BICYCLE WHEEL MOTION IN HORIZONTAL PLANE

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Abstract.

As well known, it is very difficult that free rotating bicycle wheels, before to stop, move along a rectilinear trajectory in horizontal planes. We proposed that this is mainly caused by the rolling resistance and stochastic interactions between small irregularities existent in the wheels and plane contact surfaces. This paper was written to graduate and undergraduate students of Physics and Engineering.

(I)Introduction.

In **Section1** is seen the ideal motion of a rotating wheel in an horizontal plane along a straight line submitted to a *rolling resistance*.[1-4] In **Section 2** was verified in our laboratory that it is improbable that free rolling wheels in horizontal plane move, before to stop, along a rectilinear trajectory. They deviate from the straight path and finally fall down. In **Section 3** we propose that this happens due to the rolling resistance and to stochastic effects[5] that are created by very small random irregularities of the contact surfaces. In **Section 4** are analyzed the causes for the wheel deviation and fall down.[6,7] In **Appendix** we study the case when on the wheel are applied macroscopic forces and the *countersteering* effect.[6,7]

(1)Dissipative motion of a bicycle wheel in an horizontal plane.

When, for example, elastic bicycle wheels or balls roll along horizontal planes they are submitted to a *rolling resistance* F, sometimes called *rolling friction* or *rolling drag*.[1] This resistance is created by a plastic deformation of the elastic material of the wheel or tire in contact with the hard plane. This force F is the main responsible by the dissipation of the rotational and translational energies of the wheel.

In **Figure 1** is shown the wheel motion along the x-axis with velocity V submitted to the *rolling resistance* **F**.

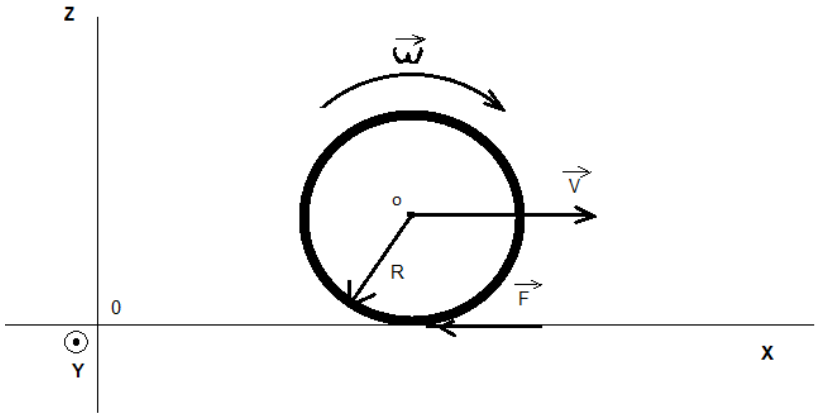

Figure 1.Wheel motion along the x-axis submitted to a rolling resistance F.

The wheel rotation plane is (x,z) and it is rotating in the positive horary sense with angular velocity **ω = -** ω **j,** where **j** is the unit vector along the y-axis. Its angular moment is **L** = -Iω**j.**

The total wheel energy E is given by[2-4]

$$E = (1/M)V^2 + (1/2)I\omega^2 \qquad (2),$$

As the rolling resistance is dissipative, dW_F due to F in a dx displacement is given by,

$$dW_F = -F\,dx = dE = M\,VdV + I\,\omega\,d\omega \qquad (3).$$

So, the dissipated power $dW_F/dt = -\,FV$ is,

$$-\,FV = MV(dV/dt) + (I/R^2)(VdV/dt) \qquad (4),$$

as $V = \omega R$ we get,

$$[M + I/R^2](dV/dt) = -\,F \qquad (5),$$

consequently, $$V(t) = V_o - \{F/[M + I/R^2]\}\,t \qquad (6),$$

where V_o is the wheel initial velocity, showing that it will stop after a certain time. **It is assumed that when V = 0 the wheel fall down.**

In the wheel case as $I = MR^2$, assuming that $F = \mu Mg$, where μ is the *rolling resistance coefficient*[1] and g the gravity acceleration, from Eq.(6) we have,

$$V(t) = V_o - \mu gt/2 \qquad (7),$$

showing that the time wheel *stability* increases when initial velocity V_o increases and μ decreases.

(2)Observed wheel trajectories in rough horizontal plane.

In **Figure 2** are shown typical motions, on the rough horizontal plane (x,y), of rotating wheels observed in our laboratory. The wheels were launched with initial velocity $\mathbf{V_o} = V_o\,\mathbf{i}$ along the x-axis and with angular velocity $\boldsymbol{\omega_o} = V_o/R\,\mathbf{j}$.

In **Figure 2** are shown the center of mass (CM) trajectories of the wheels. They begin the motion along the x-axis and after a certain time they deviate, to the right or to the left side. The wheels velocities decrease, their rotation planes begins to incline from the vertical z-axis. Finally, the wheels would be throw down, *catastrophically*.

