Michelson Interferometer and the Gravitational Waves.

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Abstract. We have done brief comments about the Michelson interferometer and the detection of gravitational waves (GW) using the Laser Interferometer Gravitational Observatories, LIGO and VIRGO.
Key words: electromagnetic waves; Michelson interferometer ; gravitational waves.

(I)Introduction.

In **Section 1** is seen a brief review about the nature of light and the electromagnetic waves. In **Section 2** we present a simple description of the Michelson interferometer. In **Section 3** was seen that, according to the Michelson-Morley experiment, the "ether" does not exist. Is also seen that that light is composed by "particles" named "photons", that accelerated charges emit electromagnetic waves and, consequently, emit photons. In **Section 4** was shown that, according to General Theory of Relativity(**GTR**) accelerated masses emit *gravitational waves*(**GW**). In 2015 these waves have been detected by the Laser Interferometer Gravitational Observatory (LIGO) which is a large Michelson interferometer. In **Appendix** is shown how the Michelson interferometer can be used to measure the air refraction index.

(1)Electromagnetic waves and the Michelson-Morley Experiment.

Up to the middle of the XVII century light was thought as composed by particles emitted by luminous sources, like Sun or by the candles flames.[1] Christian Huygens, around 1670, showed that phenomena, like interference, reflection and refraction, could be explained in a simple way by a *wave theory*.[1,2].Only in 1827, Thomas Young and Augustin Fresnel and, a few later, Leon Foucault, demonstrated that there were many optical phenomena where corpuscular model is inadequate. The second big step to explain the real nature of the light was done by James Clark Maxwell in 1873 and ,15 years later, by Heinrich Hertz, showing that light is described by Electromagnetic Waves.[1-3] In **Section 2** is analyzed a fundamental property of these waves, **interference**, taking into account the Michelson Interferometer.

(2)Michelson interferometer.

Michelson interferometer [3] is the most fundamental of two-beam interferometer (see **Figure (2.1)**) used, for instance, to measure wavelengths and refraction indexes with great precision. An initial beam of monochromatic coherent light emitted by C, propagating along an horizontal x-axis, passing by a semitransparent mirror **M** is split into two paths with lengths ℓ_1 and ℓ_2 and go to the mirrors M1 and M2. These two rays are in phase when originally split by M.

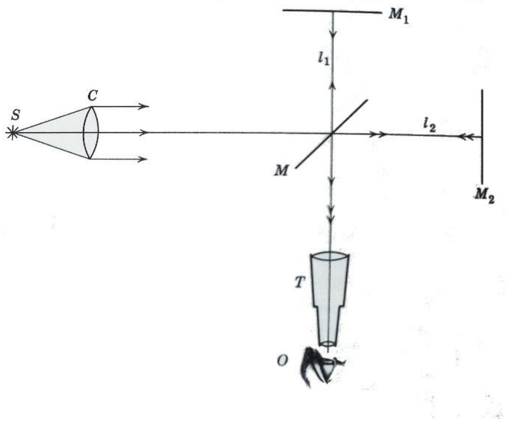
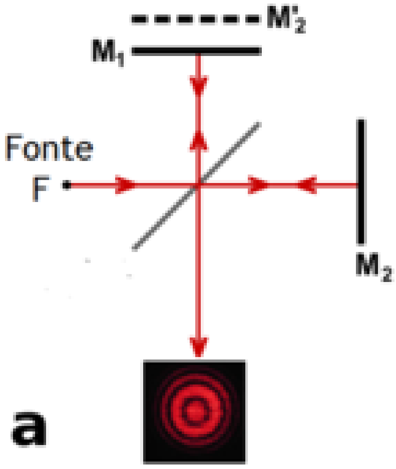


Figura (2.1). Coherent light emitted by the source S is split by a semitransparent mirror **M**. Following different paths there is an interference between them observed at the detector T.

There is an *interference* between them which is observed at T. This interference will depend on lengths ℓ_1 , ℓ_2 and on the relative orientations of **M**, **M_1** and **M_2**. As calculations of these interference patterns are very intricate they can be seen elsewhere.[4,5] We will only show figures of interference patterns in two particular cases.

