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PENETRABILITY THROUGH A THREE HUMPED BARRIER IN QUASI-CLASSICAL APPROXIMATION

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ABSTRACT

Penetrability through a three humped barrier has been calculated in quasi-classical approximation and a plausible explanation of subbarrier fission characteristics of thorium is suggested.

INTRODUCTION

During last several years, we have witnessed a great revival of interest in the study of nuclear fission. This has been due to parallel developments in experiments and theory during this period and has brought about a significant improvement in our understanding of the process. Strutinsky(1) has recently suggested the existence of a double humped barrier in fission by adding the single particle effects to the liquid drop contribution to the nuclear potential energy. This so called "Strutinsky prescription" has been used recently by several groups to calculate the potential energy surfaces for heavy and superheavy nuclei. In the actinide region, for isotopes of Thorium, the calculated(2) first saddle and second minima are about 3 MeV lower than the experimental values commonly attributed to them. This discrepancy constitutes the well known "Thorium anomaly"(2,3,4) in the recent fission literature. However Möller and Nix(5) have recently reported the possible existence of a third asymmetric minimum in the fission barrier for Thorium and they suggest it as a possible resolution of the thorium anomaly mentioned above. It is therefore of interest to investigate the behaviour of penetrability with energy through such a three humped potential barrier. It is our hope that such an investigation will be helpful in understanding the observed sub barrier fission characteristics of Thorium.

CALCULATION OF PENETRABILITY

Penetrability through a three humped barrier has been calculated in the framework of W.K.B. approximation. It is similar to a more recent calculation done by us for a double humped barrier and the details can be seen in references[6,7]. The potential barrier has been parameterized by smoothly joining five parabolas as shown in Figure 1. The potential in different regions is given as

$$
\begin{aligned}
V(x) &= E_1 - \tfrac{1}{2}\mu\omega_1^{\,2}(x - x_1)^2 && x \le x_2 \\
&= E_2 + \tfrac{1}{2}\mu\omega_2^{\,2}(x - x_3)^2 && x_2 \le x \le x_4 \\
&= E_3 - \tfrac{1}{2}\mu\omega_3^{\,2}(x - x_5)^2 && x_4 \le x \le x_6 \\
&= E_4 + \tfrac{1}{2}\mu\omega_4^{\,2}(x - x_7)^2 && x_6 \le x \le x_8 \\
&= E_5 - \tfrac{1}{2}\mu\omega_5^{\,2}(x - x_9)^2 && x \ge x_8
\end{aligned}
\qquad (1)
$$

where as shown in Figure 1; x_1, x_3, x_5, x_7 and x_9 represent the maximas and minimas in the potential energy curve with respect to the coordinate x while x_2, x_4, x_6 and x_8 are the points where different parabolic potentials join each other. x_1 is obtained by taking the potential to be zero at $x = 0$. Other values of x's are then obtained from the requirement that the potentials as well as their first derivatives match exactly at the joining points. The energies E_1, E_2, E_3, E_4, E_5 and the curvatures ω_1, ω_2, ω_3, ω_4, ω_5 are arbitrary and can be changed to obtain different desired shapes of a three humped barrier. μ has been taken as the reduced mass of the resulting fragments from the fission of Th^{232} and has been assumed constant throughout. Different a's correspond to the various classical turning points where $E = V(x)$.

As shown in Figure 1, we have a wave incident upon the barrier in region I. Part of it is reflected back and the rest is transmitted to the region VII. Taking only an outgoing wave in region VII, we calculate the corresponding W.K.B. wave function in region I. It is then straightforward to obtain transmission coefficient defined as the ratio of the transmitted flux to the incident flux. The results are given in the following.

CASE I : $E_2 \le E \le E_4$

In this energy region, the potential is only a two humped barrier with the transmission coefficient given by

$$T = \left| \frac{1}{(e^{\nu} + \frac{1}{4}e^{-\nu})\ (e^{\nu_7} + \frac{1}{4}e^{-\nu_7})e^{-i\nu_6} + (\frac{1}{4}e^{-\nu} - e^{\nu})\ (\frac{1}{4}e^{-\nu_7} - e^{\nu_7})e^{i\nu_6}} \right|^2 \tag{2}$$

where various ν's are given as

$$\nu_6 = \int_{a_2}^{a_3} K_1(x)\,dx\ , \qquad \nu_7 = \int_{a_1}^{a_2} K_2(x)\,dx$$

$$\text{and} \quad \nu = \int_{a_3}^{a_4} K_2(x)\,dx \tag{3}$$

$$\text{where} \qquad K_1(x) = \left\{ \frac{2\mu}{\hbar^2} (E - V(x)) \right\}^{1/2} = iK_2(x) \tag{4}$$

a_1, a_2, a_3, a_4 have been shown in Figure 1 on the dashed line representing the total energy E for case I.

