Liquid Droplets Deflections by Static Electric Field.

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Abstract.

We show that water droplets deflections, falling in gravitational field, are mainly due to its electrical charging by nearby objects. Effects of molecular dipole moments are shown to be very small. Droplets deflections can be seen in public sessions in the "Laboratório de Demonstrações EWH" of the Institute of Physics of the University of São Paulo(IFUSP). This text intend to be is a simple **"laboratory guide".**

Key words: droplets deflections; electrical charging; electric dipole moments.

(I)Introduction.

It is a didactical text, a **"laboratory guide",** written to students of Physics and Engineering. Only main aspects of the phenomenon will be analyzed in the particular case of liquid water droplets streams. It is assumed that droplets deflections are caused only by two different effects: **(1)** electrical charging induced in fluid streams by nearby objects and **(2)** electric dipole moments of the liquid. Are take into account two different kinds of water: (a)*tap* water [1] where are found a large number of positive and negative ions; (b)*pure* water water[2] where are present H_2O electric dipole moments and hydronium ions H_3O^+ and OH^- in equilibrium, $2H_2O \Leftrightarrow H_3O^+ + OH^-$. In **Section 1** deflections are studied assuming that droplets are electrically charged. In **Section 2** deflections are estimated assuming that deflections are ***only*** due to electric molecular dipoles. In **Appendix A**, **B** and **C** are shown experimental evidences for the electrical droplets charging by near objects.

(1)Charged Droplets.

Electrical charging of water jets by nearby charged objects has been known for well over two centuries.[1-4] This was clearly stated in 1867 by Lord Kelvin[4,5] in describing his elegantly simple *electrostatic generator*.

There is a large number of papers, videos and books analyzing the deflection, due to static electric fields, of thin liquid streams or liquid droplets in a gravitational field.[3,4] Let us consider droplets made of *tap* or *pure* water where are found positive and negative ions.[1] Let us also assume that they are falling vertically according to **Figure 1** and that the electric field is along the x-axis. In **Fig.1** is shown schematic experimental arrangement for observing deflection of water droplets in an static electric field created by a voltage terminal V. According to many works[4-7] (see **Appendix A,B and C**) when a neutral liquid jet is submitted to a static electric field applied close to the tap, polarized charges that are on the stream surface are impelled to move in the tap direction and collected by it. So, electrically charged emerging drops, close to tap, are attracted by the voltage terminal (see **Figure 1** and **Appendix A**).

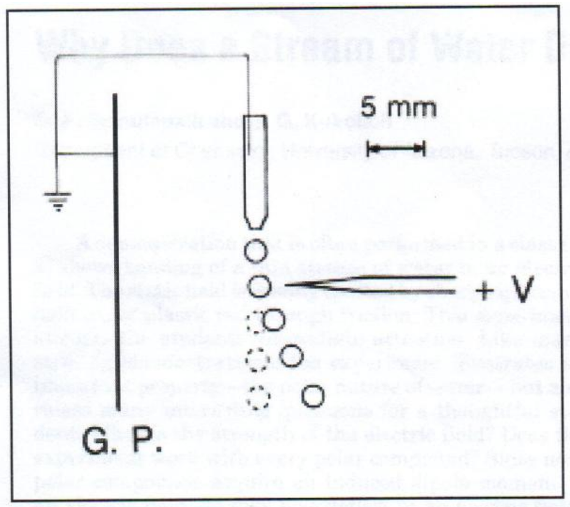

Figure 1. Schematic arrangement for observing deflection of water droplets in an electric field.[6]

The electric field **E** is created by a spherical tip with radius r_o at the point of the voltage probe. The centers of the drops, with radius R, are at a distance r* from the tip. The effective charge q_{eff} of the tip is then given by

$$k\, q_{eff} = V\, r_o \qquad (1.1)$$

where $k = 1/4\pi\varepsilon_o$. Hence,

$$E(r^*) = k\, q_{eff}/r^{*2} = V\, r_o/r^{*2} \qquad (1.2).$$

If Q is the droplet charge, the electric force $\mathbf{F}_{el}(r^*)$ on the droplet is

$$\mathbf{F}_{el}(\mathbf{r^*}) = E(r^*)Q\, \boldsymbol{i} = VQr_o/r^{*2}\, \boldsymbol{i} \qquad (1.3),$$

where $\boldsymbol{i}$ is the unit vector along horizontal x-axis. If M is the droplet mass the resultant force **F** on the droplet is given by

$$\mathbf{F} = - M g \boldsymbol{k} + E(r^*)Q \boldsymbol{i} = - M g \boldsymbol{k} + VQr_o/r^{*2} \boldsymbol{i} \qquad (1.4),$$

where $\boldsymbol{k}$ is the unit vector along the vertical z-axis.

