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"A TWO-COMPONENT MODEL FOR HIGH-ENERGY COLLISIONS"

by

H. M. França and Y. Hama
Instituto de Física - Universidade de São Paulo

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ABSTRACT:

A model for high-energy collisions is studied in which two distinct and independent mechanisms are included: a) pionization, depicted as an excitation of meson field induced by a classical source representing the incident particles in interaction and b) fragmentation, deseribed as a two-stage process consisted of an incident-particle excitation followed by its subsequent decay. It is shown that the model exihibits excellent agreement with all the ISR data on $\frac{d\sigma_{el}}{dt}$, σ_t, σ_{inel} and σ_{el} and discussed its consistency with $\omega\frac{d\sigma}{dk}$, $<n>$ and the fragmentation cross section.

1. INTRODUCTION

Nowadays, we have a reasonably large variety of experimental data conserning high-energy collisions of hadrons and, as a consequence, some common features of these interaction have been put in evidence. The aim of the present article is to discuss a particular model based on the idea of particle production as a quantized field by a time-dependent classical source and also fragmentation of the incident particles. A combination of these two mechanisms, which will be defined later, shows a possibility to accomodate all the main characteristics of high-energy reactions into a single scheme.

The experimental data suggest that, in high-energy collisions of hadrons, the finite extension of these particles plays an important role. This is most clearly seen in the diffraction pattern of $\frac{d\sigma_{el}}{dt}$, although the constancy of $\langle K_{\perp} \rangle$ and the approximately linear increase of $\langle k_{\parallel} \rangle$ with s, for the produced particles in these reactions, may also be interpreted as due to the finite and nearly constant transversal range of the interaction and to its flatter and flatter longitudinal dimension.

Eikonal models, formulated in terms of impact parameter, constitute quite appropriate devices to implement the above idea, besides presenting another interesting feature which is the (s-channel) unitarity.

It also becomes clear (1) that, if we are interested in describing collisions at energies equal or higher than those available at ISR, a mechanism which produces increasing cross-sections must be included. That is, a purely geometrical model is not satisfactory for our purpose.

The idea of making an analogy between the multiparticle production and bremsstrahlung has been proposed a long time ago by Heisenberg (2) and, since then, many authors have developped this idea and studied several aspects of the reactions (3,4,5). One can conveniently formulate the problem by considering some classical extended current, which becomes effective only during a short time of interaction between the incident particles and radiating some quantized field. This type of model predicts a series of qualitatively interesting results in the asymptotic limit when the energy (or equivalently s) goes to infinity. We could mention, for instance (see sec.2)

a) $\sigma_t, \sigma_{el}, \sigma_{inel} \underset{s\to\infty}{\sim} \ln(\ln s)$,

b) $\langle n \rangle \underset{s\to\infty}{\sim} \dfrac{\ln s}{\ln(\ln s)}$,

c) constancy of $\langle k_\perp \rangle$,

d) scaling: $\langle k_{\parallel} \rangle \underset{s\to\infty}{\sim} \sqrt{s}$ and

e) shrinking of $\dfrac{d\sigma_{el}}{dt}$.

All these properties (except the exact functional form which is not well established) are really verified at high-energies, so that this model may be looked at as a possible explanation of the origin of the energy dependence of the observable quantities.

Serious difficulties arise, however, if one tries to fit the experimental data quantitatively. Although several approximations are involved in this model, such as neglect of spins, isospins and recoils of the incident particles, we feel that the discrepancies are too large. We could resume these difficulties as follows (6):

A) If one fixes the parameters by fitting $\frac{d\sigma_{el}}{dt}$, the prediction for $\omega\frac{d\sigma}{d\vec{k}}$ becomes extremely low (by a factor of 10 ~ 20).

B) With the same parameters, the average multiplicity predicted by the model is too small and also increases very slowly (this last behavior is due to a rapid - more than expected - increase of σ_{in}).

Moreover, another important features of high-energy collisions do not appear in this model, namely:

C) The model does not predict reactions of the class $a+b \to a+\text{anything}$, with a small missing mass, which appear as a narrow peak near $x=1$ in the inclusive cross section.

D) The multiplicity distribution, σ_n, is always peaked to $n=1$, contrary to the existing data.

At this point, it seems us natural to try a model which includes explicitly the possibility of the incident-particle fragmentation, which will give a component with an approximately constant cross section, as well as the possibility of producing particles through the mechanism mentioned above. Of course, this kind of model conserves all the attractive asymptotical properties a) ~ e) mentioned above, while at smaller s values the inclusion of a constant component will allow σ_t, σ_{inel} and σ_{el} to increase much more slowly and where fragmentation particles will dominate both $\omega\frac{d\sigma}{d\vec{k}}$ and $\langle n \rangle$. It is true that in doing so, we are just including as another imput the property mentioned in C), without trying to obtain it from some principle. However, we can avoid, in this way, the difficulties A), B) and D), getting also much more satisfactory agreement of the other results with the data.

A similar attempt has been done by Henyey and Sukhatme (5), based on the idea of diffractive dissociation in a close analogy to the work by Good and Walker (7). In this way, they have been able to achieve considerable improvements over the simple classical source model, but still were unable to find a particular parametrization which resembles the important features of the data.

Here, instead, a two-component model is discussed in which the incident particles can either be excited with a subsequent decay (we call this component fragmentation) or radiate a field in analogy with bremsstrahlung (let us call this component pionization as usual, although the produced particles are not necessarily pions) or both of them.

Two-component models have been discussed by several authors in connection to the multiplicity distribution and correlation among the produced particles (8). Here, we focuss our attention mostly on σ_t, σ_{inel}, σ_{el}, $\frac{d\sigma_{el}}{dt}$, σ_{dif} and $\omega\frac{d\sigma}{d\vec{k}}$ and, by choosing a specific (eikonal) formalism satisfying unitarity, discuss its implications on these observables.

In what follows, we first describe, for the sake of completeness, a simple "classical-source" model as mentioned above, showing the properties a) through e). This model is improved in sec. 3, including also the fragmentation. The main output of the model which can be immediately derived will be shown in sec. 4, where a quantitative fit to the experimental data is tried. A discussion showing possibility of getting experimentally measured $\omega\frac{d\sigma}{d\vec{k}}$ as well as <n> is also presented there. Finally, the main conclusions of the present work are summarized in the last section, where some discussions concerning other observable quantities are also given.

. PIONIZATION: CLASSICAL-SOURCE MODEL

Let us initially describe a simple model for particle production in which the multiply-produced particles are described as a field satisfying the equation

$$(\Box+\mu^2)\varphi(x) = J(x.\vec{b}) \qquad (2.1)$$

where $J(x,\vec{b})$ is a classical source, defined appropriately in terms of the incident particles. Here, as in all the following discussion, the spins, isotopic spins and charges both of the incident and emitted particles are neglected for simplicity. In writing the equation above, no-recoil approximation is implicitly assumed and, as the notation already indicates, impact-parameter representation is used for the nucleons (let us consider p-p collisions to fix the ideas). The model we are describing in this section is essentially the same to that discussed in Ref. (4), except the way in which it is formulated.

