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Transverse-Momentum Distribution of Particles According to the Hydrodynamical Model

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ABSTRACT

A fit to the transverse-momentum distribution is performed, in the context of the hydrodynamical model. By fixing a (total-energy-independent) dissociation temperature T and a transverse fluid-rapidity distribution whose width increases logarithmically with s, the existing data can be reproduced in all the $p_\perp$ interval (where $\omega \frac{d\sigma}{d\vec{p}}$ varies by a factor of 10^{-10}) including their energy dependence. The final inclusive cross section appears to be approximately factorized in the longitudinal and the transverse rapidities, as verified experimentally.

I. INTRODUCTION

One of the most intriguing features of the large-$p_\perp$ data, when compared with the small-$p_\perp$ ones (which can be fitted by the well known exponential fall $\exp[-6p_\perp]$), is undoubtedly the change of behaviour of the cross section, which becomes much flatter and also energy dependent[1-5]. Usually, such a feature is attributed to the existence of hard-scattering processes between the incident particles or their constituents[6]. In the present paper, the same data are, instead, analysed in the context of the hydrodynamical model[7,8]. Although hard-scattering mechanism is absent in this model, the agreement with the cross-section data is better than, or at least as good as those obtained by other models based on quark-parton ideas. This means that, as far as the single-particle distribution is concerned, there is no evidence of the necessity of the hard-scattering mechanism, although this is not otherwise excluded.

II. THE MODEL

In the hydrodynamical model, it is assumed that when two high-energy particles collide, all the energy is released in a very small flat volume V in the center-of-mass frame. This highly excited system expands, then, just like a classical relativistic fluid, until it reaches some critical temperature T, when the constituents particles may be considered free.

The basic ingredients of the present calculation are, therefore, on the one hand a fluid-velocity distribution which will be discussed below and on the other hand the thermal motion of the final particles (assumed to be neutral pions) inside a fluid element[9]. The latter will be taken into account through Bose distribution

$$dn' = D \frac{d\vec{p}'}{e^{\frac{\omega'}{T}} - 1} \quad , \tag{1}$$

where D is a normalization constant, $T(\lesssim \mu_\pi)$ is the dissociation temperature and the distribution above refers to the rest frame of each fluid element.

Up to the present date, the hydrodynamical equations giving the fluid's motions have been exactly solved[10] only in one special case, namely that in which all the transverse motions are negligible. In this case, the longitudinal-velocity distribution is approximately given by

$$u^0 \frac{d\rho}{du^1} \sim e^{-C(s)\alpha^2} \quad , \tag{2}$$

where u^μ are the components of the four velocity of the fluid, α is its longitudinal rapidity and the parameter C(s) can be evaluated and behaves as $\sim (\ln s)^{-1}$.

For the three-dimensional motions of the fluid, no satisfactory result is known until now. By using a symmetry argument, Minh Duong-van and Carruthers proposed that the transverse-velocity distribution be represented by[11]

$$u^0 \frac{d\rho}{d^3u}\bigg|_{\frac{\pi}{2}} \sim e^{-B\xi^2} \quad , \tag{3}$$

where, according to their proposal, B= constant (which they estimate to be ≈ 1) and eq. (3) already represents the inclusive cross section at 90°. ξ in eq. (3) is the transverse rapidity.

We partially accept their argument, namely combining eqs. (2) and (3) we parametrize the velocity distribution of the fluid in terms of the rapidity variables α and ξ as

$$u^0 \frac{d\rho}{d^3 u} = A\, e^{-C\alpha^2 - B\xi^2} . \tag{4}$$

However, even leaving aside the question whether eq. (4) really represents the prediction of the hydrodynamical model or not, there still remain at least two points which must be carefully examined.

First, it is not reasonable to assume that B in eqs. (3) and (4) is a constant. This is because as the energy of the system increases, so does the expansion time ($t \sim \sqrt{E}$ in the case of one-dimensional expansion). Consequently, the transverse expansion is the larger the higher is the energy and the "width" B^{-1} grows with the energy in much the same way as C^{-1}. Although the exact s dependence of B can be obtained only by solving the hydrodynamical equations for the transverse motions, a task which as mentioned above not yet satisfactorily accomplished, a rough estimate shows that $B^{-1} \sim \ln s$ with a much smaller (~ 0.1) proportionality factor as compared with C^{-1} [12]. In this paper, we prefer to leave B as a free parameter and fix it for each energy using experimental data.

