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GAUGE INVARIANT WILSON OPERATORS AND RADIATIVE CORRECTIONS TO HADRONS

JOSIF FRENKEL

Instituto de Física
Universidade de São Paulo

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ABSTRACT

We discuss the renormalization of Wilson operators, which are relevant for the radiative corrections to hadrons, in covariant Lorentz gauges, for a class of non-abelian gauge theories. We find for the anomalous dimensions of the operators, which determine the asymptotic behaviour of the radiative corrections, the same gauge invariant result as the one obtained in ghost free gauges.

I. INTRODUCTION

Non-abelian gauge theories of strong interactions have the remarkable feature of being asymptotically free [1], i.e., the effective coupling constant vanishes in the deep Euclidean region. This fact allows the determination of anomalous dimensions for operators which govern the asymptotic behaviour of various physical processes. We will consider the radiative corrections in hadronic processes which are described by :

$$\Delta m = \frac{ig^2}{4\pi} \int \frac{d^4x}{x^2 - i\varepsilon} \langle B | T j_\mu(x) j_\mu(0) | A \rangle \qquad (1)$$

where j_μ are conserved currents, invariant under the usual symmetry group of strong interactions.

The study of the asymptotic properties of these processes requires two tools : Wilson's expansion [2] of the product of the currents and the renormalization group [3] which gives the behaviour of the coefficients occuring in the expansion. Of course, we require that the matrix elements of Wilson's expansion be gauge independent, since they are physical observables. Thus, the gauge independence of the divergence in (1) at $x \sim 0$ should be a consequence of an explicit evaluation of hadronic interactions in a non-abelian gauge theory.

At this point, we would like to point out that the gauge invariance in a Wilson expansion involves a basic difficulty. For calculation in a certain quantized field theory we have to abolish manifest gauge invariance by imposing a gauge condition. In the ghost-free axial gauges, where we can neglect non-gauge invariant operators, the radiative corrections have been computed and a gauge invariant result has been found [4]. However, as long as covariant Lorentz gauges are used, we are confronted with a crucial dependence on ghosts. The highly non-trivial problem of renormalization of Wilson operators in the presence of ghosts has been discussed by many authors [5] [6] and has by now been solved [7].

In this paper, following the arguments given in reference (7), we will compute the asymptotic behaviour of radiative corrections in covariant Lorentz gauges. In section II we present the model and discuss the Ward identities, which are relevant to the renormalization of Wilson operators, which are needed in the leading expansion of $T j_\mu(x)\, j_\mu(0)$. In section III we calculate, at the one-loop level, the gauge invariant matrix Z^{n}, whose eigenvalues are of interest for the Wilson expansion of gauge invariant operators. In section IV we work out the anomalous dimensions of twist-4 operators, which determine the asymptotic behaviour of radiative corrections, and find the same gauge invariant result as obtained in the ghost-free gauges [4].

II. THE MODEL

We consider a Lagrangian involving Yang-Mills fields [8] A_μ^a and fermion fields $\psi, \bar\psi$ for a group with structure constants f_{abc} and coupling constant g :

$$L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \bar\psi(\gamma_\mu \mathcal{D}_\mu + m)\psi \tag{2}$$

where we have defined :

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c \tag{2a}$$

$$\mathcal{D}_\mu = \partial_\mu - i g t^a A_\mu^a \tag{2b}$$

where the hermitian matrices t^a form a representation of the Lie algebra. For Lorentz gauges, characterized by the parameter α, one adds to L :

$$L_g = -\frac{1}{2\alpha}(\partial . A^a)^2 \tag{3a}$$

which yields the following Faddeev-Popov ghost Lagrangian :

$$L_{F.P.} = \partial_\mu \bar\omega^a (\partial_\mu \delta_{ab} - g f_{abc} A_\mu^c)\,\omega^b \equiv \partial_\mu \bar\omega^a D_\mu^{ab} \omega^b \tag{3b}$$

For the formulation of the Ward identities it is convenient to introduce sources $J, K, \bar M, M$ as follows [6] :

$$L_s = -J_\mu^a D_\mu^{ab} \omega^b + \frac{1}{2} g f_{abc} K^a \omega^b \omega^c + g \bar{M} t^a \omega^a \psi + g \bar{\psi} t^a \omega^a M \qquad (4)$$

The ghost fields $\bar{\omega}$, ω and the source J are taken to be anti-commuting quantities. Similarly, $\bar{M}$ and M anticommute with the fermion fields ψ and $\bar{\psi}$.