In **Figure (2.2a**) we have circular fringes[4,5,6] when mirrors **M_1** and **M_2** are in parallel planes. In **Figure (2.2b)** we have parallel fringes when **M1** and **M2** are in inclined planes.[4,5,6]



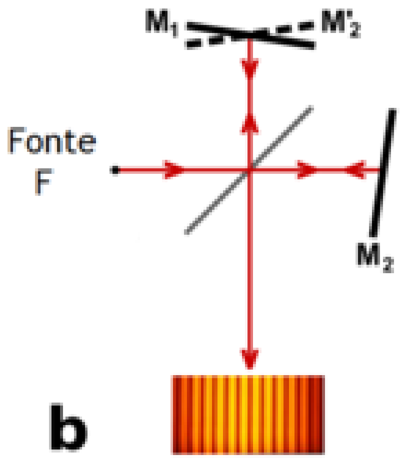


Figure (2.2) **Figure (2.3)**

Figure (2.2a). Circular rings when **M_1** and **M_2** are in parallel planes.[6].
Figure (2.2b). Parallel fringes when **M_1** and **M_2** are inclined planes.[6]

In **Figure (2.3)** is shown a small Michelson interferometer with a laser source found in our didactical laboratory.

Figure(2.3). Small Michelson interferometer of our didactical laboratory.

(3)Vacuum and Photons.

In spite of the progresses made to understand the light, seen in **Section 1** and **2**, its exact nature and the nature of the medium where it propagates still remained an unsolved problem. This unknown medium, that was named "ether"[3], assumed to fill all empty space and to penetrate all transparent materials. This medium ought to have notable qualities: extraordinary rigidity and perfect transparency. In material medium the light velocity is given by $v = c/n$, where n is the refraction index and c would be the velocity in the "ether". Experiments done by Michelson and Morley, using the Michelson interferometer[3] in 1887 shown that this "ether" does not exist. Waves propagate in "vacuum" where all material bodies in the Universe would be immersed.

In 1887 Hertz discovered the "*Photoelectric Effect*"[3] that occurs when electrons are emitted by conductors surfaces irradiated with ultraviolet light. Electromagnetic Maxwell theory could not explain the *photoelectric emission*.[3] Ten years later, in 1897, Larmor have shown[2,7] that accelerated charges emit electromagnetic waves. Einstein in 1905, based on a Planck´s idea, proposed that the electromagnetic energy transported by the wave was concentrated in corpuscles or "*photons*".[3] This statement is known as the *quantum theory* of the photoelectric effect. Consequently, as electromagnetic waves are emitted by accelerated charges, accelerated charges emit photons.

(4)General Theory of Relativity and Gravitational Waves.

According to the General Theory of Relativity(**GTR**)[8] accelerated masses would emit *gravitational waves*(**GW**) which move with velocity c in the vacuum . Since undergraduate students are not familiar with the GTR mathematical formalism, we will consider here only the physical effects produced by the **GW**.

According to many papers,[8-10] matter is deformed by *shear forces* intrinsic to GW. Let us consider, for instance, a plate in a (x,y) plane and that GW waves, propagating along the z-axis, with frequency ω, are incident on this plate. These waves, are described by functions $\sim\exp[i(kz-\omega t)]$ and depend on their polarizations.[8-10] In

Figures(4.1) are shown, at a given instant of time, the line forces $\mathbf{F}^{(+)}$ due to the **GW,** for instance**,** with polarization (+). At this time t, the forces $\mathbf{F}^{(+)}$ are increasing distances on the plate along the x-axis and decreasing distances along the y-axis.

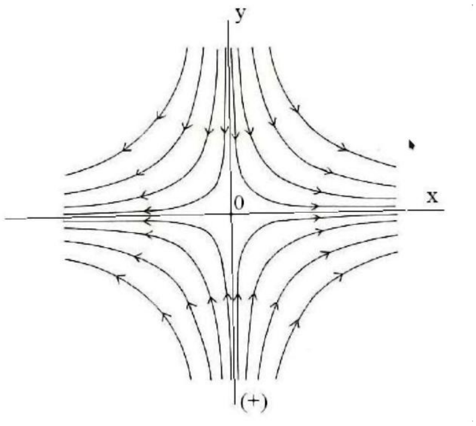


Figure (4.1). Shear forces lines $\mathbf{F}^{(+)}$ of kind (+) in a given instant of time t.[8]

In **Figure (4.2)** are shown,[8] as a function of time, deformations of a necklace of particles due to the shear forces $\mathbf{F}^{(+)}$**.** In reference [9] there are found figures where these deformations x an y are shown (in motion) as functions of the time.

