IFUSP/P-118

UNDER-THE-BARRIER ABSORPTION EFFECTS IN LOW ENERGY HI FUSION REACTIONS

Mahir S. Hussein

Instituto de Física, Universidade de S. Paulo,
C.Postal 20516 - São Paulo, S.Paulo, Brasil

UNDER-THE-BARRIER ABSORPTION EFFECTS IN LOW ENERGY HI FUSION REACTIONS

MAHIR S. HUSSEIN

Instituto de Física, Universidade de São Paulo

C.Postal 20516 - São Paulo, S.Paulo - Brasil

ABSTRACT

A separation of the transmision coefficient into a part, T_1, due to absorption in the attractive region and another, T_2, due to absorption under the barrier is made. The significance of T_2 is assessed within the WKB approximation and a condition on the diffusivity of the imaginary part of the optical potential, a_W, is imposed to diminish the effect of T_2 as recent data require.

Absorption under the barrier (AUB) was invoked[1] to interpret the seemingly sharply rising nuclear S-factor representing the fusion of $^{12}C+^{12}C$ at $E_{C.M.} \lesssim 3.2MeV$. Other nuclear systems studied did not show this anomalous behavior. For AUB to exist it was necessary to use a rather large diffusivity, a_W, of the imaginary part of the optical model potential (OMP). More recent data on $^{12}C+^{12}C$ fusion, where measurements were extended to energies $E_{C.M.} \sim 2.45MeV$[2], indicate that what was thought to be a greater rise in S was merely the beginning of the formation of another of the many peaks seen in that system. The average S-factor would seem to be not so different from what is expected of a barrier penetration phenomenon as exhibited in the other nuclear systems[3].

In this note we indicate a way of imposing an upper limit on a_W by the requirement that AUB be absent in fusion cross sections at all energies below the Coulomb barrier.

It was argued in 1) that the transmission coefficient, T, could have appreciable values in two regions, the attractive well region and, if a_W is large and the C.M. energy below a certain critical energy E_c, in the barrier region. This can easily be seen by inspecting the form of T[4]

$$T = c \int_0^\infty |\psi|^2 \, W(r) \, dr \tag{1}$$

where c is a constant, $\psi(r)$ is the optical model radial wave function and $W(r)$ is the imaginary part of the optical model potential (OMP).

The integrand $|\psi(r)|^2 W(r)$ is seen to exhibit two extrema in the barrier region. Using the WKB approximation

$|\psi(r)|^2 \sim \exp\left[2\int_{a_0}^{r} \mathrm{Re}\,K(r')dr'\right]$ where $K(r)$ is the local complex wave number and a_0 is the outer turning point, and writing for $W(r) \sim W_0\, e^{-r/a_W}$, the condition of extrema in $|\psi(r)|^2 W(r)$ yields:

$$2\,\mathrm{Re}\;K(r_0) = \frac{1}{a_W}\,,$$

$$\frac{V'(r_0)}{|W'(r_0)|} \lesseqgtr \frac{1}{\xi}\,\frac{1+\xi^2 - \sqrt{1+\xi^2}}{\sqrt{1+\xi^2}}\,, \tag{2}$$

where $\xi \equiv \dfrac{W(r_0)}{V(r_0)-E}$, $V'(r_0) = \dfrac{dV}{dr}(r_0)$ etc.

and $V(r)$ is the real part of the OMP.

The first inequality in (2) corresponds to a maximum at r_0^+ . Inspection of Eq(2) reveals that at low energies r_0^+ is larger than the position of the maximum of the barrier R_0. (Note that $V'(r_0) < 0$ for $r_0 > R_0$ and ξ is always positive). The second inequality in (2) corresponds to a minimum which occurs at $r_0^- < R_0$.

As the c.m. energy is increased to a value E_C , the maximum and minimum coalesce resulting in an inflection point at a distance r_0^C slightly below R_0. This situation corresponds to the equality in Eq(2). Since $W(r)$ attains its largest value and becomes almost constant in the well region, the integrand $|\psi|^2 W(r)$ could have a maximum in this region as the probability density $|\psi(r)|^2$ may exhibit a plateau or a maximum there. Thus for $E > E_C$ most of the absorption takes place in the region $r < r_0^C$ i.e. effectively in the attractive well region.

The above points indicate that T may be written as a sum of two terms

$$T = T_1 + T_2 \tag{3}$$

where T_1 is due to absorption taking place in the well region and represents the dominating term for $E > E_C$. T_2 accounts for absorption under the barrier.

In the absence of AUB only T_1 contributes. This situation is idealized by letting $a_w \to 0$. As a consequences of the constancy of W(r) in the well region one expects for $a_w \to 0$ only incoming waves at a distance R_b within the well region as the ontgoing waves are almost completely damped at R_b. The above condition will be normally satisfied for absorption characterized by a W(r) such that $\exp\left[2\int_0^{R_b} \mathrm{Im}\, K(r)dr\right] \gg 1$[5]. By considering the normalization, N, of the wave function $\psi(r)$ in the region $0 < r \leq R_b$, in the absence and presence of the complex barrier having respectively the values 1 and $\exp\left[-\int_{a_i}^{a_o} K(r)\, dr\right]$ where $a_i(a_o)$ is the inner (outer) turning point, one may approximately determine T_1 as being $|N|^2$. Explicitly written:

$$T_1 \sim \exp\left\{-2\sqrt{\frac{2\mu}{\hbar^2}} \int_{a_i}^{a_o} dr\left[(V(r)-E)^2 + W^2(r)\right]^{1/4} \cdot \cos\left[\frac{1}{2}\tan^{-1}\left(\frac{W(r)}{V(r)-E}\right)\right]\right\}, \tag{4}$$

where μ is the reduced mass.

