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# A LOCAL LAGRANGIAN EXHIBITING BAGS AND CONFINEMENT

by

I. Ventura(+) and G.C. Marques(++)

Instituto de Física - Universidade de São Paulo

C. Postal 20.516 - São Paulo, S.P. Brasil

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ABSTRACT

It is shown that a nonlinear field theoretical model with logarithmic nonlinearities exhibits the phenomenum of confinement. There is no elementary particle associated to the basical field of the Lagrangian. On the other hand the solitons of the model work like bags where the quantum excitation can live. Forces of the harmonic oscilator type trapps these excitations within the bags.

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## INTRODUCTION

The success of the Quark Model and the nonobservability of what would be the ultimate Constituents of matter has led to a variety of proposals for the explanation of why Quarks are not seen. However there is no Concensus among the various researchers on which mechanism prevents the Quarks of showing up in a scattering process. This question is still more difficult to be settled within realistic (three dimensional) field theoretical models.

In this paper we intend to present a Local Field theoretical model which exhibits confinement. Classical soliton-like solutions of this model exhibits features which are characteristic of the so called Bags[1]. The trapping of the constituents (Quarks) is due to an harmonic oscilator-like force which binds them together.

Section I is devoted to the introduction of the model and discussion of some of its classical solutions. In section II we discuss the classical stability of these solutions (and the related Quantum fluctuations). Although the material of these sections are not new (see for example ref. 2-4), they are important in order to understand the whole pictures of confinement which is present in section III. Section IV is left to-conclusions.

## I. The Model - Classical Solutions.

The model which will be studied here was proposed, within a different context, by Birula and Mycielski[2]. It is defined by the following Lagrangian density:

$$\mathcal{L}(\vec{x},t) = \partial_\mu \phi^* \partial_\mu \phi - \left(\frac{1}{\lambda^2} + \frac{1}{\ell^2}\right)\phi^*\phi + \frac{1}{\ell^2} \ln(\phi^*\phi\, a^{d-1})\, \phi^*\phi \tag{1.1}$$

Where $\lambda$, $\ell$ and a in (1.1) are dimensional parameters and d is the number of spatial dimensions.

The Euler-Lagrange equation resulting from (1.1) is

$$\left[\Box + \frac{1}{\lambda^2} - \frac{1}{\ell^2} \ln\left(\phi^*\phi\, a^{d-1}\right)\right]\phi = 0 \tag{1.2}$$

Two conserved quantities which we shall exploit later on are the energy and charge. They are given by:

$$E = \int d^d x \left[ 2\, \partial_t \phi^* \partial_t \phi - \mathcal{L}(\vec{x},t)\right] \tag{1.3}$$

$$Q = i \int d^d\vec{x} \left(\partial_t \phi^* \phi - \phi^* \partial_t \phi\right) \tag{1.4}$$

In ref(3) we have presented a large class of solutions of equation (1.2). Of special interest are periodic solutions of the form:

$$\phi_\omega(\vec{x},t) = A(\omega) \exp\left(-i\omega t - \frac{\vec{x}^2}{2\ell^2}\right) \tag{1.5}$$

Where $A(\omega)$ is given by

$$A(\omega) = a^{\frac{1-d}{2}} \exp\left[d + \left(\frac{\ell}{\lambda}\right)^2 - (\omega\ell)^2\right] \tag{1.6}$$

We recall that each one of these solutions describes an extended object (Hadron) in its center of mass reference frame. Such Hadron-like solutions are interpreted as Bound states[5] or resonances[4].