As is well known (9), eq. (2.1) is readily solved and the corresponding S matrix is given by

$$S_\pi(\vec{b}) = exp\left[i\int\frac{d\vec{k}}{\sqrt{2\omega}}\, j(k,\vec{b})\, a^+_{in}(k)\right] exp\left[i\int\frac{d\vec{k}}{\sqrt{2\omega}}\, j^*(k,\vec{b})\, a_{in}(k)\right]$$

$$\times\ exp\left[-\frac{1}{2}\int\frac{d\vec{k}}{2\omega}\left|j(k,\vec{b})\right|^2\right] , \qquad (2.2)$$

where

$$J(x,\vec{b}) = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\int\frac{d\vec{k}}{(2\pi)^{3/2}}\, j(k,\vec{b})\, e^{-ikx} . \qquad (2.3)$$

Let us define

$$\chi_\pi(\vec{b}) = \frac{1}{2}\int \frac{d\vec{k}}{2\omega} |j(k,\vec{b})|^2 . \tag{2.4}$$

Although not explicitly indicated, the source $J(x.\vec{b})$ may depend on the incident energy, as we actually assume, and so do $j(k,\vec{b})$ and $\chi_\pi(\vec{b})$. With this notation, the matrix element for transition from an incident $|pp\rangle$ state to a final $|pp, k_1,\ldots k_n\rangle$ state, where $k_1,\ldots k_n$ refer to momenta of the produced pions, reads

$$\langle pp, k_1, \ldots, k_n | S | pp\rangle \simeq e^{-\chi_\pi(\vec{b})} \prod_{\alpha=1}^{n} \frac{i\, j(k_\alpha, \vec{b})}{\sqrt{2\omega_\alpha}} . \tag{2.5}$$

The corresponding cross section (except the elastic one) is written

$$\sigma_n = \frac{1}{n!}\int d\vec{b}\, e^{-2\chi_\pi(\vec{b})} [2\chi_\pi(\vec{b})]^n . \tag{2.6}$$

Inclusive cross section $\omega\frac{d\sigma}{d\vec{k}}$ may be evaluated in a similar way by squaring Eq. (2.5), integrating it over all the momentum variables except one and summing the integrals over n with an appropriate statistical factor. The result is

$$\omega\frac{d\sigma}{d\vec{k}} \simeq \frac{1}{2}\int d\vec{b}\, |j(k,\vec{b})|^2 . \tag{2.7}$$

For elastic channel, we have to subtract 1 from the S matrix element in Eq. (2.5), which gives the amplitude

$$F(s,t) = \frac{i}{2\pi}\int d\vec{b}\, e^{i\vec{b}\cdot\vec{\kappa}} [1 - e^{-\chi_\pi(\vec{b})}] , \tag{2.8}$$

in terms of which the differential cross section becomes

$$\frac{d\sigma_{el}}{dt} = \pi \left| F(s,t) \right|^2 . \tag{2.9}$$

Here, as usual, we wrote $t=-\vec{\kappa}^2$ and the integration is on a plane perpendicular to the incident direction.

From Eqs. (2.8) and (2.9), it follows by t-integration

$$\sigma_{el}(s) = \int d\vec{b} \left| 1 - e^{-\chi_\pi(\vec{b})} \right|^2 . \tag{2.10}$$

Optical theorem gives also

$$\sigma_t(s) = 2\int d\vec{b} \left[1 - e^{-\chi_\pi(\vec{b})} \right] \tag{2.11}$$

and by subtracting Eq. (2.10) from Eq. (2.11), it follows

$$\sigma_{inel}(s) = \int d\vec{b} \left[1 - e^{-2\chi_\pi(\vec{b})} \right] . \tag{2.12}$$

The last equation can alternatively be derived summing σ_n given by Eq. (2.6), which shows a consistency with the unitarity requirements. The average multiplicity of pions may be calculated by using Eq. (2.6):

$$\langle n \rangle = \sum_{n=1}^{\infty} n \frac{\sigma_n}{\sigma_{inel}} = \frac{1}{\sigma_{inel}} \int d\vec{b} \left[2\chi_\pi(\vec{b}) \right] . \tag{2.13}$$

Now we come to the question of how to choose the source function $J(x.\vec{b})$ in Eq. (2.1). According to the idea of bremsstrahlung, who emits the final particles (neglecting the interaction among them) are the incident protons, but during their mutual interaction. One may write a certain hadronic-charge distribution $\rho_i(x)$ for each proton, which is assumed to be spherically symmetrical in its own system, $\rho(r)$. Looked at from an arbitrary system moving along the collision axis (see Fig. 1), such a distribution will show contracted and in movement with a velocity $\vec{\beta}_i$

$$\rho_i(x) = \gamma_i \rho\left[\gamma_i(\vec{z}-\vec{\beta}_i t) + (\vec{r}_\perp \mp \frac{\vec{b}}{2})\right] \quad . \tag{2.14}$$

However, as far as particle emission is concerned, this charge distribution will become effective only during a short time interval in which the collision takes place and if the impact parameter $\vec{b}$ is such that the particles can actually interact. We try to take this finite-range effect into account by writing

$$J(x,\vec{b}) = g\,|\vec{\beta}_1-\vec{\beta}_2|\;\rho_1(x)\rho_2(x) \quad , \tag{2.15}$$

where g is a constant proportional to the intensity of the interaction. The factor $|\vec{\beta}_1-\vec{\beta}_2|$ has been introduced in order that $J(x,\vec{b})$ become a scalar with respect to Lorentz transformation along z-axis. A similar source has been used by other authors (4,5).

Introducing the Fourier transform of the hadronic-charge density $\tilde{\rho}(\mu^2)$ by

$$\rho_i(x) = \gamma_i \int \frac{d^3u}{\sqrt{(2\pi)^3}}\, \tilde{\rho}(u^2) \exp\left\{i\vec{u}\cdot\left[\vec{r}_\perp \mp \frac{\vec{b}}{2} + \gamma_i(\vec{z}-\vec{\beta}_i t)\right]\right\} \tag{2.16}$$

and calculating $j(k,\vec{b})$ from Eq. (2.3), we have, in the C.M. system,

$$\begin{aligned} j(k,\vec{b}) &= \frac{1}{(2\pi)^{3/2}} \int d^4x\, e^{ikx} J(x,\vec{b}) \\ &= \frac{g}{\sqrt{2\pi}} \int d^2u_\perp\, \tilde{\rho}(u^2)\tilde{\rho}(u'^2) \exp\left[-i\vec{b}\cdot(\vec{u}_\perp - \frac{k_\perp}{2})\right] \quad , \end{aligned} \tag{2.17}$$

where

$$\begin{cases} \vec{u} \equiv \left[\frac{1}{2\gamma}\left(k_{\parallel} + \frac{k_c}{\beta}\right), \ \vec{u}_{\perp} \right] \\ \vec{u}' \equiv \left[\frac{1}{2\gamma}\left(k_{\parallel} - \frac{k_c}{\beta}\right), \ -\vec{u}_{\perp} + \vec{k}_{\perp} \right] \end{cases}$$

The equation above shows that $k_{\perp}$-dependence of $j(k,\vec{b})$ is more or less independent of the incident energy, while $k_{\parallel}$ dependence appears through the variable $k_{\parallel}/\gamma$. This property together with Eqs. (2.4) and (2.7-2.13) guarantees all the asymptotic behaviors a) ~ e), mentioned in the introduction. Let us illustrate this point, by assuming a simple parametrization for the hadronic charge density

$$\tilde{\rho}(u^2) = e^{-\alpha u^2} \tag{2.18}$$

Putting this $\tilde{\rho}(u^2)$ into Eq. (2.17), we get for $s\to\infty$

$$j(k,\vec{b}) = \frac{g}{2\alpha}\sqrt{\frac{\pi}{2}}\exp\left\{ -\frac{4\alpha m^2}{s}k_{\parallel}^2 - \frac{\alpha \vec{k}_{\perp}^2}{2} - \frac{\vec{b}^2}{8\alpha} \right\} \tag{2.19}$$

and the substitution of this into Eq. (2.7) gives

$$\omega\frac{d\sigma}{d\vec{k}} = \frac{g^2\pi^2}{4\alpha}\exp\left\{ -\frac{8\alpha m^2}{s}k_{\parallel}^2 - \alpha\vec{k}_{\perp}^2 \right\}, \tag{2.20}$$

which shows the constancy of $\langle k_{\perp}\rangle$ and the scaling of $\omega\frac{d\sigma}{d\vec{k}}$.

