Tesla Radiation and Dipole Antenna

C.H.Furukawa (furukawa@if.usp.br), F.D.Saad (fuad@if.usp.br) and
M.Cattani (mcattani@if.usp.br)
Institute of Physics of the University of São Paulo (IFUSP)

Abstract. This is a didactical text written to students of Physics and Engineering to explain Tesla radiation and Dipole Antenna radiation of electromagnetic waves. These phenomenon can be observed in lessons and public sessions in the "Laboratório de Demonstrações EWH" of the Institute of Physics of the University of São Paulo (IFUSP).
Key words: *Tesla radiation; dipole antenna; essential aspects.*

(I)Introduction.

A large number of excellent papers and books have been written about the Tesla radiation and dipole antenna emission of electromagnetic waves. So, it is not our intention give detailed descriptions of these phenomenon. We have written a simple text, a **"laboratory guide",** where are shown essential aspects of the mentioned effects. In **Section 1** is analyzed the Tesla Radiation. In **Section 2** and **3** are seen the dipole antennas which can be described mathematically in simple way using the Electrodynamics. They are commonly composed by two identical conductive elements such as metal wires or rods. The driving current from the transmitter is applied between the two halves of the antenna. Only two kinds of antennas have been investigated. In **Section 2**, Hertzian Dipole Antenna. In **Section 3,** Half-Wave Center-Fed Dipole Antenna.

(1) Tesla Radiation.

A Tesla coil[1] is a radio frequency oscillator that drives an air-core double-tuned resonant transformer to produce high voltages at low currents. Tesla's original circuits as well as most modern coils use a simple spark gap to excite oscillations in the tuned transformer. More sophisticated designs use transistor or thyristor switches or vacuum tube electronic oscillators to drive the resonant transformer.

Tesla coils can produce, for instance, output voltages from 50 kV to several million volts for large coils. The alternating current output is in the low radio frequency range, usually between 50 kHz and 1MHz. Although some oscillator-driven coils generate a continuous alternating current, most Tesla coils have a pulsed output; the high voltage consists of a rapid string of pulses of radio frequency alternating current.[1]

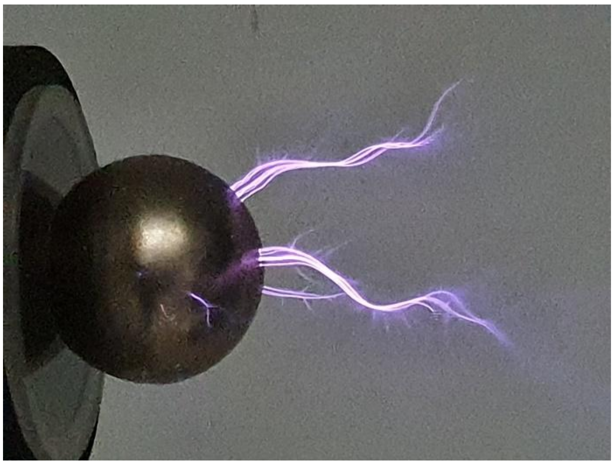


Figure (1.1). Tesla coil in operation, showing brush discharges from the sphere.[1]

High electric field causes the air around the high voltage terminal to ionize and conduct electricity, allowing electricity to leak into the air in colorful corona discharges, brush discharges and streamer arcs [see **Figure (1.1)].**Tesla coils are used for entertainment at science museums and public events, and for special effects in movies and television.[1]

(1.a)Tesla Radiation from Metallic Sphere.

Let us assume that we have a metallic sphere with charge Q and radius R submitted to a voltage $V(R,t) = V_o \sin(\omega t)$, where

$$V_o = Q/4\pi\varepsilon_o R \qquad (1.1),$$

Q = Ne and N the number of free electrons on the surface.

The total electromagnetic power emitted by these electrons will be given by P,[2]

$$P(t) \sim N\zeta(e^2/c^3)(d\mathbf{a(t)}/dt)^2 \qquad (1.2),$$

where $\zeta = 1/6\pi\varepsilon_o$.

The electron acceleration is $\mathbf{a}(t) = e\mathbf{E}(R,t)/m$, where e and m are its charge and mass, respectively, and $\mathbf{E} = \mathbf{E}(R,t) = \mathbf{E_o}\sin(\omega t)$ is the normal electric field to the sphere.

