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THE KLEIN PARADOX AND THE MASSES AND LIFETIMES OF CHARMONIA

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In numerous previous works[1,2] it has been proposed that the narrow resonances at 3.105 and 3.695 GeV are charmed quark-antiquark bound states. With this physical picture in mind several simple models of these bound states have been developed in order to calculate their structures and other related properties. Masses and decay widths of these resonances have been calculated with relativistic and non relativistic approaches. Several long range forces of quark confinement have been considered[3] with particular emphasis on gluon exchange, analogous to linear and harmonic potentials.

In this paper we assume that the quarks interact via a static neutral vector gluon field $V_\mu(x)$ [3] where $\vec{V} = 0$ and $V_4 = V(r)$ is given, for instance, by $V(r) = (K/2)\, r^2$. In a first approximation we use Schrödinger's equation to obtain the bound states of the quark-antiquark system. So, the masses of the resonances $M^{(n,\ell)}$ should be given by

$$M^{(n,\ell)} c^2 = 2\, m c^2 + \hbar\omega\, (2n + \ell + 3/2) \qquad (1)$$

where $\underline{m}$ is the quark mass and ω is the frequency of the harmonic vibration. The quark-antiquark system produced through a single photon channel is interpreted as having $\ell = 0, 2,$ in a

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(1) B. Humpert: Dynamical Concepts on Scaling Violation and the New Resonances in e^+e^- Annihilation. Springer Verlag. Berlin - Heildelberg - New York (1976).

(2) S. Okubo: Rochester University Preprint UR-616 (1977).

(3) H.J.W. Müller - Kirsten: Phys. Rev. D, 12, 1103 (1975).

spin triplet configuration. For the ortho-charmonium I (3.105 GeV) it is assumed that $\ell = 0$ and $n = 0$, whereas for ortho-charmonium II (3.695 GeV) which is considered as an S wave radial excitation of the system, $\ell = 0$ and $n = 1$. Under these conditions, using eq. (1), we obtain $mc^2 = 1.331$ GeV and $\omega \cong 0.22\ mc^2/\hbar$.

Taking into account the vibration energy and the masses of the quarks, we see that a relativistic treatment seems to be necessary to describe quark dynamics[3,4]. However, as is well known, the Klein Paradox or its manifestations[3,5] are present in the relativistic approach: when the potential energy becomes larger and larger with the distance, creation and annihilation processes become increasingly important. We verify that instead of bound states one has "resonances" for the quark-antiquark system and this probably occurs due to the instabilities generated by the vacuum polarization processes. In these processes, decay channels appear, for which pairs of quarks are created in order to prevent free quark escape.

In the present work we will show that reasonable values for masses and lifetimes of the quark-antiquark system are found considering Klein's Paradox as the main mechanism for the decay. Our intention is to obtain only the general features of the process and not to find an exact solution of the problem. In

(4) T. Goldman and S. Yankielowicz: Phys. Rev. D, 12, 2910 (1975).

(5) J.F. Gunion and L.F. Li: Phys. Rev. D, 12, 3583 (1975).

the framework of our calculations (first quantization) it is not possible to study in detail the mechanism of pair creation. As will be seen in what follows, essentially the existence of a finite amplitude at infinity for the wavefunction of the quark-antiquark system will be used to evaluate the masses and lifetimes of the resonances. Under these conditions we can treat the problem by using a reduced mass Dirac equation[4] (if the calculations are performed in the spirit of the shell model, i.e., introducing separately the quark and the antiquark into a fixed potential, very similar results are obtained). Of course, these approximations are far from being satisfactory[6], but for our purposes, they are quite sufficient to bring out the essential physical behavior of the system.