## II. Classical Stability and Quantum Fluctuations

After one has discovered a Classical extended object a relevant question which must be answered is concerning to its stability. This problem can be placed in the following way: suppose that

$\phi(\vec{x},t)$ is another Classical solution of equation (1.2) which differs from $\phi_\omega(\vec{x},t)$ by a fluctuation $e^{-i\omega t}\eta(\vec{x},t)$ (supposed to be small at least for t=0)

$$\phi(\vec{x},t) = \phi_\omega(\vec{x},t) + e^{-i\omega t}\,\eta(\vec{x},t) \tag{2.1}$$

We say that $\phi_\omega(\vec{x},t)$ is stable if $\eta(\vec{x},t)$ remains bounded in time. In accordance with the criteria of infinitesimal stability one has to check if the solutions for the linearized equation for $\eta(\vec{x},t)$ satisfies this requirement. The linearized stability equation is obtained by plugging (2.1) into equation (1.2) and retaining, (in the Taylor expansion around $\phi_\omega(\vec{x},t)$) up to linear terms in $\eta(\vec{x},t)$.

The procedure sketched above leads us to the following linearized stability equation for the fluctuation[4].

$$\left[\partial_t^2 - 2i\omega\,\partial_t - \partial_{\vec{x}}^2 + \frac{\vec{x}^2}{\ell^4} - \frac{(d+1)}{\ell^2}\right]\eta = \frac{1}{\ell^2}\,\eta^* \tag{2.2}$$

We would like to call the attention of the reader to the resemblance of equation (2.2) with the harmonic oscilator equation in Quantum Mechanics. In Ref.(4) we have obtained the solutions of the stability equation. They can be written in the form:

$$\eta_{K_1\cdots K_d} = A\{k_i\} \prod_{j=1}^{d} h_{k_j}(x_j/\ell)\,\cdot$$
$$\cdot\left\{\left(2K-1-2\omega\ell^2\gamma_{\{k_i\}}-\ell^2\gamma^2_{\{k_i\}}\right)^{1/2} e^{i\gamma_{\{k_i\}}t} + \left(2K-1+2\omega\ell^2\gamma_{\{k_i\}}-\ell^2\gamma^2_{\{k_i\}}\right)^{1/2} e^{-i\gamma_{\{k_i\}}t}\right\} \tag{2.3}$$

Where A{ki} is a real constant, $h_{k_i}$ is the Ki-th order normalized eigenfunction of the harmonic oscilator, while

$$K = \sum_{i=1}^{d} k_i$$

$\gamma_{\{k_i\}}$ in (2.3) stands for the so called stability angles[6].

They are given by

$$\left(\gamma^{\pm}_{\{kij\}}\right)^2 = \frac{2}{\ell^2}\left\{\left[(\omega\ell)^2 + k - \tfrac{1}{2}\right] \pm \sqrt{\left[(\omega\ell)^2 + (k-\tfrac{1}{2})\right]^2 - (k-\tfrac{1}{2})^2 + \tfrac{1}{4}}\right\} \tag{2.4}$$

With respect to the stability of the solutions (1.5) we should say that the main features which emerges from solutions (2.3 ) are:

There are stable solutions and unstable ones (Some of those corresponding to k=0 in (2.4)). Instead of discarding those unstable solutions we have suggested[4] that they are Classical manifestations of resonances.

All stability angles are discrete. As we will discuss shortly, this is a consequence of confinement. There are no asymptotic states of the basical field of our Lagrangian.

We mention that before quantization the fluctuations given by (2.3) will be related to the quantum excitations around the soliton. Since these fluctuations are eigenfunctions of an harmonic oscilator like equation it is clear that they are confined near the soliton.

## III. Confinement:

We intend to show in this section that there are no particles associated to the "elementary" field $\phi$ which appears in the Lagrangian (1.1). This is, essentially, due to the fact that one cannot introduce quantum fluctuations around the Vaccuum $\phi=0$.

In order to see this, we shall place our systems in a "cubic" box of volume $L^d$. We shall impose also periodic Boundary conditions. Later on we will perform the Thermodynamic Limit $L\to\infty$.