So, as $da/dt = (e\omega/m)E_o \cos(\omega t) = (e\omega/m)V_o \cos(\omega t)/R$

and $Q = Ne = 4\pi\varepsilon_o RV_o$ we get,

$$P(t) \approx N(e^2/6\pi\varepsilon_o c^3)\{(e\omega/m)V_o\cos(\omega t)/R\}^2 =$$

$$\approx (4/6)\, RV_o(e/c^3)\{(e\omega/m)V_o\cos(\omega t)/R\}^2$$

and its time average value $< P(t) >$ is given by

$$< P(t) > \approx 7.10^{-28} V_o^3 \omega^2/R \quad \text{watts} \qquad (1.5).$$

If V_o goes from 50 kV up to 1 MV we see that $< P(t) >$ goes from

$$< P(t) > \sim \{9.10^{-14} \omega^2/R\} \text{ watts up to } < P(t) > \sim \{7.10^{-10} \omega^2/R\} \text{ watts} \quad (1.6),$$

and that the electric field $E_o = V_o/R$ is in the interval,

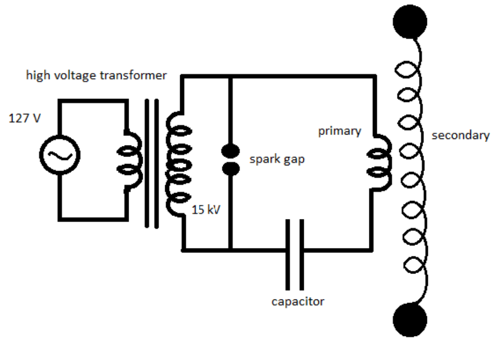
$$E_o = \{50/R\} \text{ kV/m} \quad \text{up to} \quad E_o = \{10^6/R\} \text{ kV/m} \qquad (1.7).$$

Using Eqs.(1.6) and (1.7), for R ~1 m and frequencies $\omega \sim 10^9$ Hz with V_o going from 50 kV up to 1 MV, we verify that $< P(t) >$ goes from 9.10^4 watts up to 7.10^8 watts and E_o goes from 0.5 kV/cm up to 10^4 kV/cm.

The above peak output voltage V_o is only achieved in coils in which air[1] discharges do not occur; in coils which produce sparks, like entertainment coils, the peak voltage on the terminal is limited to the voltage at which the air breaks down and becomes conductive. As the output voltage increases during each voltage pulse, it reaches the point where the air next to the high voltage terminal ionizes and creates corona, brush discharges and streamer arcs, break out from the terminal. This happens when the electric field strength exceeds the dielectric strength of the air, about 30 kV/cm [see **Figure (1.1)**].

Since the electric field is greatest at sharp points and edges, air discharges start at these points on the high voltage terminal. The voltage on the high voltage terminal cannot increase above the air breakdown voltage, because additional electric charge pumped into the terminal from the secondary winding just escapes into the air.[1] The output voltage of open-air Tesla coils is limited to a few million volts by air breakdown, but higher voltages can be achieved by coils immersed in pressurized tanks of insulating oil.[1]

High voltage production (Tesla coil schematics).[1]



In **Figure (1.2)** above (Typical circuit configuration) the spark gap shorts the high frequency across the first transformer that is supplied by alternating current. An inductance, not shown, protects the transformer. This design is favored when a relatively fragile neon sign transformer is used.

In **Figure (1.3)** is seen the Tesla coil of the "Laboratório de Demonstrações EWH" of the IFUSP.