As emerging droplets are formed with null vertical velocities, after the interaction time τ with g and E they would have a momentum $\mathbf{P}(\tau) \sim Mg\tau\, \mathbf{k} + F_{el}(r^*)\, \tau\, \boldsymbol{i}$. So, the deflection angle θ can be estimated by

$$\tan\theta \sim F_{el}(r^*)/Mg \qquad (1.5).$$

If $V = 10^3$ V, $Q \sim 10^{-9}$ C (according to **Appendix A**), $r_o = 0.5$ mm, $r^* = 1.5$ mm we have, $F_{el}(r^*) = VQr_o/r^{*2} = 2.22\ 10^{-4}$ N and $Mg = 6\ 10^{-4}$ N. Thus, using Eq.(1.5),

$$\tan\theta \sim F_{el}(r^*)/Mg \sim 0.37, \quad \text{that is,} \quad \theta \sim 20^o \qquad (1.6).$$

(2)Droplets only with Electric Dipole Molecules.

Now let us analyze an **ideal experiment** assuming that water is composed **only** by molecules with the H_2O electric dipoles (effects of hydronium ions[2]are not considered). As well known, applied electric fields in liquids not only orient dipolar molecules but also induce dipole moments in them. If the liquid has molecules with permanent electric dipole moment μ the net average dipolar molecular moment is given by[6]

$$< \mu_{net} > = \alpha E + (\mu^2/3K_BT)E \qquad (2.1),$$

where E is α the molecular polarizability, E the applied electric field **inside** the droplet, T the droplet temperature and K_B the Boltzmann constant. For nonpolar molecules $\mu = 0$ and the induced dipole moment is given by α E.

Let us assume that the droplets are falling vertically according to **Figure 1** and that the electric field is along the x-axis. So, in this context, the distance between the spherical tip of the voltage probe and the droplet center will be put equal to x. **O**ne can show that the interaction energy $\Xi(x)$ of a dipolar molecule with an electric field E(x) is given by[6]

$$\Xi(x) = \{\alpha + \mu^2/3K_BT\}E(x)^2/2 \qquad (2.2).$$

Thus, the electric force $\mathbf{f_{ele}}(x)$ on a dipolar molecule is written as,

$$\mathbf{f_{ele}}(x) = - (d\Xi/dx)\, \boldsymbol{i} = -\{\alpha + \mu^2/3K_BT\}E(dE/dx)\, \boldsymbol{i} \qquad (2.3).$$

From **Eq. (2.3)** we see that polar and nonpolar molecules are not affected by uniform electric fields.

If the tip radius is r_o = 0.5 mm and that V $=10^3$ V we have

$$k\, q_{eff} = V\, r_o = 0.5 \text{ Vm} \qquad (2.4).$$

So, the electric field (in the **vacuum**) at x = 1.5 mm, in the drop center, is given by

$$E(x) = k\, q_{eff}/x^2 = 2\ 10^5 \text{ V/m} \qquad (2.5)$$

and the field gradient

$$dE(x)/dx = 2\, k\, q_{eff}/x^3 = 3\ 10^8 \text{ V/m}^2 \qquad (2.6).$$

Let us estimate the electric force (2.3) for water droplets. So[6] we have μ =1.85 D = 6.16 10^{-30} Cm, α = 1.61 10^{-40} $C^2J^{-1}m^2$ and the dielectric constant $\varepsilon \sim 8$.**[#]** Thus, the electric force given by Eq.(2.3), on each water molecule, would be $f_{ele} \sim 2.5\ 10^{-27}$ N.[7]. As the gravitational force on each molecule is mg = 3 10^{-25} N for each one we have a resultant force **f** ,

$$\mathbf{f} = -3\ 10^{-25}\, \boldsymbol{k} + 2.5\ 10^{-27}\, \boldsymbol{i} \ \text{ N} \qquad (2.7)$$

and that the deflection angle θ is very small, given by

$$\tan\theta \sim f_{el}/Mg \sim 8.3\ 10^{-3}, \quad \text{implying that,} \quad \theta \sim 1^o \qquad (2.8).$$

(3)Conclusions.