Now we will seek for plane wave type solutions of (1.2) (we recall that these are solutions associated to the excitations of the vaccuum) with finite charge. We write

$$\phi_{\vec{k},\omega}(\vec{x},t) = A(\vec{k},\omega)\, e^{i(\vec{k}\vec{x}-\omega t)} \tag{3.1}$$

After substituting (3.1) into the equation of motion (1.2)we will get the following restriction for $A(\vec{k},\omega)$

$$\omega^2 = \vec{k}^2 + \frac{1}{\lambda^2} - \frac{1}{\ell^2} \ln\left(a^{d-1}|A|^2\right) \tag{3.2}$$

The energy associated to (3.1) is

$$E(\omega,\vec{k}) = |A|^2 L^d \left(2\omega^2 + \frac{1}{\ell^2}\right) \tag{3.3}$$

On the other hand, the charge associated with solutions of the form (3.1) is

$$Q = |A|^2 L^d\, 2\omega \tag{3.4}$$

expressing$|A|^2$ in terms of the charge - By using (3.4) - and subs tituting in (3.2) we shall get

$$\omega^2 = \vec{k}^2 + \frac{1}{\lambda^2} - \frac{1}{\ell^2} \ln\left(\frac{a^{d-1}Q}{2L^d\omega}\right) \tag{3.5}$$

While the expresion for the energy in terms of the charge is

$$E = \frac{Q}{2\omega}\left(2\omega + \frac{1}{\ell^2}\right) \tag{3.6}$$

If we require that the charge Q be kept finite in the $L\to\infty$ limit, there are only two possible behavior for $\omega$ compatible with such a requirement. Namely:

$$\omega \underset{L\to\infty}{\sim} \frac{1}{\ell}\left[\ln\left(\frac{L}{a}\right)\right]^{1/2} \longrightarrow \infty \tag{3.7}$$

or

$$\omega \underset{L\to\infty}{\sim} \frac{1}{L^d}\left(\frac{a^{d-1}Q}{2}\right) \exp\left(-\vec{k}^2\ell^2 - \frac{\ell^2}{\lambda^2}\right) \longrightarrow 0 \tag{3.8}$$

The point which we would like to emphasize is that whatever the case ($\omega\to\infty$ as in (3.7) and $\omega\to 0$ as in (3.8)). The energy will allways be infinite if the charge is kept finite (see (3.6)).Therefore we conclude that there are no (free) observable elementary particles associated to the basic field of the Lagrangian.

The confinement of the "constituents" of matter,in the above mentioned sense, is associated to the nonexistence of the weak field limit, which on its turn is a consequence of the nonanaliticity of the Lagrangian interaction at $|\phi|^2 = 0$.

## IV. Conclusion:

We have shown that there are no free particles having finite charge and energy associated to the elementary fields. That leads us to the possibility - in terms of particles - of having only composite particles in the theory in such a way that the constituents does not show up freely (confinement). Such Hadrons indeed exist, and their manifestation at the Classical level are the so called solitons.

In order to complete our picture of the soliton as a Bag we shall analyse the nature of the fluctuations around the solitons. We would like to recall that inspite the nonexistence of Quantum excitations around the Vaccuum ($|\phi|^2 = 0$), there are Quantum fluctuations (represented by $\eta(\vec{x},t)$ in (2.1)) around the soliton whose main features are:

1) these Quantum fluctuations are practically zero for any region of the space outside the region delimited by the soliton (which in our example is similar to an hypersphere of radius $\ell$)

2) the spectrum of these Quantum excitations is a discrete

one. That means that there are no scattering states of the constituents (whose evidence is manifested by the continuum in the stability angle spectrum).

In this way the soliton works like a Bag. It holds the constituents inside a region of space delimited by its "Boundaries".

This strong Binding has its origin in the Harmonic Oscilator nature of the forces trapping the quantum fluctuations. At this point it is worth mentioning that there are many papers in the scientific literature which tries to confine quarks putting by hand the Harmonic Oscilator potentials. Within our approach this picture emerges in a very natural way.

In resume, we can say that the main features of a Bag, within our semi-classical description are:

- Free "elementary" particles does not show up.
- Quantum excitations are also confined within the Bag.
- The discrete nature of the stability angles (this is not dissociated of the earlier property). Within semi-classical approximation this implies a discrete spectrum for the excitations.

A similar picture is achieved within the quantized (semi-classical) nonrelativistic version of this model[7].

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6) Actually the stability angle as defined in ref. (5) differes from $\gamma$ by a multiplicative factor of

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