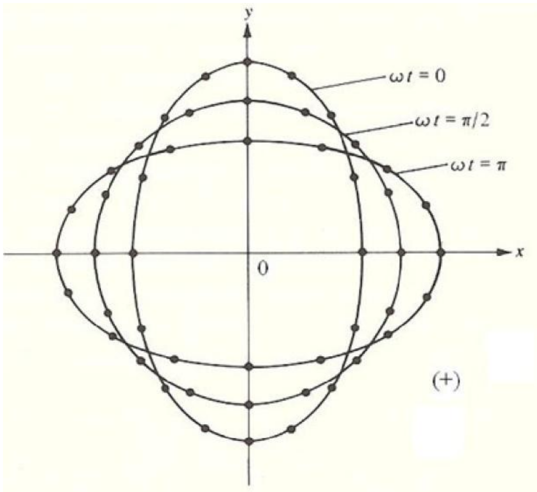


Figure (4.2). Deformations along the x-axis and y-axis, as functions of the time, of a necklace of particles due shear forces $\mathbf{F}^{(+)}$ created by GW with polarization (+).[8]

Very large Michelson interferometers (LIGO[11] and VIRGO[12]) used to detect GW are seen in a schematic form in **Figure(4.3).** Inside the arms along the x-axis and y-axis , which have $L_x = L_y = 4$ Km (Ligo) and $L_x = L_y = 3$ Km(Virgo), there are *optical delay lines* where the light beam perform an "effective optical length " around 150 km. Multiple passes are made in each arm to increase the optical path.

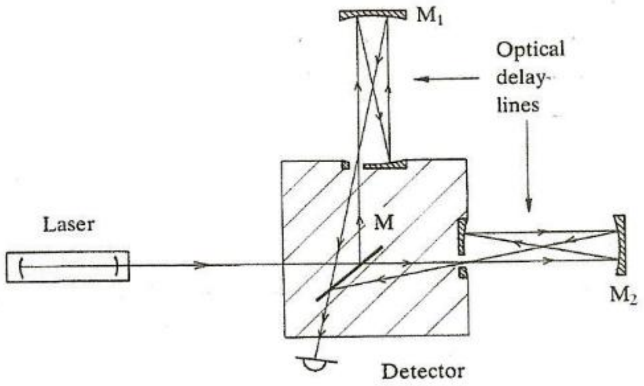


Figure (4.3). Schematic LIGO and VIRGO interferometers.

In 14 September of **1965** was done the first direct detection of GW using a very large Laser Michelson Interferometers or **LIGO**[11] constructed at Livingston, Louisiana, and at Hanford ,Washington, United States. These were named **LIGO**, that is, **Laser Interferometer Gravitational -Wave Observatory.** These GW were generated by the merger ("coalescence") of two black holes (of a BBH binary) with masses of m_1 = 29 and m_2 = 36 solar masses (**Figure (4.4)),** with an initial separation r ~10^4 km and distant ~10^9 light-years from the Earth.

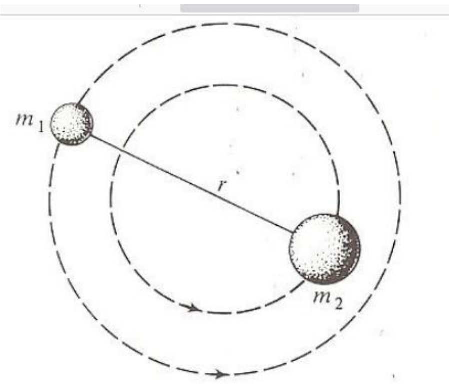


Figure (4.4). Black hole binary(BBH) with masses m_1 = 29 and m_2 = 36 solar masses