Consider then the Lagrangian :

$$L_t = L + L_g + L_{F.P} + L_s = \qquad (5)$$

$$= -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \bar{\psi} (\not{D} + m) \psi - \frac{1}{2\alpha} (\partial . A^a)^2 +$$

$$+ (\partial_\mu \bar{\omega}^a - J_\mu^a) D_\mu^{ab} \omega^b + \frac{1}{2} g f_{abc} K^a \omega^b \omega^c + g \bar{M} t^a \omega^a \psi + g \bar{\psi} t^a \omega^a M$$

In this model we expect the occurence of gauge invariant operators with positive charge-conjugation property and twist 4 :

$$O_1 = F_{\mu\nu}^a F_{\mu\nu}^a \qquad\qquad O_2 = \bar{\psi} \not{D} \psi \qquad (6a)$$

in the leading terms of the expansion of $T j_\mu(x) j_\mu(0)$ at small distances x :

$$T j_\mu(x) j_\mu(0)_{x \sim 0} = \sum_i \frac{C_i (\mu^2 x^2)}{x^2 - i\epsilon} O_i(0) + \cdots \qquad (6b)$$

where μ is a (arbitrary) parameter with dimensions of mass, which sets the scale of the theory.

There are situations, like the electromagnetic mass differences of hadrons, where only non-singlet operators are relevant. Such operators can be obtained from O_2 by insertion of the Gell-Mann matrices λ between the quark states $\bar{\psi}\psi$. Of course, in this case no mixing with the singlet operators (6a) occurs.

Remark that the gauge invariant operators O_i do not vanish by the equation of motion. ($m\bar{\psi}\psi$ is related to O_2 via the field equations) Following reference (7), we will introduce them in the theory with the help of an infinitesimal external source ϕ, which in our case is a dimensionless scalar quantity :

$$\phi \tilde{L} = \phi \alpha_1 O_1 + \phi \alpha_2 O_2 \qquad (7)$$

We now consider the renormalized "transverse" Lagrangian :

$$\hat{L}_R = L_t + \frac{1}{2\alpha}(\partial\cdot A^a)^2 + \phi\tilde{L} = \hat{L} + \phi\tilde{L} \tag{8a}$$

and define the corresponding renormalized transverse action :

$$\hat{S}_R = \int d^4x \; \hat{L}_R(x) \tag{8b}$$

The Ward identities tell us that both the renormalized (S_R) and bare (S_B) actions will obey [6] :

$$\partial_\mu \frac{\delta \hat{S}}{\delta J^a_\mu} = \frac{\delta \hat{S}}{\delta \bar{\omega}^a} \tag{9}$$

and

$$\int d^4x \left\{ \frac{\delta\hat{S}}{\delta A^a_\mu}\frac{\delta\hat{S}}{\delta J^a_\mu} + \frac{\delta\hat{S}}{\delta\omega^a}\frac{\delta\hat{S}}{\delta K^a} + \frac{\delta\hat{S}}{\delta\psi^a}\frac{\delta\hat{S}}{\delta\bar{M}^a} + \frac{\delta\hat{S}}{\delta M^a}\frac{\delta\hat{S}}{\delta\bar{\Psi}^a} \right\} = 0 \tag{10}$$

where all fields and sources are renormalized quantities.

The renormalized action (8b) is not the most general function which satisfies (9) and (10), whereas the bare action will be. Consequently, we must search for the most general renormalized Lagrangian which satisfies (9) and (10). It will have the form :

$$\hat{L}_{RG} = \hat{L}_t + \phi\,\tilde{L}_G \tag{11a}$$

where

$$\tilde{L}_G = \tilde{L} + \Delta \tag{11b}$$

The operators Δ, which arise in the renormalization procedure, must satisfy (from (10)) :

$$W\Delta = \frac{\delta\hat{L}_t}{\delta A^a_\mu}\frac{\delta\Delta}{\delta J^a_\mu} + \frac{\delta\hat{L}_t}{\delta J^a_\mu}\frac{\delta\Delta}{\delta A^a_\mu} + \frac{\delta\hat{L}_t}{\delta\omega^a}\frac{\delta\Delta}{\delta K^a} + \frac{\delta\hat{L}_t}{\delta K^a}\frac{\delta\Delta}{\delta\omega^a} + \frac{\delta\hat{L}_t}{\delta\psi}\frac{\delta\Delta}{\delta\bar{M}} + \frac{\delta\hat{L}_t}{\delta M}\frac{\delta\Delta}{\delta\bar{\Psi}} + \frac{\delta\Delta}{\delta\psi}\frac{\delta\hat{L}_t}{\delta\bar{M}} + \frac{\delta\Delta}{\delta M}\frac{\delta\hat{L}_t}{\delta\bar{\Psi}} = 0 \tag{12}$$