So, for $\vec{V} = 0$ and $V(r) = (K/2)\,r^2$, the reduced mass Dirac equation, that we interpret as an equation for a single particle of mass μ and spin 1/2 in an external potential $V(r)$, is given by

$$\frac{df(r)}{dr} = \frac{\chi}{r}\, f(r) + \left[\frac{\mu c^2 - E^{(\mu)}}{\hbar c} + \frac{V(r)}{\hbar c}\right] g(r) \qquad (2)$$

$$\frac{dg(r)}{dr} = -\frac{\chi}{r}\, g(r) + \left[\frac{\mu c^2 + E^{(\mu)}}{\hbar c} - \frac{V(r)}{\hbar c}\right] f(r) \qquad (3)$$

where $g(r)$ and $f(r)$ are the large and small components, respectively, $\mu = m/2$ is the reduced mass of the system, $\chi = -$

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(6) M.E. Rose: Relativistic Electron Theory. J. Wiley. New York - London (1961).

$\chi = -(\ell+1)$ if $j = \ell + 1/2$ and $\chi = +\ell$ if $j = \ell - 1/2$ and $E^{(\mu)}$ is the energy eigenvalue of the reduced mass system. Note that the total angular momentum J of the meson states is obtained in our scheme by coupling the 1/2 unit of spin to the total angular momentum $j = \ell \pm 1/2$. Putting $K = \mu\omega^2$ and defining $\xi = (\mu\omega/\hbar)^{1/2}\, r$ and $E^{(\mu)} = \eta\hbar\omega + \mu c^2$, equations (2) and (3) become:

$$\frac{df(\xi)}{d\xi} = \frac{\chi}{\xi} f(\xi) + (\varepsilon_- + A\xi^2)\, g(\xi) \qquad (4)$$

$$\frac{dg(\xi)}{d\xi} = -\frac{\chi}{\xi} g(\xi) + (\varepsilon_+ - A\xi^2)\, f(\xi) \qquad (5)$$

where $\varepsilon_- = -\eta\varepsilon$, $\varepsilon_+ = \eta\varepsilon + 2/\varepsilon$, $A = \varepsilon/2$ and $\varepsilon = (\hbar\omega/\mu c^2)^{1/2}$. One easily verifies that for $\varepsilon \to 0$ (7) the non relativistic limit is obtained, namely, $f(\xi) \to 0$ and $g(\xi)$ obeys the radial equation for the non relativistic harmonic oscillator:

$$\left[\frac{d^2}{d\xi^2} - \frac{\ell(\ell+1)}{\xi^2} - \xi^2 + 2\eta\right] g(\xi) = 0$$

The total energy E of the system is given by $Mc^2 = E = E^{(\mu)} - \mu c^2 + 2mc^2$, that, in the non relativistic limit, becomes $E = 2mc^2 + \hbar\omega\,(2n + \ell + 3/2) = M^{(n,\ell)}c^2$.

If we consider μ and ω obtained from the non relativistic approximation we see that $\varepsilon = (\hbar\omega/\mu c^2)^{1/2} \cong 0.67$

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(7) K. Nikolsky: Zeits. Phys. 62, 677 (1930).

(8) M. Cattani and N.C. Fernandes: Lett. Nuovo Cimento 18, 324 (1977).

M. Cattani and N.C. Fernandes: University of São Paulo Preprint - 104 (1977).

which means that relativistic effects are not completely negligible.

Equations (4) and (5) can be solved by expanding $g(\xi)$ and $f(\xi)$ into power series[8] that are summed numerically.

For large values of ξ, let us say, larger than critical value ξ_c, we verify that $g(\xi) = i\, f(\xi) = \varphi \exp\left[i \left(\frac{A}{3} \xi^2 + \theta \right) \right]$, meaning that there is no bound state for the quark-antiquark system (Klein Paradox) and consequently the energy spectrum $E^{(\mu)} = \eta\hbar\omega + \mu c^2$ is continuous. The amplitude $|\varphi|^2$ of the asymptotic function depends on ε and η, going to zero as ε tends to zero.

For $\xi < \xi_c$, when $\varepsilon \lesssim 1$, the large component $g(\xi)$ is given approximately by the wavefunctions of the non relativistic harmonic oscillator when η is equal to $(2n + \ell + \frac{3}{2})$. In this region the average amplitude of the small component is very small in comparison with the average amplitude of the large component.