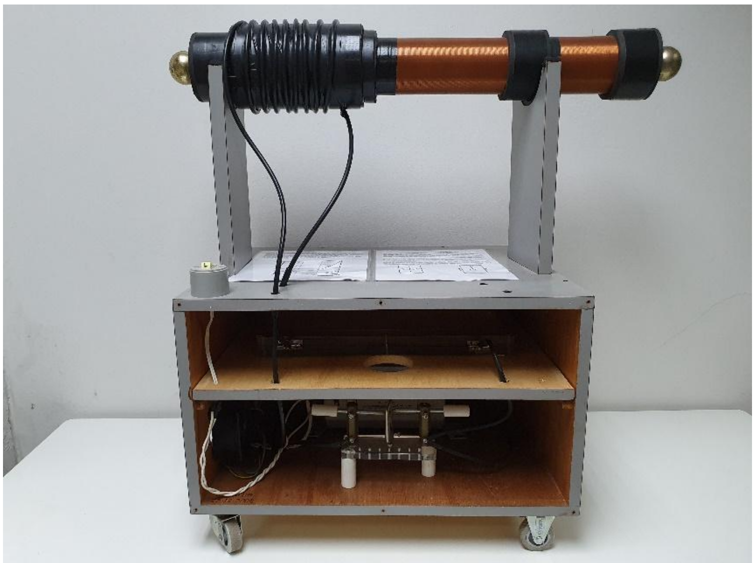


Figure (1.3) Tesla coil of “Laboratório de Demonstrações EWH” - IFUSP.

It has the following characteristics:

- Input circuit transformer: V = 15 kV, I = 30 mA
- Number of turns of the secondary circuit coil: 950
- Number of turns of the primary circuit coil: 8
- Diameter of the secondary coil ball: 2R = 5 cm
- Oscillation frequency (resonance): $\omega = 250$ kHz
- Output voltage: $V_o \sim 200$ kV
- Electric Field $E_o = V_o/R$: $E_o \sim 8\ 10^4$V/cm

With these data, using Eqs.(1.5)-(1.6), we get $< P(t) > \sim 14$ W.

(2) Hertzian Dipole Antenna.

The typical Hertzian dipole [3] appears in **Figure (2.1)** where a sinusoidal current $I_\omega(z,t)$, with a single frequency ω, along a short wire with length L, radiates fields detected by an observer at a point r >> L.

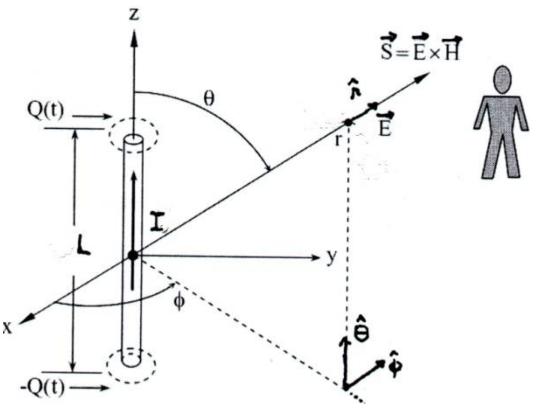


Fig.(2.1). The dipole current $I_\omega(z,t)$ is along the z-axis.

Conservation of charge uniquely relates the current $I_\omega(z,t)$ to the oscillating charges Q(t) at the ends of the short dipole, where we assume that these two charges have a zero sum. The accelerated charges in the "antenna", located in a small wire volume V(**r´**), emit radiation with wavelength λ assuming that

$$(1)\ \ \lambda >> r' = |\mathbf{r}'| \qquad (2.1),$$

and that these are detected at a distance r very far from this volume, that is,

$$(2)\ r >> r' \quad \text{and that} \quad (3)\ r >> \lambda \qquad (2.2).$$

The accelerated charges are in the **"near zone"** (antenna region) and the detected waves are in the **"radiation zone".**

Indicating by $\mathbf{J}_\omega(\mathbf{r}',t) = \mathbf{J}_\omega(\mathbf{r}')e^{-i\omega t}$ and $\rho(\mathbf{r}',t) = \rho(\mathbf{r}')\, e^{-i\omega t}$ the densities of current and charge, respectively, localized in a system with volume V´, the potential $\mathbf{A}_\omega(\mathbf{r})$ generated by this system is given by[2]

$$\mathbf{A}_\omega(\mathbf{r}) = (\mu_o/4\pi) \int_{V'} \mathbf{J}_\omega(\mathbf{r}')d^3\mathbf{r}'\, e^{ik|\mathbf{r}-\mathbf{r}'|}/|\mathbf{r}-\mathbf{r}'| \qquad (2.1),$$

where k = ω/c =2π/λ.