Since W obeys the following identity [7] :

$$W\,W = 0 \tag{13a}$$

any operator Δ which is given by :

$$\Delta = W\,F \tag{13b}$$

will satisfy equation (12). It can be shown [7], that (13b) is also the most general form that Δ can take. Note that, if we assign to the ω field a ghost-number equal to 1, then $J, M, \bar{M}$ will have ghost-number equal to -1, while the ghost number of K will be -2. Since W rises the ghost-number by one unity, F must have ghost-number equal to -1. Furthermore, F must be a scalar with dimension 3. The most general expression, linear in the sources, which satisfies these requirements, is given by (*) :

$$F = \beta_1 J^a_\mu A^a_\mu + \beta_2 K^a \omega^a + \beta_3 \bar{M}^a \psi^a + \beta_4 M^a \bar{\psi}^a \tag{14}$$

Using (11), (12), (13) and (14), we obtain :

$$\begin{aligned}\hat{L}_{RG} = {} & \hat{L}_t + \phi\alpha_1 O_1 + \phi\alpha_2 O_2 + \\ & + \phi\beta_1 [D^{ab}_\nu F^b_{\nu\mu} A^a_\mu + J^a_\mu \partial_\mu \omega^a + ig\bar{\psi}\gamma_\mu t^a A^a_\mu \psi] + \\ & + \phi\beta_2 [J^a_\mu D^{ab}_\mu \omega^b - \frac{g}{2} f_{abc} K^a \omega^b \omega^c + g\bar{M} t^a \omega^a \psi + g\bar{\psi} t^a \omega^a M] + \\ & - \phi(\beta_3+\beta_4)\bar{\psi}(\not{D}+m)\psi - 2\phi\beta_3 g\bar{M} t^a \omega^a \psi + 2\phi\beta_4 g\bar{\psi} t^a \omega^a M\end{aligned} \tag{15}$$

Since (15) is the most general expression which obeys (12), it follows that the general bare (transverse) Lagrangian $\hat{L}_{0G}$ must have the same form. It is convenient to express $\hat{L}_{0G}$ in terms of renormalized fields and sources. We have (9) :

$$A^a_{0\mu} = Z_3^{1/2} A^a_\mu \qquad \alpha_0 = Z_3 \alpha$$
$$\psi_0 = Z_F^{1/2}\psi \qquad \bar{\psi}_0 = Z_F^{1/2}\bar{\psi} \qquad m_0 = Z_m m \tag{16a}$$
$$\omega_0 = \tilde{Z}_3^{1/2}\omega \qquad \bar{\omega}_0 = \tilde{Z}_3^{1/2}\bar{\omega}$$

From the Ward identities it follows that :

$$g_0 = Z_3^{-1/2}\frac{Z_1}{Z_3} g = Z_3^{-1/2}\frac{\tilde{Z}_1}{\tilde{Z}_3} g \tag{16b}$$

where Z_1 and $\tilde{Z}_1$ are the renormalization constants for the three-point vertices AAA and $\bar{\omega}A\omega$, respectively. Since equation (10) must hold for both renormalized and unrenormalized actions, we must have :

$$A^a_{0\mu} J^a_{0\mu} = Z A^a_\mu J^a_\mu \qquad \omega^a_0 K^a_0 = Z\omega^a K^a \qquad \psi_0 \bar{M}_0 (\bar{\psi}_0 M_0) = Z\psi\bar{M}(\bar{\psi}M) \tag{17}$$

(*) - In consequence of the Ward identity (9), it is to be understood that J^a_μ always appears with the ghost field $\bar{\omega}^a$ in the combination $(J^a_\mu - \partial_\mu \bar{\omega}^a)$.