Eq. (2.19) also implies that $\chi_{\pi} \sim \ln s$ as $s\to\infty$. This can immediately be seen by noting that the energy dependence of χ_{π} arises from the $k_{\parallel}$ integration of Eq.. (2.4) which is roughly $\sim \int \frac{dk_{\parallel}}{k_{\parallel}}$, the upper limit being determined by the cutoff factor $|j(k,\vec{b})|^2$ which expands proportionally to $\sqrt{s}$. Detailed integration of Eq. (2.4) gives

$$\chi_\pi(\vec{b}) \underset{s\to\infty}{\simeq} A\,[\ln s - B]\, e^{-\frac{b^2}{4\alpha}} \,, \tag{2.21}$$

with

$$A = \frac{\pi^2 g^2}{32\alpha^3} \quad \text{and} \quad B = \gamma + \ln(2\alpha m^2 \mu_\perp^2)\,, \tag{2.22}$$

where $\mu_\perp^2 = \langle \mu^2 + k_\perp^2 \rangle$

and γ in Eq. (2.22) is Euler's constant.

Substituting Eq. (2.21) into Eq. (2.11), we have

$$\sigma_t(s) = 2\int d\vec{b}\,[1 - e^{-\chi_\pi(\vec{b})}] = -2\sum_{n=1}^{\infty} \frac{1}{n!} \int d\vec{b}\,[-\chi_\pi(\vec{b})]^n$$

$$\simeq -2\sum_{n=1}^{\infty} \frac{[-A(\ln s - B)]^n}{n!} \int d\vec{b}\, e^{-\frac{nb^2}{4\alpha}}$$

$$= -8\alpha\pi \sum_{n=1}^{\infty} \frac{[-A(\ln s - B)]^n}{n\, n!}$$

$$\therefore\ \sigma_t(s) \simeq 8\alpha\pi \left\{ \gamma + \ln[A(\ln s - B)] - Ei[-A(\ln s - B)] \right\}. \tag{2.23}$$

The leading term of this equation is

$$\sigma_t(s) \underset{s\to\infty}{\longrightarrow} 8\alpha\pi \ln(\ln s)\,. \tag{2.23a}$$

Analogously, the substitution of Eq. (2.21) into Eqs. (2.10) and (2.12) gives

$$\sigma_{el}(s) \simeq 4\alpha\pi \left\{ (\gamma - \ln 2) + \ln[A(\ln s - B)] - 2E_i[-A(\ln s - B)] + E_i[-2A(\ln s - B)] \right\} \tag{2.24}$$

and

$$\sigma_{inel}(s) \simeq 4\alpha\pi\left\{\gamma + \ln[2A(\ln s - B)] - Ei[-2A(\ln s - B)]\right\} \qquad (2.25)$$

whose leading terms are

$$\sigma_{el}(s) \underset{s\to\infty}{\longrightarrow} 4\alpha\pi \ln(\ln s) \qquad (2.24a)$$

and

$$\sigma_{inel}(s) \underset{s\to\infty}{\longrightarrow} 4\alpha\pi \ln(\ln s) \ . \qquad (2.25a)$$

The average multiplicity <n> is obtained from Eq. (2.13), by using Eqs. (2.21) and (2.25).

$$\langle n\rangle = \frac{2}{\sigma_{inel}}\int d\vec{b}\,\chi(\vec{b}) = \frac{8\pi\alpha A[\ln s - B]}{\sigma_{inel}} \underset{s\to\infty}{\longrightarrow} \frac{2A\ln s}{\ln(\ln s)} \ . \qquad (2.26)$$

Finally, the shrinking of $\frac{d\sigma_{el}}{dt}$ is evident from Eqs. (2.8) and (2.9), for $\chi_\pi(\vec{b})$ is now an increasing function of s.

3. TWO COMPONENT MODEL

In the preceding section, a simple classical source model has been described, which exihibits several interesting asymptotic properties. However, as already mentioned in Introduction (A) - D)), it meets serious difficulties as soon as a quantitative fit is attempted (6). Here, we present a possible improvement to that model, which conserves all its desireable features, while avoiding the difficulties mentioned in Introduction.

Suppose the high-energy collisions between two particles occur through the following two mechanisms or a combination of both (let us consider pp collisions as in preceding section):

i) Excitation of one or both of the incident particles, which decay subsequently. We call this process fragmentation and assume "independent" of the energy.

ii) Particles (or clusters) may be irradiated in analogy to bremsstrahlung. We call this component pionization and which will exihibit an energy dependence.

These two mechanisms are assumed to be independent, which may be expressed by writing the S-matrix in the product form

$$S = S_1 \, S_2 \, S_\pi \quad , \tag{3.1}$$

where S_1, S_2 and S_π commute. Here, S_i refers to fragmentation of the i-th incident particle, S_π to pionization and the impact parameter of the incident particles is fixed.

Physically, independence of the two mechanisms means first that pionization causes negligible changes on the incident particles so that fragmentation, which we are assuming independent of the incident energy in the sense of "limiting fragmentation hypothesis" (10), occurs as if no particle has been emitted. Secondly, it means that during the short time of interaction, the hadronic-charge distribution remains constant, even when there is some excitation of the incident particles and, consequently, the source responsible for the pionization stays the same regardless of whether the incident particles suffer fragmentation or not. Also, no-recoil approximation is implicit in using fixed impact parameters for protons.

The unitarity of the S matrix together with the above assumption require that each factor S_i and S_π be unitary. This condition leads, for instance, to the following useful equality which will be employed later:

$$\langle p|S_i^+ S_i|p\rangle = \sum_f |\langle f|S_i|p\rangle|^2 = 1 \qquad (i = 1, 2) \quad , \quad (3.2)$$

where $|p\rangle$ represents the incident proton state and the sum is over all the possible final states including the elastic channel.

The pionization is described in the way already drawn in the preceding section, where the source function $J(x,\vec{b})$ in Eq. (2.1) is now assumed to be independent of the proton fragmentation, in accordance with the discussion above. Using the same notation as in sec. 2 , the matrix element for transition from an incident two-proton state $|pp\rangle$ to a final state $|f_1, f_2; k_1, \ldots, k_n\rangle$, where f_1, f_2 refer to the incident proton fragmentation and $k_1, \ldots, k_n$ to momenta of the pionization particles, reads

$$\langle f_1, f_2; k_1, \ldots, k_n | S | pp \rangle \simeq \langle f_1 | S_1 | p \rangle \langle f_2 | S_2 | p \rangle e^{-\chi_\pi(\vec{b})} \prod_{\alpha=1}^{n} \frac{i j(k_\alpha, \vec{b})}{\sqrt{2\omega_\alpha}} . \tag{3.3}$$

The corresponding cross section is written

$$\sigma_n^{f_1, f_2} = \frac{1}{n!} \int d\vec{b}\, e^{-2\chi_\pi(\vec{b})} (2\chi_\pi(\vec{b}))^n \left| \langle f_1 | S_1 | p \rangle \langle f_2 | S_2 | p \rangle \right|^2 . \tag{3.4}$$

If we look at the production of n pionization particles without worrying about what happens with the incident protons, we have to sum Eq. (3.4) over f_1 and f_2, recovering, on account of Eq. (3.2), the familiar expression already given as Eq. (2.6). In an entirely similar way, contributions to inclusive cross-section coming from pionization is shown to be given by Eq. (2.7).

