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by

M. Carolina Nemes* A. F. R. T. Piza
Instituto de Física, Universidade de São Paulo

and

R. P. J. Perazzo
Comisión Nacional de Energia Atómica, B. Aires

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Comisión Nacional de Energia Atómica, Buenos Aires, Argentina

ABSTRACT

Scattering properties and the decay of prepared states in a schematic potential landscape are studied by means of a coupled-channel calculation. Structure related to the topography of the potential is found in inelastic spectra and fragment energy distributions.

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Potential effects in the collision of two complex nuclei (or the related phenomenon of binary nuclear fission) must be described in terms of more or less complicated potential landscapes which involve at least a number of fragment intrinsic collective degress of freedom in addition to the main scattering coordinate which goes asymptotically into the relative coordinate between their centers of mass [1]. Although these potential landscapes are particular at least to each mass number and charge, there are some general qualitative features that are shared by many of them. One such feature is the presence of "winding" or "misaligned" valleys corresponding to different minimum potential paths for processes leading inwards ("fusion") or outwards ("fission") respectively [2]. In any case, the topography of the potential landscapes is important to determine the type and the strength of the couplings of the different degrees of freedom. It is, in this sense, one of the important ingredients to determine the features of inelastic yields in a scattering situation. In this note we report on a study of the qualitative scattering features produced by such potential surfaces by considering in some detail the scattering solutions of a soluble schematic model. It consists essentially of a two-dimensional enrichment of the simple potential model studied some time ago by McVoy, Heller and Bolsterli [3], which is conveniently handled in terms of a set of coupled equations whose solution can be reduced to a problem of matrix inversion. A suitable adaptation of the treatment allows as well for a study of the decay of a prepared state in the two-dimensional potential surface. Among other results, interesting effects emerge in the spectra for the energy of relative motion in each case.

The model we consider is best defined by the Hamiltonian

$$H_M = H_r + H_x + H_b =$$

$$= \left[\frac{p^2}{2m} + V(r)\right] + \left[\frac{\pi^2}{2\mu} + \frac{1}{2}\mu\omega^2 x^2\right] + Aaf(x)\delta(r-a)$$

The first **bracket**, H_r, contains a one-body kinetic energy (with constant, scalar, mass parameter m) and a potential V(r). This potential includes an infinite potential barrier at the origin (r=0). This makes this part of the problem equivalent to s-wave scattering by the spherical potential V(r). Although a generalization to arbitrary partial waves in three dimensions is straightforward, we felt that it would be inadequate in view of the form chosen for the last term H_b of H. The second bracket, H_x, is a one-dimensional harmonic oscillator with mass μ and frequency ω. It describes qualitatively bound intrinsic excitations of the colliding particles. Its coordinate x must therefore be seen as being associated qualitatively to some sort of intrinsic (e.g., deformation) degree of freedom with a discrete spectrum. Finally, the last term presents a schematic barrier to the colliding particles at r=0 with a barrier height that depends, through the function f(x), on the oscillator degree of freedom. This function couples the different channels that correspond to the eigenstates of the oscillator. The implication of this is that the existence of many channels reflects the existence of intrinsic degrees of freedom whose behavior affects also the transmission amplitudes through the potential barrier [1,3]. The main advantage of the thin (δ-function) barrier is in the simplicity gained in the treatment of the model. Although quantities like transmission coefficients, etc., have for this barrier a much less pronounced energy dependence as compared with more realistic thick barriers, we still expect to get an adequate qualitative picture of the main scattering patterns for potentials of this type. Our choice for the coupling function f(x) has been

$$f(x) = 1 - \alpha \exp\left[- \rho (x-x_o)^2\right]$$

so that it differs from unity by a gaussian dent of depth α and width $\rho^{-1/2}$, centered about the value x_o of the oscillator coordinate. This is intended to simulate the existence of a fission pass in the potential landscape which is not aligned with the main fusion valley (given here by x=0). In this respect our potential model extends the topography of the two-dimensional fission model studied recently by Massmann, Ring and Rasmussen [4], in which the channel coupling is caused by a localized symmetrical narrowing of a straight valley. This situation would correspond qualitatively to setting $x_o=0$ in our model.

