Forces of Viscous Fluids on Balls, Cylinders and Wings.

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Abstract. Are displayed main aspects of viscous fluids forces on balls, cylinders and wings. In our didactical laboratory are found only simple experiments involving air flows around these solids.
key words: *viscous fluid flows; balls, cylinders; wings.*

(I) Introduction.

Calculations and measurements of viscous forces on solid bodies like, for instance, balls, cylinders and wings, are very difficult to perform.[1] In our didactical laboratory are seen only essential aspects necessary to visualize these effects. In **Section 1** is taken into account the force on a **non rotating** ball immersed in viscous fluid with uniform flow velocity **V**. Similarly, in **Section 2** is shown the force on a **non rotating** cylinder. In **Sections 3** and **4** are shown forces on **rotating** balls and cylinders. In **Section 5** are analyzed forces on wings immersed in this fluid**.**

(1)Non Rotating Balls.

It can be shown[1] that the "drag" force $\mathbf{F_d}$ or "resistance" force, on a non rotating ball with radius a immersed in viscous fluid with a "small" velocity **V** is given by the **Stokes Law,**

$$\mathbf{F_d} \approx - 6\pi\eta a\mathbf{V} \qquad (1.1),$$

where η is the fluid viscosity.

As for "large" velocities is extremely difficult (or "impossible") to obtain analytical expressions for this force it is generically written as,[1,2]

$$F_d = C_d\,(\rho V^2 A/2) \qquad (1.2),$$

where ρ is the fluid density, $A = \pi a^2$ the sphere "frontal area" (area projected on the normal plane perpendicular to **V)** and C_d is the "**drag coefficient**" that depends on the **Reynolds** number defined by $Re = 2aV/\eta$.

In **Figure 1**[2] are shown the C_d values as a function of Re for smooth and rough spheres. We verify that for Re > 10^5 drag forces are different for smooth and rough spheres.

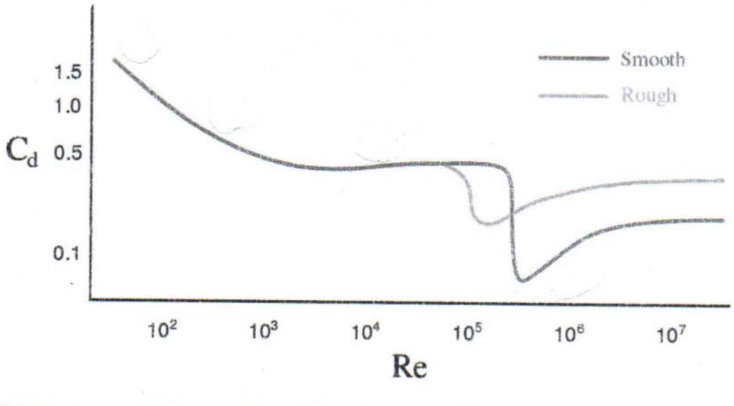

Figure 1. Drag coefficient C_d for spheres as a function of Re.

(2)Non Rotating Cylinders.

Now, let us consider a **non rotating** cylinder, with length L and circular radius a, immersed in the viscous fluid. The cylinder axis is perpendicular to the flow velocity **V** = V **n.** Similarly to spheres, in the general case, the "drag" force F_d on the cylinder is given by[3]

$$\mathbf{F_d} = -C_d\,(\rho V^2 A/2)\,\mathbf{n} \qquad \mathbf{(2.1).}$$

where $A = \pi a^2 L$ is the cylinder "frontal area" (area projected on the normal plane perpendicular to **V**) and C_d is the "drag" coefficient that depends on the Reynolds number Re = $2aV/\eta$.

In **Figure 2** are shown the C_d coefficients as a function of Re.

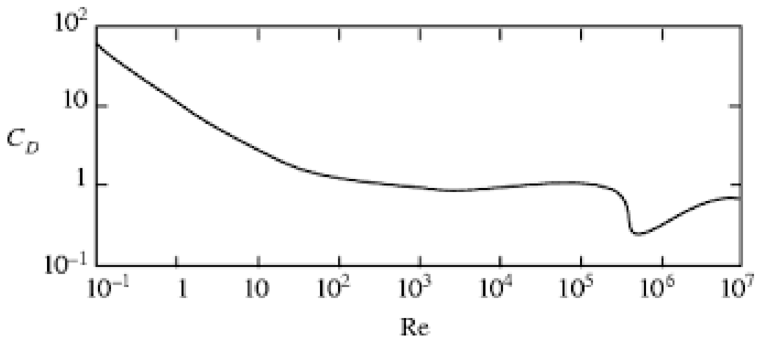

Figure 2. Drag coefficient C_d for cylinders as a function of Re.

