Magnetic Levitation: Faraday´s Law and Eddy Currents.

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Abstract.

Will be analyzed the magnetic levitation (**maglev**) or magnetic suspension process which occurs when objects are suspended by magnetic forces. It is employed, for instance, in **maglev trains,** to stop rotating power tools quickly when they are turned off, in contactless melting and in *magnetic bearings*.[1,2]

Key words: Faraday law of induction; Foucault or eddy currents; magnetic levitation.

(I)Introduction.

In **Section A**, following basic physics texts[3.4] are shown **conductor** rings with currents and interactions between them. It is valid only for *usual* metallic conductors like, for instance, aluminum, copper and zinc. It cannot be applied for conductors with magnetic properties like, for instance, *magnets*[5] or "imãs" (see **Appendix I**). In Section **B** are analyzed interactions of rings with *magnets* and levitation effects. In **Section C** is shown the *magnetic gun* which is a consequence of repulsive forces between two rings. In **Section D** is studied the magnetic levitation between a solenoid[3,4] and a metallic plate due to eddy currents.

Section A. Conductors rings.

Following basic physics texts[3,4] are shown essential aspects of magnetic charges in linear motion, in conductor rings with currents and interactions these between rings.

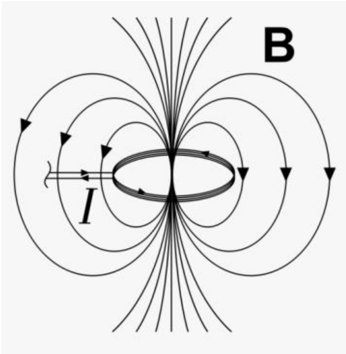
According to Ampere´s law the magnetic field **B(r,t)** due to straight line current I(t) is given by [3,4]

$$\mathbf{B}(\mathbf{r},t) = \{\mu_o I(t)/2\pi r\}\, \boldsymbol{\theta} \qquad \mathbf{(A.1),}$$

showing that **B(r,t)** is tangent to circles with radius r around the line current.

On the other hand, if the current I(t) is along a circular conductor ring with radius ρ, the magnetic field **B**(**r**,t) = μ_o**H**(**r**,t) would have a geometrical configuration seen in **Figure (A.1).**

N



S

Figure (A.1). **B**(x,y,z,t) due to a conductor ring with current I. **N** and **S** indicate, respectively, the NORTH and SOUTH poles of the **B** field.

Will be avoided the intricate mathematical formalism necessary to get an exact description of levitation processes. Are adopted simple descriptions of the phenomena according to basic physics texts.[3,4] In this way, are shown in **Figures (A.2)** only the north and south poles of magnetic fields **B***created by horary and anti-horary currents I in circular rings.

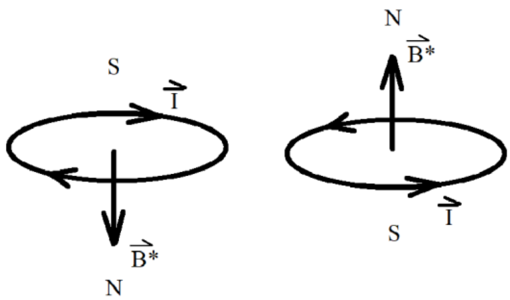


Figure (A.2.a) **Figure(A.2.b)**

Figures (A.2). South (**S**) and north (**N**) poles **B***of magnetic fields created by circular rings**.** In **(A.2.a)** we have a current I in *horary sense* and in **(A.2.b)** in *anti horary sense*.

In **Appendix II** is shown the magnetic field B(x) created by a circular ring with radius R and current I at a point P at a distance x of the origin O of the ring.[3]

In **Figures (A.3)** are displayed rings put close. In **Figure (A.3.a),** when poles **N and S** are near, there is attraction between the rings. On the other hand, in **Figures (A.3.b)** and **(A.3.c)** when north poles **N** or **S** poles are near there is repulsion between rings.