We observe that, for few particular values of η, $|\varphi|^2$ becomes very small compared with the average amplitude of $|g(\xi)|^2$ for $\xi < \xi_c$. It means that for these particular values of the mass $Mc^2 = \eta\hbar\omega + 2mc^2$ the system presents "resonances" or "virtual levels" in analogy with the nuclear virtual levels. Note that we are interpreting the amplitude $|\varphi|^2$ to be proportional to the probability to found the system in an unbound state.

In Figure 1 are seen our predictions for the masses of the resonances that have been obtained putting $mc^2 = 1.260$ GeV and $\varepsilon \cong 0.81$ (we must note that these values are only slightly different from these obtained with Schrödinger's equation) to obtain the resonances at 3.105 and 3.695 GeV that are well established experimentally.

Insert Figure 1

Note that in our scheme it is not possible to distinguish the triplet state $^3\ell_J$ from single state $^1\ell_J$. Our predictions are now compared with the experimental results[9,10].

In the region near 3.50 GeV where the states generically named by χ have been observed experimentally, we have predicted two states, one at 3.45 GeV and another at 3.36 GeV.

The recent experimentally observed resonance[10] at 3.772 GeV could be identified with our predictions at 3.76 GeV with $\ell = 2$. In the region where we have predicted the states at 3.90 and at 3.96 GeV, a broad shoulder near to 3.95 GeV and an extremely rise near to 4.0 GeV have been found experimentally.

The predicted resonances at 4.15 GeV ($\ell = 2$), 4.17 GeV ($\ell = 0$) and 4.24 GeV ($\ell = 2$) could be identified with several enhancements found around 4.1 GeV. On the other hand, it is difficult to decide whether the state observed near to 4.4 GeV corresponds to our resonance at 4.42

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(9) V. Lüth: Stanford Linear Accelerator Center Preprint SLAC - PUB - 1873 (1977).

(10) L.A. Rapidis et al. Phys. Rev. Lett. 39, 526 (1977).

GeV ($\ell = 1$) since the spin configuration and the parity of the former have not been measured yet. The state at 4.37 GeV ($\ell = 1$) could be identified with one narrow peak that seems to exist near to 4.4 GeV, but to confirm this it should be necessary a more detailed experimental results in this region.

For higher levels as 4.60 GeV ($\ell = 2$), 4.61 GeV ($\ell = 0$), 4.68 GeV ($\ell = 2$) and 4.84 GeV ($\ell = 1$), which are almost in the limit of accuracy of our calculation, we are unable to make any comments due to the lack of experimental data.

It is interesting to note that very similar results for the masses of the resonances have been obtained by Harrington et al.(11) using a non relativistic Schrödinger equation with a linear potential.

Making an analogy with the theory of alpha-decay of nuclei(12) we have estimated the lifetimes τ of the resonances assuming, as before, that $|\varphi|^2$ is proportional to the probability to find the system in an unbound state. Following a similar procedure used in that theory, with relativistic currents and probability densities, we have calculated the values of τ that are quoted (in brackets) in Figure 1. Our results (as occurs also in nuclear physics where these calculations have been originally applied) do not agree exactly with experimental data.

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(11) B.J. Harrington, S.Y. Park and A. Yildiz: Phys. Rev. Lett. 34, 706 (1975).

(12) G. Gamow: Structure of Atomic Nuclei and Nuclear Transformations. Oxford University Press (1937).

Probably these results for τ could be improved, if an additional mechanism for the formation of pairs is included. Nevertheless, the calculated τ that range from 10^{-22} s up 10^{-20} s give good estimates for the observed lifetimes.

Using the same formalism very good results for the masses of the vector mesons ρ and ϕ have been obtained(13).

