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KINEMATICAL CONSTRAINTS AND DYNAMICAL ASSUMPTIONS

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SUMMARY

It is shown how the energy-momentum correlation sum rules lead to the inconsistency of a class of uncorrelated jet models. This is an amusing example of how dynamical models can be rejected as violating kinematical constraints.

1 - INTRODUCTION

In the past uncorrelated jet models for multiple production enjoyed a wide popularity due to their oversimplified structure of the multiparticle transition matrix element for the process $a + b \rightarrow 1 + 2 \ldots\ldots + n$.

In what follows we restrict ourselves to those models that obey factorization property

$$T(a+b\rightarrow 1+2+\cdots+n)=\prod_{i=1}^{n} T(a,b,i) \tag{1.1}$$

where all particles are assumed to be identical spinless objects and where each symbol denotes the fourmomentum p_i of the corresponding particle. In eq.(1), $T(a,b,i)$ is some unespecified function of the scalar variables that can be constructed with the fourmomenta p_a, p_b, p_i.

We wish to show that when the Energy Momentum Correlation Sum Rules for inclusive cross sections[1] (EMCSR) are enforced on eq.(1.1), the only choice for the elementary function $T(a,b,i)$ compatible with these sum rules is

$$T(a,b,i)\equiv 0 \tag{1.2}$$

While it is fairly possible that more general classes of uncorrelated jet models could be proposed which would not lead to any incompatibility, we think that this example provides an interesting illustration of how kinematical constraints can eventually rule out models based on some oversimplified dynamical assumption.

2 - Definitions and notations : the EMCSR

To show how eq.(1.2) comes about, we remind that the inclusive cross section of order k for the process in which only k of the n final particles are produced ($n \geq k$) is defined as

$$\sigma^{(k)}_{inc.} = \frac{d\sigma^{(k)}_{inc.}}{dp_1 \ldots dp_k} = \sum_{n=k}^{\infty} \frac{1}{(n-k)!} \int dp_{k+1} \ldots$$

$$\ldots dp_n \left| T(a+b \to 1+2+\ldots+n) \right|^2 \delta^4\left(P - \sum_{i=1}^{n} p_i\right) \qquad (2.1)$$

where dp_i is the invariant infinitesimal volume in phase space of particle i

$$dp_i \equiv \frac{d^3 p_i}{2E_i} \qquad (2.2)$$

and $p = p_a + p_b$ is the total fourmomentum of the initial state.

The production cross section $\sigma_n(s)$ is given by

$$\sigma_n(s) = \frac{1}{n!} \int dp_1 \ldots dp_n \left| T(a+b \to 1+2+\ldots+n) \right|^2$$

$$\delta^4\left(P - \sum_{i=1}^{n} p_i\right) \qquad (2.3)$$

while the total cross section $\sigma_t(s)$ is

$$\sigma_t(s) = \sum_{n=2}^{\infty} \sigma_n(s) \equiv \sum_{n=2}^{\infty} \frac{1}{n!} \int dp_1 \ldots\ldots dp_n$$

$$\left| T(a+b \to 1+2+\ldots+n) \right|^2 \delta^4 \left(P - \sum_{i=1}^{n} p_i\right) \tag{2.4}$$

Quite in general one has

$$\int dp_1 \ldots dp_k \, \sigma_{inc.}^{(k)} = \sum_{n=k}^{\infty} \frac{1}{(n-k)!} \int dp_1 \ldots dp_n$$

$$\left| T(a+b \to 1+2+\ldots+n) \right|^2 \delta^4 \left(P - \sum_{i=1}^{n} p_i\right) = \tag{2.5}$$

$$= \sum_{n=k}^{\infty} \frac{n!}{(n-k)!} \sigma_n(s)$$

and the most general formulation of energy momentum conservation has been shown to imply the EMCSR[2]

$$\left(e^{-izP} - 1\right) \sigma_t = \sum_{k=1}^{\infty} \frac{1}{k!} \int dp_1 \ldots dp_k$$

$$\left(e^{-ip_1 z} - 1\right) \ldots\ldots \left(e^{-ip_k z} - 1\right) \sigma_{inc.}^{(k)}$$

$$\tag{2.6}$$

$$\left[e^{-i\left(P - \sum_{i=1}^{k} p_i\right)z} - 1\right] \sigma_{in}^{(k)} = \sum_{l=1}^{\infty} \frac{1}{l!} \int dp_{k+1} \ldots dp_{k+l}$$

$$\left(e^{-ip_{k+1} z} - 1\right) \ldots\ldots \left(e^{-ip_{k+l} z} - 1\right) \sigma_{inc.}^{(k+l)} \tag{2.7}$$

The general properties of eqs.(2.6,7) have been studied elsewhere[3] in order to extract the widest set of mathematically independent relations.