The magnetic induction $\mathbf{B}_\omega(\mathbf{r})$ and electric field $\mathbf{E}_\omega(\mathbf{r})$**,** when the observation point r is at a distance r ~R >> r´, are given by:

$$\mathbf{B}_\omega(\mathbf{r}) = \mu_o\mathbf{H}_\omega = (\mu_o/4\pi)\, \mathrm{rot}(\mathbf{A}_\omega) = -\, i(\mu_o/4\pi)(e^{ikR}/R) \int_{V'}(\mathbf{J}_\omega \times \mathbf{k})\, e^{-i\mathbf{k}.\mathbf{r}'}dV$$

and, where $\mathbf{k} = k\,\boldsymbol{r}$ and $\boldsymbol{r} = \mathbf{r}/r$,

$$\mathbf{E}_\omega\mathbf{(r)} = (1/4\pi\varepsilon_o)(i/k)\mathrm{rot}(\mathbf{B}_\omega) \qquad (2.2).$$

The time average of the Poynting vector $S(\mathbf{r},t)$ is given by[2]

$$< S(\mathbf{r},t) > = (1/2)\mathrm{Re}(\mathbf{E \times H^*}) = (1/2)\zeta_o|\mathbf{H}|^2\,\boldsymbol{r},$$

where $\zeta_o = [\mu_o/\varepsilon_o]^{1/2} \approx 377\ \Omega$ is the "vacuum impedance". The total power emitted by the antenna dP/dt is given by,[2]

$$dP/dt = \int_\Sigma < S > \bullet d\mathbf{A} = (\zeta_o/32\pi^2) \int_\Sigma \left|\int_{V'} (\mathbf{J}_\omega \times \mathbf{k})\, e^{-i\mathbf{k.r}'} dV'\right|^2 d\Omega \qquad (2.3),$$

where Σ is a surface, very far from the volume V´, involving the antenna in the "radiation zone", that is, for $r >> \lambda/2\pi$.

Assuming that the current distribution $\mathbf{J}_\omega(z')$ is given by

$$\mathbf{J}_\omega(z') = J_o \cos(2\pi z'/\lambda)\, \mathbf{e_3} \qquad (2.4),$$

where $\mathbf{e_3}$ is the unitary vector along the z´-axis. The integral along the z´-axis, from - L/2 up to + L/2, is given by[2]

$$k\, I_o \sin(\theta) \int \cos(2\pi z'/\lambda)dz' = k\, I_o\, L\, [\sin(\pi L/\lambda)/\pi L/\lambda] \sin(\theta) \qquad (2.5),$$

where I_o is the current along the wire, θ is the angle between $\mathbf{J}_\omega$ and z´goes from -L/2 up to L/2. Therefore, from Eqs.(2.3) we get,[2]

$$dP/dt = (\pi/6)\zeta_o\, I_o^2(L/\lambda)^2\, [\sin(\pi L/\lambda)/(\pi L/\lambda)]^2 \ \text{watts} \qquad (2.6),$$

For $\lambda >> L$ we have,

$$dP/dt \approx (\pi/6)\zeta_o\, I_o^2(L/\lambda)^2 \qquad \text{watts} \qquad (2.7).$$

To evaluate the *efficiency* of the antenna it is usually taken into account the radiation resistance $\mathbf{R_{radiation}}$ defined by

$$dP/dt = (1/2)I_o^2\, \mathbf{R_{radiation}} \qquad \text{watts} \qquad (2.8).$$

When $\lambda = L/2$, that is, for the *half-wave dipole antenna* (hwda) we have, using Eq.(2.7):

$$R_{radiation}^{(hwda)} \approx (\pi/3)\, \zeta_o\, (L/\lambda)^2 \approx 394\, (L/\lambda)^2 \quad \Omega \qquad (2.9).$$

Any antenna has a resistive component $R_d + R_{radiation}$, where R_d consists of dissipative losses in the antenna structure itself, and the radiation resistance $R_{radiation}$ which corresponds to power which is radiated away and dissipated remotely. It gives an idea of the antenna efficiency.[3,4]

Note that, thin linear conductors of length ℓ are in fact resonant at any integer multiple of a half-wavelength ℓ where *n* is an integer.

From Eq.(2.6) we see that the fundamental resonance of a thin linear conductor with length L occurs for a wavelength $\lambda = 2L$. Dipole antennas are frequently used when this condition is satisfied and are known as half-wave dipole antennas.