The elastic amplitude, Eq. (2.8) is modified to

$$F(s,t) = \frac{i}{2\pi} \int d\vec{b}\, e^{i\vec{b}\cdot\vec{K}} \left[1 - e^{-\chi_\pi} \langle p | S_1 | p \rangle \langle p | S_2 | p \rangle \right] , \tag{3.5}$$

where an additional factor representing the beam attenuation due to fragmentation appears now. Analogous change in Eqs. (2.10) - (2.12) gives

$$\sigma_{el}(s) = \int d\vec{b} \left| 1 - e^{-\chi_\pi} \langle p | S_1 | p \rangle \langle p | S_2 | p \rangle \right|^2 , \tag{3.6}$$

$$\sigma_t(s) = 2 \int d\vec{b} \left[1 - e^{-\chi_\pi} \langle p | S_1 | p \rangle \langle p | S_2 | p \rangle \right] \tag{3.7}$$

and

$$\sigma_{inel}(s) = \int d\vec{b} \left[1 - e^{-2\chi_\pi} \left| \langle p | S_1 | p \rangle \langle p | S_2 | p \rangle \right|^2 \right] . \tag{3.8}$$

It is also easily seen that Eq. (2.13) corresponds now to the contribution from pionization to the average pion multiplicity, provided σ_{inel} is appropriately reinterpreted as given by Eq. (3.8).

Let us now consider a (pure) fragmentation, i.e., an inelastic process in which no pionization particle is emitted. For single fragmentation, by Fourier-transforming Eq. (3.3) with n=0 and one of f_i equal to p, we have

$$G(f_1;s,t) \simeq -\frac{i}{2\pi}\int d\vec{b}\, e^{i\vec{b}\vec{\kappa}} e^{-\chi_\pi} \langle f_1|S_1|p\rangle\langle p|S_2|p\rangle \tag{3.9}$$

and

$$\frac{d\sigma(f_1)}{dt} = \pi \left| G(f_1;s,t)\right|^2 \tag{3.10}$$

with a similar result for $\frac{d\sigma(f_2)}{dt}$. Here, and in the following equations, we neglect the small shift in t threshold.

In the corresponding way, double-fragmentation amplitude is written

$$G(f_1,f_2;s,t) \simeq -\frac{i}{2\pi}\int d\vec{b}\, e^{i\vec{b}\vec{\kappa}} e^{-\chi_\pi} \langle f_1|S_1|p\rangle\langle f_2|S_2|p\rangle \tag{3.11}$$

and the cross section

$$\frac{d\sigma(f_1,f_2)}{dt} = \pi \left| G(f_1,f_2;s,t)\right|^2 . \tag{3.12}$$

The integration of Eqs. (3.10) and (3.12) over t gives us

$$\sigma(f_1) \simeq \int d\vec{b}\, e^{-2\chi_\pi} \left|\langle f_1|S_1|p\rangle\langle p|S_2|p\rangle\right|^2 \tag{3.13}$$

and

$$\sigma(f_1,f_2)\simeq\int d\vec{b}\, e^{-2\chi_\pi}\left|\langle f_1|S_1|p\rangle\langle f_2|S_2|p\rangle\right|^2 . \tag{3.14}$$

In actual problems, however, one is often more interested in a sum over several states in the mass range from M to M+dM and not in a transition into a single state specified by f_i. As a matter of fact, what the states f_i are is often not clear in real problems. Thus, we assume the matrix element $<f_i|s_i|p>$ a function of mass M only (and so are $G(f_1,s,t)$ and $G(f_1,f_2;s.t)$) and, by introducing a density of states $n(M)$, write

$$\frac{d^2\sigma}{dt\,dM_i}\simeq\pi\left|G(M_i,s,t)\right|^2 n(M_i) \tag{3.10a}$$

and

$$\frac{d^3\sigma}{dt\,dM_1dM_2}\simeq\pi\left|G(M_1,M_2;s,t)\right|^2 n(M_1)\,n(M_2) . \tag{3.12a}$$

The corresponding formulas to Eqs. (3.13) and (3.14) are

$$\frac{d\sigma}{dM_1}\simeq\int d\vec{b}\, e^{-2\chi_\pi} n(M_1)\left|\langle M_1|S_1|p\rangle\langle p|S_2|p\rangle\right|^2 \tag{3.13a}$$

and

$$\frac{d^2\sigma}{dM_1dM_2}\simeq\int d\vec{b}\, e^{-2\chi_\pi} n(M_1)\left|\langle M_1|S_1|p\rangle\right|^2 n(M_2)\left|\langle M_2|S_2|p\rangle\right|^2 . \tag{3.14a}$$

The total single- and double-fragmentation cross-section are then, on account of Eq. (3.2)

$$\sigma_f^1 = \sigma_f^2 = \int d\vec{b}\, e^{-2\chi_\pi} |\langle p|S_2|p\rangle|^2 \int dM_1\, n(M_1) |\langle M_1|S_1|p\rangle|^2$$

$$= \int d\vec{b}\, e^{-2\chi_\pi} |\langle p|S_2|p\rangle|^2 \left[1 - |\langle p|S_1|p\rangle|^2\right] \qquad (3.15)$$

and

$$\sigma_f^{1,2} = \int d\vec{b}\, e^{-2\chi_\pi} \left[1 - |\langle p|S_2|p\rangle|^2\right]\left[1 - |\langle p|S_1|p\rangle|^2\right] . \qquad (3.16)$$

In the following, the parametrization

$$|\langle p|S_1|p\rangle|^2 = |\langle p|S_2|p\rangle|^2 = e^{-2\chi_f(\vec{b})} \qquad (3.17)$$

will be used. A particular choice of χ_f for a quantitative comparison with the experimental data will be considered in the next section. Here, we argue that χ_f is expected to be approximately proportional to $\langle F^2 \rangle$, the two-dimensionl Fourier transform of the square of the proton form factor. This follows if we accept Chou-Yang model (11) as valid for fragmentation. At intermediate energies ($p_{lab} \sim 20$ Gev/c) such that bremsstrahlung is still negligible, although high enough to allow a geometrical description of collisions, the above assumption may be approximately satisfied, which can be inferred from the excellent results obtained in their works.

Introducing the above notation, we summarize here some of the formulas derived above, which will be employed in the next section. Namely, Eqs. (3.5) - (3.8) and (3.15) and (3.16) may be rewritten

$$F(s,t) = \frac{i}{2\pi} \int d\vec{b}\, e^{i\vec{b}\cdot\vec{\kappa}} \left[1 - e^{-\chi_\pi - 2\chi_f}\right] , \qquad (3.18a)$$

$$\sigma_{el}(s) = \int d\vec{b}\, \left| 1 - e^{-\chi_\pi - 2\chi_f} \right|^2 \quad , \tag{3.18b}$$

$$\sigma_t(s) = 2\int d\vec{b} \left[1 - e^{-\chi_\pi - 2\chi_f} \right] \quad , \tag{3.18c}$$

$$\sigma_{inel}(s) = \int d\vec{b} \left[1 - e^{-2(\chi_\pi + 2\chi_f)} \right] \quad , \tag{3.18d}$$

$$\sigma_f^{1} = \sigma_f^{2} = \int d\vec{b}\, e^{-2(\chi_\pi + \chi_f)} \left[1 - e^{-2\chi_f} \right] \; , \tag{3.18e}$$

$$\sigma_f^{1,2} = \int d\vec{b}\, e^{-2\chi_\pi} \left[1 - e^{-\chi_f} \right]^2 \; . \tag{3.18f}$$

In Eqs. (3.18), χ_f is assumed to be constant (for sufficiently high energy), whereas χ_π is given by E_q. (2.21) and increases logarithmically with s.

4. COMPARISON WITH EXPERIMENTS

The results obtained in the preceding section are compared, in this section, with experimental data.

First, we notice that there exist certain quantities, like those given by Eqs. (3.18), which depend only on χ_f and χ_π . These quantites can easily be compared with the data and this will be done explicitly in the present section.

