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"LEVEL-LEVEL CORRELATION AND ABSORPTION IN NUCLEAR REACTIONS"

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ABSTRACT

Level-level correlation (LLC) in nuclear reactions is discussed in general and it is shown that in the presence of LLC, $N_\mu = \frac{\sum_a |g_{\mu a}|^2}{\Gamma_\mu} > \tilde{\tau}$, where $\tilde{\tau}$ is the average absorption in the eigen channels.

It is of interest to acquire a better understanding of the different types of correlations in nuclear reactions. The problem of channel-channel correlation has received a recent extensive discussion[2] Level-level correlation has also been discussed[1]. We show in this letter that there is a difficulty inherent in the interpretation of the conclusions reached by[1].

The result of [1] are valid only in the weak-absorption-in-all-channel limit.

To fix the notation as well as make this letter as self-contained as possible we shall sketch in what follows the derivation of reference[1] but in a slightly different form. Nuclear reaction theory gives for the S-matrix element in the absence of intermediate structure the following

$$S_{ab} = S_{ab}^{(0)} - i \sum_{\mu} \frac{g_{\mu a}\, g_{\mu b}}{E - E_{\mu} + \frac{i}{2}\Gamma_{\mu}} \qquad (1)$$

where $S_{ab}^{(0)}$ describes direct non-resonant reactions and the second term is the compound nucleus contribution with $g_{\mu a}$ being the partial width amplitude. E_{μ} the position of the compound resonance μ with the corresponding width Γ_{μ}. The sum in (1) extends over all the compound levels of interest.

One speaks of level-level correlation if $g_{\mu a}$ is connected to the partial width amplitudes pertaining to other compound level. Channel-channel correlation comes into the picture as a result of the nondiagonal nature of $S_{ab}^{(0)\dagger}$. In order to exhibit the presence of both types of correlation in nuclear reactions we make use of analytic unitarity.

$$S(E)\, S^{\dagger}(E^{*}) = 1 \qquad (2)$$

† No intermediate structure is assumed present

Evaluating (2) at a pole $E = \mathcal{E}_\mu^*$ one finds, assuming slow energy dependence of all the parameters in (1),

$$N_\mu \, g_{\mu a} = \sum_b S_{ab}^{(o)} \, g_{\mu b}^* + \sum_\nu g_{\nu a} \, X_{\nu\mu} \tag{3}$$

$$; \quad \mathcal{E}_\mu^* = E_\mu + i \frac{\Gamma_\mu}{2}$$

where $N_\mu = \dfrac{\sum_a |g_{\mu a}|^2}{\Gamma_\mu}$ and the level-level correlation matrix $X_{\mu\nu} = -i \sum_b \dfrac{g_{\mu b} \, \Lambda_{\mu\nu} \, g_{\nu b}^*}{E_\nu - E_\mu + \frac{i}{2}(\Gamma_\mu + \Gamma_\nu)}$, here $\Lambda_{\mu\nu} = 1 - \delta_{\mu\nu}$.

Equation (3) is the desired one as it relates $g_{\mu a}$ to other ones associated with different channels as expressed by the first term on the right-hand side of Eq.(3) (channel-channel correlation) as well as to $g_{\nu a}$ corresponding to a different compound levels as exhibited by the last term in (3) (level-level correlation). One should note that generally the compound states that define the partial width amplitudes are bi-orthogal implying that a simple linear relationship between $g_{\mu a}$ and its complex conjugate $g_{\nu a}^*$ can be defined only in the absence of level-level correlation where the last term in (3) vanishes. This, however, results in an apparent puzzle since in such a case one has

$$g_{\mu a} = \sum_b S_{ab}^{(o)} \, g_{\mu b}^*$$

implying that $N_\mu = 1$. But a second glance at Eq.(4) and assuming $S_{ab}^{(o)} = e^{2i\delta_a} \tau_a \, \delta_{ab}$ where τ_a is the absorption in channel a, shows that weak absorption in all channels ($\tau_a \simeq 1$) is implicitly assumed. The puzzle is resolved if one has $N_\mu = \tau_a$. To see this in more details we multiply equation 3 from the left by

N_μ getting:

$$N_\mu^2 g_{\mu a} - \sum_b S_{ab}^{(0)} N_\mu g_{\mu b}^* = N_\mu F_{\mu a} = N_\mu \sum_\nu g_{\nu a} X_{\nu\mu} \quad (4)$$

Taking the adjoint of (3) and substituting for $N_\mu g_{\mu b}^*$ in (4) one finally gets:

$$N_\mu^2 g_{\mu a} - \sum_b S_{ab}^{(0)} \sum_c S_{cb}^{(0)*} g_{\mu c} = \sum_b S_{ab}^{(0)} F_{\mu b}^* + N_\mu F_{\mu a} \quad (5)$$