From (9) we see that we must have :

$$J_{o\mu}^{a} = \tilde{Z}_3^{1/2} J_{\mu}^{a} \tag{18a}$$

so that $Z = Z_3^{1/2} \tilde{Z}_3^{1/2}$. It follows that :

$$K_o^a = Z_3^{1/2} K^a \qquad M_o^a(\bar{M}_o^a) = Z_F^{-1/2} Z_3^{1/2} \tilde{Z}_3^{1/2} M^a(\bar{M}^a) \tag{18b}$$

We then obtain for the general bare transverse Lagrangian the expression :

$$\begin{aligned} \hat{L}_{o6} = {} & \hat{L}_{ot} + \phi\alpha_{o1} Z_3 \bar{F}_{\mu\nu}^{a} \bar{F}_{\mu\nu}^{a} + \phi\alpha_{o2} Z_F \bar{\Psi}\bar{\not{D}}\Psi + \\ & + \phi\beta_{o1}[Z_3 \bar{D}_{\nu}^{ab} \bar{F}_{\nu\mu}^{b} A_{\mu}^{a} + \tilde{Z}_3 J_{\mu}^{a}\partial_{\mu}\omega^{a} + Z_F i\bar{g}\bar{\Psi}\gamma_{\mu}t^{a}A_{\mu}^{a}\Psi] + \\ & + \phi\beta_{o2}\tilde{Z}_3[J_{\mu}^{a}\bar{D}_{\mu}^{ab}\omega^{b} - \tfrac{\bar{g}}{2} f_{abc} K^{a}\omega^{b}\omega^{c} + \bar{g}\bar{M}t^{a}\omega^{a}\Psi + \bar{g}\bar{\Psi}t^{a}\omega^{a}M] + \\ & - \phi(\beta_{o3}+\beta_{o4})Z_F\bar{\Psi}(\bar{\not{D}} + m_o)\Psi - 2\phi\beta_{o3}\bar{g}\tilde{Z}_3\bar{M}t^{a}\omega^{a}\Psi + 2\phi\beta_{o4}\bar{g}\tilde{Z}_3\bar{\Psi}t^{a}\omega^{a}M \end{aligned} \tag{19}$$

In this expression, $\bar{F}$, $\bar{\not{D}}$ and $\bar{D}$ are respectively F, $\not{D}$ and D with g replaced by $\bar{g}$, where :

$$\bar{g} = \frac{Z_1}{Z_3} g = Z_3^{1/2} g_o \tag{19a}$$

The bare constants α_{oi} and β_{oi} are related to their renormalized counterparts by :

$$\alpha_{oi} = Z_{ij}^{11}\alpha_j + Z_{im}^{12}\beta_m \tag{20a}$$

$$\beta_{om} = Z_{mj}^{21}\alpha_j + Z_{mn}^{22}\beta_n \tag{20b}$$

III. THE GAUGE INVARIANT MATRIX Z^{11}

We will now proceed to the calculation of the gauge invariant matrix Z^{11}, whose eigenvalues are relevant for the Wilson expansion of gauge invariant operators. In this computation we will use the dimensional regularization scheme with the minimal renormalization prescription [10], where the counterterms contain only poles in $\varepsilon = n-4$. As argued in reference (7), α_{oi} are, when expressed in terms of the renormalized parameters, independent of the gauge parameters α and of the parameters β, which arise in the renormalization procedure. Due to this fact, we can set $\beta_i = 0$ in $\hat{L}_{RG}$. In this case, when we calculate the one-loop divergence with $\hat{L}_{RG}$, we will find that the counter-terms contained in $\hat{L}_c = \hat{L}_{oG} - \hat{L}_{RG}$ include (see equations (15), (19) and (20)) the following terms :

$$\begin{aligned}\tilde{L}_c = {} & \phi\{[4(Z^{11}_{11} - Z_3^{-1}) - 2Z^{21}_{11}]\alpha_1 + [4Z^{11}_{12} - 2Z^{21}_{12}]\alpha_2\} Z_3 \times \\ & \times \frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \phi(Z^{21}_{21}\alpha_1 + Z^{21}_{22}\alpha_2) g \tilde{Z}_1 f_{abc} J^a_\mu A^b_\mu \omega^c + \\ & + \phi[(Z^{21}_{11} + Z^{21}_{21})\alpha_1 + (Z^{21}_{12} + Z^{21}_{22})\alpha_2]\tilde{Z}_3 J^a_\mu \partial_\mu \omega^a + \\ & + \phi Z_F \bar{\psi}\{[(Z^{11}_{21} - Z^{21}_{31} - Z^{21}_{41})\alpha_1 + (Z^{11}_{22} - Z_F^{-1} - Z^{21}_{32} - Z^{21}_{42})\alpha_2]\not{\partial} + \\ & - [(Z^{21}_{31} + Z^{21}_{41})\alpha_1 + (Z^{21}_{32} + Z^{21}_{42})\alpha_2] Z_m m\}\psi \end{aligned} \tag{21}$$

which are sufficient for the calculation of the gauge invariant matrix Z^{11}.