Besides, there are other quantites, the computation of which requires more detailed knowledge of fragmentation, such as the off-diagonal elements of S_i, the state density $n(M)$ and the momentum distribution of the decay products of these states. The cross sections $\frac{d^2\sigma}{dtdM}$, $\frac{d^3\sigma}{dtdM_1dM_2}$, $\frac{d\sigma}{dM}$ and $\frac{d^2\sigma}{dM_1dM_2}$ given by Eqs. (3.10a) ~ (3.14a), as well as $\omega\frac{d\sigma}{d\vec{k}}$, σ_n and $\langle n\rangle$, are such observables. An attempt to calculate $\frac{d^2\sigma}{dMdt}$ has been done in one of our works (12), though restricted to the small-mass region. As expected, absorption effect, which is embodied through the exponential factor $e^{-\chi_f}$, gives more peripheral impact-parameter amplitude (profile function) or equivalently shrinking of the forward peak as compared to the elastic channel. In this paper, however, we are not going to rediscuss or improve the previously presented results, but will simply give a discussion about the possibility of reconcile the data on $\omega\frac{d\sigma}{d\vec{k}}$ and $\langle n\rangle$ with our scheme.

Let us begin by fixing the function χ_f. As discussed in the previous section, we expect that χ_f be approximately proportional to $\langle F^2\rangle$ and constant with energy. The purpose of this paper is not to get the "best-fit" of the experimental data, but to verify whether a simple mechanism as discussed here allows to reproduce <u>all</u> the essential features of high-energy collisions. Thus, we are not interested in determining the exact form of χ_f, nor $\rho(x)$ for pionization. We prefer, instead, to parametrize them in a way which allows an easy computational manipulation. We write

$$\chi_f = C e^{-\beta b^2} \tag{4.1}$$

with C and β two adjustable parameters. A comparison with $<F^2>$ gives an estimate of β and in the following calculation we take

$$\frac{1}{\beta} = 13 \, (GeV/c)^{-2} . \tag{4.2}$$

This value of β corresponds to an average in the interval $0 < \kappa^2 < 0.8$ GeV^2 and, as a consequence of this parametrization, we don't expect a good agreement of $\frac{d\sigma_{el}}{dt}$ in the large-t region (see Fig.2).

The parameter C in Eq. (4.1) could be evaluated by using, for instance, data on $\frac{d\sigma_{el}}{dt}$ at intermediate energies (p_L ~ 30 GeV/c), where, according to our description, pionization is not yet important. A slightly smaller value of C is taken, however, considering the over-all agreement of the results with the data, namely

$$C = 0,5 \tag{4.3}$$

Once, χ_f is fixed, it remains only to determine the two parameters, for instance g and α , in Eqs. (2.21) and (2.22), in order that a numerical comparison of the quantities given by Eqs. (3.18) with experiments can be carried out. We do this by fitting $\frac{d\sigma_{el}}{dt}$ for $-t<0.4$ $(Gev/c)^2$, at the highest energy where such data are available. As is known, $\frac{d\sigma_{el}}{dt}$ at these energies and at such t intervals shows two distinct components with different slope parameters (see Fig. 2). In our description, the wider component corresponds roughly to fragmentation, whereas the narrower one is due to pionization. This interpretation is consistent with the experimental evidence that the large-t part is very little dependent on energy, while the small-t peak shrinks continuously with energy (see Fig.3).

At $s=2809$ Gev^2, a good fit is obtained with

$$\begin{cases} \alpha = 8.5\ (\text{Gev/c})^{-2} \\ x = A(\ln s - B) = 0.290 \end{cases} \tag{4.4}$$

from which and by using Eq. (2.22) we find

$$\begin{cases} A = 4.44 \times 10^{-2} \\ B = 1.41 \end{cases} \tag{4.5}$$

Now we are ready to compute χ_π at any energy value and, together with χ_f, calculate all the observables listed in Eqs. (3.18). In Fig. 3, the so calculated $\frac{d\sigma_{el}}{dt}$ at $s=2809\text{GeV}^2$ is shown together with the experimental data. At other values of s, we have similar curves and the agreement is excellent for all the ISR data. Fig. 4 shows the slope parameter at $t=0$. The total, inelastic and elastic cross sections as function of s are plotted in Fig. 5.

In spite of a very simple parametrization of χ_f and χ_π, it is seen that the agreement of these results with the available data is more than satisfactory in all the ISR energy range and down to $s \simeq 200$ GeV^2. For $s < 200$ Gev^2, a deviation occurs which, in our opinion, is due essentially to the simplified choice of χ_f we have adopted. Looking at Fig.2, we see that, at $-t < 0.2$, a considerable deviation of F^2 from our curve occurs. If we have taken $\chi_f \sim \langle F^2 \rangle$, this would of course allow to form a more pronounced peak in $\frac{d\sigma_{el}}{dt}$ at low energies without destroying the good agreement seen in Fig.3 for $-t \gtrsim 0.15$. On the other hand, this would evidently cause a flattening of σ_t, σ_{inel} and σ_{el} as s decreases below ISR energies. At the same time, it would force α to become smaller, which is desireable, since our $\tilde{\rho}(u^2)$ in Eq.(2.18) with α fixed above is much narrower than $F(\kappa^2)$, although there is a priori no reason why they must coincide each other. In writing the source density, Eq. (2.15), we intended to consider not only the finite extension of protons' hadronic charge but also the finite range of their mutual interaction, which may be different from the former.

As to σ_f^1 and $\sigma_f^{1,2}$, very few data are available and moreover these are frequently contradictory. Here we just mention that these quantities are slowly decreasing with s, due to the increase of χ_π and the prediction for σ_f^1 at $s=2809 Gev^2$ is

$$\sigma_f^1 = 5{,}79 \text{ mb} \tag{4.6}$$

Later we will return to discuss this result, in connection with $\omega\frac{d\sigma}{d\vec{k}}$ and $\langle n \rangle$.

Having been shown the most immediate results predicted by the model, let us now turn to a discussion of the other observables such as $\omega\frac{d\sigma}{d\vec{k}}$ and $\langle n \rangle$. As is stated in Introduction, our purpose is to construct a model which conserves the nice features of bremsstrahlung analogy and which eliminates the difficulties A) to D) mentioned there.

Quantities $\omega\frac{d\sigma}{d\vec{k}}$ and $\langle n \rangle$ are now given as a sum

$$\omega \frac{d\sigma}{d\vec{k}} = P_f^1\left(\omega \frac{d\sigma_f^1}{d\vec{k}}\right) + P_f^2\left(\omega \frac{d\sigma_f^2}{d\vec{k}}\right) + P_\pi\left(\omega \frac{d\sigma_\pi}{d\vec{k}}\right)$$

$$= P_f\left(\omega \frac{d\sigma_f^1}{d\vec{k}} + \omega \frac{d\sigma_f^2}{d\vec{k}}\right) + P_\pi \omega \frac{d\sigma}{d\vec{k}} \qquad (4.7)$$

and

$$\langle n \rangle = P_f^1 \langle n_1 \rangle + P_f^2 \langle n_2 \rangle + P_\pi \langle n_\pi \rangle$$

$$= 2 P_f \langle n_f \rangle + P_\pi \langle n_\pi \rangle \qquad (4.8)$$

where $P_f^1 = P_f^2 \equiv P_f$ and P_π are respectively the fragmentation probability of the proton 1 and 2 and that for pionization. Here, $P_f^1 + P_f^2 + P_\pi = 2P_f + P_\pi > 1$, since mixed events are also possible and, in these cases, a double or triple counting is made.