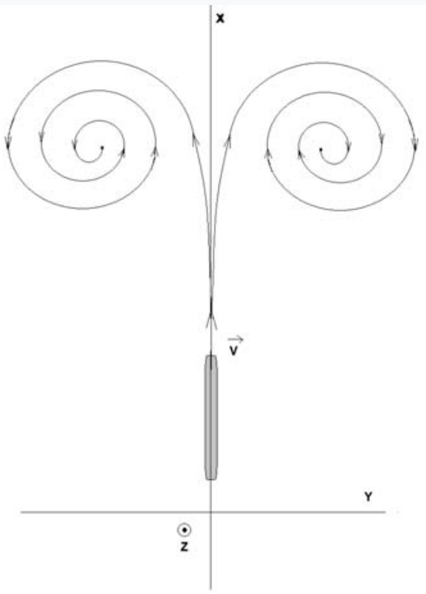

Figure 2. **Typical observed deviations of the wheel center of mass, to left or right, from the initial trajectory with velocity $\mathbf{V_o} = V_o\,\mathbf{i}$.**

(3) Analysis of wheel motion in rough horizontal plane.

We will try to explain the *catastrophic* wheels motions assuming that they are due to the rough contact surfaces between them and the horizontal plane (x,y). Both surfaces have very small, but non negligible irregularities. Besides the macroscopic rolling resistance due to F it is now necessary to take into account the effect of the small surface irregularities. So, due to the random surfaces irregularities, myriad chaotic forces $\mathbf{f_n}$ arise between the plane (x,y) and the wheel surface.[5] These $\mathbf{f_n}$, generating infinitesimal torques $\boldsymbol{\tau}_n = \mathbf{R} \times \mathbf{f}_n$,would change the wheel angular momentum **L**. Consequently, we would have angular displacements $\delta\varphi_n$ of the wheel center of mass from the vertical plane (x,z) and angular deviations $\delta\theta_n$ of the wheel plane from the x-axis trajectory.

(A) Torques of normal components of $\mathbf{f}_n$**,** that is, along the radius R, are null. They do not alter the angular momentum **L**.

(B) On the other hand, the (x,y) components of $\mathbf{f}_n$ create torques $\boldsymbol{\tau}_n$ in (x,y) plane, parallel to **L**. These random torques modify **L**. So, they would be given by $\mathbf{L}_n = \mathbf{L}\,\mathbf{j} \pm \delta\mathbf{L}_n$, where $\delta\mathbf{L}_n$, due to $\boldsymbol{\tau}_n$, have components along **i** and **j**. In this way, they would be responsible not only to stochastic[5] angular deviations $\delta\theta_n$ of the wheel rotation plane, but also to decrease the wheel angular momentum. These torques, would be also responsible by small angle inclinations $\delta\varphi_n$ of the wheel plane from the vertical z-axis. So,would have torques $\tau_\varphi \sim RMg\delta\varphi$ deviating the wheel plane from the vertical position.

Consequently, in the time interval δt, due to these $\delta\boldsymbol{\tau}_n$ the wheel rotation plane would be deviated from x-axis by an angle $\delta\theta \sim \beta\,\delta t$ and inclined from the vertical axis by an angle $\delta\varphi \sim \lambda\,\delta t$.

In **Appendix,** to help students to understand the microscopic angular wheel deviations, these effects are shown when macroscopic forces are applied on the wheel.

(4) Wheel deviation and fall down. Conclusions.

The above analysis show that the wheel motion depends on:
(a) rolling resistance F which is responsible by the decrease of the wheel velocity V and, consequently, of its angular momentum **L**;
(b) stochastic torques due to superficial chaotic irregularities responsible by the **L** modifications. They would be create deviations of the wheel plane from its rectilinear trajectory and by inclinations of the wheel plane from vertical z-axis.

With the increasing time, due to rolling dissipative effects, the velocities V and ω decrease. So**,** **L** becoming smaller it will be more sensible to the perturbing stochastic effects. According to **stochastic theories,[5]** deviations from the initial state **increases with time.** That is, the wheel plane deviations from the x-axis and from the vertical z-axis would grow with time. Finally, due to non-linear effects that are present in rolling processes[6,7] the wheel would be throw down, *catastrophically*.

Appendix. **Macroscopic forces applied on the wheel; countersteering[6,7]effect.**

Let us consider the wheel moving in the horizontal plane (x,y) along the x-axis with speed **V**. At a given instant t is applied, in a short period of time Δt, a force **F*** on the wheel axle, shown in **Figure 3.** This force $\mathbf{F}^*$ produce a torque $\mathbf{T}^* = \boldsymbol{\varepsilon} \times \mathbf{F}^*$, where **ε** is the distance from the wheel center of mass O.