The Schrödinger equation for H_M can be very simply solved for a variety of potentials V(r) and of coupling functions f(x). The wavefunction $\Psi(r,x)$ (with appropriate scattering boundary conditions) is conveniently expanded in the eigenstates X_n of the harmonic oscillator

$$\Psi(r,x) = \sum_n u_n(r)\, X_n(x)$$

and the amplitudes $u_n(r)$ satisfy equations coupled at r=a by the matrix elements, in the oscillator basis, of the coupling function f(x). For simple choices of V(r) the amplitudes u_n are given analytically both in the internal (i.e., r<a) and in the external region, and the coupled equations problem reduces to that of a linear system driven by a term related

to the incident waves specified in the boundary conditions. We use the standard boundary conditions with incident waves in a given oscillator channel n_o and outgoing waves in all channels ("fusion" calculation).

As an extension of this calculation we also treat the case in which a given resonant state in the r<a part of the potential landscape is fed through an independent channel, orthogonal to those described by H_M. This state is allowed to decay and the yield distribution in the oscillator channels is studied ("fission" calculation). This case differs from the preceding scattering situation in that the oscillator channels are now driven by a source which is in a definite oscillator state in the "inside" region of V(r). The general framework adopted for such an extension is as follows. We first add to the system as described by H_M another (orthogonal) channel described by a Hamiltonian H_1 and coupled to the degrees of freedom of H_M by separable terms of the form

$$H_{1M} = |v_1\rangle\, g^* \langle w_M| = H^+_{M1}$$

where $|v_1\rangle$ and $|w_M\rangle$ are states in the space of H_1 and of H_M respectively, and g is a coupling constant. The extended model is now described by the Hamiltonian

$$H = H_M + H_1 + H_{1M} + H_{M1}$$

The amplitudes corresponding to the new and old channels satisfy the coupled equations

$$(E - H_M)\ |\Psi_M\rangle = |w_M\rangle\ g\ \langle v_1|\ \Psi_1\rangle$$

$$(E - H_1)\ |\Psi_1\rangle = |v_1\rangle\ g^* \langle w_M|\ \Psi_M\rangle$$

and it is now possible to use an incident wave in the newly introduced channel and to require outgoing waves only in all other channels. These waves are fed by the effective source term, proportional to $|w_M\rangle$, on the right hand side of the first of the two coupled equations. The special separable form chosen for H_{1M}, on the other hand, allows for the complete formal solution of these equations in closed form. One gets

$$g\langle v_1|\Psi_1\rangle = \frac{g\langle v_1|\phi_1\rangle}{1 - \langle w_M|\frac{1}{E+i\eta-H_M}|w_M\rangle |g|^2 \langle v_1|\frac{1}{E+i\eta-H_1}|v_1\rangle}$$

where $|\phi_1\rangle$ is the scattering state for the uncoupled channel described by H_1. This form guarantees, in particular, that unitarity is properly maintained for the complete scattering problem.

There is still much freedom in the choice of H_1 and of the ingredients of the coupling term H_{1M}. In order to study the properties of the potential landscape of H_M without additional effects brought in by the new channel, we chose the latter to be "featureless" in the following sense. We take $g\langle v_1|\phi_1\rangle = \gamma_1$, constant with E, and consistently with this,

$$|g|^2 \langle v_1|\frac{1}{E+i\eta-H_1}|v_1\rangle = -i\pi|g|^2|\langle v_1|\phi_1\rangle|^2 = -i\pi|\gamma_1|^2$$

One still has to specify $|w_M\rangle$. This normalized source state can be defined as the product of an eigenstate of the oscillator with the appropriate eigenstate of H_r in the internal region, confined by an infinite potential barrier at r=a. As it is known, this choice essentially exhausts the residue of the

Green's function of H_r at the corresponding resonance pole [5].