Our choice of parameters defining X_π and X_f, Eqs. (4.2) and (4.4), when considered together with Eqs. (3.18), indicates that even at the highest ISR energy, fragmentation is dominant over pionization, that is $P_f^i > P_\pi$. Thus, in the energy range where data are available, $\omega\frac{d\sigma}{d\vec{k}}$ and $\langle n \rangle$ are much more influenced by fragmentation than pionization and they won't be given by correcting simply Eqs. (2.7) and (2.13). Rather, contributions from pionization shall be regarded as small corrections to the basic results which correspond to fragmentation. Thus, a complete analysis of the problem requires detailed description of how fragmentation takes place. This will, however, be left to another occasion and here just an argument will be given showing that an appropriate account of fragmentation leads probably to experimentally consistent results.

Descriptions of fragmentation have been given by several authors in the past years, but here the version of Jacob and collaborators (13) will be considered. In those papers, they succeeded in reproducing both <n> and $\omega\frac{d\sigma}{d\vec{k}}$ for several final particles $\pi^{\pm}$, $k^{\pm}$, p, n,...., by taking a conveniently defined $\rho(M,p_{\perp}^2)$, the differential cross section for producing a cluster of mass M and transverse momentum $p_{\perp}$; n(M), the average multiplicity for each kind of particle as a function of the cluster mass M; and finally $\frac{dD}{d\vec{q}}$, the normalized decay distribution. The agreement with experiments is excellent, so we feel that with a similar parametrization we can obtain, also in our model where fragmentation is dominant below ISR energy, $\omega\frac{d\sigma}{d\vec{k}}$ and <n> which are as good as in their calculation. Of course, the multiplicity distribution σ_n will become now much broader.

In Jacob's calculation, the cross section

$$\sigma \quad = \quad \int\int \rho(M,p_{\perp}^2)\, dM dp_{\perp}$$

which gives a good fit for other quantities, when compared with usually reported σ_{dif} (14) is too large. However, a care must be exercised in carrying out such a comparison. According to our notation

$$\sigma = \int d\vec{b}\left[1 - e^{-2\chi_f}\right] \tag{4.9}$$

would be such a cross section, while σ_{dif} should be compared to σ_f^1 of Eq. (3.18e), or better twice this value, where an additional factor $e^{-2(\chi_\pi+\chi_f)}$ appears due to the beam absorption corresponding both to the fragmentation of the other particle and to pionization. In our calculation, σ = 12.7 mb, which is close to the Jacob's value (σ = 15mb), whereas σ_f is given by Eq. (4.6) which is much smaller, favouring our model.

5. CONCLUSIONS

In this paper, a model for high-energy collisions has been studied, which includes two distinct and independent particle-production mechanisms: a) pionization, depicted as an excitation of meson field by a classical source representing the incident particles in interaction and b) fragmentation, described as a two-stage process consisted of an incident-particle excitation followed by a subsequent decay of this. The use of eikonal approximation allowed us to write S matrix satisfying (S-channel) unitarity, which is convenient for studying absorption effects.

In spite of its simplicity, the model reproduces quite satisfactorily all the ISR data on $\frac{d\sigma_{el}}{dt}$, σ_t, σ_{inel} and σ_{el} and predicts σ_f^1 which is comparable to the experimentally measured σ_{dif}. Taking the success of fragmentation models in reproducing <n> and $\omega\frac{d\sigma}{d\vec{k}}$ into consideration, it is concluded that the present scheme is also able to incorporate such observables.

As discussed in preceding section, these results may be improved further both at lower energies and at larger t values for $\frac{d\sigma_{el}}{dt}$, if we take a more realistic parametrization for χ_f.

Also, once fragmentation is well defined it seems straightforward though laborious the extension of the model to calculations of other quantities such as correlations and multiplicity distributions.

As to the large-$k_\perp$ distribution of particles, we think that it depends strongly on the momentum-energy conservation constraints which we have not appropriately taken into account. In consequence, we have a constant (energy independent) distribution, which is actually verified only in the small-$k_\perp$ range.

Finally, we recognize that the work is of course incomplete to the extent that we simply borrowed some results on fragmentation, without elaborating it with more detail. This question is being studied now and will be discussed in another occasion.

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FIGURE CAPTIONS

Fig. 1) Definition of coordinate system.

Fig. 2) The square of the eletromagnetic form factor as a function of κ^2. The approximate curve used in our calculation is also shown (broken like). The dotted line is the exponential extrapolation of $F^2(\kappa^2)$ at small κ^2.

Fig. 3) $\frac{d\sigma_{el}}{dt}$ calculated at $s=2808$ Gev^2 compared with the experimental data (15).

Fig. 4) The slope parameter B(s) at t=0, predicted by our model. The experimental data have been taken from Ref. (15) where references to all the data are given.

Fig. 5) σ_t, σ_{inel} and σ_{el} as function of s, together with experimental data (1).