In the second place, the distribution given by eqs. (3) or (4) should not be compared directly with the inclusive cross section, as done by those authors, but the thermal motions should also be taken into account. This would cause an additional widening of the distribution given by eq. (4) and, for a very narrow transverse-velocity distribution (B large) such as the one expected intuitively, would affect especially the small-$p_\perp$ region. It is easily seen from eq. (1) that, there (but $p_\perp \gtrsim \mu_\pi$) the inclusive cross section would fall exponentially like $\exp[-p_\perp/T]$, in complete accordance with experiments.

In short, we propose to represent the inclusive cross section as a convolution product of the fluid's velocity distribution given by eq. (4) with the thermal motions given by eq. (1). Since comparisons of this kind of calculation with experimental data for small $p_\perp$ and along the collision axis have already been carried out by other authors[8,9], showing excellent agreements of the model with the existing data, we fix our attention only to the large-angle data, especially to those at 90° in the center-of-mass frame.

Before proceeding our presentation, it is worth while to call the readers attention that the parameters C and T in eqs. (1) and (4) are essentially fixed by the hydrodynamical model, whereas the product AD becomes uniquely determined once B is chosen. That is, the only free parameter in the present calculation is B.

III. EVALUATION OF $\omega \frac{d\sigma}{d\vec{p}}$

First, we rewrite the four-velocity u^μ in eq. (4) in terms of α and ξ and the azimuthal angle ϕ:

$$u^0 = \mathrm{ch}\,\alpha\,\mathrm{ch}\,\xi\ ,\ u^1 = \mathrm{sh}\,\alpha\,\mathrm{ch}\,\xi\ ,\ u^2 = \mathrm{sh}\,\xi\cos\varphi\ ,\ u^3 = \mathrm{sh}\,\xi\sin\varphi\ . \tag{5}$$

It is seen that the relation $u^\mu u_\mu = 1$ is automatically guaranteed by this parametrization. From eqs. (5), one obtains

$$\frac{d^3u}{u^0} = sh\xi\, ch\xi\, d\xi\, d\alpha\, d\varphi \; . \tag{6}$$

By Lorentz transforming eq. (1) to the center-of-mass frame, expressing all the quantities in terms of α, ξ and ϕ given above and doing the convolution of this with eq. (4), the inclusive cross-section is written as

$$\omega \frac{d\sigma}{d\vec{p}} = AD \int_{-\infty}^{\infty} d\alpha \int_0^{\infty} d\xi \int_0^{2\pi} d\varphi\; sh\xi\, ch\xi\; e^{-B\xi^2 - C\alpha^2}$$

$$\times \frac{(\omega\, ch\,\alpha - p_{\parallel}\, sh\,\alpha)\, ch\xi - p_{\perp} \cos\varphi\, sh\xi}{\exp\left\{\frac{1}{T}\left[(\omega\, ch\alpha - p_{\parallel} sh\alpha) ch\xi - p_{\perp}\cos\varphi\, sh\xi\right]\right\} - 1} \; . \tag{7}$$

Next, we substitute the energy and the momentum of particles in terms of their longitudinal and transverse rapidities $y_{\parallel}$ and $y_{\perp}$:

$$\omega = \mu\, ch\, y_{\parallel}\, ch\, y_{\perp} \;, \quad p_{\parallel} = \mu\, sh\, y_{\parallel}\, ch\, y_{\perp} \;, \quad p_{\perp} = \mu\, sh\, y_{\perp} \; . \tag{8}$$