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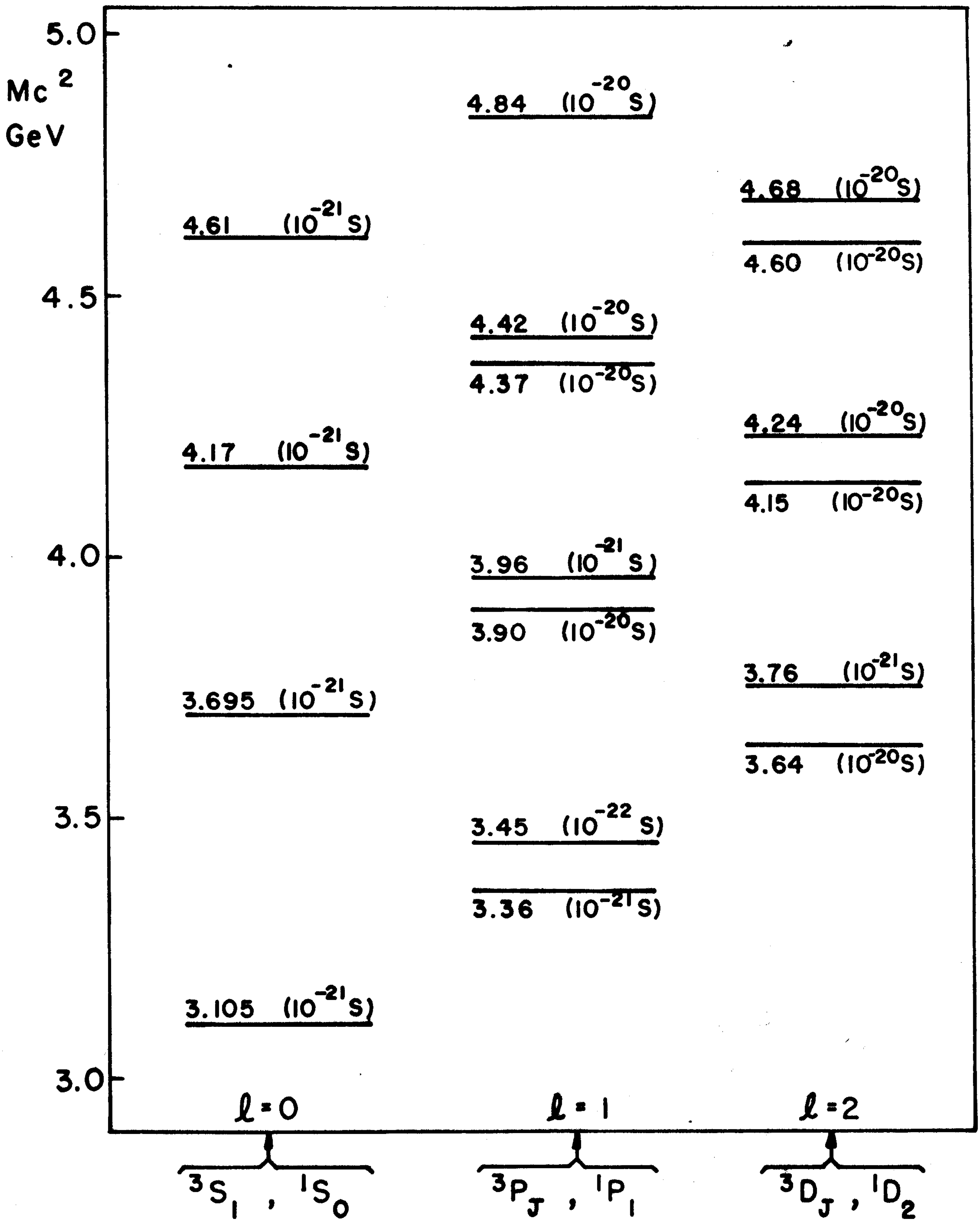
(13) M. Cattani and N.C. Fernandes. To be published.



5.0
4.5
4.0
3.5
3.0
Mc 2
GeV
4.61 (10-21 S)
4.17 (10-21 S)
3.695 (10-21 S)
3.105 (10-21 S)
4.84 (10-20 S)
4.42 (10-20 S)
4.37 (10-20 S)
3.96 (10-21 S)
3.90 (10-20 S)
3.45 (10-22 S)
3.36 (10-21 S)
4.68 (10-20 S)
4.60 (10-20 S)
4.24 (10-20 S)
4.15 (10-20 S)
3.76 (10-21 S)
3.64 (10-20 S)
ℓ=0
ℓ=1
ℓ=2
3S1, 1S0
3PJ, 1P1
3DJ, 1D2

FIGURE CAPTIONS

Figure 1 - The masses of our predicted resonances. The corresponding lifetimes are indicated in brackets.