The Feynman diagrams which contribute to the vertex $(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2$ are shown in Figure 1 :

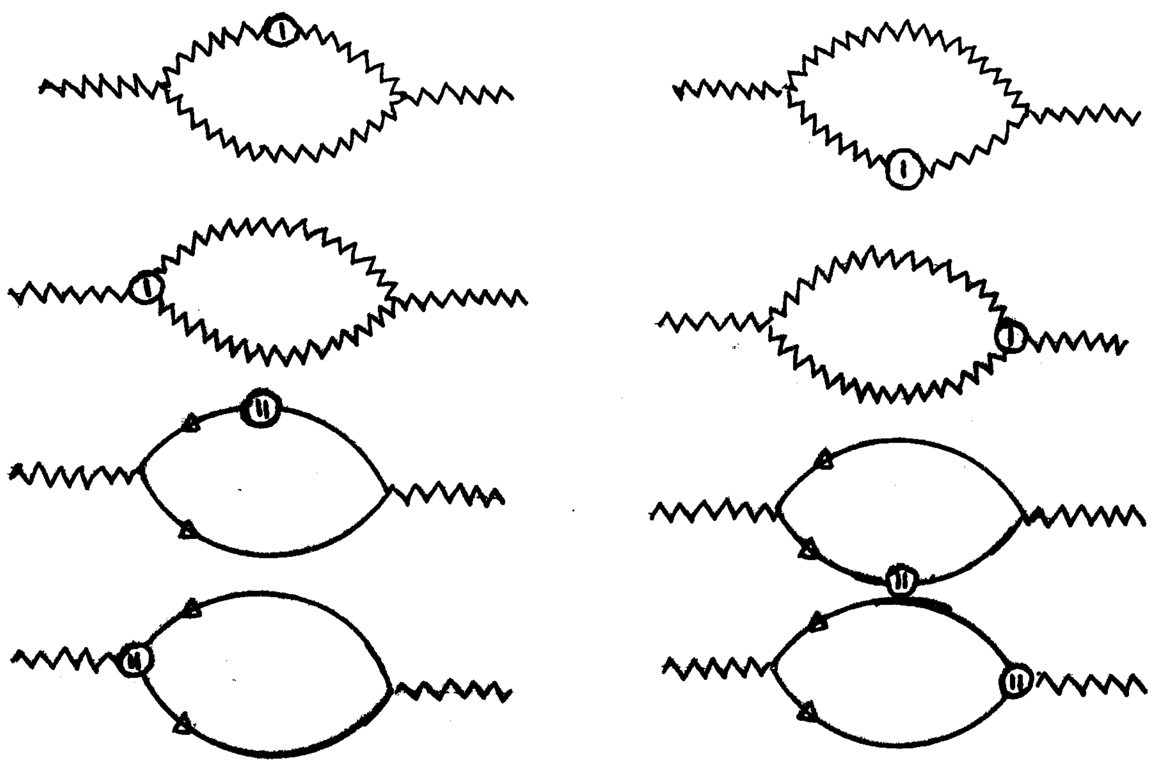

Figure 1

In the figures, wavy lines represent vector bosons, full lines stand for fermions and the special vertices (I) and (II) denote the insertion of $\phi\alpha_1 O_1$ and $\phi\alpha_2 O_2$, respectively. From these graphs, one finds that the total pole part arising from the insertion of $\phi\alpha_2 O_2$ cancels, while the insertion of $\phi\alpha_1 O_1$ yields the result (see also reference (7)) :

$$F_1^{\,p} = -\phi\alpha_1 \frac{g^2}{16\pi^2} C\alpha \frac{1}{n-4} \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a \right)^2 \tag{22}$$

where α is the gauge parameter, n is the dimension of space-time and

$$\delta_{ab} C = \sum_{cd} f_{acd} f_{bcd} \tag{22a}$$

The Feynman diagram which contributes in lowest order to the vertex $J_\mu^a \partial_\mu \omega^a$ is shown in Figure 2, where dashed lines represent ghost fields and the dotted line stands for the source J.