(3)Laminar and Turbulent Flow.

In next figures are shown flows for different Re values for balls and cylinders. We are taking into account that, in a first approach, flow lines for balls and cylinders are quite similar.

In **Figure 3** we have laminar flow with Re << 1 or very small V.

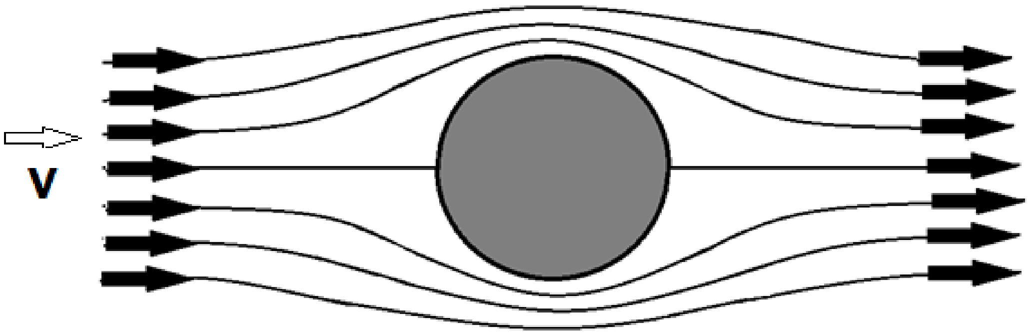

Figure 3. Laminar flows for Re << 1 or very small V.

For these laminar flows the "drag" forces are given by $F_d \approx - 6\pi\eta aV$ for spheres and $F_d \approx 58\ (\rho V^2 A/2)$ for cylinders.

For large velocities, that is, for Re > 1, flows ceases to be laminar; **"vortices"** are created in the fluid. For Re >> 1 the flows become **"turbulent"**. In **Figure (4)** is shown the flow, for instance, when R ~10^4.

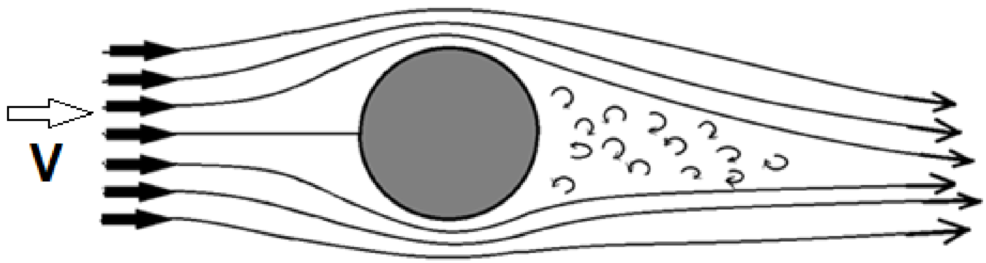

Figure 4. Turbulent flows for R ~10^4**.**

Vortices that are created on surfaces of balls or cylinders are violently ripped from there. They are dragged along by the fluid and compressed between lines of flow. The dragged vortices are confined in a region called "**vortex stream**".

Since for these conditions there are no rigorous theoretical estimations for "drag" forces the coefficients C_D are obtained using semi-empirical methods based on experimental results.[1]

(4) Rotating Balls and Cylinders. Magnus Effect.

When a solid, independently of its form, is translating and rotating inside a viscous fluid it is deviated from its trajectory. This happens because there is a combination between rotation and translation that creates a force that will be responsible by the deviation. This phenomenon is called **Magnus Effect**.

Will analyzed here only cases of rotating solids in turbulent flows forces. According to **Section 3**, when **non rotating** balls or cylinders move in a viscous fluid with velocity **V** and Re = $2aV/\eta > 1$ we can have a **turbulent flow** creating a vortex stream. Due to rotation and viscous forces between solid surfaces and fluids, flow lines close to their surfaces are deflected as shown in **Figure 5.** The fluid entrained by the solid greatly decreases the vortex current which becomes narrower.