When a black hole (BBH) binary is unstable, the BHs tend to merge (to "coalesce").When this happens an immense amount of energy should be given off as GW and thus offer a good chance of directly detecting such waves. These sources have been finally recently observed. Only a very small fraction of this emitted energy (~$1/10^{22}$) arrived at LIGO. The entire merge evolution of the BBH can be divided into three stages: "inspiral", "merger" (or "plunge") and "ringdown", seen in **Figure (4.5)**. The stages are shown as functions of the time (along the horizontal line) and of the "strain"[10] defined by **strain** ~ $\Delta L/(\lambda/2)$, where $\Delta L = L_x - L_y$ is the difference of length between the two paths due to the incidence GW, and λ is laser wavelength.

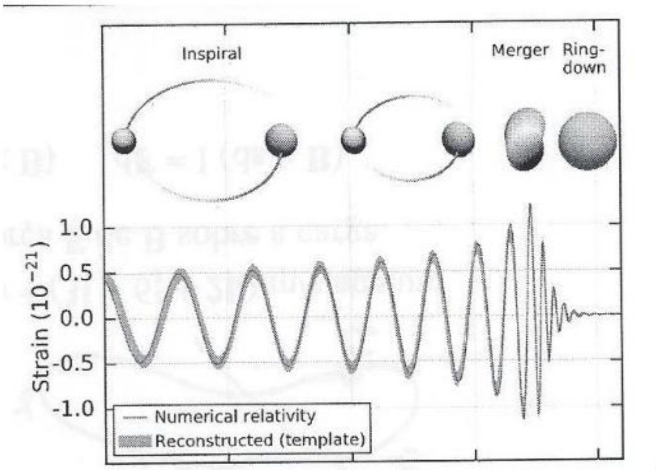


Figure (4.5). BBH merge stages: inspiral, merger and "ringdown" as a function of the "strain" and of the time detection of the GW.

The "inspiral" is the first stage of the BBH life which resembles a gradually shrinking orbit and take a longer time, and emitted GW are very weak. As the BH orbit shrinks, the speed increases , and the GW emission increases. This is followed by a plunging orbit and the BH will "merge" once they are close enough. At this time the GW amplitude reaches its peak. Once merged, this time the single hole settles down to a stable form, via a stage called “ringdown", where any distortion in the shape is dissipated as more gravitational waves. In **Figure (4.6)** is shown the observed BBH merge stages as function of the detection time and strain. It is important to note that Hanford and Livingston data are in excellent agreement.

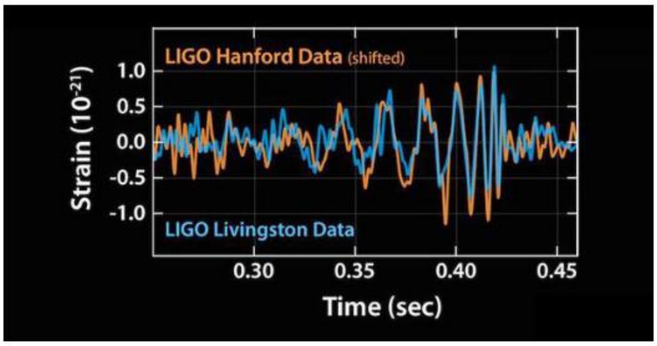


Figure (4.6). The GW strains measured simultaneously , as a function of the time, by the two LIGO observatories.

The detected signals increase in frequency from 35 to 250 Hz[10-12] over 10 cycles (5 orbits) as it rose in strength for a period of 0.2 second. The total energy produced in the coalescence process was ~ 10^{49} W. Only a very small fraction ~$1/10^{20}$ of this emitted energy arrives at the Earth.

In 2017, using similar interferometer technique, GW have been detected by the **VIRGO - Observatory** constructed at Cascina, close to Pisa, in Italy.[12]

In 2017, Rainer Weiss, Kip Thorne for Barry Barish were awarded with the Nobel Prize of Physics for their contributions to the GW detection.

In **Figures (4.3)** and **(4.4)** are shown, for instance, the observatories LIGO at Livingston (USA) and VIRGO in Italy.