From the above theoretical analysis and experimental observations seen in **APPENDIX A,B** and **C**, we can conclude that pure and tap water droplets deflections are essentially due to charging effects. Molecular dipoles effects are negligible.

[#] ***Note that in reference 6, as pointed out in reference 7, was not taken in account the water dielectric constant to estimate the electric force f_{ele}.***

APPENDIX A. **Electrical charging of water droplets.**

Now, using simple experimental techniques, we show how static electric fields induce polarization and separation of electric charges in liquid streams and droplets.

The experiment illustrated in **Figure (A.1)** is an evidence for the water droplets electrization. There is shown an electroscope with two aluminum foils inside a glass vessel put below the water droplets stream.

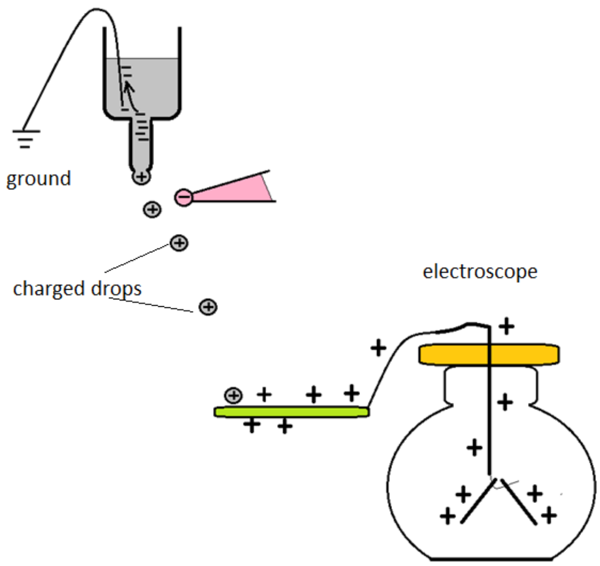

Figure (A.1). Experimental evidence for the droplets electric charging.

Due to the negative probe put close to the tap, polarized negative free charges induced on droplets surfaces are impelled along the grounded wire. Thus, the falling droplets becoming positive are attracted by the probe and deviated from the vertical. They are deposited on a metallic plate connected with aluminum foils of the electroscope. The separation of the foils increases as the number of collected drops increases. One can verify that the electroscope is charged with positive charges, that is, contrary to the probe charges.

In **Figure (A.2)** is the photo of the experimental apparatus shown in the diagram of **Figure (A.1)**

Figure (A.2). **Photo showing charged droplets being collected by the electroscope.**

Note that in all analyzed cases the grounded system plays a fundamental role in the droplets charging and, consequently, on the electroscope charging.

Comments on electroscope charging for pure water droplets.

Each falling droplet collected by the electroscope has a volume ~ $6\ 10^{-5}$ L. Taking into account that concentrations of (H_3O^+) and (OH^-) ions in pure water, at ambient temperature, is ~ $6\ 10^{15}$ ions/L[11] each droplet would carry ~ $3{,}6\ 10^{11}$ ions.

Let us assume that only falling drops with positive ions would be collected by electroscope foils. As each drop has ~$1.8\ 10^{11}$ positive ions it would be charged with ~$2.8\ 10^{-8}$ C. Since the electroscope is well charged with ~10^{-10} C **only one** droplet would be sufficient to its charging.

However, as it is very difficult or almost impossible to create only positive droplets it is necessary **various** falling droplets, that have a mixture negative e positive charges, for the complete electroscope charging.

Thus, it is plausible that "pure water" droplets would have significant trajectory deviations as occurs with the "tap water" droplets.

APPENDIX B. Influence of the probe distance from the tap.

In **Figure (B.1)** there is a photo showing the droplets deviation due to charged negative probe put close to the tap.