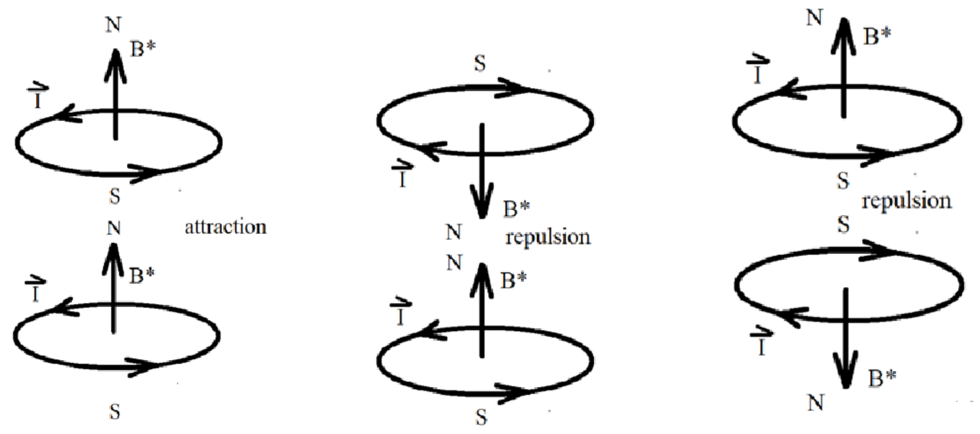


Figure (A.3.a) **Figure (A.3.b)** **Figure (A.3.c)**

Figures (A.3). In (A.3.a) there is attraction between the rings**.** In **(A.3.b)** and **(A.3.c)** there is repulsion between them.

(B)Magnet crossing a ring area A with velocity v(t).

Let us consider the case when a magnet[5] (**See Appendix)** with velocity **v** is crossing a ring, which is at rest, with area **A** and initially with no current (**Figure (B.1)**). The magnetic field $\mathbf{B}_{mag}$ crossing **A** creates a magnetic flux Φ(t) given by $\Phi(t) = \mathbf{B}_{mag}(t) \bullet \mathbf{A}$.

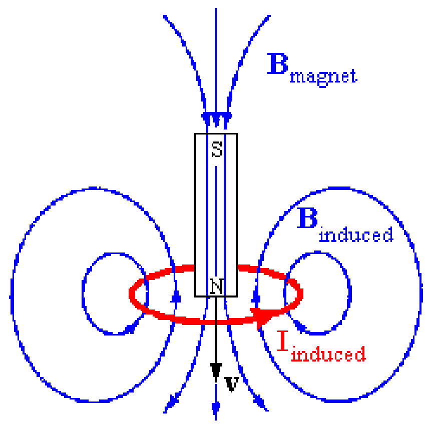


Figure (B.1). Magnet[5] passing by a ring, at rest, with area **A**.

According to Faraday´s induction law, [3,4] appears in the ring an electromotive force ε (emf) given by,

$$\varepsilon(t) = - d\Phi(t)/dt \qquad \textbf{(B.1).}$$

Thus, the magnet creates in the ring an induced current $I_{induced}(t)$, represented by a *red circle*, given by $I_{induced}(t) = R/\varepsilon(t)$, where R is the ring resistance. As $I_{induced}(t)$ has an *anti-horary sense*[3,4] it would be responsible by a magnetic field $\mathbf{B}_{induced}$ (indicated by *blue lines*) which is contrary to the $\mathbf{B}_{magnet}$. So, according to **Figure (A.3.b)** would appear on the magnet an induced force $\mathbf{F}_{induced}$. Being contrary to the **v**, it would prevent the penetration of the magnet through the area **A.** If the magnet (with mass m) is, for instance, in a gravitational field it would be submitted to a net force,

$$F = mg - F_{induced} \qquad \textbf{(B.2).}$$

When this $F_{induced}$ upward force equals the weight mg it is called "**levitation force".**[1,2] When $mg > F_{induced}$ it will pass by **A** and now would be attracted by the ring with a force equal to $F_{induced}$.

