UNIVERSE EXPANSION AND THE QUANTUM VACUUM

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Abstract.

This is a didactical paper written to graduate and postgraduate students of Physics. An unpretentious and speculative analysis is performed connecting the Universe expansion with the quantum vacuum. We take for granted that Einstein´s Gravitational Field Equations are able to give a faithful description of gravitational and cosmological effects of the Universe. These equations show that there is a medium with negative mass density responsible by the Universe expansion. We are predisposed to believe that this medium is the quantum vacuum.
Key words: Einstein gravitation theory; universe expansion; quantum vacuum.

(I) Introduction.

In **Section 1**, assuming that the Universe was created by a Big Bang[1] are shown the basic assumptions of our paper. We like to remember that the Big Bang theory is still not universally accepted. It is, however, the most promising theory to explain how our Universe came to be (if inflation is included), and hence forms part of the current concordance model for cosmology. In **Section 2** are seen essential aspects of the Riemann geometry. Theoretically, the starting point of investigations about the Universe expansion [2] begins with the Einstein´s Gravitational Field Equations. [2-5] In **Section 3** is briefly shown how to obtain Einstein´s equations taking into account geometrical and physical arguments. Detailed calculations are omitted because these can be found in many text books and didactical papers on the subject.[2-6] In **Section 4** is analyzed the observed Universe taking into account Einstein´s predictions and the cosmological constant Λ. In **Section 5** and in **Appendix,** following the Big Bang model,[1] is tried to justify that the quantum vacuum is a strong candidate to explain the Universe expansion.

(1) Big Bang Model.

According to the Big Bang model[1] the total energy of the Universe was concentrated in a very small sphere with radius R_o. At the initial time t = 0 this sphere explodes due to extremely high energy processes and the energy begins to expand. Here, we introduce a **new hypothesis** for times t very far from the "inflation period":[1] *in the expansion, the quantum vacuum in the interstitial particles regions liberates energy".* This energy would be absorbed, for instance, by the particles immersed in the vacuum.

As, for times very far from the inflation period the Universe average matter density $\rho \sim 10^{-30}$ g/cm^3 we can assume, to analyze the expansion process, that it is immersed in a 4-dim Minkowski space. Observing, for instance, the expansion from the Earth, if E_o is the initial energy of the Universe, we must have

$$E_o = E_{matter}(t) + E_{luminous}(t) + E_{vacuum}(t) \approx E_{matter}(t) + E_{vacuum}(t), \quad (1.1)$$

assuming that the luminous energy $E_{luminous}(t) \sim 10^{-5} E_{matter}(t)$.[1]

As $dE_o/dt = 0$,

$$dE_{matter}/dt = - dE_{vacuum}(t)/dt \qquad (1.2).$$

This means that, if the vacuum is losing energy, that is, $dE_{vacuum}(t)/dt < 0$ we must have $dE_{matter}/dt > 0$. That is, the energy liberated by the vacuum will be used to increase the particles energies, for instance, to accelerate them in the radial expansion . In what follows, the energy densities of matter and vacuum will be indicate by ρ and ρ_v, respectively.

(2)Main Aspects of the Riemannian Geometry.

As shown, for instance, by S. Chandrasekhar,[2] the deduction of the Einstein´s Gravitational Field Equations can be done using a combination of reasonable physical arguments and mathematical simplicity. Of course, it would be simple if one knows basic concepts of Riemannian geometry. These can be found, for instance, in didactical papers published elsewhere[2] and many text books.[3-6] In metric geometries the basic equation is the *fundamental quadratic form* defined by,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad (2.1),$$

that gives the infinitesimal "distance" ds between two points in a n-dimensional Riemannian space, where $g_{\mu\nu}$ is the metric tensor.

In this space are defined the *Christoffel symbols*,

$$\Gamma^{\alpha}_{\mu\nu} = (g^{\alpha\beta}/2)(\partial_{\mu}g_{\lambda\nu} + \partial_{\nu}g_{\lambda\mu} - \partial_{\lambda}g_{\mu\nu}) \qquad (2.2)$$

and the *curvature tensor* or *Riemann- Christoffel tensor*

$$R^{\lambda}_{\sigma\mu} = \partial_{\nu}\Gamma^{\lambda}_{\mu\nu} - \partial_{\mu}\Gamma^{\lambda}_{\nu\sigma} + \Gamma^{\lambda}_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma} - \Gamma^{\lambda}_{\alpha\mu}\Gamma^{\alpha}_{\sigma\nu} \qquad (2.3).$$