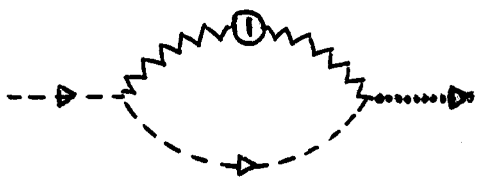

Figure 2

We obtain the result :

$$F_2^{\,p} = \phi\alpha_1 \frac{3g^2}{8\pi^2} C \frac{1}{n-4} J_\mu^a \partial_\mu \omega^a \tag{23}$$

The Feynman diagrams which in the one-loop level contribute to the vertex $g f_{abc} J_\mu^a A_\mu^b \omega^c$ are shown in Figure 3.

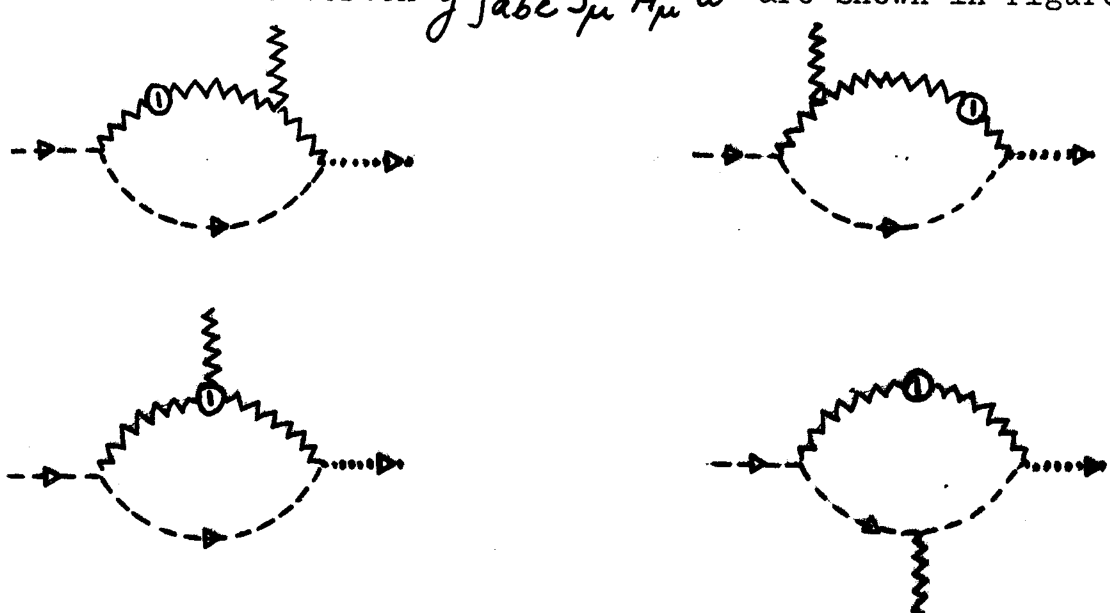

Figure 3

From these graphs we find that the pole terms cancel :

$$F_3^{\,p} = 0 \tag{24}$$

Using the fact that (1) :

$$Z_3 = 1 + \left(\frac{13}{3} - \alpha\right)\frac{Cg^2}{16\pi^2}\frac{1}{n-4} - \frac{8}{3}\frac{Tg^2}{16\pi^2}\frac{1}{n-4} \tag{25}$$

where

$$\delta_{ab}\, T = Tr\; t^a t^b \tag{25a}$$

we obtain, with the help of (21), (22), (23) and (24) the result :

$$Z^{11}_{11} = 1 - \frac{11}{3}\frac{Cg^2}{8\pi^2}\frac{1}{n-4} + \frac{4}{3}\frac{Tg^2}{8\pi^2}\frac{1}{n-4} \tag{26a}$$

$$Z^{11}_{12} = 0 \tag{26b}$$

which is gauge invariant.