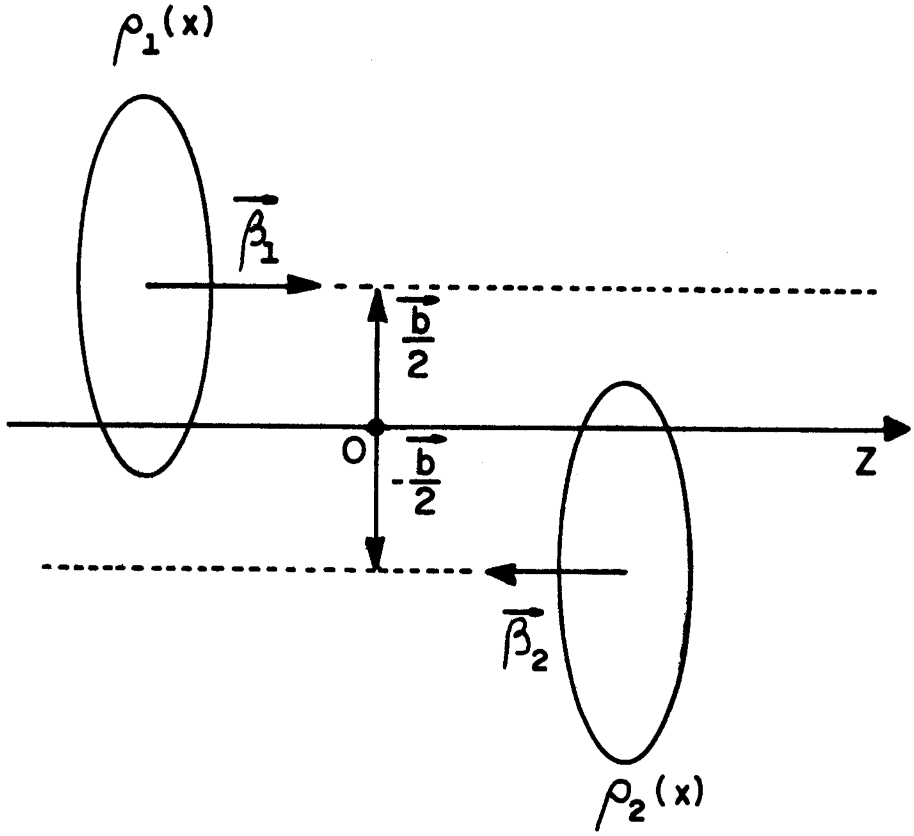


Fig. 1

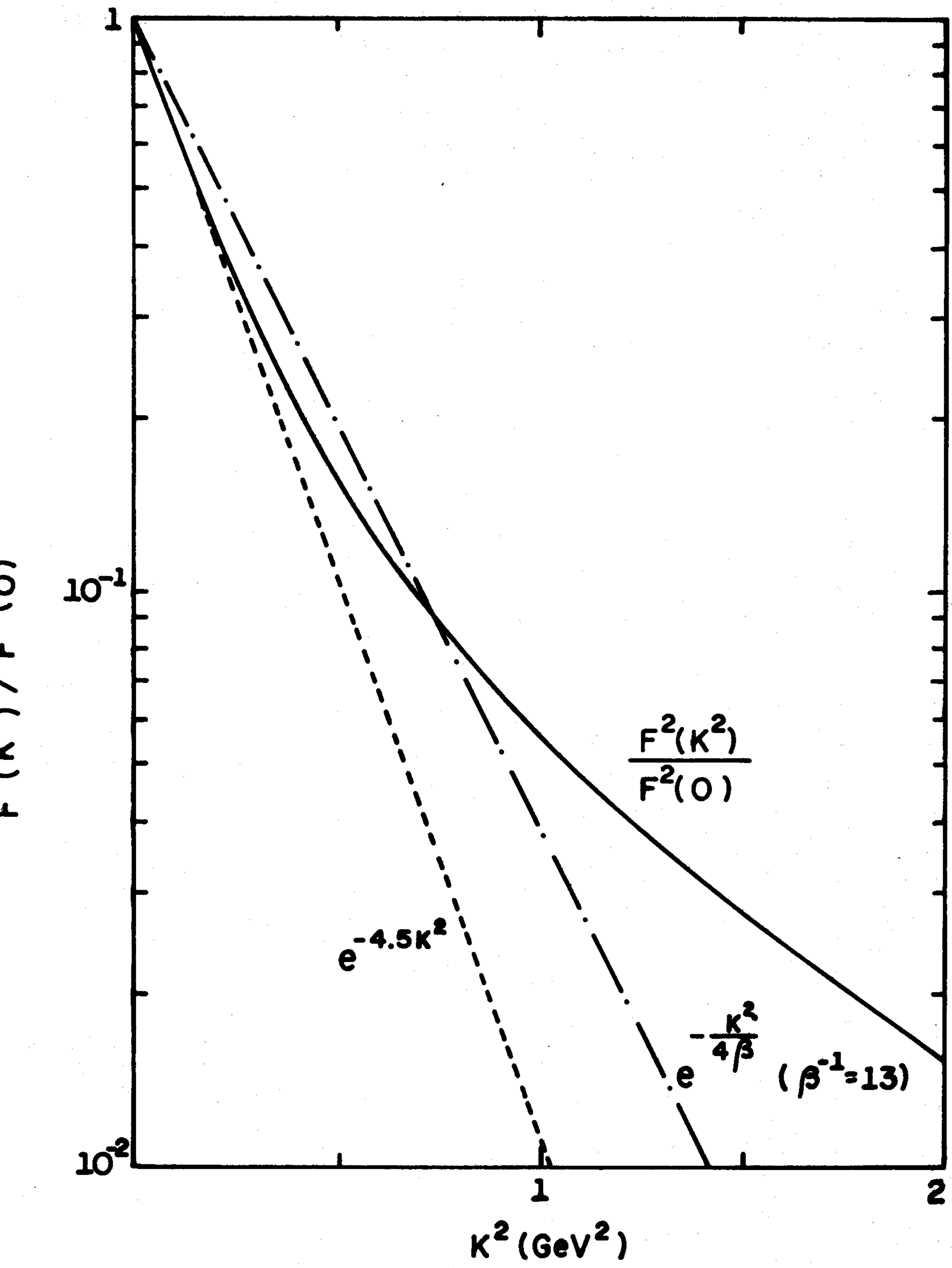

1
10^{-1}
10^{-2}
$F^2(K^2)\,/\,F^2(0)$
$\frac{F^2(K^2)}{F^2(0)}$
$e^{-4.5K^2}$
$e^{-\frac{K^2}{4\beta}}$ $(\beta^{-1}=13)$
1
2
K^2 (GeV^2)

Fig. 2

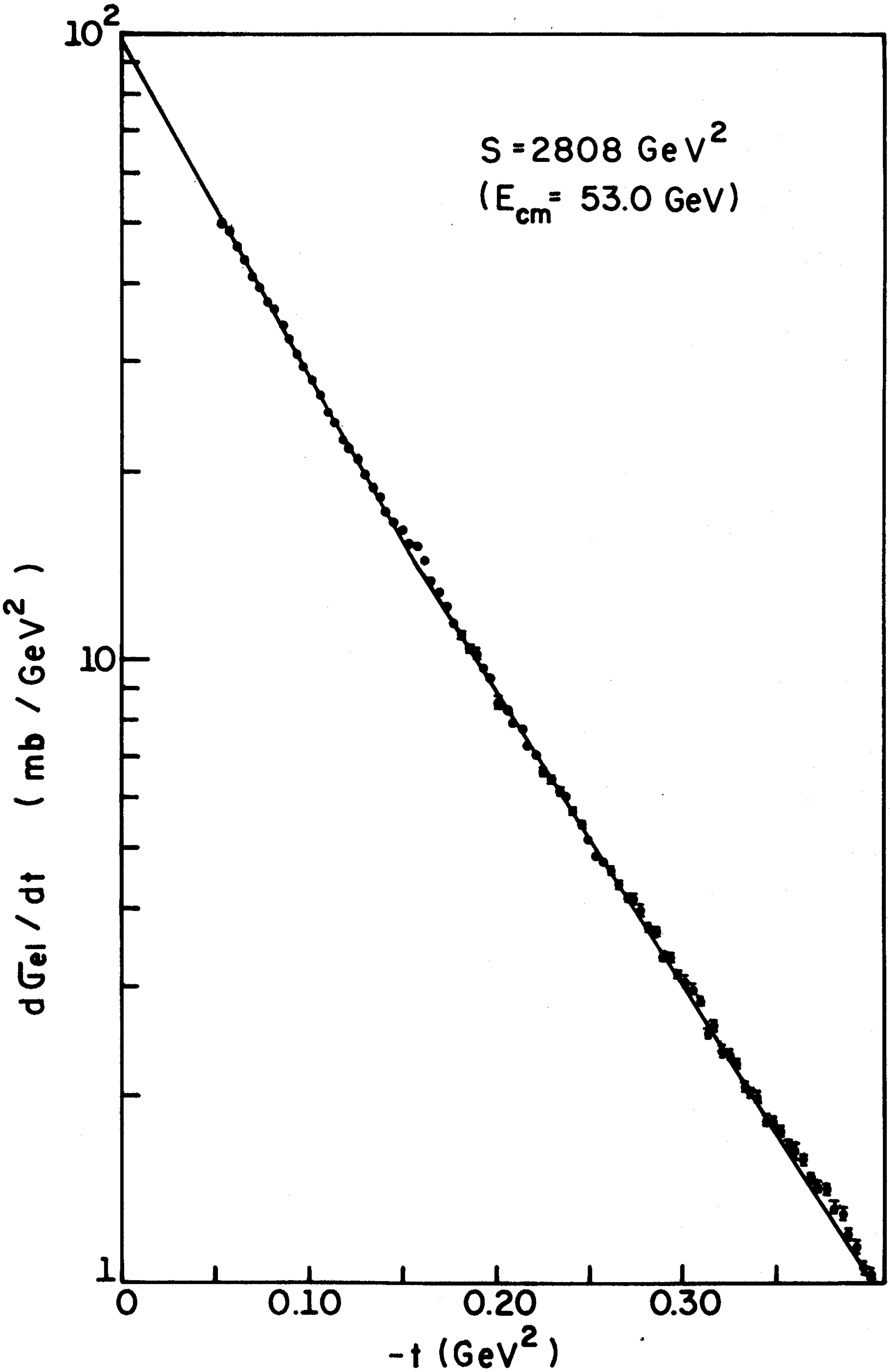

S = 2808 GeV^2
(E_{cm} = 53.0 GeV)
10^2
10
1
0
0.10
0.20
0.30
$d\sigma_{el}/dt$ (mb / GeV^2)
$-t$ (GeV^2)