We report here on several "fusion" and "fission" calculations for potentials with V(r)=0. We used arbitrarily a reduced mass m=20 AMU and a radius a=1 fm. The oscillator frequency was chosen to be 0.6 MeV and, for each value of the energy, all channels having positive kinetic energy on either side of the thin barrier were taken into account. In the cases shown the barrier height A was 25 MeV, and the parameters of the coupling function f(x) were α=0.8 and ρ=0.8. The value of the pass displacement x_o was varied near the value x_o=2. The last two parameters are in units such that the oscillator parameter b=1. This choice for the parameters in f(x) implies that the dent in the potential barrier is somewhat wider than the ground state of the oscillator and is displaced away from the origin by distances of the order of its width.

In a fusion calculation elastic scattering and transition amplitudes for all open channels were obtained as a function of total energy in order to study resonant behavior in the two-dimensional potential landscape. For the sake of comparison, the pure elastic scattering obtained by neglecting nondiagonal elements of the coupling matrix based on f(x) was also calculated. Results for pure elastic, coupled channel elastic and total reaction cross sections are shown in fig. 1. Here x_o=2.0, and incident waves contained the ground state of the oscillator. A strong coupling to inelastic channels in this case essentially removes the resonant peak from the coupled-channel elastic cross-section. An Argand plot of the corresponding elastic S-matrix element shows that inelastic processes, together with many-channel unitarity requirements, cause the resonance loop to shink until it nearly touches the origin [3].

In a fission calculation fragment yields in all open oscillator channels were obtained. A suitably small value was taken for γ_1 in order not to broaden the prepared state appreciably through the coupling to the feeding channel. Single oscillator channel results involving no off-diagonal matrix elements of f(x) were also obtained. This, together with the total fission yield in the many-channel case is shown in fig. 2 for the same resonance as in fig. 1. The source involves the ground state of the oscillator, and the broadening of the fission line due to coupling to excited oscillator channels can be clearly seen.

We show next, in fig. 3, spectra at different values of total energy for the fusion and for the fission calculation. The potential is again the same as that of fig. 1. They show structure as a function of fragment excitation Q. This structure is not determined by resonant effects. It varies only slowly with total energy (in the scale set by the resonance width) and appears also in the transmission matrix obtained in a "one-dimensional" treatment of the r degree of freedom, in which the waves transmitted to the left of the barrier are allowed to travel undisturbed to $r=-\infty$. This "one-dimensional" transmission matrix is, of course, related to the "three-dimensional" (s-wave) scattering amplitudes via an infinite series of successive reflections, as done in ref. [3]. Fig. 4 shows the variation of the near-resonance structure for different values of the parameter x_o of the coupling function. The minima in the spectra are seen to move to higher excitation energies as the pass moves away from the main valley, while new peaks appear eventually in the first channels.

The results of this schematic model seem thus to indicate that, appart from the expected broadening of resonant peaks by inelastic effects in the potential landscape [6] , structure may be generated in spectra, as a function of Q, by the channel couplings produced by the topography of the potential landscape. This structure could be strongly damped, however, by non-potential absorptive or "viscous" effects, involving the coupling to additional non-collective degrees of freedom on the potential surface. A fuller investigation of these effects is presently under way.

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FIGURE CAPTIONS

FIG. 1 - Pure elastic (dash), coupled channel elastic (full) and total reaction (dot) cross-sections as a function of energy. Potential parameters are as shown, with $V(r) = 0$.

FIG. 2 - Single oscillator channel (dash) and many-channel total fission yields for the same potential of fig. 1.

FIG. 3 - Spectra for fusion (circles) and for fission (crosses) calculations. Potential parameters are those of fig. 1. Full curves correspond to "on-resonance" total energy (9.40 MeV). "Off resonance spectra (at 8.6 MeV) are shown in the dashed curves. Black rectangles show values of the "one-dimensional" transmission amplitudes.

FIG. 4 - Variation of the structure in fusion spectra with the pass displacement parameter x_o.