The radiation pattern of the antenna $< S(\mathbf{r},t) >$ (see **Figure (2.2))** is given by[3,4]

$$< S(r,\theta) > = (\zeta_o/2)\,(I_oL/2\lambda)^2 \sin^2\theta/r^2 \qquad (2.9)$$

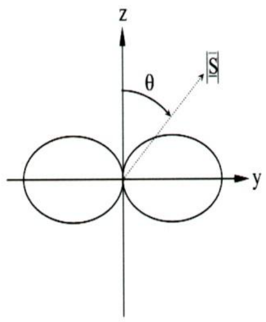


Figure (2.2) Doughnut-shaped radiation pattern of the z-oriented Hertzian dipole

Our above analysis refers only to a monochromatic source. When many frequencies are present see comments in reference [2].

(3)Half-Wave Center-Fed Antenna (Short Dipole Antenna).

The fundamental resonance of a thin linear conductor with size L occurs at a frequency wavelength $\lambda/2 = L$. Dipole antennas are frequently used at around that frequency and thus termed generically half-wave dipole antennas. [3,4] It is made with two conductors with sizes ~ $\lambda/4$ placed end to end for a total length ~ $\lambda/4$. The two wires are powered by an alternating current generator (see **Figure 3).**

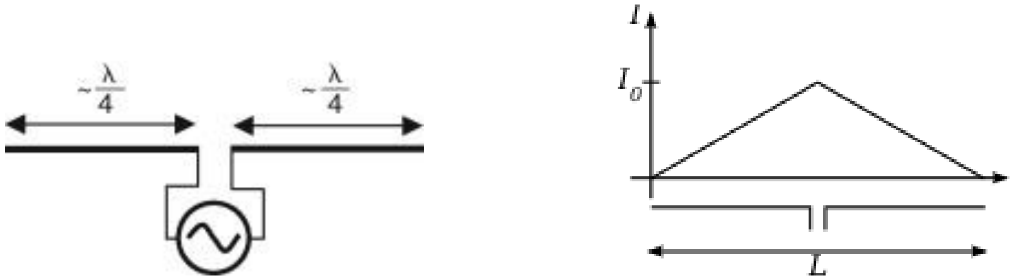


Figure 3. Schematic representation of a **center-fed half-wave dipole antenna** or, for simplicity**, short dipole antenna,** . The right scheme roughly shows the current in the arms: it increases from zero up to I_o at L/2 (feedpoint) and after decreases from I_o up to zero at L.[3,4]

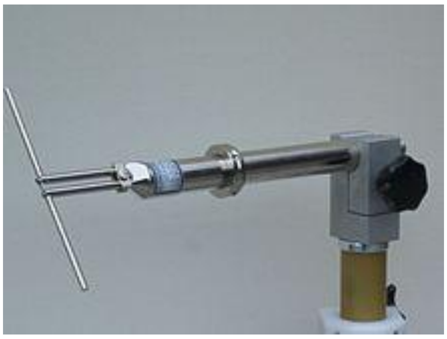


Figure 4. Typical UHF **center-fed half-wave dipole antenna**.[4]

Let us assume that the currents $\mathbf{J}_\omega(\mathbf{r}',t)$ are confined in wires with small sizes L in volumes V(**r´**) such as

$$r' \sim L \ll \lambda \text{ and that the waves are detected at } r \gg L \qquad (3.1).$$

To calculate the power emitted by the dipole we use a coordinate system similar to that seen in **Figure (2.1)**. The dipole wires are disposed along the z-axis and the origin of coordinates taken at the dipole feedpoint. When the conditions given above by (3.1) are obeyed we verify that [4,5] the electromagnetic potential $\mathbf{A}_\omega(\mathbf{r})$ is given by

$$\mathbf{A}_\omega(\mathbf{r}) = (\mu_o/4\pi).(e^{ikr}/r) \int_{V'} \mathbf{J}_\omega(\mathbf{r}')d^3\mathbf{r}' \qquad (3.2).$$