(C) Magnetic gun.

Now, let us consider the case of a aluminium *ring* which is over a *solenoid* [3,4] (**Figure (C.1))** fixed on a table in our laboratory**,** as shown in **Figure (C.2).**

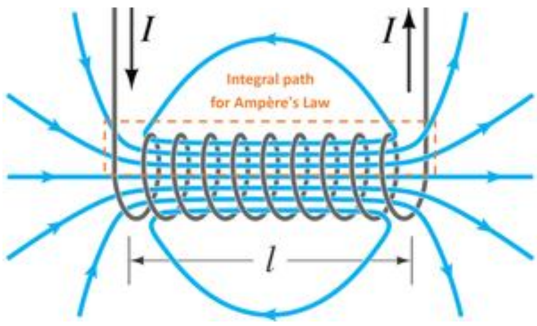


Figure (C.1). Magnetic field **B(r,**t) of a solenoid

Initially there is no current in the solenoid.

Figure (C.1). Aluminum ring at rest over a solenoid at our laboratory**.**

Now, let us assume at the instant $t = t_o$ passes by the solenoid a current I(t), in a very short time interval δt, with the maximum value $I(t_o) = I_o$. This current creates a magnetic field $\mathbf{B}_{coil}(x,y,z,t)$. The flux of $\mathbf{B}_{coil}(x,y,z,t)$ through the area $\mathbf{A}_{ring}$ would generate a magnetic flux $\Phi_{ring}(t)$ on the ring given by[3,4]

$$\Phi_{ring}(t) = \int \mathbf{B}_{coil}(x,y,z,t) \bullet d\mathbf{A}_{ring}. \qquad \text{(C.1).}$$

According to Faraday´s induction law,[3,4] this flux would appear at $t = t_o$ an *efm* in the ring $\varepsilon_{ring}(t_o)$ given by

$$\varepsilon_{ring}(t_o) = - \{d\Phi_{ring}(t)/dt\}_{t = to} \qquad \text{(C.2).}$$

Thus, similarly to what happened with the falling magnet in **Section 2**, appears on the ring a magnetic field $\mathbf{B}_{ring}(t_o)$ which is contrary to the applied field $\mathbf{B}_{solenoid}(t_o)$. This induced $\mathbf{B}_{ring}(t_o)$ would be responsible by an "instantaneous" *repulsive magnetic* force on the ring. When this instantaneous levitation force is very strong we have a "**magnetic gun**".[6]

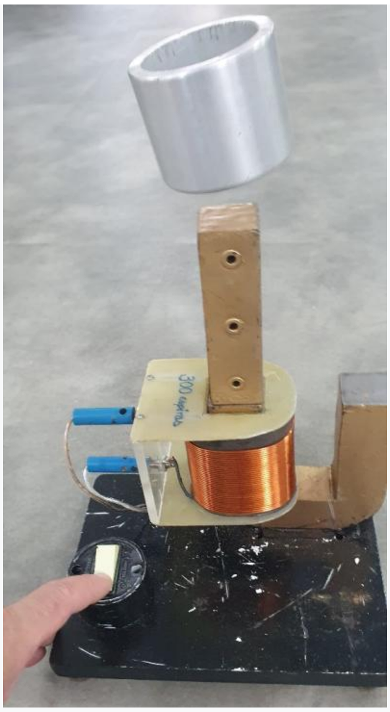


Figure (C.2). Instantaneous repulsive magnetic force on the ring.

(D)Solenoid levitation on a metallic plate.