For "plane" spaces where the components $g_{\mu\nu}$, written in rectilinear coordinates, are constants we have $R^{\lambda}_{\sigma\mu} = 0$, independently of the coordinate system. The scalar,

$$R = g^{\lambda\sigma} R_{\sigma\lambda} \qquad (2.4)$$

is named *"scalar curvature"* or "*curvature invariant*", where $R_{\lambda\mu}$ is the "Ricci tensor" defined by

$$R_{\lambda\mu} = R_{\mu\lambda} = \partial_{\mu}\Gamma^{\sigma}_{\lambda\sigma} - \partial_{\sigma}\Gamma^{\sigma}_{\lambda\mu} + \Gamma^{\alpha}_{\lambda\sigma}\Gamma^{\sigma}_{\lambda\mu} - \Gamma^{\alpha}_{\lambda\mu}\Gamma^{\alpha}_{\alpha\sigma} \qquad (2.5).$$

In the case of a 2- dimensional spherical surface with radius r we see that $R = -2/r^2$ which is proportional to the Gaussian curvature of a sphere with radius r, given by $1/r^2$.

For instance, a j-*covariant derivative* of a tensor A_{ik} is given by[3-6]

$$D^{j}A_{ik} \equiv A_{ik;j} = \partial A_{ik}/\partial x_j - \Gamma^{m}_{ij} A_{mk} - \Gamma^{m}_{kj} A_{im} \qquad (2.6).$$

It can be shown[3-6] that the most general symmetric second-order tensor $G_{\mu\nu}$ which has **purely geometrical properties**, that can be built with $R_{\mu\nu}$ and $g_{\mu\nu}$ and obeying the condition $D^{\mu} G_{\mu}^{\ \nu} = G_{\mu\ ;\nu}^{\ \nu} = 0$ is given by

$$G_{\mu\nu} = a R_{\mu\nu} + b g_{\mu\nu} R + \Lambda g_{\mu\nu} \qquad (2.7),$$

where a, b and Λ are constants. Eqs.(2.7) written in mixed components are

$$G_{\mu}^{\ \nu} = a R_{\mu}^{\ \nu} + b \delta_{\mu}^{\ \nu} R + \Lambda g_{\mu}^{\ \nu} \qquad (2.8).$$

As $g_{\mu\ ;\nu}^{\ \nu} = 0$ and $G_{\mu\ ;\nu}^{\ \nu} = 0$ [3-5,8] we verify, from Eqs.(2.8), that

$$(a R_{\mu}^{\ \nu} + b g_{\mu}^{\ \nu} R)_{;\nu} = 0 \qquad (2.9).$$

Taking into account the Bianchi identity $\{R_{\mu}^{\ \nu} - (1/2) g_{\mu}^{\ \nu} R\}_{;\nu} = 0$ [3] we conclude that $b = -a/2$. Consequently, from Eq.(2.8) we have:

$$G_{\mu\nu} = a (R_{\mu\nu} - g_{\mu\nu} R/2) + \Lambda g_{\mu\nu} \qquad (2.10).$$

(3)Einstein´s Gravitational Field Equations.

Till now it was only taken into account geometrical properties of metric Riemannian spaces. How to connect Geometry with Physics? At this **crucial point**, Einstein formulated the *Mach Principle* as a connection between metric and the energy-momentum tensor $T_{\mu\nu}$ of the matter. As gravitation involves macroscopic bodies he proposed that the spacetime geometry is entirely determined by the energy-momentum tensor $T_{\mu\nu}$ of a **macroscopic** ideal fluid[2-5] defined by

$$T_{\mu\nu} = \rho\, v_\mu v_\nu + (v_\mu v_\nu - g_{\mu\nu})p/c^2 = (\rho + p/c^2)\, v_\mu v_\nu - g_{\mu\nu}\, p/c^2 \qquad (3.1),$$

where ρ is the matter density, p its scalar pressure and v_k the 4-vector velocity of the fluid. Note that $D^\nu T^\mu{}_\nu = T^\nu{}_{\mu;\nu} = 0$.[3-6]