The integral seen in Eq. (3.2) becomes, by integration by parts,

$$- \int_{V'} \mathbf{r}'[\mathrm{div}\mathbf{J}_\omega(\mathbf{r}')]d^3\mathbf{r}' = -i\omega \int_{V'} \mathbf{r}'\rho(\mathbf{r}')d^3\mathbf{r}' \qquad (3.3),$$

remembering that, from the continuity equation, $i\omega\rho = \mathrm{div}(\mathbf{J})$. Thus,

$$\int_{V'} \mathbf{J}_\omega(\mathbf{r}')d^3\mathbf{r}' = I\,\boldsymbol{k} = -i\omega\mathbf{p} \qquad (3.4),$$

where $\boldsymbol{k}$ is unit vector along the z-axis and **p** is the **electric dipole moment**

$$\mathbf{p} = \int_{V'} \mathbf{r}'\rho(\mathbf{r}')d^3\mathbf{r}' \qquad (3.5).$$

Consequently, the vector potential $\mathbf{A}_\omega(\mathbf{r})$ is written as,

$$\mathbf{A}_\omega(\mathbf{r}) = (\mu_o/4\pi)\mathbf{p}(e^{ik\mathbf{r}}/r) \qquad (3.6),$$

In the radiation zone for r >> d and λ >>d we get, using Eq. (3.6),

$$\mathbf{B}_\omega(\mathbf{r},t) = (\mu_o/4\pi)\,\text{rot}[\mathbf{A}_\omega(\mathbf{r},t)] = (\mu_o/4\pi)\,k^2\,(\boldsymbol{r}\times\mathbf{p})\,e^{i(k\mathbf{r}-\omega t)}/r \qquad (3.7)$$

and $$\mathbf{E}_\omega(\mathbf{r},t) = \mathbf{B}_\omega(\mathbf{r},t)\times\boldsymbol{r} \qquad (3.8),$$

where $\boldsymbol{r} = \mathbf{r}/r$ is the unit radial vector. Omitting the index ω,the vectors $\mathbf{H}(\mathbf{r},t) = \mathbf{B}_o(\mathbf{r},t)/\mu_o$ and $\mathbf{E}(\mathbf{r},t)$ are at right angles to each other.

In terms of the unitary vectors **θ** and **ø**, shown in **Figure (2.1),** Eqs.(3.7) and (3.8) are therefore,

$$H_ø(\mathbf{r},t) = i(I\,L\,k/4\pi r)\,e^{i(k\mathbf{r}-\omega t)}\sin(\theta)$$

$$E_\theta(r,t) = \zeta_o H_ø = i(\zeta_o I\,L\,k/4\pi r)\,e^{i(k\mathbf{r}-\omega t)}\sin(\theta) \qquad (3.9).$$

As $\mathbf{H}(\mathbf{r},t) = \mathbf{B}_o(\mathbf{r},t)/\mu_o$ and $\mathbf{E}(\mathbf{r},t)$ are at right angles to each other the time average of the Poynting vector **S = E x H** is given by[2]

$$<\mathbf{S}> = (1/2)R_e(\mathbf{E}\times\mathbf{H}^*) = (1/2)\zeta_o|\mathbf{H}|^2\,\boldsymbol{r} \qquad (3.10),$$

where $\zeta_o = [\mu_o/\varepsilon_o]^{1/2} \approx 377\ \Omega$ is the "vacuum impedance". The average value of Eq.(3.10) becomes written as[4,5]

$$<\mathbf{S}(r,\theta)> = (1/2)\,E_\theta\,H_ø{}^*\boldsymbol{r} = (1/2)\zeta_o\{I\,Lk/4\pi r\}^2\sin^2(\theta)\,\boldsymbol{r} \qquad (3.11),$$

that was estimated, in a first approach, assuming that I = constant along the wire with length L. However, the current is not constant; thus taking, for instance, the average value $I \approx I_o/2$ and $k = 2\pi/\lambda$ we have a doughnut radiation pattern, seen in **Figure (2.2)**,

$$<\mathbf{S}(r,\theta)> = (\zeta_o/32)\,I_o^2\,(L/\lambda)^2\sin^2(\theta)/r^2\ \boldsymbol{r} \qquad (3.12).$$