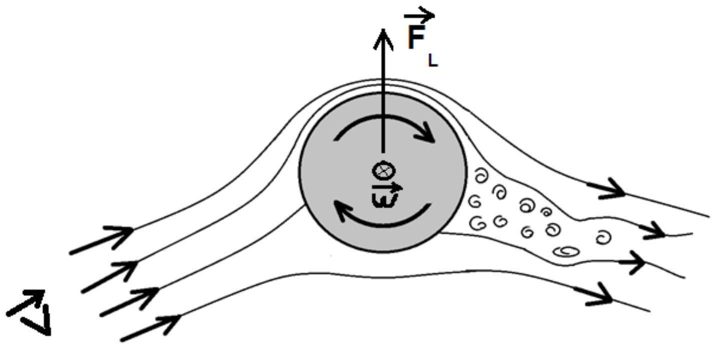

Figure 5. Flow lines for rotating sphere or cylinder. Lift force F_L on the rotating solid.

According to calculations mentioned in reference [1], besides the "**drag force**" $\mathbf{F}_d$, appears on the solid a **"lift force" $\mathbf{F}_L$**. This force would be responsible by the deflection of the trajectory of the solid. It is essentially due to difference in pressure between opposing sides of the object scaled by the cross-sectional area A.[3] For spheres it can be estimated by,[3]

$$\mathbf{F}_L = (4/3)\pi\rho a^3 (\boldsymbol{\omega} \times \mathbf{V}) \qquad (4.1),$$

where $\boldsymbol{\omega}$ is the ball angular velocity. When $\boldsymbol{\omega}$ is perpendicular to the flow velocity **V** the lift force is perpendicular to **V**.

For cylinders F_L can be estimated by[1]

$$F_L = C_L \rho V^2 a \qquad (4.2),$$

where $C_L \approx \pi\omega a/V$ in the interval $4 \geq \omega a/V > 1$.

Some aircraft have been built to use the Magnus effect to create lift with a rotating cylinder instead of a wing, allowing flight at lower horizontal speeds.[3] Rotor ships have been built using mast-like cylinders, called Flettner rotors, for propulsion. These are mounted vertically on the ship's deck (see **Figure 7**)

Figure 7. Flettner rotors ships.

When the wind blows from the side, the Magnus effect creates a forward thrust. Thus, as with any sailing ship, a rotor ship can only move forwards when there is a wind blowing. The effect is also used in a special type of ship stabilizer consisting of a rotating cylinder mounted beneath the waterline and emerging laterally. By controlling the direction and speed of rotation, strong lift or downforce can be generated.[3]

(5) Wings.

To abbreviate our analysis let us consider only wings in a small turbulent flow(see **Figure 8**).[1]

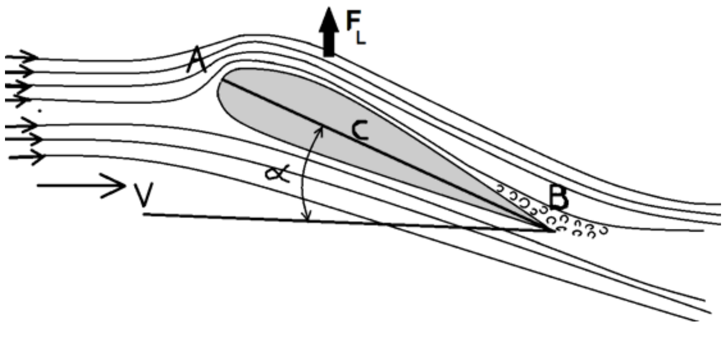

Figure 8. Wing in a small turbulent flow.

In this case the wing will be submitted to a lift force F_L given by[1]

$$F_L = 4\pi\, a\, \rho V^2 \sin(\alpha - \beta) = C_L\, (1/2)\rho V^2 a \qquad (5.1),$$

where the "lift coefficient" $C_L = 2\pi \sin(\alpha - \beta)$, a is the "chord", that is, the distance between A and B and the parameter β depends on the wing profile.

In **Figure 9** is seen the "**lift coefficient**" C_L and "drag coefficient" C_D as a function of the inclination α[1] for a profile with β = 4,5°.