CASE II : $E_4 \le E \le E_3$

In this region, we encounter a three humped potential barrier with the transmission coefficient as

$$T = \left| \frac{1}{\left[(e^{\nu_1} + \frac{1}{4} e^{-\nu_1})(-e^{\nu_3} + \frac{1}{4} e^{-\nu_3})(\frac{1}{4} e^{-\nu_5} - e^{\nu_5}) e^{i(\nu_4 - \nu_2)} + (\frac{1}{4} e^{-\nu_1} - e^{\nu_1})(e^{\nu_3} + \frac{1}{4} e^{-\nu_3})(\frac{1}{4} e^{-\nu_5} - e^{\nu_5}) e^{i(\nu_2 + \nu_4)} + (e^{\nu_1} + \frac{1}{4} e^{-\nu_1})(e^{\nu_3} + \frac{1}{4} e^{-\nu_3})(e^{\nu_5} + \frac{1}{4} e^{-\nu_5}) e^{-i(\nu_2 + \nu_4)} + (\frac{1}{4} e^{-\nu_1} - e^{\nu_1})(-e^{\nu_3} + \frac{1}{4} e^{-\nu_3})(e^{\nu_5} + \frac{1}{4} e^{-\nu_5}) e^{-i(\nu_4 - \nu_2)} \right]} \right|^2$$

where various ν's are given by (5)

$$\nu_1 = \int_{a_5}^{a_6} K_2(x)\,dx \quad , \quad \nu_2 = \int_{a_4}^{a_5} K_1(x)\,dx$$

$$\nu_3 = \int_{a_3}^{a_4} K_2(x)\,dx \quad , \quad \nu_4 = \int_{a_2}^{a_3} K_1(x)\,dx$$

$$\text{and} \quad \nu_5 = \int_{a_1}^{a_2} K_2(x)\,dx \qquad (6)$$

where $K_1(x)$ and $K_2(x)$ are defined above in equation (4). Here a_1, a_2, a_3, a_4, a_5 and a_6 are various classical turning points as shown in Figure 1 on the dashed line representing the total energy E for case II.

RESULTS:

The transmission coefficient for a three humped barrier (Fig.1) has been plotted versus energy in Figure 2. In the low energy region (case I), one obtains resonances corresponding to states only in the second well. These occur at energies given by

$$E_{n_{II}}^{(0)} = E_2 + (n_{II} + \frac{1}{2}) \hbar w_2 \qquad (7)$$

where $n_{II} = 0,1,2.....$

At higher energies (case II), in addition to the states in the second well, there are also peaks corresponding to states in the third well occuring at energies given by

$$E_{n_{III}}^{(0)} = E_4 + (n_{III} + \frac{1}{2}) \hbar w_4 \qquad (8)$$

where $n_{III} = o,1,2...$

It is to be noted that due to finite nature of the potential wells, the energies for the higher lying states are slightly lower than those given by equations (7) and (8). Also, depending upon the barrier parameters, a state in the second well can lie very close to another state in the third well. This can be seen in our results in Figure 2 where the two peaks near 9.25 MeV are separated only by 20 KeV. Such states will become indistinguishable when a broadening of pure transmission resonances is introduced in any of the physical models[6,7,8] to calculate cross sections etc.

POSSIBLE INTERPRETATION OF SUBBARRIER FISSION CHARACTERISTICS OF THORIUM:

From the above study of penetrability through a three humped barrier, we can see that if the innermost barrier and the second minimum are not well formed (Figure 3), the only possibility of a shape isomeric state would be as the ground state in the third minimum. The excitation energy of such an isomer will then be high enough to make its life time extremely small and thus difficult to observe with the present available techniques. Furthermore, the isomer may also preferably decay by gamma deexcitation because of a more penetrable inner barrier. This may explain why no fission isomer has been observed to date for thorium isotopes. On the other hand, the existence of subbarrier fission resonances in neutron induced fission of Th^{230} [9], in photofission of Th^{232} [10] and in direct reaction induced fission of Th^{234} [11] can be understood in terms of vibrational states in the third minimum. It is therefore plausible that a potential barrier similar to the one shown in Figure 3 may exist for thorium isotopes and that attempts should be made to obtain information about the parameters of such a barrier from the observed subbarrier fission characteristics of thorium isotopes.