Figure (4.3)LIGO, Livingston,USA

Figure (4.4)VIRGO, Pisa, Italy

Both LIGO and VIRGO interferometers have arm lengths $\ell = 4$ km for LIGO and $\ell = 3$ km for VIRGO, with very potent lasers with ~750 KW and using astonishing precise technology, able to detect lengths variations less than 10^{-15} cm, that is, 10^{-2} smaller than the proton radius. Due to the extremely small intensities of GW the interferometer must be optically perfect and extremely well isolated from the rest of the world (seismic isolation, cosmic rays, electromagnetic fields, etc.). Lasers must be very powerful, extremely monochromatic ($\lambda = 500$ nm) and stable, the mirrors must have high reflectivity, the light path must be done over large tubes (see **Figure (4.4))**with extreme high vacuum. They continue up to day to detect GW emitted for many different sources.[13] They are, essentially, "**dipolar detectors"** of GW.

Figure (4.4). Very large tubes for the lasers paths.

(5) Appendix. **Michelson Interferometer of our Laboratory.**

We can use the Interferometer (see **Figure (A.1**)) to determine small variations in the optical path. Moving one of the mirrors by means of a micrometer will be altering the distance traveled by one of the light beams. This beam passes twice through the path between the moving mirror and the semi-transparent mirror. Therefore, if the mirror is shifted by a quarter of a wavelength, the optical path will change by half a wavelength. This can be observed in the "*interference pattern*" seen in **Fig.(A.1**).

When the distance is changed by half a wavelength, the region that was light in the interference pattern (maximum light intensity) becomes dark and vice versa.

Moving the mirror a quarter wavelength, we get the same original interference pattern again. Therefore, whenever the mirror moves a distance d. The same pattern of initial interference fringes is obtained when,

$$2d = m\lambda \qquad \textbf{(A.1)}$$

where λ is the light wavelength in air and m = 1, 2, 3, ...The m value are determined by counting the number of times when the interference pattern is restored.

Thus, by measuring the distance d we can determine the value of λ. Or, knowing the value of λ, we can determine the value of displacement d.

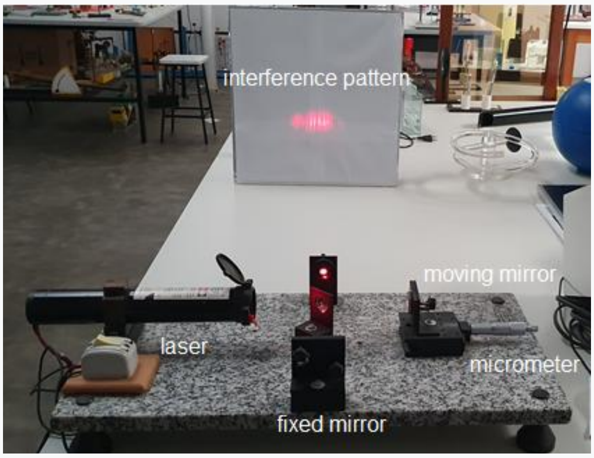


Figure (A.1). Michelson interferometer of our Laboratory.

The Michelson interferometer can also be used to determine the air refractive index n as a function of pressure P. This can be done with a chamber(see **Figure (A.2)**) with transparent windows and connected to a vacuum pump or a syringe containing air. This apparatus is schematically shown in **Figure (A.3).** Usual interferometers are able to detect lengths variations less than 10^{-8} m.

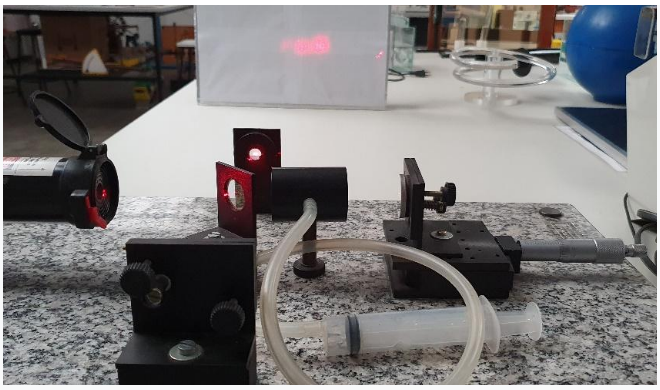


Figure (A.2). Interferometer with chamber and syringe.