FIGURE 1

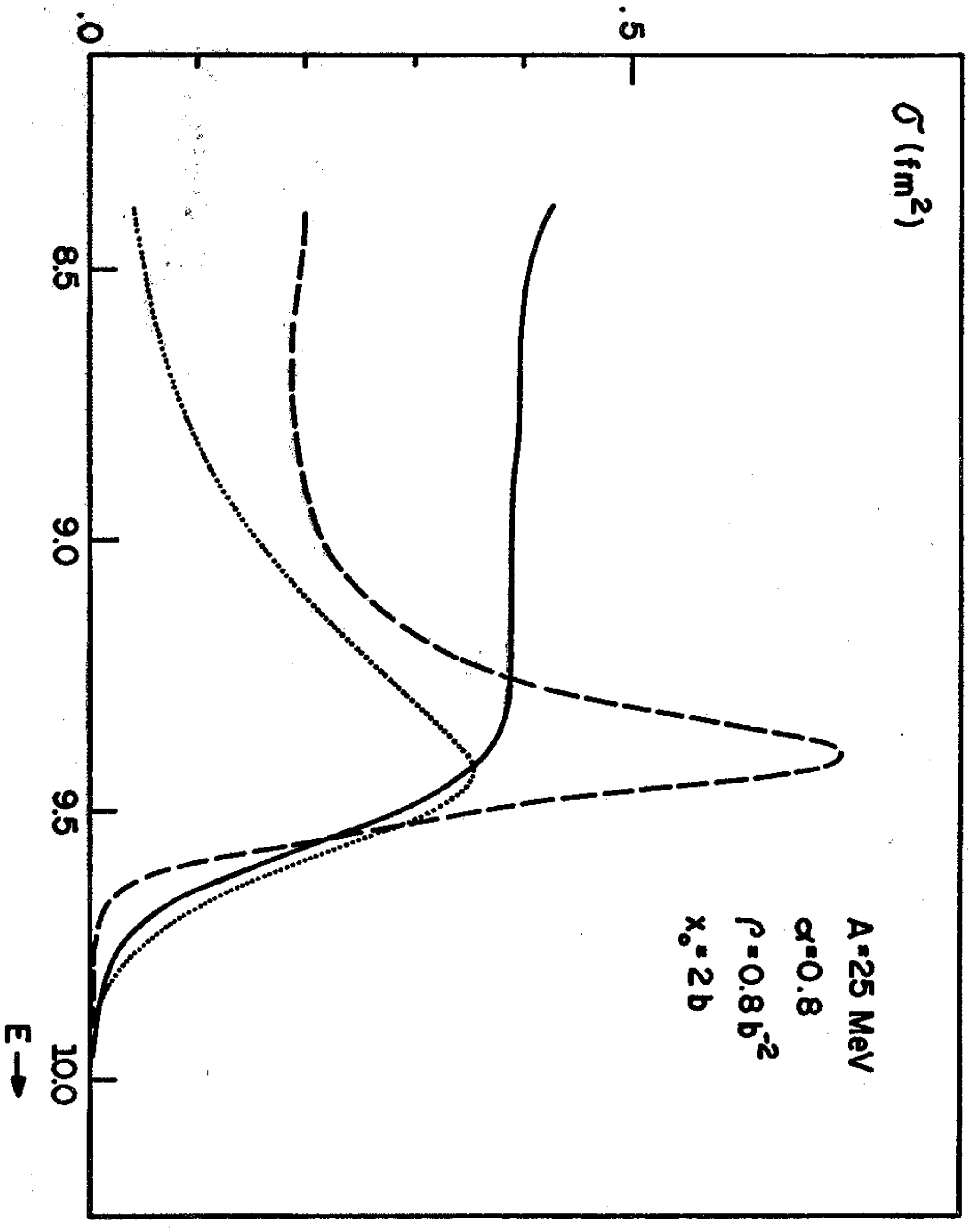


FIGURE 2

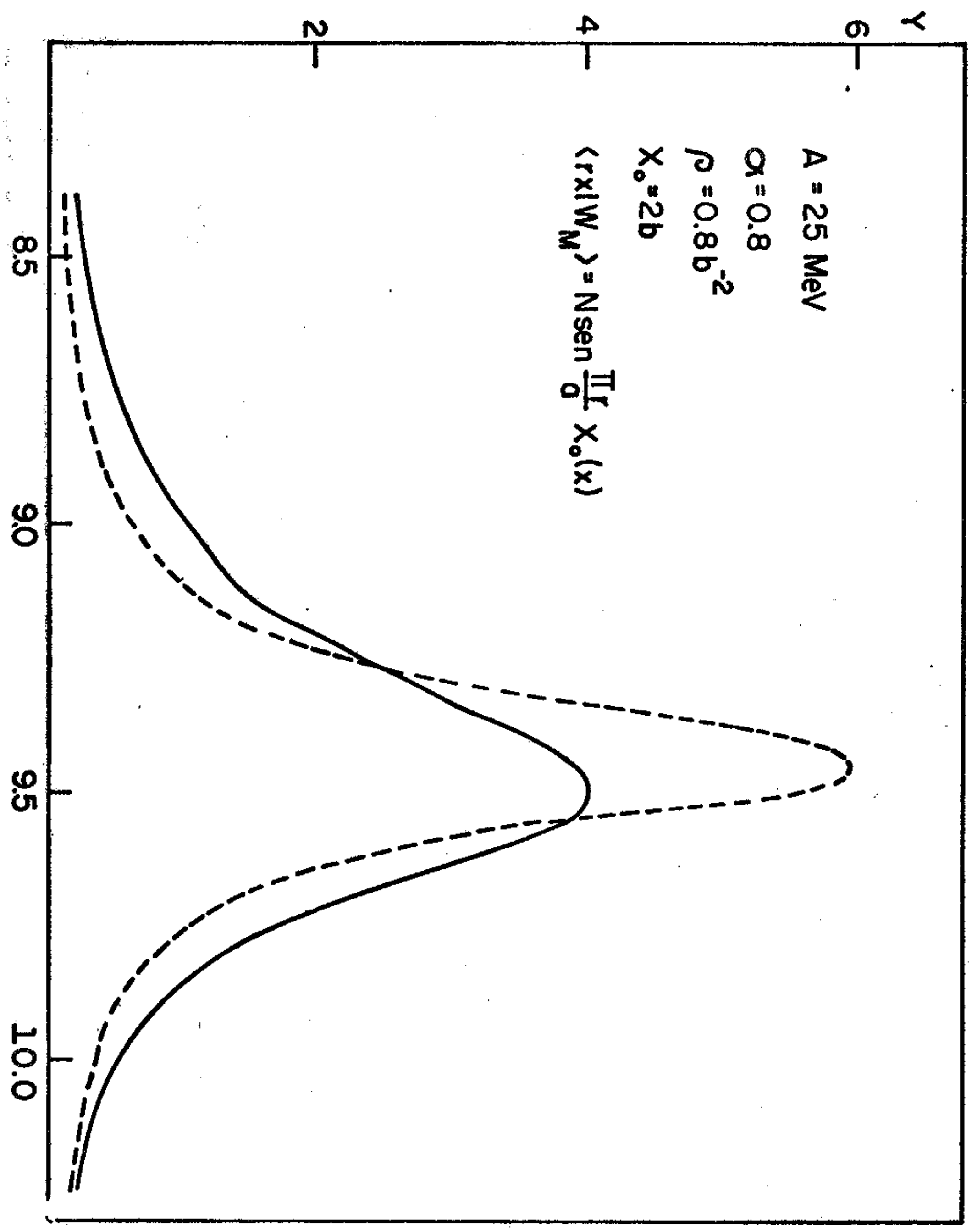