So , Einstein proposed[3] that $G_{\mu\nu} = \Theta T_{\mu\nu}$, that is,

$$a\, R_{\mu\nu} + b\, g_{\mu\nu} R + \Lambda\, g_{\mu\nu} = \Theta T_{\mu\nu} \qquad (3.2),$$

where a, b, Λ and Θ are constants to be determined. This equation according to Eqs.(2.10) becomes written as,

$$a\,(R_{\mu\nu} - g_{\mu\nu} R/2) + \Lambda\, g_{\mu\nu} = \Theta T_{\mu\nu} \qquad (3.3),$$

remaining now a, Λ and Θ to be determined. To do this it is necessary to assume that Einstein´s theory for $c \to \infty$ becomes Newton´s theory. In this limit it can be shown[3] that $\Lambda = 0$, $a = 1$ and $\Theta = 8\pi G/c^4$, resulting

$$R_{\mu\nu} - g_{\mu\nu} R/2 = (8\pi G/c^4) T_{\mu\nu} \qquad (3.4).$$

Einstein originally introduced the constant Λ in 1917[3,7] to counterbalance the effect of gravity and achieve a static universe, a notion which was the accepted view at the time. Einstein abandoned this constant in 1931[7] after Hubble's confirmation of the expanding universe.[3,7] From the 1930s until the late 1990s, most physicists agreed with Einstein's retraction, assuming the cosmological constant Λ to be equal to zero.[3] This changed with the surprising discovery in 1998 that the expansion of the universe is accelerating, implying the possibility of a positive nonzero value for Λ[3,7]. It was taken as non zero and positive[3,7] and Einstein´s equations with $\Lambda \neq 0$ have been extensively investigated.[3,6]

Note that Eqs.(3.4) (where $\Lambda = 0$) have described successfully events where relativistic gravitational effects are significant.[3-6,8,9] For sufficiently small Λ values Eqs.(3.4) agree very well with the equation for Newtonian potential. However, even a small value of Λ could have **drastic effects** on the evolution of the Universe.[3,6]

(4)Observed Universe.

In regions where $\rho \approx 0$, that is, when $T_{\mu\nu} \approx 0$, as a consequence of the field equations with $\Lambda \neq 0$, the static Newtonian potential Φ is given by[3]

$$\nabla^2\Phi(r) = - \Lambda \qquad (4.1).$$

It shows that in the cosmic medium the parameter Λ would have an effect equivalent to that created by an *effective mass* density ρ_{eff} given by,

$$\rho_{eff} = - \Lambda/4\pi G \qquad (4.2),$$

where Λ can be positive or negative. Note that this effective mass, up to now, has only a **geometrical origin.** In other words, Einstein´s equations predicted that in absence of real matter, that is, when $\rho = 0$, could exist a medium which has negative or positive mass density. At a first sight the cosmological medium could be taken as the *quantum vacuum* because it can have negatives or positives densities of mass and energy.[9,10] The quantum vacuum is an underlying medium that we believe to exist in space throughout the entire Universe.

If in spherical coordinates we arbitrarily set $\Phi = 0$ at the origin, then Eq.(4.1) has the solution

$$\Phi(r) = - \Lambda r^2/6 \qquad (4.3),$$

which is responsible for an harmonic force,

$$F(r) = \Lambda r/3 \qquad (4.4).$$

This force indicates that between any two "particles" of the vacuum acts an **attractive** force if $\Lambda < 0$ and **repulsive** if $\boldsymbol{\Lambda > 0}$.

So, as from the available (**cosmological**) observation data[3,7] there is an accelerated Universe expansion, we must expect that Λ is **positive**.

From cosmological data one can set the limit[3,7]

$$|\Lambda| < 10^{-35}/\text{sec}^2 \qquad (4.5),$$

obtaining from Eq.(4.2):

$$\rho_{eff} < - 10^{-25}\ \text{g/cm}^3 \qquad (4.6).$$

In what follows we assume that the cosmological medium is the quantum vacuum,[9,10] according to **Section 1**, and write $\boldsymbol{\rho_v = \rho_{eff}}$.

Calculations done with quantum field theories (**QFT**)[3,7] give an infinite value or at least ~10^{120} larger values than $\rho_v = \rho_{eff} \sim -10^{-50}$ erg/cm^3. Surprise is rather that the energy density is only $\rho_{eff,} \sim -10^{-50}$ erg/cm^3. Due to this, many authors[3,7] do not believe that the ρ_{eff} shown by Eq.(4.6) represents the quantum vacuum. See conclusions in **Section 5.**

(5)Comments, Discussions and Conclusions.

The "reality" and great importance of the "quantum vacuum" is confirmed by innumerable papers and books where are described, for instance, spontaneous emission, Casimir effect, Lamb Shift, quantum electrodynamics, high energy physics and so on...[9,10] As well known, Einstein´s equations and quantum vacuum approaches have explained successfully myriad of observed phenomena.

Intricate models and quantum field theories (**QFT**) calculations have been developed to explain the Universe expansion.[11,12] Guided by the **Ockham's razor (***"the simplest explanation is usually the best one"* **),** considering the successes of the quantum vacuum theory and of the Einstein´s equations we are inclined to believe that quantum vacuum is responsible by the accelerated expansion.