The expansion of the integrand in series gives, then

$$\omega \frac{d\sigma}{d\vec{p}} = -AD \frac{d}{d(\frac{1}{T})} \sum_{n=1}^{\infty} \frac{1}{n} \int_{-\infty}^{\infty} d\alpha\, e^{-C\alpha^2} \int_0^{\infty} d\xi\, sh\xi\, ch\xi\, e^{-B\xi^2}$$

$$\times \int_0^{2\pi} d\varphi\, \exp\left\{-\frac{n\mu}{T}\left[ch\, y_{\perp}\, ch\xi\, ch(\alpha - y_{\parallel}) - sh\, y_{\perp}\, sh\xi \cos\varphi\right]\right\} . \tag{9}$$

The integration in ϕ gives

$$2\pi\, I_0\left(\frac{n\mu}{T} sh\, y_{\perp}\, sh\xi\right) .$$

In order to evaluate the integral in α, observe that $C \ll 1$, so that the integrand behaves essentially as

$$\text{const.} \times \exp\left\{-\frac{n\mu}{T}\,\mathrm{ch}\,y_\perp\,\mathrm{ch}\,\xi\,\mathrm{ch}(\alpha - y_\parallel)\right\}.$$

Having this in mind, we substitute $e^{-C\alpha^2}$ by $e^{-Cy_\parallel^2}$ and, after this, the use of the known result

$$\int_{-\infty}^{\infty} d\alpha \exp\left\{-\frac{n\mu}{T}\,\mathrm{ch}\,y_\perp\,\mathrm{ch}\,\xi\,\mathrm{ch}(\alpha - y_\parallel)\right\} = 2K_0\left(\frac{n\mu}{T}\,\mathrm{ch}\,y_\perp\,\mathrm{ch}\,\xi\right)$$

transforms eq. (9) into

$$\omega \frac{d\sigma}{d\vec{p}} = -4\pi A D\, e^{-Cy_\parallel^2} \frac{d}{d\left(\frac{1}{T}\right)} \sum_{n=1}^{\infty} \frac{1}{n} \int_0^{\infty} d\xi\,\mathrm{sh}\,\xi\,\mathrm{ch}\,\xi\, e^{-B\xi^2}$$

$$\times I_0\left(\frac{n\mu}{T}\,\mathrm{sh}\,y_\perp\,\mathrm{sh}\,\xi\right) K_0\left(\frac{n\mu}{T}\,\mathrm{ch}\,y_\perp\,\mathrm{ch}\,\xi\right). \tag{10}$$

Notice that, in this approximation, $\omega\frac{d\sigma}{d\vec{p}}$ appears factorized with respect to the variables $y_\parallel$ and $y_\perp$.

It is clear from eq. (9) that this approximation ceases to be valid when $y_\parallel$ becomes large.

After the differentiation is effected, the right-hand member of eq. (10) is numerically integrated for the sake of comparison with the experimental data.

IV. COMPARISON WITH THE DATA AT 90° IN c.m.s.

In the hydrodynamical model, the dissociation temperature is fixed by imposing the condition

$$V_0\, n \lesssim 1 \tag{11}$$

at the moment of dissociation. Here, v_o is the volume of a pion, which is approximately

$$V_o = \frac{4}{3}\pi\left(\frac{\hbar}{\mu_\pi c}\right)^3 ,$$

and n is the pion density. From the condition (11), it follows[7] that the dissociation temperature is $T \lesssim \mu_\pi$ and, usually[7-9], this is taken to be $T = \mu_\pi$. Although this value is also acceptable in our fit, a better agreement is obtained with a little lower value, namely $T = \mu_\pi/1.5$, which means that the mean distance between two neighbouring pions in the system at the moment of dissociation is about 1.5 times their diameter.

Figures 1 and 2 show $\omega \frac{d\sigma}{d\vec{p}}\Big|_{\frac{\pi}{2}}$ calculated in this way together with corresponding data. In the large-$p_\perp$ region, π^o-production data obtained by the Aachen-CERN-Heidelberg-Munich Collaboration[4] have been used. In the same $p_\perp$ domain, there are other π^o-production experiments[2,3], all of them in agreement within quoted errors. Thus, there is no special reason in preferring this particular experiment over the others, the use of the latter instead of the first ones amounting just in a small modification in the choice of parameters. In the small-$p_\perp$ region, an average of π^+- and π^--production data[1] has been employed. However, these data show a small but not negligible systematic discrepancy in the region where both of them exist and what has been done is renormalize one of them. In Figs. 1 and 2, $\pi^\pm$ data appear divided by a constant factor 1.72.