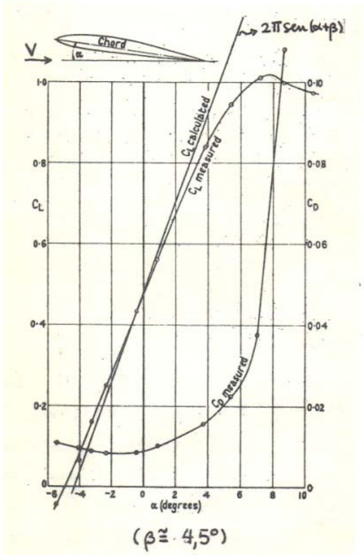

Figure 9. Lift and drag coefficients C_L and C_D for α_c = -4.5°.

For angles $\alpha \geq \alpha_c = 4.5°$ turbulence increases and the lift forces decreases abruptly (**Figure 10**). This happens because, when α becomes larger than the critical value $\alpha_c = 4.5°$, the drag resistance increases. Consequently, the airplane loosing velocity begins to "stall".

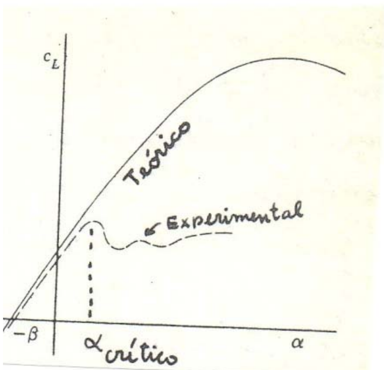

Figure 10. Lift coefficient as function of α, where $\alpha_{critical} = 4.5°$.

APPENDIX. Photos showing Lift Forces and Magnus Effect.

(A)Air flows around cylinders

Will be shown air flows from hair dryers with a thin plastic tape glued to the air outlet around **c**ylindrical glass containers (**Figure (A.1)**).

Figure (A.1). Cylindrical glass and the hair dryer with plastic tape.

In the **Figure (A.2)** is seen the hair dryer on, with the airflow away from the cylinder.

Figure (A.2). The airflow passing far from the cylinder.

In **Figure (A.3)** the air flow is passing close around to the cylinder surface.

Figure (A.3). The airflow passing close around the cylinder.

(B) Moving a cart with a perpendicular airflow.

In **Figure (B.1)** is seen an hair dryer and a cart where is affixed a plastic cylindrical.

Figure (B.1). Hair dryer and cart with cylindrical pot.

In **Figure (B.2)** is seen the hair dryer flow passing far from the cylinder.

Figure (B.2). The airflow passing far from the cylinder.

In **Figure (B.3)** is seen the card moving forward due to the force created by the air stream passing close perpendicularly to the rotating cylinder.

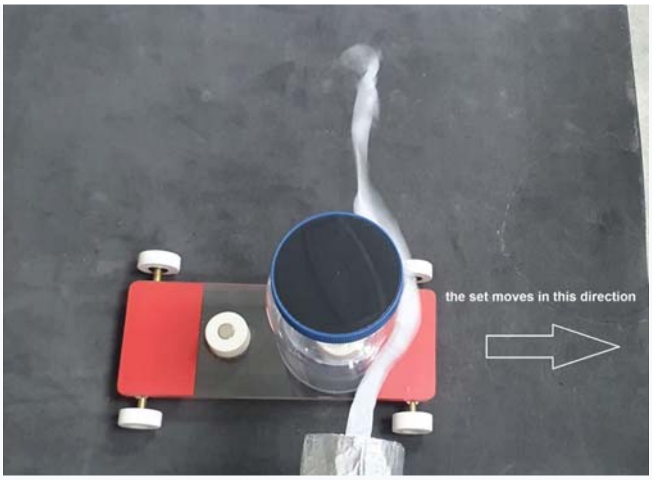

Figure (B.3). Plastic tape showing the wind path around the cylinder surface producing a net force that moves the cart forward.

(C) Air flows around Wings.

With hair dryer and the plastic tapes are shown airflow trajectories over wings. In **Figure (C.1)** the airflow is passing away from the wing

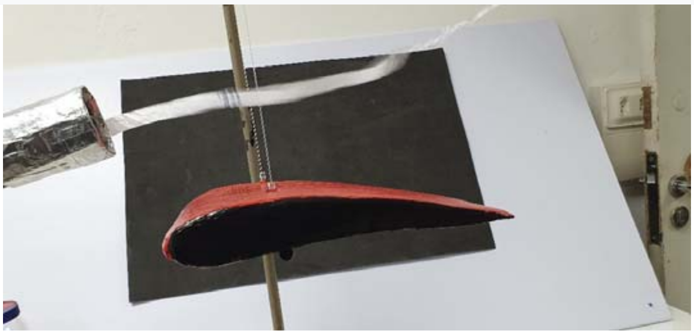

Figure (C.1) Airflow passing far from the wing. .