As magnetic levitation is created by eddy (Foucault) currents in conductors we will show, avoiding intricate calculations, how to estimate these currents. Mathematical estimations of this effect are presented elsewhere.[7]

So, let us consider a solenoid [3,4], which is very far of any material(**Figure (D.1)).** It is carried with an alternating current I(t) with frequency ω creating a magnetic field **B(r,t)** (blue lines**),**

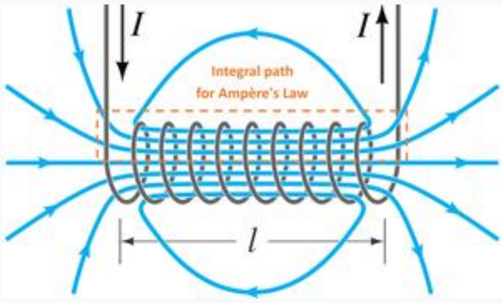
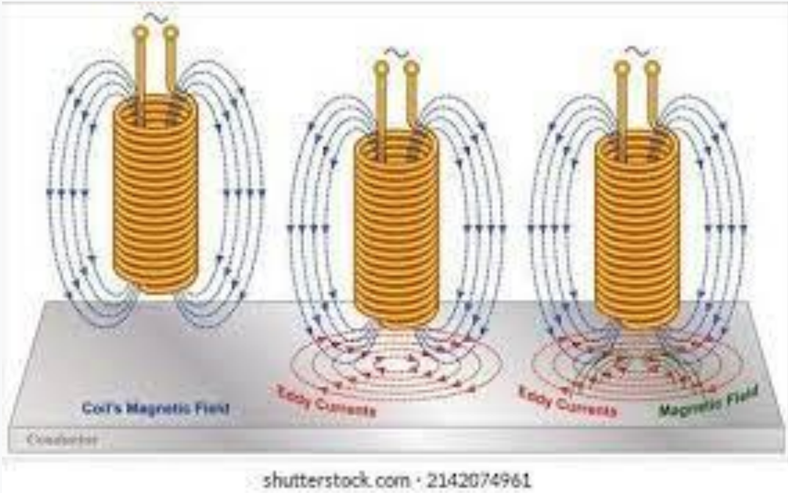


Figure (D.1). Magnetic field **B(r,**t**)** of a solenoid.

In **Figure (D.2.a)** the solenoid is very far from the metallic surface. When it approaches the metallic surface(**Figures (D.2.b)** and **(D.2.c)** are created **induced currents** $I_c(\mathbf{r},t)$ inside the conductor indicated by red lines. These are called "**eddy**" or "**Foucault currents**".[7,8] In the **Figure (D.2.c)** is also shown the magnetic field, represented by yellow lines, created by the eddy currents.[7,8] They will be analyzed in what follows.



Figures (D.2.a), (D.2.b) and **(D.2.c)** show the interactions of the solenoid magnetic field (blue lines) with the plane metallic surface.[7,8]

In **Figures (D.3) are** better seen what happens when the solenoid it is **far** and when it is **close** to the metallic surface.[7,8]

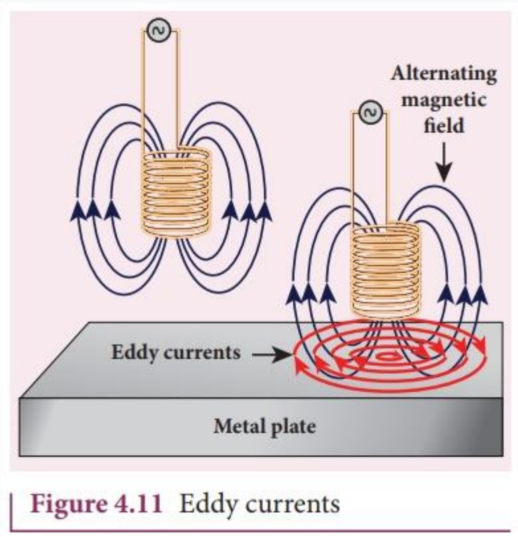


Figure (D.3). Solenoid far (left) and close (right) to the metallic plate.[8]

In **Figure(D.4)** is seen in details the case when the solenoid is close to the conductive plate.