Now changing the order of summation the second term in (5) becomes

$$\sum_c \left(S^{(0)} S^{(0)\dagger}\right)_{ac} g_{\mu c}$$

If one assumes weak absorption in all channels then one recovers equation (14) in reference [1]

$$(N_\mu^2 - 1)\, g_{\mu a} = \sum_b S_{ab}^{(0)} F_{\mu b}^* + N_\mu F_{\mu a} \quad (6)$$

However in general $\sum_c \left(S^{(0)} S^{(0)\dagger}\right)_{ac} g_{\mu c}$ is smaller than $g_{\mu a}$ due to absorption which comes from the compound nucleus formation via unitarity. This is easiest seen if there was no channel-channel correlation i.e $S_{ab}^{(0)}$ is diagonal in the channel indices $S_{ab}^{(0)} = e^{2i\delta_a} \tau_a \delta_{ab}$:

$$\left(N_\mu^2 - \tau_a^2\right) g_{\mu a} = \sum_b S_{ab}^{(0)} F_{\mu b}^* + N_\mu F_{\mu a} \quad . \quad (7)$$

In the case that $F_{\mu a} = \sum_\nu g_{\nu a} X_{\nu\mu} = 0$ (no level-level correlation) and none of the $g_{\mu a}$ is zero we multiply (7) by $g_{\mu a}^*$

and sum over a getting $N_\mu = \tau$ where $\tau = \dfrac{\sum_a \tau_a |g_{\mu a}|^2}{\sum_a |g_{\mu a}|^2}$ i.e absence of level-level correlation implies that N_μ is given by the average absorption present in all channels . Only when one approximates this absorption to be non existent ($\tau = 1$) would one then get the result of ref[1).]

In the presence of channel-channel correlation one can derive a similar result by using the Engelbrecht-Weidenmüller (EW) transformation that leaves the transmission matrix $T_{ab} = \delta_{ab} - \sum_c \langle S_{ac}\rangle \langle S^*_{bc}\rangle$ diagonal in channel indices. Where $\langle S_{ab} \rangle$ is the energy-averaged S-matrix. This same transformation diagonalizes $\langle S_{ab} \rangle$ and $\langle S^{(o)}_{ab} \rangle$. Thus denoting the transformation matrix by U.

$$U U^\dagger = 1$$

$$(U P U^\dagger)_{ab} = p_a \delta_{ab} \tag{8}$$

$$\text{and:} \quad (U S^{(o)} U^T)_{ab} = e^{2i\tilde{\delta}_a} \tilde{\tau}_a \delta_{ab}$$

where $\tilde{\tau}_a$ is the absorption in " eigenchannel " a,(or "eigen" absorption) and $\tilde{\delta}_a$ is the corresponding "eigen"-phase. Calling $Ug \equiv v$ we obtain after multiplying equation (7) from the left by U:

$$N_\mu^2 v_{\mu a} = \sum_c (U S^{(o)} S^{(o)\dagger} U^\dagger)_{ac} v_{\mu c} + \sum_b (U S^{(o)} U^T)_{ab} f^*_{\mu b} + N_\mu f_{\mu a} \tag{9}$$

Where

$$f_{\mu a} = (UF)_{\mu a} = \sum_\nu v_{\nu a} X_{\nu\mu}$$

But $\left(U S^{(0)} U^T U^* S^{(0)\dagger} U^\dagger\right)_{ac} = \tilde{\tau}_a^2 \delta_{ac}$

then eq (9) reduces to:

$$\left(N_\mu^2 - \tilde{\tau}_a^2\right) v_{\mu a} = e^{2i\tilde{\delta}_a} \tilde{\tau}_a f_{\mu a}^* + N_\mu f_{\mu a} \qquad (10)$$

Equation (10) contains both channel-channel correlation ($U \neq 1$) and level-level correlation $F_{\mu\nu} \neq 0$ both conspicuously exhibited in $f_{\mu a}$ on the right-hand side. Again assuming no level-level correlation makes $f_{\mu a} = 0$ and thus for all non zero $v_{\mu a}$ we obtain $N_\mu = \tilde{\tau}$ where $\tilde{\tau} = \frac{\sum_a \tilde{\tau}_a |v_{\mu a}|^2}{\sum_a |v_{\mu a}|^2}$. Thus the condition that $N_\mu = 1$ in the absence of level-level correlation is only an upper limit valid when "eigen" absorption in all channels is weak ($\tilde{\tau} = 1$).

The necessary and sufficient condition for the presence of level-level correlation in nuclear reactions can thus be obtained, following the argument of [1]:

$$N_\mu > \tilde{\tau} \qquad (11)$$

Any discussion of level-level correlation in nuclear reactions is thus intimately connected with that of absorption.

References:

1)A. Sevgen, Phys. Lett. 52B,(1974), 306.

2) C.A.Engelbrecht and H.A.Weidenmüller, Phys. Rev.C8,(1973)85[illegible]