In **Figure (C.2)** the flow is passing near the top of the wing.

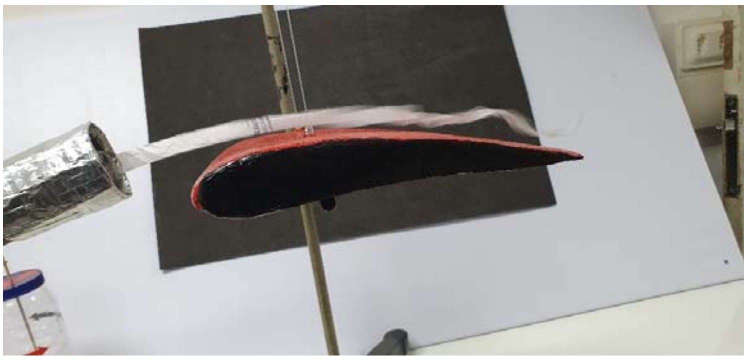

Figure (C.2). Air flow passing near the top of the wing.

Now, let us visualize the lift force on the wing using a fan blowing in front of the wing. The wing was made using styrofoam board, cut and sanded into the shape of an airplane wing and then painted. The wing has two holes through which two thin nylon lines pass, so that it can move freely in the vertical direction.

In **Figure (C.3)** wing profile is in front of a stopped fan.

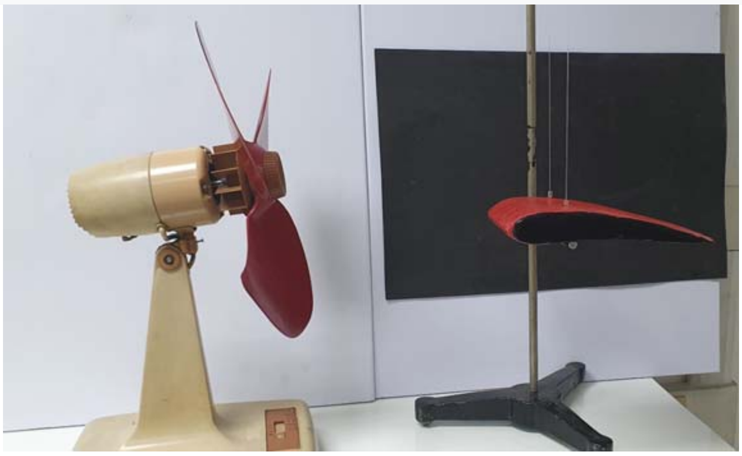

Figure (C.3). Wing profile in front of the stopped fan.

In **Figure (C.4)** is shown that the wing can move freely in the vertical direction.

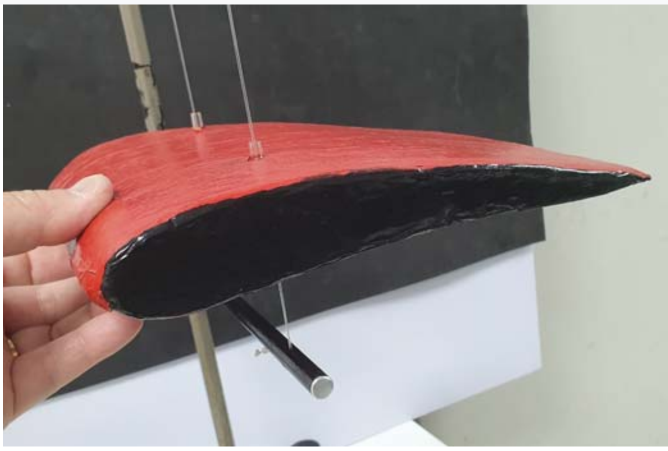

Figure (C.4). Free wing motion along the vertical direction.

In **Figure (C.5a)** the fan is off and the wing is at rest on its cylindrical base.