Figure 3 shows the inverse of the parameter B, plotted against the total energy squared, where one can see its logarithmical increase with s.

V. FIXED ANGLE CROSS SECTION

As mentioned below eq. (10), the single-particle distribution is approximately factorized in $y_{\parallel}$ and $y_{\perp}$ variables. One can summarize eq. (10) in the form

$$\omega \frac{d\sigma}{d\vec{p}} = e^{-Cy_{\parallel}^2} f(y_{\perp}) = e^{-Cy_{\parallel}^2} F(p_{\perp}) . \tag{12}$$

Now, if one writes $y_{\parallel}$ in terms of ω and $\vec{p}$ by using eqs. (8) and takes into account that $C \ll 1$, one can show that, excepting the region where simultaneously θ is small and p large, the factor $e^{-Cy_{\parallel}^2}$ is essentially equal to one. That is

$$y_{\parallel} = \frac{1}{2} \ln \frac{\omega + p_{\parallel}}{\omega - p_{\parallel}} = \ln \frac{\omega + p_{\parallel}}{\sqrt{p_{\perp}^2 + \mu^2}} = \ln \frac{\sqrt{1 + (\frac{\mu}{p})^2} + \cos\theta}{\sqrt{\operatorname{sen}^2\theta + (\frac{\mu}{p})^2}} .$$

$$\exp(-Cy_{\parallel}^2) = \exp\left[-C \ln^2 \frac{\sqrt{1+(\frac{\mu}{p})^2} + \cos\theta}{\sqrt{\operatorname{sen}^2\theta + (\frac{\mu}{p})^2}}\right] \simeq 1 . \tag{13}$$

This means that, for large angles,

$$\omega \frac{d\sigma}{d\vec{p}}\bigg|_{\text{fixed } \theta} \simeq \omega \frac{d\sigma}{d\vec{p}}\bigg|_{\frac{\pi}{2}} \tag{14}$$

as function of $p_{\perp}$. This result is in excellent agreement with experiments[4].

VI. ADDITIONAL REMARKS

We have presented in this paper a simple parametrization of the single-particle distribution based on Landau's hydrodynamical model. There remained still the question whether eq. (4) really represents the velocity distribution predicted by the hydrodynamical model. While we are not able to answer this question yet, it is clear, nevertheless, that, as discussed below eq. (4), whatever the form of the distribution is, its transversal width increases showly with the energy (perhaps like $\ln s$) and is much narrower than the longitudinal width. These features are enough to guarantee the qualitative reproduction of the experimental characteristics mentioned at the beginning.

Another remark concerns the existence of the leading-particle effect. The fact that a non-negligible fraction of events are accompanied by a leading particle shows that, although the agreement of the multiplicity and the single-particle distribution predicted by the conventional hydrodynamical model with data is surprisingly good, it is, in general, not justified this direct comparison. Obviously, a comparison with such data has any sense only after taking all kind of events into account. We think, however, that if one assumes that in the so-called high-mass diffractive dissociation the excited state decays in the way described by the usual hydrodynamical model[13)], all the relevant results may remain valid. In this case, however, the excitation probability as function of the invariant mass M must be computed outside the framework of hydrodynamics.