Equation (4) corresponds to the penetration of a complex barrier. Most of the contribution to T_2 is due to absorption taking place at r_o^+. Thus

$$T_2 \propto \int_{\text{barrier}} |\psi(r)|^2 W(r)\, dr \tag{5}$$

Expanding the integrand around its maximum value at r_o^+ and using the stationary phase method we obtain

$$T_2 = \beta(E)\, T_1 \,,$$

$$\beta(E) \propto A(E)\, e^{-r_o^+/a_W}\, e^{+\int_{a_i}^{r_o^+} \mathrm{Re}\, k(r)\, dr} \,, \tag{6}$$

where A is a slowly varying function of E. In obtaining (6) use has been made of eq (4) to estimate T_1.

The point r_o^+ is determined from eq. (2). Now since $r_o^+ \gtrsim R_o$ and therefore $\frac{W(r_o^+)}{V(r_o^+) - E} \ll 1$, r_o^+ may be determined from the simpler equation:

$$2\sqrt{\frac{2\mu}{\hbar}}\left(V(r_o^+) - E\right)^{1/2} = \frac{1}{a_W} \tag{7}$$

The critical energy, E_C , is determined from (7), as the c.m. energy at which r_o^+ coincides with the inflection point r_o^C Approximating $V(r_o^C) \approx E_B$, the height of the Coulomb barrier, we find

$$E_c \cong E_B - \frac{\hbar^2}{2\mu}\left(\frac{1}{2a_W}\right)^2 \tag{8}$$

Michaud and Vogt[1),6)] have been able to fit the original Pennsylvania data on the $^{12}C+^{12}C$ system with an optical model potential which has $a_W = 0.5$ fm. With this value Eq(8) predicts E_C=3.18 MeV which is very close to the energy, E=3.2 MeV, below which the average nuclear S-factor representing those data starts to increase above the smooth barrier penetration value.

At $E \ll E_C$ T_1 becomes very small due to the large thickness of the barrier. Thus T_2 dominates at these energies. In the vicinity of E_c both T_1 and T_2 may have comparable values.

It is clear that for β (E) to exceed unity by a large amount, as is the case in AUB, the penetration factor, $\exp\left[+2\int_{a_i}^{r_o^+} \mathrm{Re}\, K(r)\, dr\right]$, must dominate over the absorption factor $\exp\left[-\frac{r_o^+}{a_W}\right]$. This would be the case if the difference $r_o^+ - a_i$ attains an optimum value for a given $E_{c.m.}$. Since a_i is basically determined by a_v, the diffusivity of the real part of the OMP, the above condition on $r_o^+ - a_i$ implies a stringent condition on the value of a_v which might be at variance with elastic scattering information.

Setting $E_c=0$ in eq. (7) we obtain a critical value for a_W namely

$$a_c = \frac{\hbar}{\sqrt{8\mu E_B}} \tag{9}$$

Calculation with $a_W \leq a_c$ would give rise to a negligible contribution of T_2 at all energies and the resulting fusion cross sections would show no AUB. It is clear from (9) that a_c would be small for all light ion systems that have been studied recently e.g. for $^{12}C+^{12}C$, $a_c=0.36$ fm. (It should be noticed that eq. (8) gives a value for a_c which is slightly smaller than the value at which the exact E_c vanishes). Needless to say values of $a_W > a_c$ may be needed to describe absorption due to transfer reactions at energies $E \gtrsim E_B$. Those transfer reactions with optimal Q-values persist at lower energies with cross sections much larger than fusion[7] thus necessitating the use of $a_W > a_c$ even at $E < E_B$

Therefore even though the W(r) which is used to calculate total reaction cross sections at low energies may lead to absorption below the barrier due to the optimal Q-value transfer reactions, fusion cross sections at energies well below the barrier should be calculated either by an incoming wave boundary condition model[8] or by an optical model with $a_W < a_C$.

ACKNOWLEDGEMENTS

The author is indebted to Professor Aage Winther for arousing his interest in fusion problems and for generous advice and help that made the writing of the paper in its present form possible. The author would like to thank Professors Aage Winther and Aage Bohr for hospitality at the Niels Bohr Institute in the winter of 1976 and Professors C.A.Barnes, W.A.Fowler and T.A.Tombrello for hospitality at the Kellogg Radiation Laboratory at Caltech in the winter of 1977. Thanks are also due to Dr. E.Plagnol for a careful reading of the manuscript.

A travel grant from FAPESP is also acknowledged.

REFERENCES

1) G.Michaud et.al., Phys.Rev.C3 (1970) 864; G.Michaud and E.W.Vogt, Phys.Rev. C5 (1972) 350; G.Michaud, Phys.Rev. C8 (1973) 525.

2) K.U.Kettner et.al., Munster preprint (1976).

3) R.G.Stokstad et.al., Phys.Rev.Lett. 37 (1976) 888.
H.Feshbach, M.I.T. - CTP nº 580, October 1976.

4) A.M.Preston, The Physics of the Nucleus, (Addison Wesley Publishing Co. Inc., Reading, Mass., 1962).

5) G.H.Rawitcher, Nucl.Phys. 35 (1963) 337

6) M.Mazarakis and W.E.Stephens, Phys.Rev. C7 (1973) 1280

7) Z.E.Switkowski et.al., Phys.Rev.Lett. 33 (1974) 840;
Z.E.Switkowski et.al., Lemon Aid Preprint LAP - 157, November (1976).

8) P.R.Christensen and Z.E.Switkowski - Lemon Aid Preprint LAP - 155, August (1976).