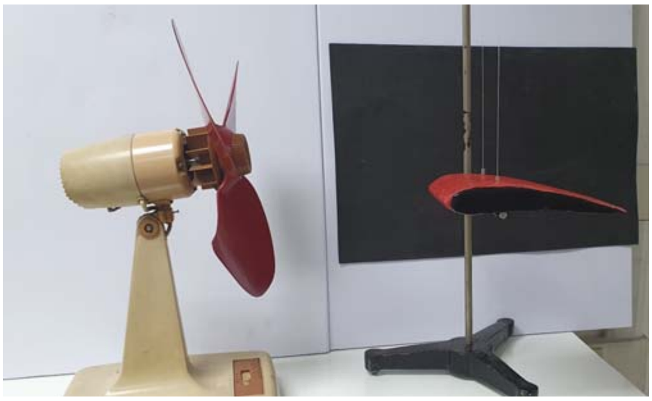

Figure (C.5a). Wing when the fan is off.

In **Figure (C.5b)** fan is on and the wing suspended vertically by the lift force $\mathbf{F_L}$.

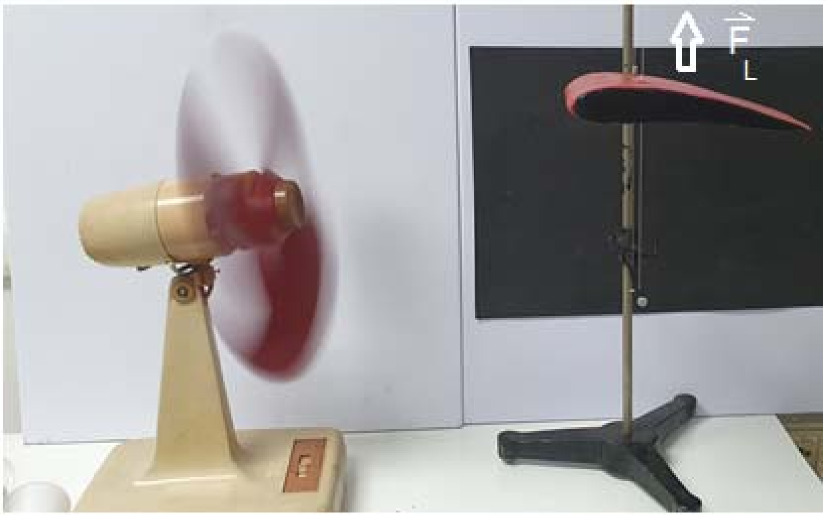

Figure (C.5b). Fan is on and wing suspended by the lift force $\mathbf{F_L}$.

(D) Cart powered by the Magnus effect.

In what follows we use a plastic cylindrical pot that can rotate coupled to an electric motor. In **Figure (D.1)** we have the assembly with non rotating pot and with the engine off.

Figure (D.1). Cart with cylindrical plastic pot and hair dryer.

In next **Figure (D.2)** the motor is on and the cylindrical pot is rotating. The hair dryer flow is passing in **front** of the pot which is rotating in **anti-horary** sense. One can see, looking the plastic tape, that the air flow is deflected.

Figure (D.2). Motor on and pot rotating in the anti-horary sense. Air is deflected.

In **Figure (D.3)** is shown the air flow passing **behind** close to the pot which is again rotating in the **anti-horary** sense. Now the wind flow is circulating around the cylindrical pot. In this situation due to the Magnus force the cart moves forward, in the direction perpendicular to the incident air flow.

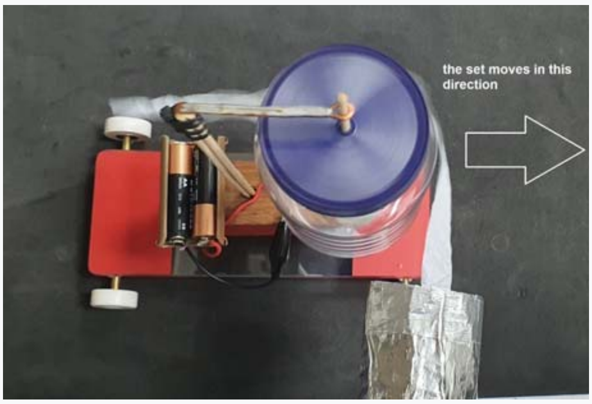

Figure (D.3). Cylindrical pot rotating in anti-horary sense and air flow close to the cylinder. In these conditions, due to Magnus force, cart moves forward.

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[2]https://lambdageeks.com/drag-coefficient-of-sphere/
[3]https://en.wikipedia.org/wiki/Magnus_effect.