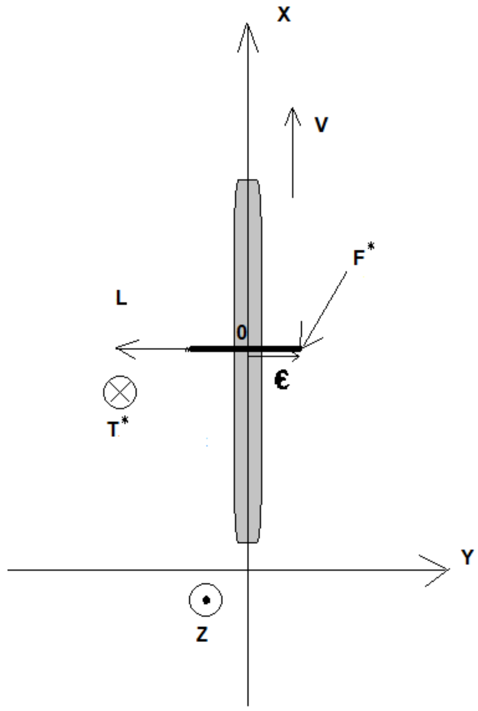

Figure 3. **Force F* applied at the wheel axle at a short time interval Δt. L is the wheel angular moment and V its velocity along the x-axis.**

This torque would be given by

$$\mathbf{T}^* = \boldsymbol{\varepsilon} \times \mathbf{F}^* = \varepsilon\, \mathbf{j} \times (F_x^*\, \mathbf{i} + F_y^*\, \mathbf{j} + F_z^*\, \mathbf{k}) = \varepsilon F_x^* \mathbf{k} - \varepsilon F_z^* \mathbf{j} \qquad \textbf{(A.1).}$$

As $\mathbf{T}^* = \Delta\mathbf{L}/\Delta t$ we see that

$$\Delta\mathbf{L} = -(\varepsilon F_z \Delta t)\, \mathbf{j} + (\varepsilon F_x \Delta t)\, \mathbf{k} = \Delta\mathbf{L}_y + \Delta\mathbf{L}_z \qquad \text{(A.2),}$$

showing that there is simultaneously a wheel deviation from the y-axis and an inclination respect z-axis (see **Section 3**).

After this transient period Δt, due to the wheel inclination from the z-axis (see **Figure 4**), arises a torque $\mathbf{T}_P = \mathbf{R} \times \mathbf{P}$, created by the wheel weight $\mathbf{P} = -Mg\mathbf{k}$, that now would govern the wheel motion,

$\mathbf{T}_P = \mathbf{R} \times \mathbf{P} = (-R_y\,\mathbf{j} + R_z\,\mathbf{k}) \times (-Mg\mathbf{k}) = MgR_y\,\mathbf{i} = MgR\sin\alpha\,\mathbf{i} \quad$ (A.3),

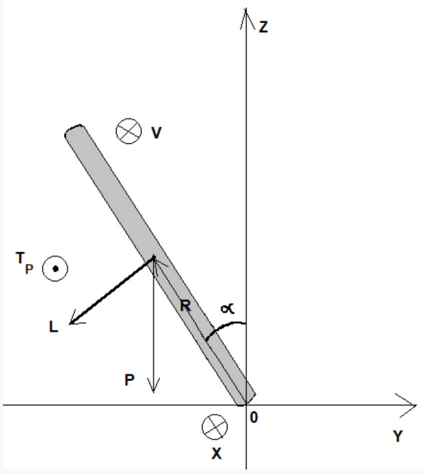

Figure 4. The wheel plane inclination by an angle α and weight torque T_P.

The torque $\mathbf{T}_P = \mathbf{R} \times M\mathbf{g}$ due to the wheel weight, where $\mathbf{R} = R\cos\alpha\,\mathbf{k} + R\sin\alpha\,\mathbf{j}$, is given by,

$$\mathbf{T}_P = \mathbf{R} \times M\mathbf{g} = RMg\sin\alpha\,(\mathbf{k} \times \mathbf{j}) = RMg\sin\alpha\,\mathbf{i} \qquad \text{(A.3).}$$

This torque, in a time interval δt, being along the x-axis, will alter **L** by a factor

$$\boldsymbol{\delta}\mathbf{L} = RMg\sin\alpha\,\delta t\,\mathbf{i}, \qquad \textbf{(A.4),}$$

which is perpendicular to **L**, in the (y,z) plane. In this way, the wheel would turn around the z-axis in the anti-horary sense, or in the "left sense".

This "non-intuitive" effect is very important for motorcyclists, who use this "trick" to turn at high speeds. To lean the motorcycle and make a left turn, simply turn the steering wheel to the right side. This "trick" is called ***countersteering.***[7]

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