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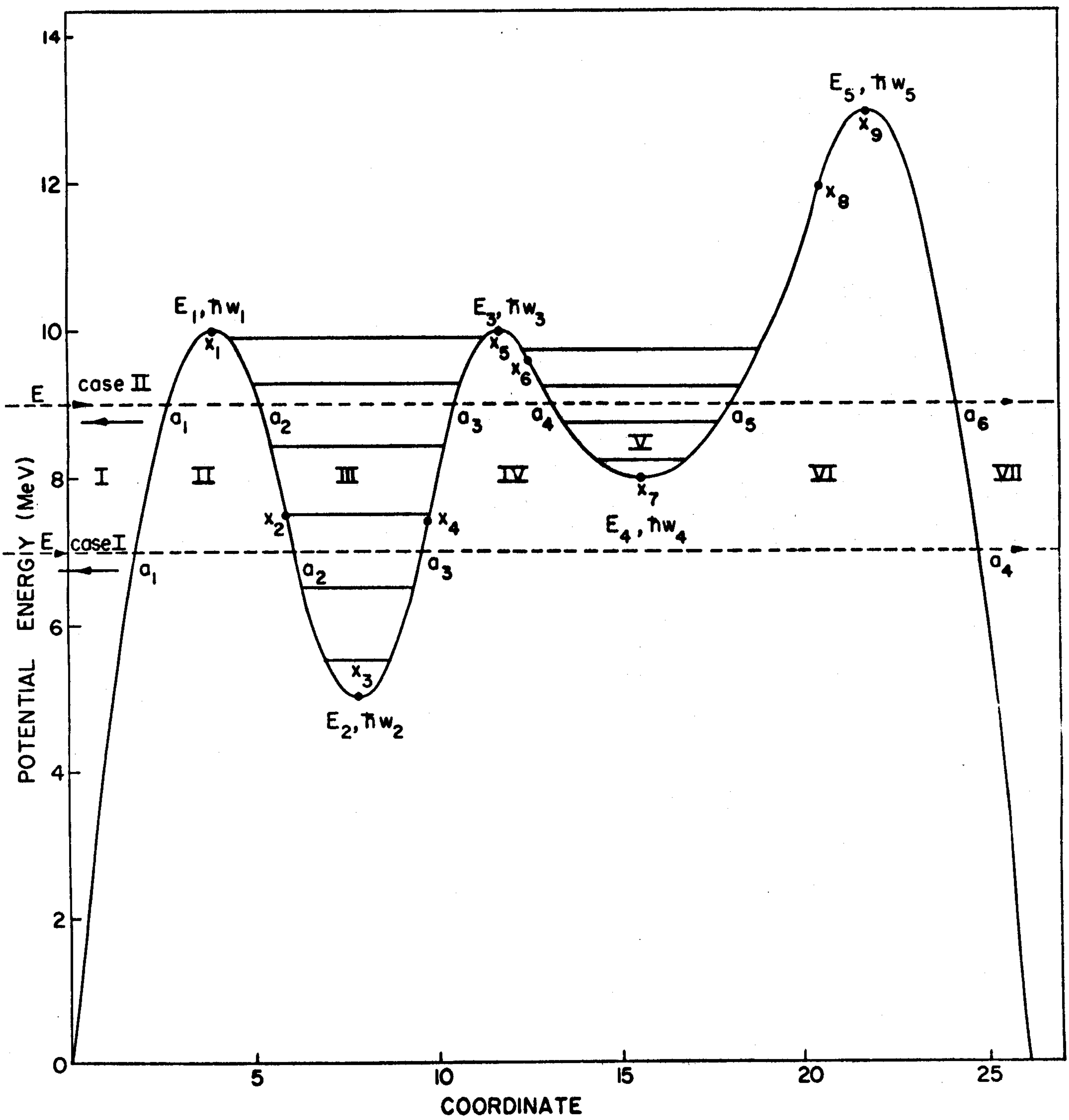
FIGURE CAPTIONS

Figure 1 - An illustration of a three humped barrier parameterized by smoothly joining five parabolas. The barrier parameters are: E_1 = 10.0 MeV, $\hbar\omega_1$ = 1.0 MeV, E_2 = 5.0 MeV, $\hbar\omega_2$ = 1.0 MeV, E_3 = 10.0 MeV, $\hbar\omega_3$ = 1.0 MeV, E_4 = 8.0 MeV, $\hbar\omega_4$ = 0.5 MeV, E_5 = = 13.0 MeV, $\hbar\omega_5$ = 1.0 MeV. The quasi-bound levels in the second and third well are indicated by the solid lines.

Figure 2 - A logarithmic plot of the calculated transmission coefficient through the three humped barrier of Figure 1. Note the sharp resonances at the positions of the quasi-bound levels in the second and third wells.