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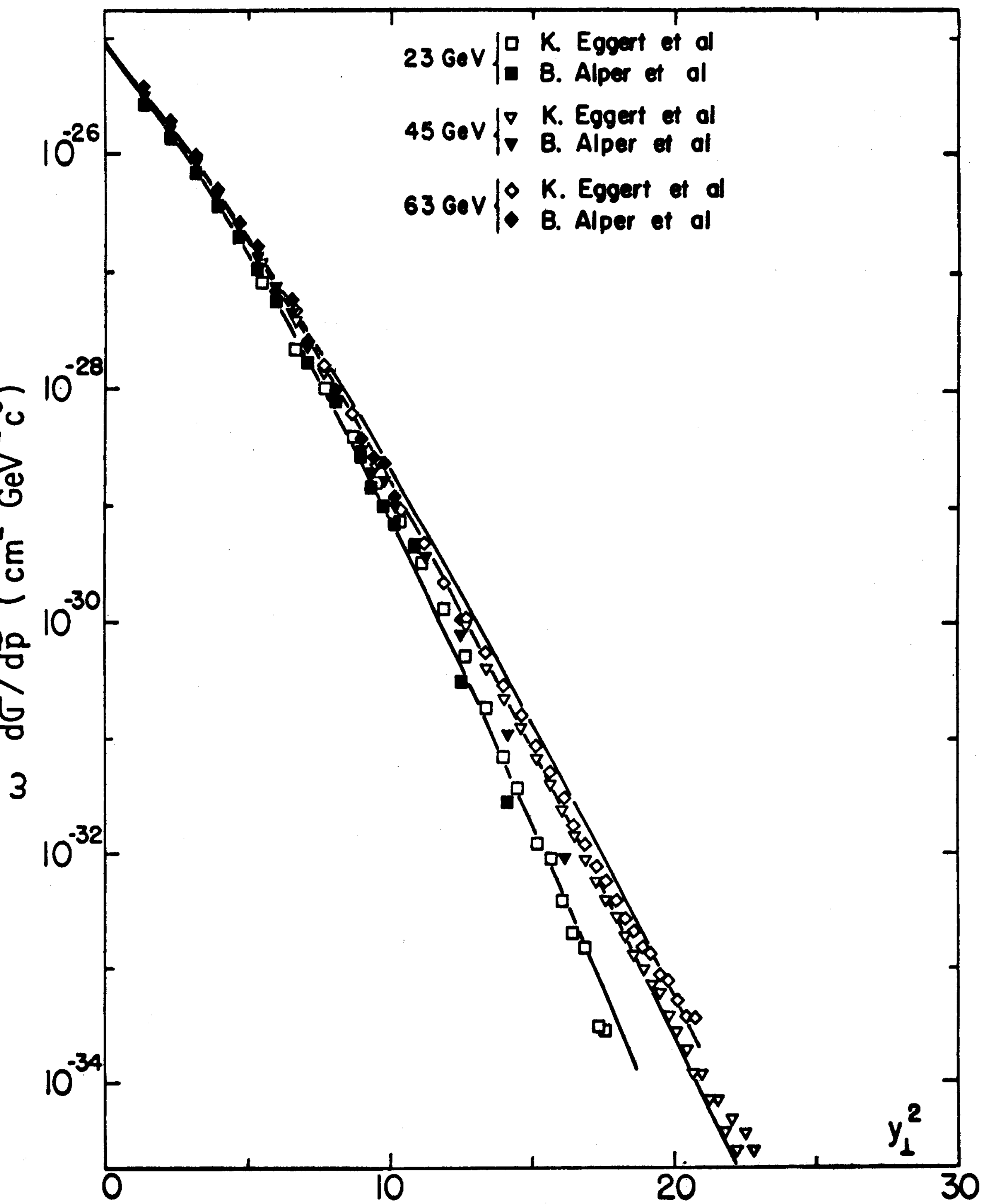
FIGURE CAPTIONS

FIG. 1 - Single-particle distribution at 90° in the center-of-mass frame at $\sqrt{S}$=23, 45 and 63 GeV.