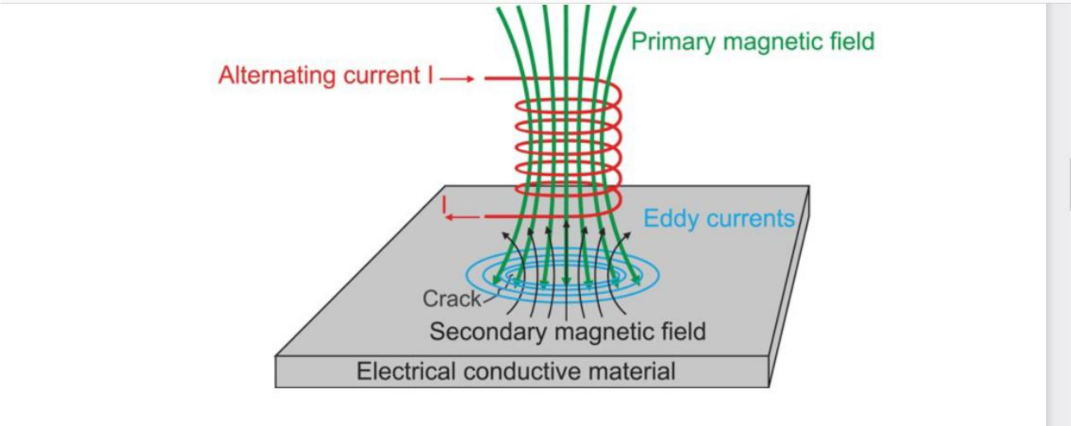


Figure (D.4). Solenoid close to the plane conductor.[8]

In recent paper[7] we have shown that the *eddy current* density $\mathbf{J}_c$ inside an ideal plane conductor is given by

$$\mathbf{J}_c = \sigma\, \mathbf{E}_c = (1/\delta)\, H_{//}(h,\rho)\, \exp(-z/\delta)\sin(\omega t)\boldsymbol{\varphi} \qquad \textbf{(D.1),}$$

where σ is the metal conductivity, δ the metal skin depth, h the distance of the solenoid basis from the metal surface, $H_{//}(h,\rho)$ is magnetic field generated by the current that circulates at a distance ρ from the z-axis and **φ** is the tangent unitary vector. $H_{//}(h,\rho)$ is shown explicitly in reference [7]. From **Eq.(D.1)** we verify that Foucault currents $I_c(t)$ are induced inside the conductor in a metallic layer with thickness ~ δ (**Figure (D.3)**).

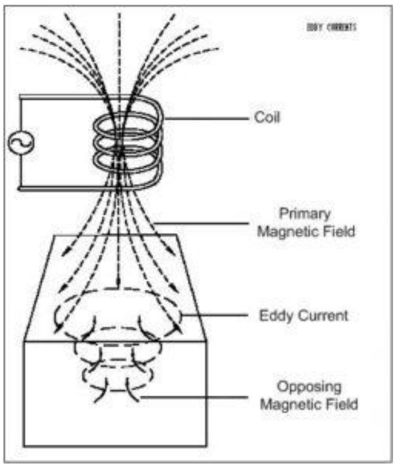


Figure (D.3). Eddy currents are induced inside the conductor.[7,8]

The solenoid current I(t) and the eddy currents I_c (t) inside the conductor circulate in contrary senses, according to the Faraday´s law.[3,4] (see **Section A**). The eddy *efm*, ε_{ed} , would be estimated by $\varepsilon_{ed} = - d\Phi_{ed}/dt$, where Φ_{ed} is the flux of the eddy currents that are inside the metal up a depth δ. This *efm* is responsible by a **repulsive** force F_z between the solenoid and the conducting plate. F_z is the **levitation force** when $F_z = mg$, where m is the solenoid mass.

In our laboratory can be observed the levitation of a metallic plate which is over an induction oven.[10] See **Figures (D.4).**

Figure (D.4.a) Induction oven, aluminum plate (kitchen aluminum foil) and a plastic holder with wooden pin.

Figure (D.4.b). Induction oven turned off with aluminum foil over.