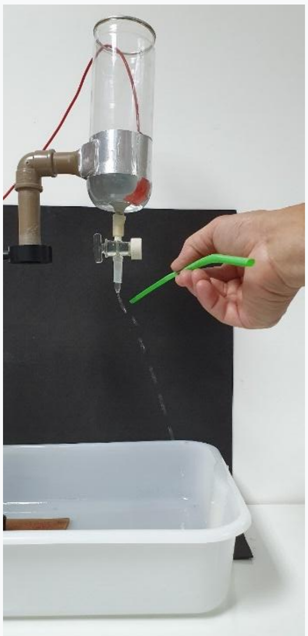

Figure (B.1). Negative probe tip put close to the tap.

In **Figure (B.2)** is shown the scheme of droplets trajectories when the probe tip is far from the tap. The droplets deviations are very small.

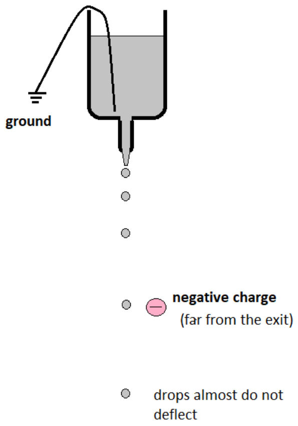

Figure (B.3) shows the photo when the probe is very far from the tap with a very small deviation of the droplets stream.

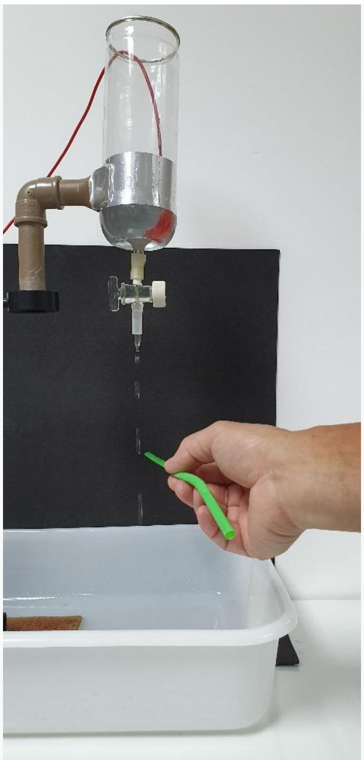

Figure (B.3). Probe very far from the tap and a very small deviation of the droplets.

Appendix C. **Water droplets charged with the Van de Graaff stick.**

Here we see how to enhance the droplets charging using the "Van de Graaff stick" which can stored a large quantity of charges.[10]

In **Figure (C.1),** shows an ungrounded vessel plain of water in contact with the charged "Van de Graaff stick". This induces a negative charges density at the right side of the recipient. In this way, as shown below in **Figure (C.1)**, the falling droplets become positively charged.

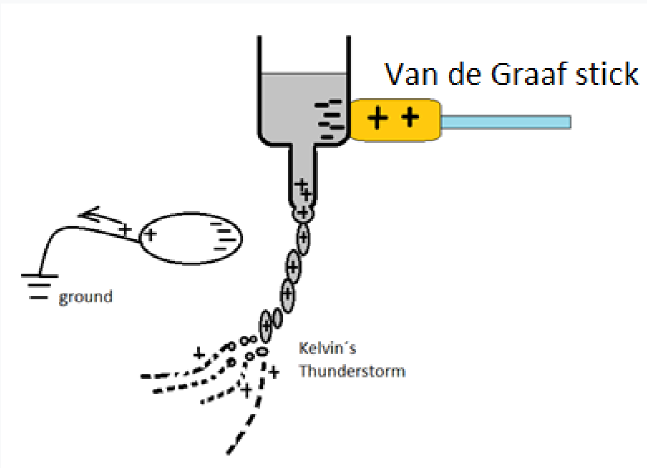

Figure (C.1). In the vertical fall the charged stream, due to the electric repulsion, is divided into smaller droplets originating the "Kelvin´s Thurderstorm." [5,6]

In **Figure (C.2)** we have the photo of the van de Graaf stick in contact with the ungrounded vessel. The hand has a grounding function and is attracting the water drops. The "Kelvin´s Thurderstorm" is almost invisible in the photo.

Figure (C.2). Photo of the ungrounded vessel stick in contact with the van de Graaf stick, grounded hand and the subtle "Kelvin´s Thurderstorm".

Acknowledgements. *The authors thank Dr.Alvaro Vannucci for helpful discussion on the problem*. *The authors also thank the library Virginia de Paiva for the invaluable assistance for publication of this paper*

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