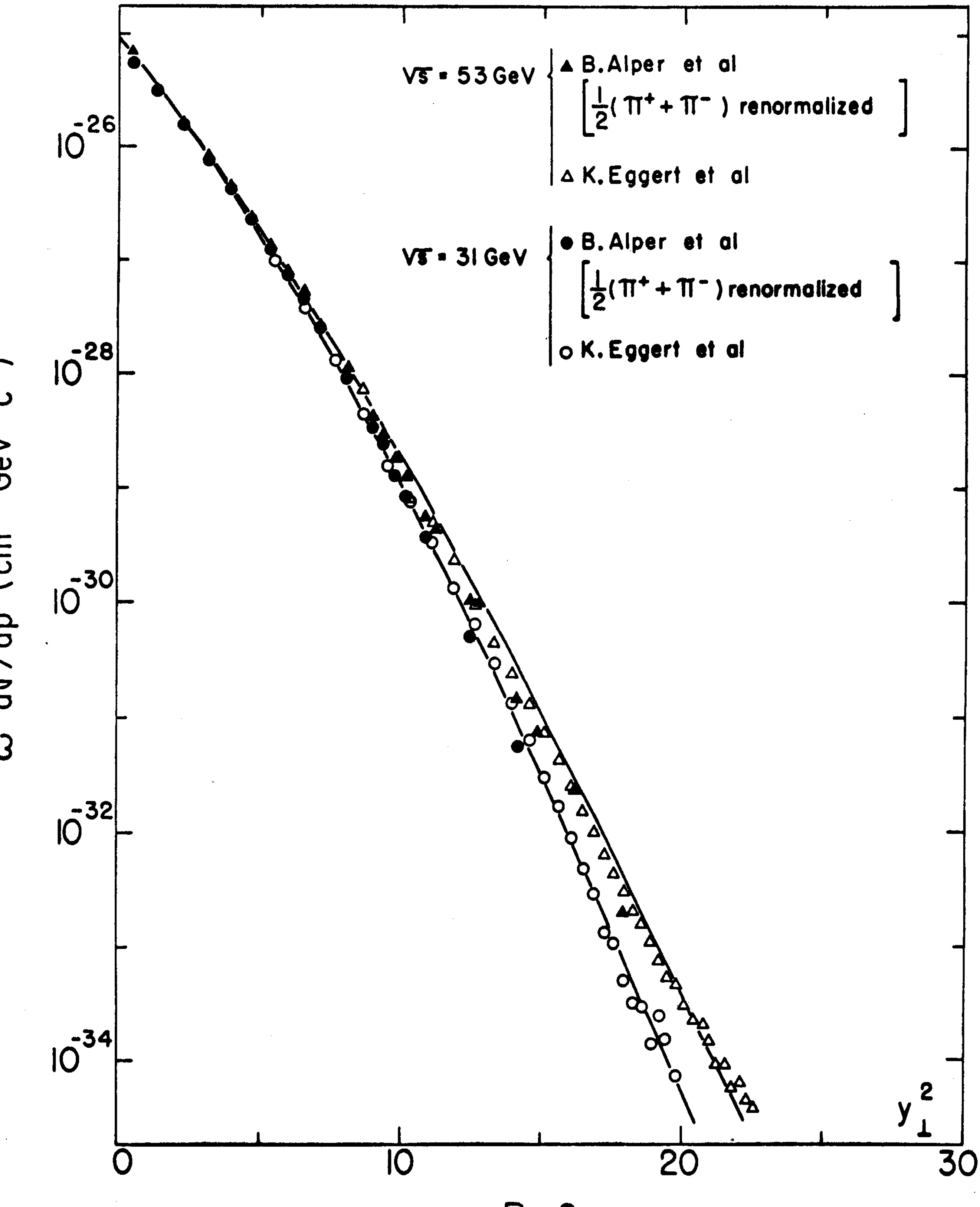
FIG. 2 - Single-particle distribution at 90° in the center-of-mass frame at $\sqrt{S}$=31 and 53 GeV.

FIG. 3 - Transverse "width" as explained in the text against the square of the total energy s. The curve is just to guide the eyes.



23 GeV
K. Eggert et al
B. Alper et al
45 GeV
K. Eggert et al
B. Alper et al
63 GeV
K. Eggert et al
B. Alper et al
10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-34}
$\omega\ d\sigma/d\bar{p}$ ($cm^2\ GeV^{-2}\ c^3$)
$y_\perp^2$
0
10
20
30

Fig. 1



$\sqrt{s}$ = 53 GeV
▲ B. Alper et al
$\left[\frac{1}{2}(\pi^+ + \pi^-)\right.$ renormalized $\left.\right]$
△ K. Eggert et al
$\sqrt{s}$ = 31 GeV
● B. Alper et al
$\left[\frac{1}{2}(\pi^+ + \pi^-)\right.$ renormalized $\left.\right]$
○ K. Eggert et al
$\omega\, d\sigma/d\vec{p}$ (cm^2 GeV^{-2}c^3)
10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-34}
$y_\perp^2$
0
10
20
30