3. Implication of the EMCSR on the model Sec.1

To analyze the implications of eqs.(2.6,7) on the elementary amplitude of the uncorrelated jet model as defined in Sec.1, i.e. to show the validity of eq.(1.2) we introduce the Fourier transform of $|T(a,b,i)|^2$

$$\tau(a,b,y) = \int dp_i \, e^{-ip_i y} \left|T(a,b,i)\right|^2 \equiv \tau(y) \qquad (3.1)$$

and we use the integral representation

$$\delta^4\left(P - \sum_{i=1}^{n} p_i\right) = \frac{1}{(2\pi)^4} \int d^4y \, e^{iy\left(P - \sum_{i=1}^{n} p_i\right)} \qquad (3.2)$$

Inserting eqs.(3.1,2) into the definitions (2.1,2,3) we get:

$$\sigma_n(s) = \frac{1}{(2\pi)^4} \frac{1}{n!} \int d^4y \, e^{iyP} \tau^n(y) \qquad (3.3)$$

$$\sigma_t(s) = \frac{1}{(2\pi)^4} \int d^4y \, e^{iyP} \left\{ e^{\tau(y)} - 1 - \tau(y) \right\} \qquad (3.4)$$

$$\sigma_{inc.}^{(k)} = \frac{1}{(2\pi)^4} \int d^4y \, e^{iy\left(P - \sum_{i=1}^{k} p_i\right)} e^{\tau(y)}, \qquad (3.5)$$

$$\prod_{i=1}^{k} \left|T(a,b,i)\right|^2$$

The insertion of eq.(3.5) into eq.(2.7) implies the following compatibility relation for $\tau(y)$:

$$\left[e^{-i(P-\sum_{i=1}^{k} p_i)z} - 1\right] \int d^4y \; e^{iy(P-\sum_{i=1}^{k} p_i)} e^{\tau(y)} =$$

$$= \int d^4y \; e^{iy(P-\sum_{i=1}^{k} p_i)} \left[e^{\tau(y+z)} - e^{\tau(y)}\right] \tag{3.6}$$

Upon performing the translation $y + z \to y$ in the first term on the right hand side, we see that eq.(3.6) is identically satisfied and so is, accordingly, the corresponding EMCSR eq.(2.7) irrespective of the choice of $\tau(y)$ (and, therefore of T (a,b,i)).

If we now insert eqs.(3.4,5) into the first EMCSR, eq.(2.6) we get the other compatibility relation that $\tau(y)$ must satisfy

$$\left(e^{-iPz} - 1\right) \int d^4y \; e^{iyP} \left[e^{\tau(y)} - 1 - \tau(y)\right] =$$

$$= \int d^4y \; e^{iyP} \left[e^{\tau(y+z)} - e^{\tau(y)}\right] \tag{3.7}$$

Upon performing the translation $y + z \to y$ on the first term of the right hand side, eq.(3.7) becomes:

$$\left(e^{-iPz} - 1\right) \int d^4y \; e^{iPy} \left[\tau(y) + 1\right] = 0 \tag{3.8}$$

Since the following identity holds

$$\left(e^{-iPz}-1\right)\int d^4y\, e^{iPy}=0 \tag{3.9}$$

we are left with the condition

$$\left(e^{-iPz}-1\right)\int d^4y\, e^{iPy}\,\tau(y)=0 \tag{3.10}$$

for $\tau(y)$.

Barring the unphysical case $P\mu \equiv 0$, this requires

$$\int d^4y\, e^{iPy}\,\tau(y)=0 \tag{3.11}$$

which, in turn implies

$$\tau(y)=0 \tag{3.12}$$

from which eq.(1.2) follows.

4 - Conclusions

While it is quite obvious that even sligth variations on the factorization hypothesis (1.1) could easily spoil the proof so that other versions of the uncorrelated jet model may not violate the EMCSR, we think that the above exercise is fairly interesting as it provides an explicit example of how "dynamical" assumptions may be in conflict with simple kinematical constraints. Such a possibility is much too often overlooked in high energy physics.

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2) The fourvector Z_μ introduced in Ref.1 has here been relabeled $-iZ_\mu$.

3) A.Ballestrero, R.Nulman and E.Predazzi : Nuovo Cimento <u>10A</u> 311 (1972).