Fig.3

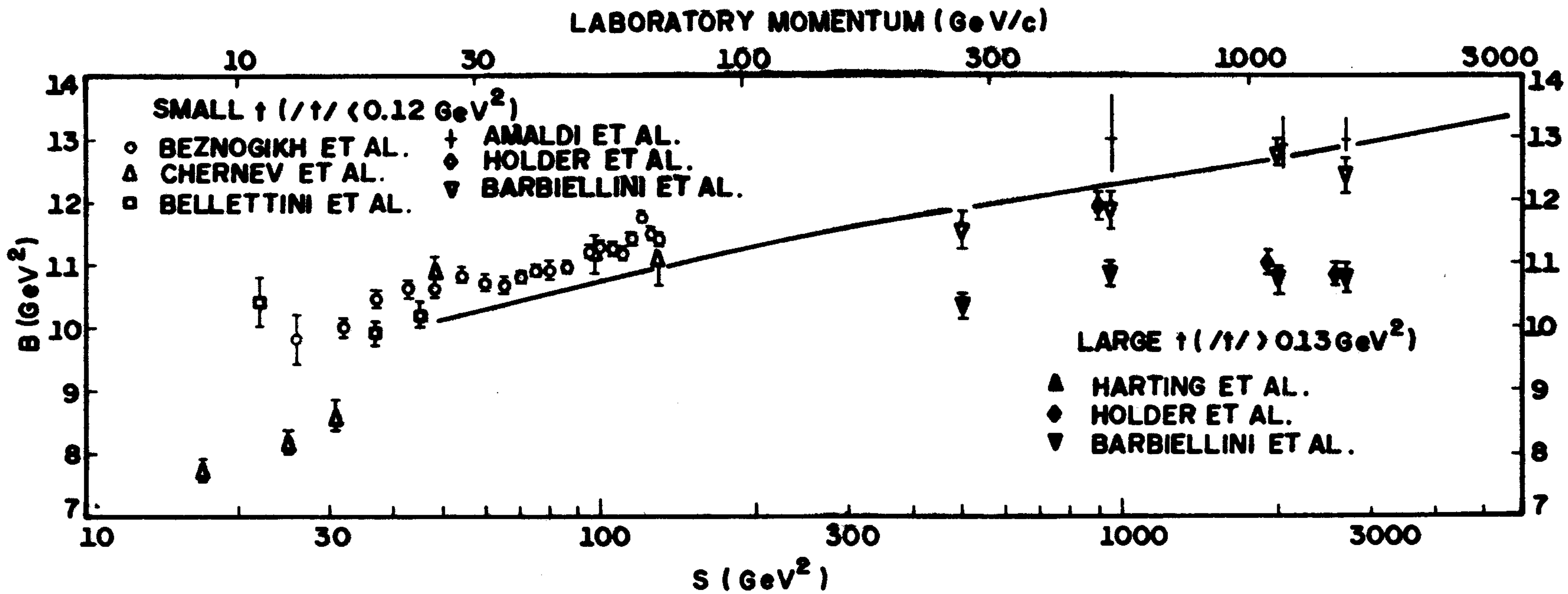
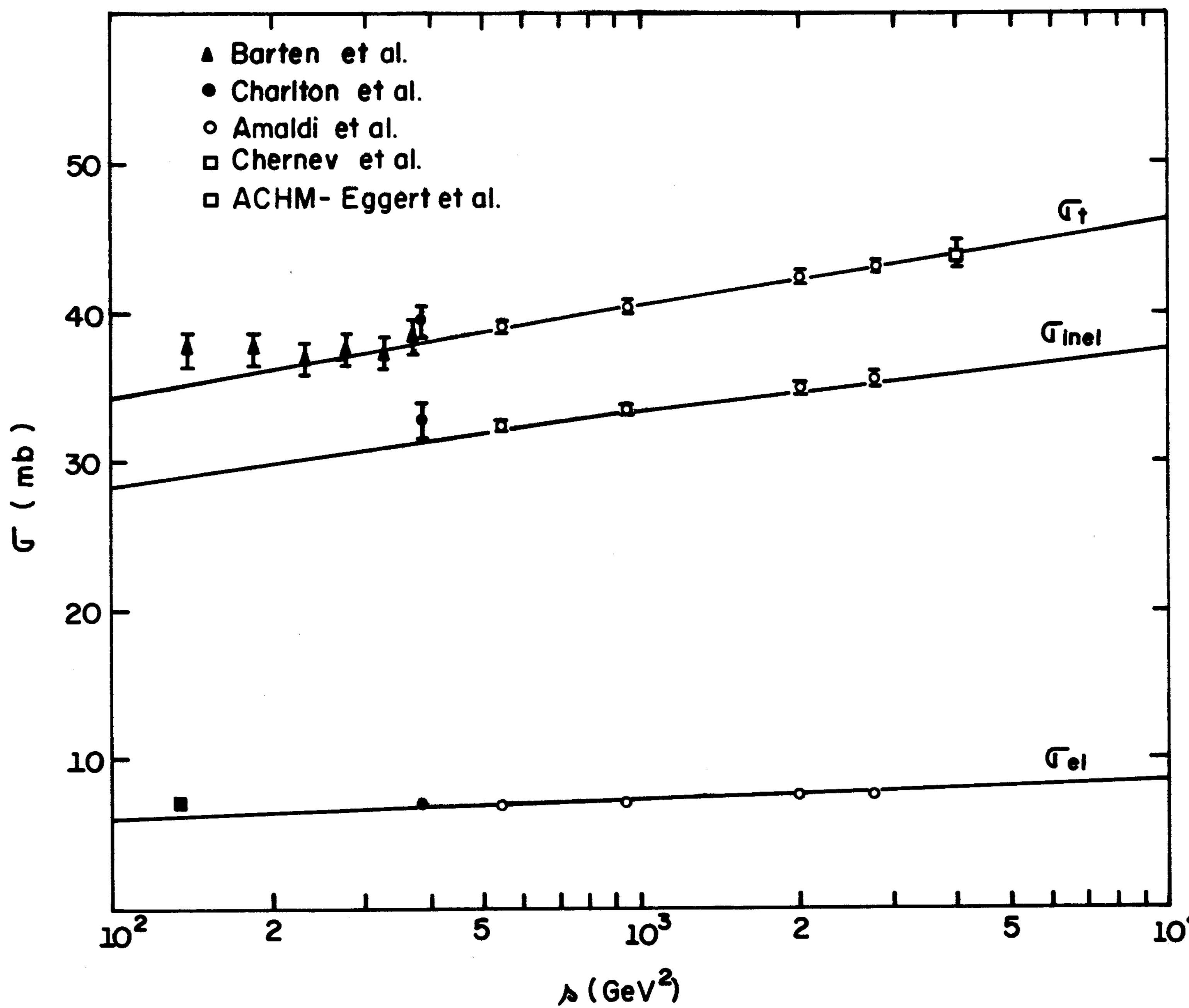

LABORATORY MOMENTUM (GeV/c)
SMALL t (/t/ < 0.12 GeV²)
BEZNOGIKH ET AL.
CHERNEV ET AL.
BELLETTINI ET AL.
AMALDI ET AL.
HOLDER ET AL.
BARBIELLINI ET AL.
LARGE t (/t/ > 0.13 GeV²)
HARTING ET AL.
HOLDER ET AL.
BARBIELLINI ET AL.
B (GeV²)
S (GeV²)

Fig. 4



Barten et al.
Charlton et al.
Amaldi et al.
Chernev et al.
ACHM- Eggert et al.
50
40
30
20
10
σ (mb)
σ_t
σ_inel
σ_el
10^2
2
5
10^3
2
5
10^4
s (GeV^2)

Fig.5