Figure 3 - An illustration of a plausible three humped barrier for thorium. The barrier parameters are: E_1 = 1.2 MeV, $\hbar\omega_1$ = 0.2 MeV, E_2 = 1.0 MeV, $\hbar\omega_2$ = 0.5 MeV, E_3 = 6.0 MeV, $\hbar\omega_3$ = 1.1 MeV, E_4 = 4.5 MeV, $\hbar\omega_4$ = = 0.9 MeV, E_5 = 7.0 MeV, $\hbar\omega_5$ = 0.8 MeV.



POTENTIAL ENERGIY (MeV)
COORDINATE
0
2
4
6
8
10
12
14
5
10
15
20
25
E₁, ħw₁
E₂, ħw₂
E₃, ħw₃
E₄, ħw₄
E₅, ħw₅
case II
case I
E
I
II
III
IV
V
VI
VII

Figure 1

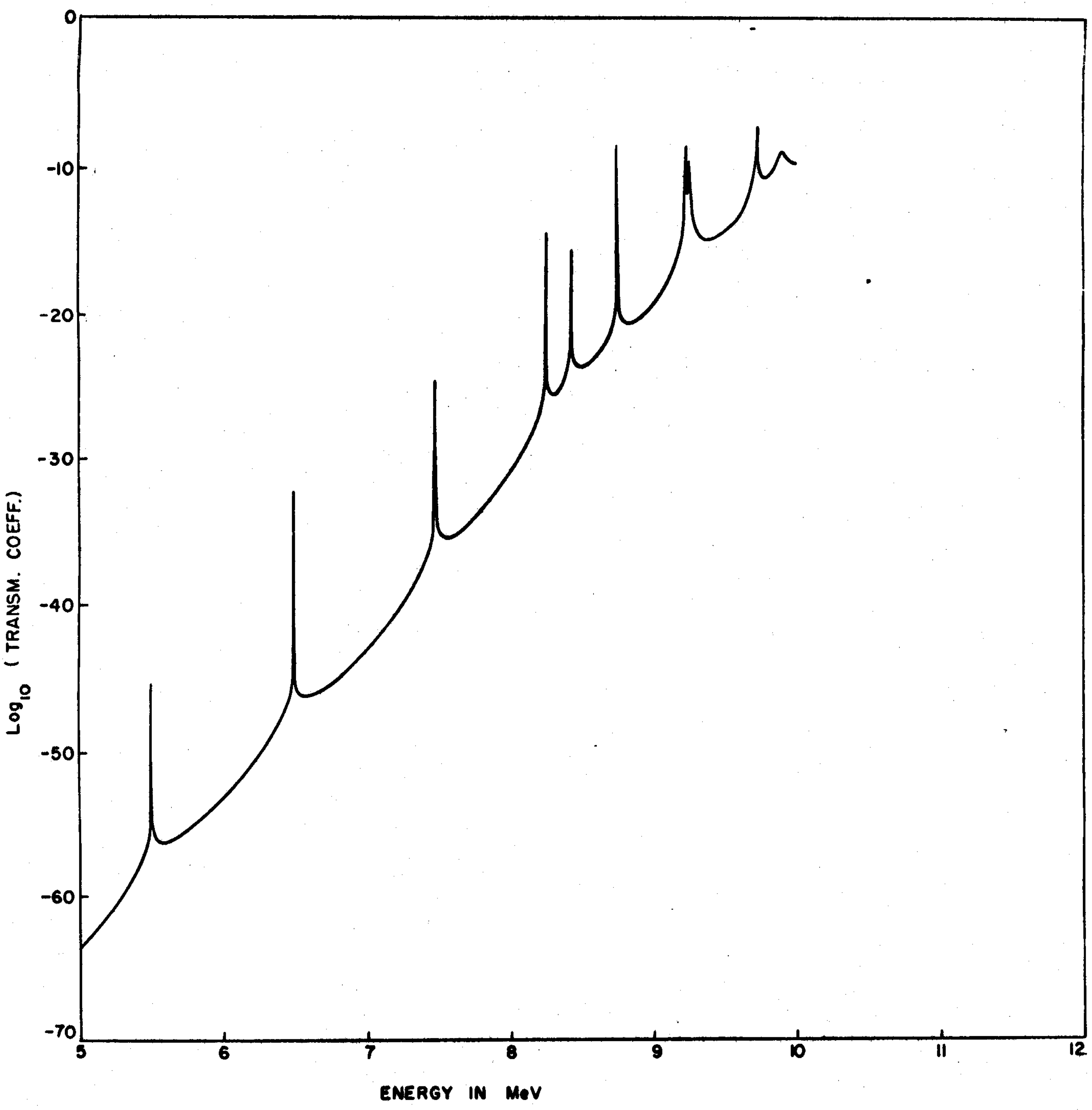
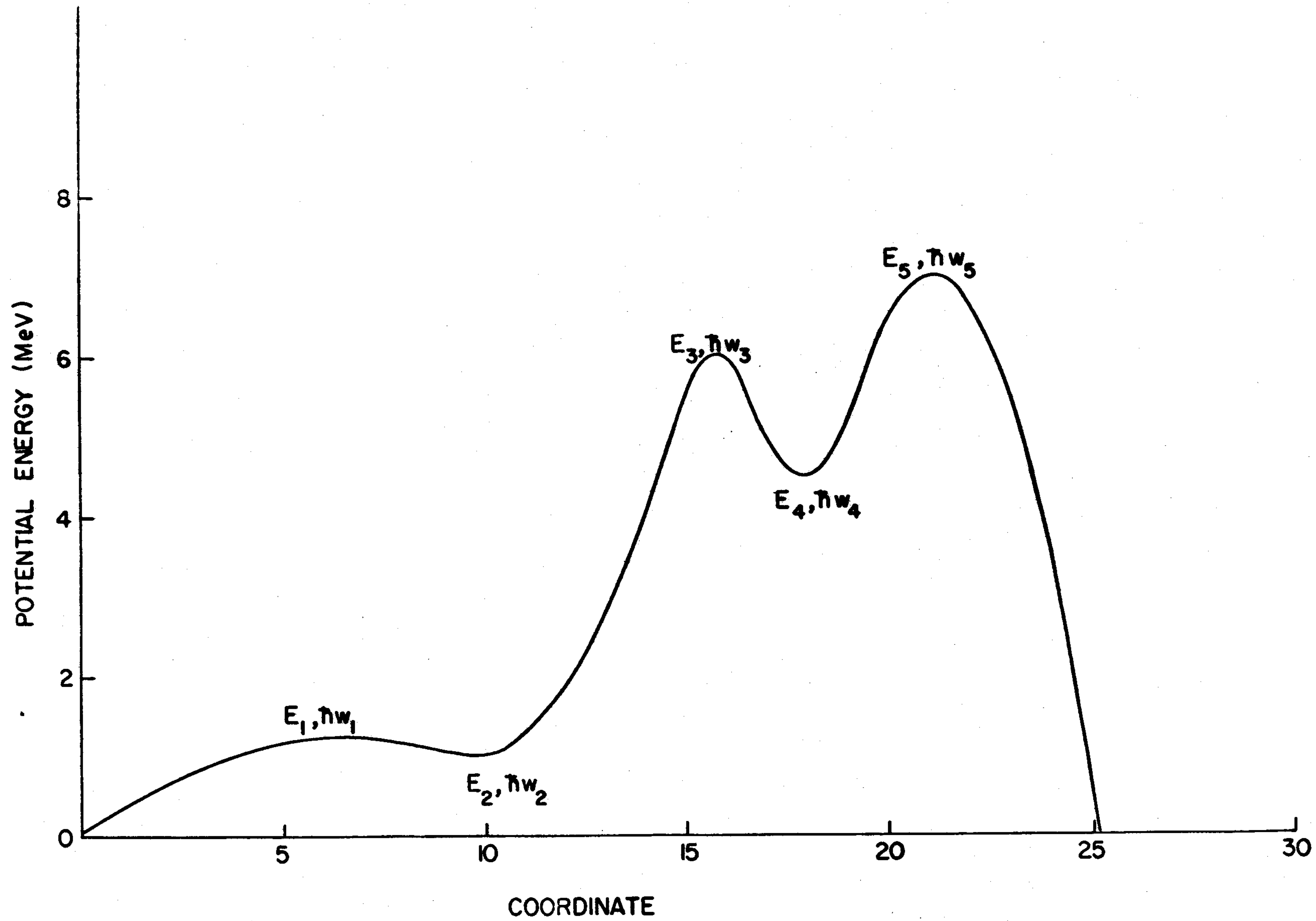

0
-10
-20
-30
-40
-50
-60
-70
Log10 (TRANSM. COEFF.)
5
6
7
8
9
10
11
12
ENERGY IN MeV

Figure 2



POTENTIAL ENERGY (MeV)
0
2
4
6
8
5
10
15
20
25
30
COORDINATE
$E_1, \hbar w_1$
$E_2, \hbar w_2$
$E_3, \hbar w_3$
$E_4, \hbar w_4$
$E_5, \hbar w_5$

Figure 3