The total radiate power $dP/dt = \int_\Sigma <S>\bullet d\mathbf{A}$ now is given by,

$$dP/dt = (\pi/12)\,\zeta_o I_o^2\,(L/\lambda)^2 \qquad (3.13).$$

Thus, for the **short dipole** antenna we have

$$dP/dt = (1/2)\, Io^2\, \mathbf{R_{radiation}}^{(sd)} = (\pi/12)\, \zeta_o I_o^2\, (L/\lambda)^2 \quad \text{watts} \quad (3.14),$$

where,

$$R_{radiation}{}^{(sd)} = (\pi/6)\, \zeta_o\, (L/\lambda)^2 \approx 197\, (L/\lambda)^2 \quad \Omega \qquad (3.15).$$

Setting $L = \lambda/2$ we verify that $R_{radiation}{}^{(sd)} \approx 49\ \Omega$. On the other hand, according to Eq. (2.9), the radiation resistance for the **half-wave dipole** antenna would be given by

$$R_{radiation}{}^{(hwda)} \approx 394\, (L/\lambda)^2 \approx 98\ \Omega,$$

that is, around two times larger than that given by the short-dipole antenna.

With a slightly different approach,[4] taking $I(z) = I_o \cos(\omega t) \cos(kz)$ it was shown that for $L = \lambda/2$ the predicted resistance radiation for the short dipole would be improved, obtaining $R_{radiation}{}^{(sd)} \approx 73.1\ \Omega$, instead of $49\ \Omega$.

For an AM radio station broadcasting at 1 MHz ($\lambda = 300$ m) with an antenna of effective length L_{eff} ~1 m; Eq.(2.14) yields $\mathbf{R_{radiation}} \sim 0.01\Omega$ ohms, which is very badly mismatched if the amplifier impedances of 50-100 ohms are found in long-wavelengths radio receivers. Equation (2.14) suggests that the effective length L should be somewhat greater than one-quarter wavelength, that is, $L \geq \lambda/4$, in order to achieve high radiation efficiencies. However, at these lengths Eq. (3.13) is beginning to lose validity because was assumed $L \ll \lambda/2\pi$. To mitigate this problem most sensitive resonant structures or transformers must be employed by long-wavelength radio receivers.[3,6]

(3.1) Power emitted by Tesla Radiator and Dipole Antenna.

The time average power emitted by the Tesla Radiator and by the Dipole Antenna are given, respectively, by Eqs. (1.6) and (3.13):

$$\langle P(t) \rangle_{Tesla} \sim 7\ 10^{-28}\, V_o^3\, \omega^2/R \quad \text{watts} \qquad \text{and}$$

$$\langle P(t) \rangle_{dipole\ antenna} \sim (\pi/12)\, \zeta_o I_o^2\, (L/\lambda)^2 \sim 10^2\, I_o^2\, (L/\lambda)^2 \quad \text{watts}$$

From these equations,

$$\langle P(t) \rangle_{Tesla} / \langle P(t) \rangle_{dipole\ antenna} \sim 2.5\ 10^{-7}\, V_o^3/(I_o^2 R L^2) \qquad (3.16).$$

For $R \sim L \sim 1$m, $I_o = 50$A and $V_o = 50$ V we get

$$\langle P(t) \rangle_{Tesla} \sim 10^4 \langle P(t) \rangle_{dipole\ antenna.} \qquad (3.17),$$

and for $V_o = 10^6$ V $\longrightarrow$ $\langle P(t) \rangle_{Tesla} \sim 10^8 \langle P(t) \rangle_{dipole\ antenna.}$ (3.18).

REFERENCES

[1]https://en.wikipedia.org/wiki/Tesla_coil

[2]W.K.H.Panofsky and M.Phillips."**Classical Electricity and Magnetism**" Addison Wesley(1956).

[3]https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-661-receivers-antennas-and-signals-spring-2003/readings/ch3new.pdf

[4]https://en.wikipedia.org/wiki/Dipole_antenna

[5]J.D.Jackson**."Classical Electrodynamics"**. John Wiley &Sons(1962).

[6]https://www.google.com/search?q=How+does+current+flow+in+a+dipole+antenna?&tbm=isch&chips=q:how+does+current+flow+in+a+dipole+antenna,online_chips:coax:iT8UxcXxv0E%3D&client=firefox-b-d&hl=pt-BR&sa=X&ved=2ahUKEwj37uvol-XxAhV_pZUCHfSZDmIQ4lYoAnoECAEQFQ&biw=1072&bih=444#imgrc=KuRH3i0agpbDtM