Ar^2$$

Using Eqs. (3.15) and (3.16) and assuming that $\rho_e(r)$ = constant = $\rho_{e,}$

$$dL_{cr}(\omega) = -\, \pi\rho_e\{B_o{}^2R^8v^{@2}\}\{q^4/m^2c^3\}\, d\omega_{sr}/3Ar^2 \qquad (A.4)$$

Since $1/Ar^2 = \omega^{2/3}/A^{2/3}$ Eq.(A.4) becomes,

$$dL_{cr}(\omega) = -\, (\pi/3)\rho_e\{B_o{}^2R^8v^{@2}\}\{q^4/m^2c^3\}\, \omega^{2/3}d\omega/A^{5/3} \qquad (A.5).$$

Remembering that the $A = R^3\omega_{max} = R^3\omega(R) = qB_oR^3/m$,

$$dL_{cr}(\omega)/d\omega = -\, (\pi/3)\rho_e\{B_o{}^2R^3v^{@2}\}\{q^4/m^2c^3\}\, \omega^{2/3}/\omega_{max}{}^{5/3} \qquad (A.6).$$

Taking into account that $v^{@2} \approx 3.8\ 10^7(m/s)^2$ we see that

$$dL_{cr}(\omega)/d\omega \approx -\, \pi\rho_e\{T\, B_o{}^2R^3\}10^{-39}\, \omega^{2/3}/\omega_{max}{}^{5/3} \qquad (A.7).$$

As, seem before, according to Eq.(3.16), the L_{sr} would be obtained integrating ω from 0 up to ω_{max} :

$$L_{cr} \approx \pi\rho_e\{T\, B_o{}^2R^3\}10^{-39} \qquad (A.8),$$

in good agreement with L_{cr} given by Eq.(3.15).

Now, with Eq.(A.6) we can calculate the frequency contributions to the luminosity in intervals like, for instance, $\omega_2 - \omega_1$, where $\omega_2 \leq \omega_{max}$:

$$\delta L_{12} \approx \pi\rho_e\{T\, B_o{}^2R^8\}10^{-39}\{\omega_2{}^{5/3} - \omega_1{}^{5/3}\}/\omega_{max}{}^{5/3} \qquad (A.9).$$

Taking into account that $\omega_{max} = \omega(R) = qB_o/m$, Eq.(3.23) becomes

$$\delta L_{12} \approx \pi\rho_e\{T\, B_o{}^2R^3\}10^{-39}\{\omega_2{}^{5/3} - \omega_1{}^{5/3}\}/(qB_o/m)^{5/3}$$

$$\approx \pi\rho_e\{T\, B_o{}^{1/3}R^3\}(m/q)^{5/3}10^{-39}\{\omega_2{}^{5/3} - \omega_1{}^{5/3}\} \qquad \text{, that is,}$$

$$\delta L_{12} \approx \pi\rho_e\{T\, B_o{}^{1/3}R^3\}\, 10^{-39}\{\omega_2{}^{5/3} - \omega_1{}^{5/3}\} \qquad (A.10).$$

From these equations we note that the luminosity depends only of three independent parameters ρ_e, B_o, T; it does not depend of the intrinsic angular velocity(spin) of the NS and of the rotational velocity $\boldsymbol{\Omega}$ of the dipole moment $\boldsymbol{\mu}$ about an external axis**.**

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