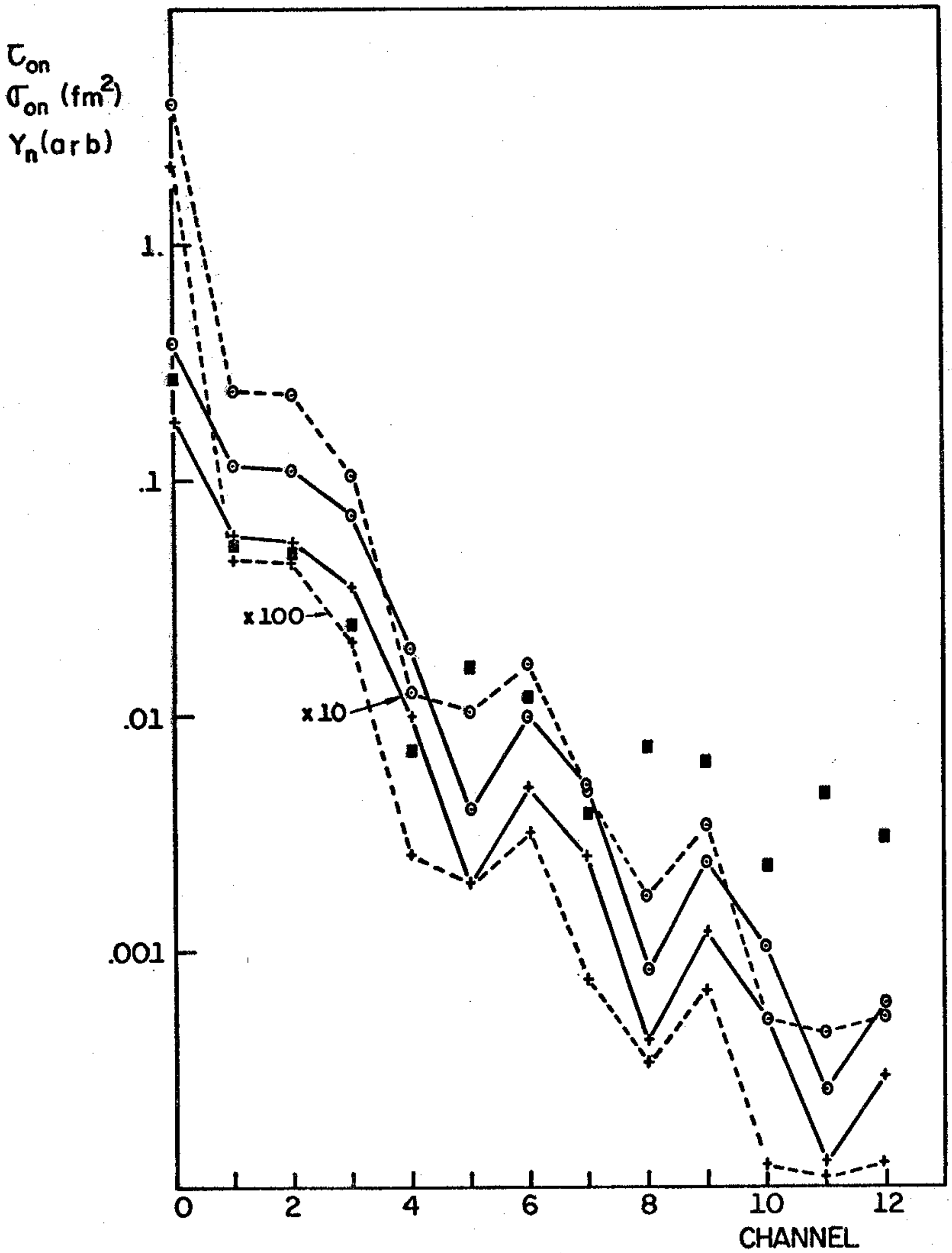
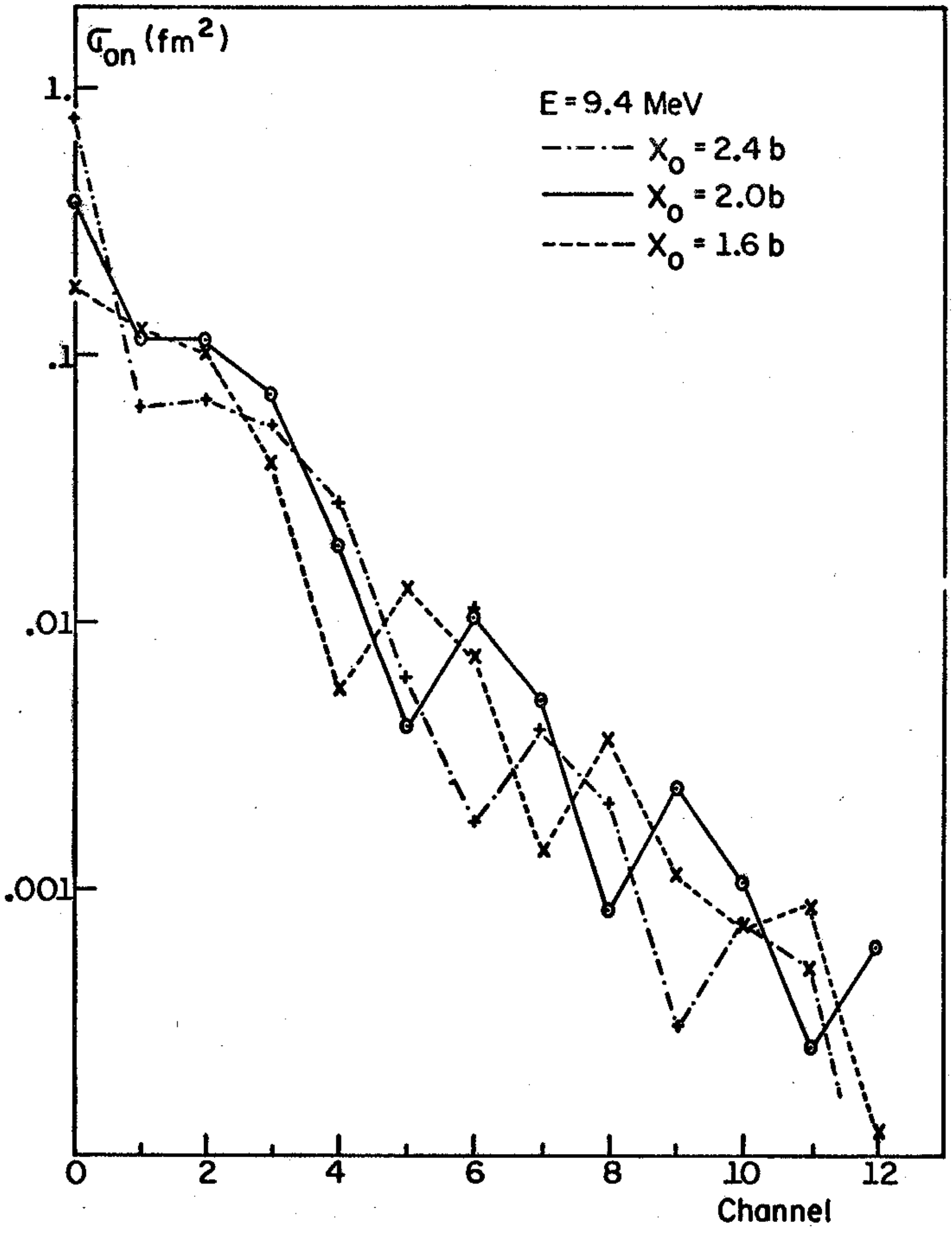


FIGURE 3



τ_{on}
σ_{on} (fm^2)
Y_n(arb)
1.
.1
.01
.001
x100
x10
0
2
4
6
8
10
12
CHANNEL

FIGURE 4



σ_{on} (fm^2)
1.
.1
.01
.001
E = 9.4 MeV
X_o = 2.4 b
X_o = 2.0b
X_o = 1.6 b
0
2
4
6
8
10
12
Channel