To support this hypothesis we estimated in **Appendix** the vacuum density ρ_{vac} of the Universe at now days taking into account the Big Bang model.[11,12] We have found $\rho_{vac} \sim \rho_{eff} \sim -10^{-28}$ g/cm^3 in fair agreement with predictions obtained in **Section 4**.

As vacuum energy is $E_{vac} = (4\pi/3)\, \rho_{vac} R^3$ and $\rho_{vac} < 0$ we see that $dE_{vac} = -(4\pi)\, |\rho_{vac}|\, R^2 dR$. So, in the Bag Bang expansion, when $dR > 0$, the vacuum energy decreases (negatively). This energy transferred to matter, according **Section 1**, would cause its accelerated expansion. The vacuum accelerated expansion would occur because $\Lambda > 0$ (see **Eq.(4.4)**).

APPENDIX. The Universe Radius and the Big Bang Model.

Is performed here a rough estimation of the vacuum mass density ρ_{vac} in the Universe at now days. It is done taking into account the time evolution of the Universe mass density $\rho(t)$ according to the Big Bang model.[11-13] In **Figure A**[13] is seen the radius R(t) of the Universe as a function of the time.

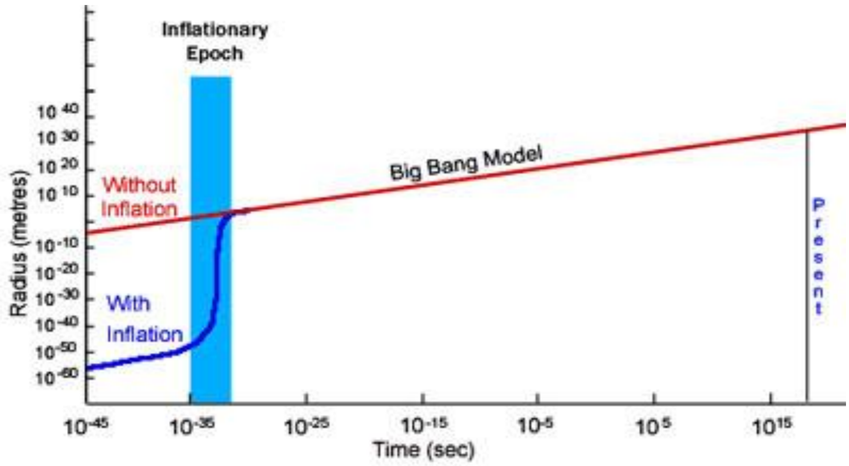

Figure A.[13] Universe radius R(t) according to the Big Bang Model as a function of the time.

According to **Section 1,** where t and R are defined in a 4-dim Minkowski space, we get from Eq.(1.1),

$$(3/4\pi)E_o = \text{constante} = \rho(t)R_u(t)^3 + \rho_{vac}(t)R_u(t)^3 \qquad (A.1),$$

where $\rho_u(t)$, $M_u(t)$, $R_u(t)$ and $\rho_{vac}(t)$ are, respectively, mass density, mass, radius and vacuum density of the Universe at times t very far from inflation epoch. In the absence of an exact value of E_o[4] inside the General Relativity context we take $E_o \approx M_o c^2$, $M_o = (4\pi/3)\rho_o R_o^3$, where ρ_o and R_o are, respectively, the mass density and radius of the Universe at t = 0.

Taking into account that[1] $\rho_o \sim 10^{93}$ g/cm^3, from Eq.(A.1) we obtain

$$\rho_{vac}(t) = 10^{93}[R_o/R_u(t)]^3 - M_u(t)/R_u(t)^3 \quad \text{g/cm}^3 \qquad (A.2),$$

that at now days, that is, when $t^* \sim 10^{20}$ s, $R_u(t^*) \sim 10^{10}$ pc $\sim 10^{28}$ cm and $M_u(t^*) \sim 10^{56}$ g [11] gives,

$$\rho_{vac}(t^*) = 10^9 R_o^3 - 10^{-28} \qquad \text{g/cm}^3 \qquad (A.3).$$

As, according to Eq.(4.6), $\rho_{vac}(t^*) \sim \rho_{eff}(t^*) < -10^{-25}$ g/cm^3, we verify that this condition is satisfied for $R_o < 10^{-14}$ m. That is, for all R_o values *inside the inflation period* (see **Figure A**).

So, from Eq.(A.3), we verify that $\rho_{vac}(t^*) \sim -10^{-28}$ g/cm^3. This result shows that $\rho_{eff}(t^*) < -10^{-25}$ g/cm^3, predicted in **Section 4**, would be a faithful value for the **vacuum density** at now days.

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