Fig. 2

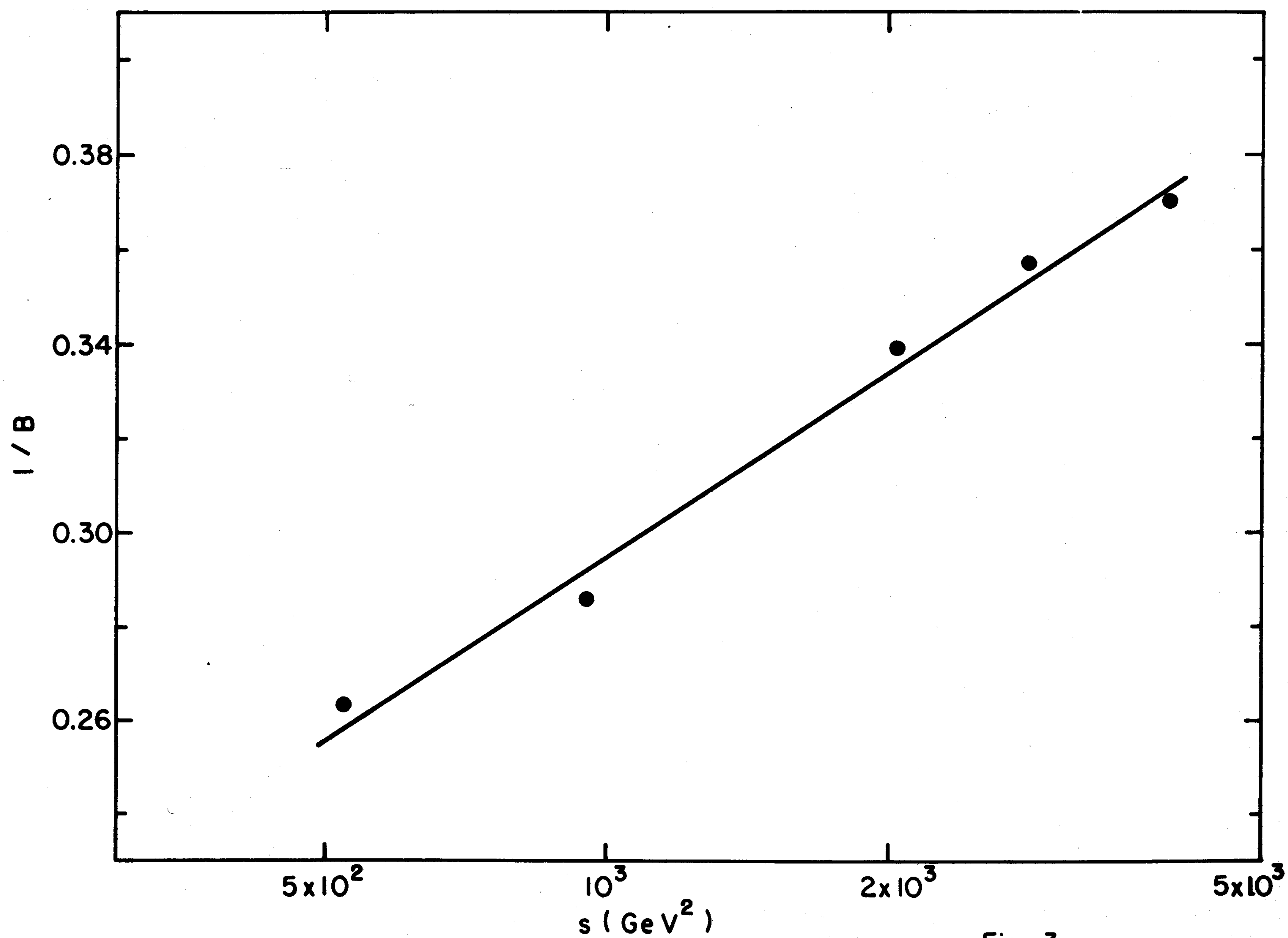


Fig. 3