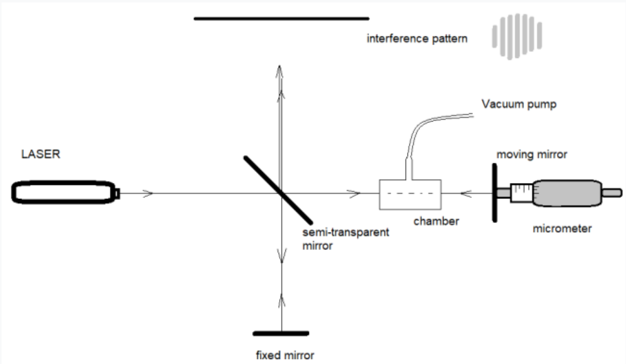


Figure (A.3). Interferometer assembly diagram with chamber with length d.

Outside the chamber, the optical paths remain unchanged. However, inside the chamber the wavelength and speed of light change according to air pressure.

Let us consider a chamber of length d containing air at pressure P_1. In this situation the wavelength of light through the air is λ_1. As the beam passes through the chamber twice, the number of wavelengths m_1 inside is given by

$$m_1 = 2d/\lambda_1 \quad \text{(A.2)}$$

Changing the air pressure to P_2, the number m_2 of λ_2 in the chamber is,

$$m_2 = 2d/\lambda_2 \quad \text{(A.3)}$$

If n_1 and n_2 are the refractive indexes of air at pressures P_1 and P_2,respectively, and λ_0 the light wavelength in vacuum. So,

$$\lambda_1 = \lambda_0/n_1 \text{ and } \lambda_2= \lambda_0/n_2 \quad \text{(A.4)}$$

From these equations we can obtain:

$$n_1 - n_2 = \lambda_0(m_1 - m_2)/2d = \lambda_0 m/2d \quad \text{(A.5)}$$

where $m = m_1 - m_2$ is the number of interference fringes counted when the gas pressure goes from P_1 to P_2. From Eq.(A.5) we see that

$$\Delta n = \lambda_0 m/2d \quad \text{(A.6).}$$

The refractive index n for most gases is very close to 1.[14] For air and other ideal gases ,the difference between the refractive index and 1 is proportional to the pressure P of the gas. Thus, it is defined by

$$n = 1 + k\,P, \qquad (A.7),$$

where k is an unknown constant.

So, when P is changed, we have $\Delta n = k\,\Delta P$, that is, $k = \Delta n/\Delta P$. From **Eqs.(A.6)** and **(A.7)** we now can relate the number m of fringes shifted with the change of pressure $\Delta P = \Delta n/k$ obtaining,[14]

$$n = 1 + m\lambda_0 P/2d\Delta P \qquad (A.8).$$

In **Figure (A.4)** are shown, as a function of the pressure P, refraction indexes n for some ideal gases.

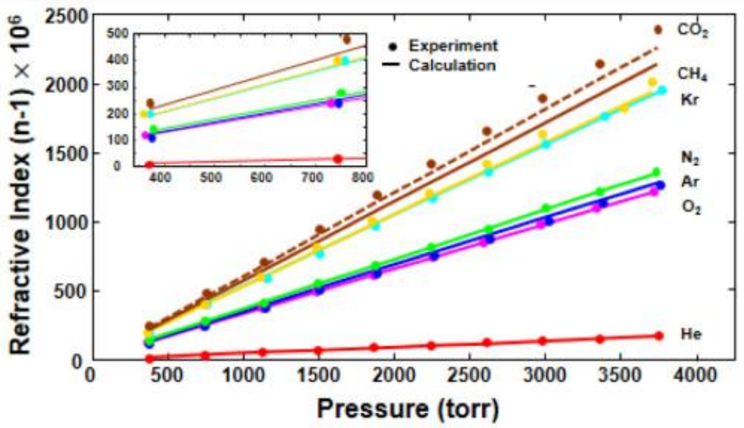


Figure (A.4). Refractive indexes n for some ideal gases as a function of the pressure.

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