In order to calculate Z^{11}_{2i} we consider the Feynman diagrams shown in Figure 4 :

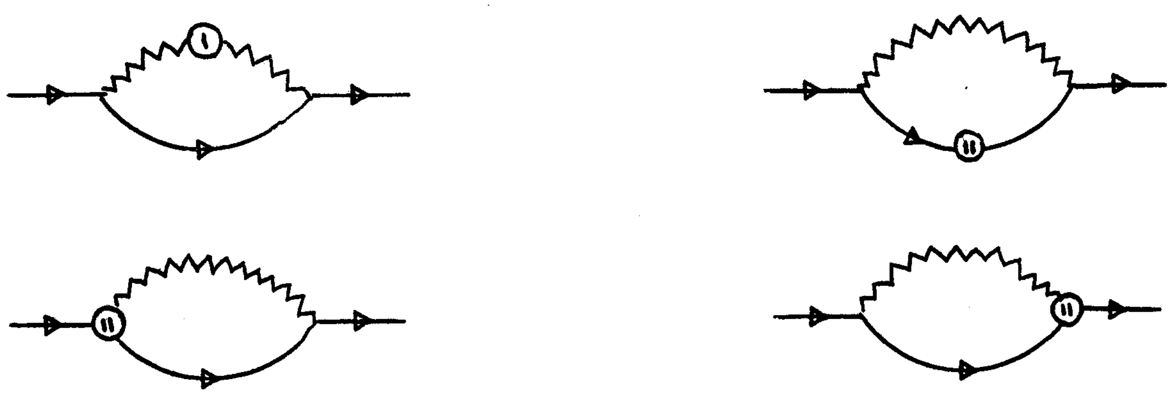

Figure 4

From these graphs we obtain for the pole part the expression :

$$F_4^{\,p} = -\phi\,\bar{\psi}\left\{\alpha_2 \frac{Tg^2}{8\pi^2}\frac{1}{n-4}\,\alpha\not\partial + \alpha_1 \frac{12\,Tg^2}{8\pi^2}\frac{1}{n-4}\, m\right\}\psi \tag{27}$$

Finally, from the Feynman diagram shown in Figure 5 :

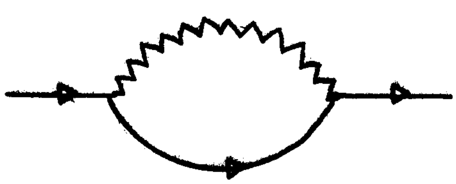

Figure 5

it follows that :

$$Z_F = 1 + \alpha \frac{Tg^2}{8\pi^2} \frac{1}{n-4} \tag{28a}$$

$$Z_m = 1 + \frac{3Tg^2}{8\pi^2} \frac{1}{n-4} \tag{28b}$$

Comparing equations (21) and (27) and using (28), we obtain the gauge invariant result :

$$Z_{21}^{11} = - \frac{12Tg^2}{8\pi^2} \frac{1}{n-4} \qquad Z_{22}^{11} = 1 \tag{29}$$

Therefore, the gauge invariant renormalization matrix has the form :

$$Z^{11} = \begin{bmatrix} 1 - \frac{11}{3} Cg^2 Z_0 + \frac{4}{3} Tg^2 Z_0 & 0 \\ & \\ -12Tg^2 Z_0 & 1 \end{bmatrix} \tag{30}$$

where $Z_0 = 1/8\pi^2(n-4)$.

IV. DISCUSSION

The determination of the leading behaviour of the form factors C_i in (6b) at $x \sim 0$ is based on the renormalization group equation for the coefficient functions in the Wilson expansion.(11) In our case where the anomalous dimensions of the conserved currents j_μ vanish, this equation simplifies to :

$$\left\{ \left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] \delta_{ij} - \gamma_{ij} \right\} C_j = 0 \tag{31}$$

where

$$\gamma_{ij} = \mu \frac{\partial}{\partial \mu} \log Z_{ij}^{11} \tag{31a}$$

and in lowest order of perturbation theory (1) :

$$\beta = -\frac{1}{2} b \frac{g^3}{8\pi^2} = -\frac{1}{2} \left(\frac{11}{3} C - \frac{4}{3} T \right) \frac{g^3}{8\pi^2} \tag{31b}$$

with $b > 0$ for asymptotically free gauge theories.

Equation (31) has the solution :

$$\lim_{t\to\infty} C_j(t,g) = \sum_{\ell} K_{ji}^{\ell}\, C_i[0,\bar{g}(t)]\,\left|\log \mu^2 x^2\right|^{-\left(\frac{g^2}{16\pi^2}\right)^{-1}\cdot\frac{\gamma_\ell}{2b}} \tag{32}$$

with $t \equiv -\frac{1}{2}\log\mu^2 x^2$. In this equation γ_ℓ are the eigenvalues of the matrix γ_{ij}. The effective coupling constant $\bar{g}(t)$ vanishes logarithmically as $t\to\infty$, while the constant matrix K_{ji}^{ℓ} is in general an unknown quantity.

In the dimensional regularization scheme, we have (12) (see equation (31a)) :

$$\gamma_{ij} = \lim_{n\to 4}\left[\frac{1}{2}(n-4)\, g_0 \frac{\partial}{\partial g_0}\log Z_{ij}^{11}\right] \tag{33}$$

where g_0 is the bare coupling constant. Using this relation together with equation (30), we find for the anomalous matrix of the singlet operators the expression :

$$\gamma_{ij} = \begin{bmatrix} -b\frac{g^2}{8\pi^2} & 0 \\ -12T\frac{g^2}{8\pi^2} & 0 \end{bmatrix} \tag{34}$$

whose eigenvalues are given by :

$$\gamma_1 = -b\, g^2/8\pi^2 \qquad\qquad \gamma_2 = 0 \tag{35}$$

Inserting (6b) together with C_i given by (32) and (35) into equation (1), we find for the singlet part of the radiative corrections the following asymptotic behaviour :

$$\Delta m_{singlet} \sim \int_{x\simeq 0} \frac{d^4x}{x^4}\log \mu^2 x^2 \tag{36}$$

which shows that the hadronic interactions do, in fact, increase (for the singlet part), the divergence above the logarithmic one due to the weak and electromagnetic interactions (see also reference (4)).

On the other hand, we remark that the vanishing of γ_{22} (and γ_{12}) implies, as noted by Weinberg (13), the vanishing of the anomalous dimension for the non-singlet operators $O_3^i \equiv \bar{\Psi}\lambda_i \not{\partial}\Psi$, since in this case no mixing occurs. Then, from equations (32) and (1), we see that the strong interactions do not affect (for the non-singlet part) the divergences already present in the theory.

We observe from equations (31b) and (35) that we have the relation :

$$\beta = \frac{1}{2} g \gamma_1 \tag{37}$$

which coincides, as it should, with the result obtained for ghost-free gauges [4] since β and γ_1 are gauge independent quantities. As shown in reference (4), in these gauges, γ_1 is identical to the anomalous dimension γ_A of the vector boson field A_μ^a. Then, relation (37) follows from the generalized Slavnov-Taylor identity [9], which holds in the absence of Faddeev-Popov ghosts.

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REFERENCES

(1) - D.J.Gross and F.Wilczek, Phys. Rev. D8 (1973) 3633 ;
D9 (1974) 980 .
H.Georgi and H.D.Politzer, Phys. Rev. D9 (1974) 416 .

(2) - K.G.Wilson, Phys. Rev. 179 (1969) 1499 .

(3) - C.G.Callan, Phys. Rev. D5 (1972) 1670 .
K.Symanzik, Comm. Math. Phys. 23 (1972) 49.

(4) - W.Kainz, W.Kummer and M.Schweda, Nucl. Phys. B79 (1974) 484.

(5) - J.A.Dixon and J.C.Taylor, Nucl. Phys. B78 (1974) 552 .
S.Sarkar and H.Strubbe, Nucl. Phys. B90 (1975) 45 .

(6) - H.Kluberg-Stern and J.B.Zuber, Saclay Preprints
D.Ph.T/74-56 ; D.Ph.T/74-83 .

(7) - J.A.Dixon and J.C.Taylor, Oxford Preprint 74-74 .
W.S.Deans and J.A.Dixon, Oxford Preprint 20-75 .

(8) - C.N.Yang and R.L.Mills, Phys. Rev. 96 (1954) 191 .

(9) - J.C.Taylor, Nucl. Phys. B33 (1971) 436 .
A.A.Slavnov, Theor. and Math. Phys. 10 (1972) 99 .
B.W.Lee and J.Zinn-Justin, Phys. Rev. D5 (1972) 3121 .

(10)- G.'t Hooft and M.Veltman, Nucl. Phys. B44 (1972) 189 .

(11)- N.Christ, B.Hasslacher and A.Mueller, Phys. Rev D6 (1972) 3543.

(12)- G.'t Hooft, Nucl. Phys. B61 (1973) 455 .

(13)- S.Weinberg, Phys. Rev. Letters 31 (1973) 494 .