A magnetic field perpendicular to the oven metal plate is created when it is on. This magnetic field inducing eddy currents in the metallic plate is responsible by its levitation**.(Figure D.4.c)**

Figure (D.4.c). Aluminum foil levitating over the induction oven on.

APPENDIX I. Magnetism and Magnets.[5]

Simple explanations about magnetism and magnets can be found in many basics physics books[4] A naïve interpretation for the magnetism phenomenon in solids was given by Ampère.[4] It was done after the discovery that electric current also cause magnetic effects. He proposed[4] that magnetic properties of a "**magnet**" would be created by electric currents in elementary rings located inside the body. See **Figure (I.1).**

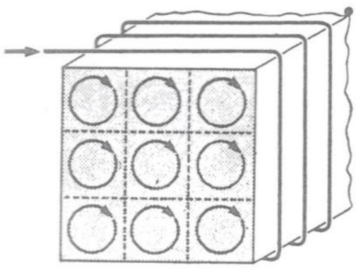


Figure (I.1). Are seen the elementary ring currents inside the body.

These elementary currents were supposed to be permanent: there were no resistance and, consequently, no dissipation. In the **non magnetized** body, the rings would be randomly oriented.

The magnetization process would consist of orienting these rings with their parallel planes and with currents all circulating in the same direction, as shown in **Figure (I.1)**. Now we know that this naïve theory is essentially correct, that is, the elementary currents being formed by electrons revolving around atomic nuclei.[11]

Additional explanations about magnetism are given, for instance, by R. Feynman.[11].

APPENDIX II. Magnetic field B due to a circular ring.

Let us consider coaxial circular rings as seen in **Figures (A.3)**. The magnetic field **B**(x) due to one ring with radius R and current I at a point P, distant x from the center of another ring is given by,[3]

$$\mathbf{B}(x) = \mu_o I R^2/\{2(R^2 + x^2)^{3/2}\}\ \mathbf{i} \qquad \textbf{(II.1),}$$

where the unit vector **i** is along the co-axial axis. The force **F**(x) of this ring on the other ring *r* at a point x is given by[3]

$$\mathbf{F}(x) = -\ \mathbf{i}\ d\{\boldsymbol{\mu}_r \bullet \mathbf{B}(x)\}/dx, \qquad \textbf{(II.2),}$$

where $\boldsymbol{\mu}_\mathbf{r}$ is the *magnetic dipole moment* (see **APPENDIX III**) of the r-ring defined by $\boldsymbol{\mu}_\mathbf{r} = I_r \mathbf{A}_\mathbf{r}$ and $\mathbf{A}_\mathbf{r}$ the r-ring area.

APPENDIX III. Magnetic dipole moment.

Let us consider a conductor loop that can rotate around a fixed axis, with area **A** and with a constant current I. When this loop is submitted to an uniform magnetic field **B** it can be shown that will have on this loop a torque given by

$$\tau = I\ \mathbf{A} \times \mathbf{B} \qquad \textbf{(III.1),}$$

independently of the loop form. The area **A** is vector normal to the loop plane. Defining ***magnetic dipole moment*** of the loop by $\boldsymbol{\mu} = I\ \mathbf{A}$, **Eq.(III.1)** will be written as

$$\tau = \boldsymbol{\mu} \times \mathbf{B} \qquad \textbf{(III.2).}$$

The above **Eqs. (III.1)** and **(III.2)** are deduced for particular cases of rectangular loops.[3]

The work dW = dU done by the applied electromagnetic field during the loop rotation d**θ** around the fixed axis is given by

$$dU = \tau \bullet d\boldsymbol{\theta} \qquad \textbf{(III.3)}$$

From **Eqs.(III.2)** and **(III.3)** we can calculate the potential magnetic energy of the system integrating dU from 90° up to θ:

$$U(\theta) = \int \tau \bullet d\boldsymbol{\theta} = \boldsymbol{\mu} B \int \sin(\theta)\, d\theta = -\mu B \cos(\theta),$$

that is,

$$U(\theta) = -\boldsymbol{\mu} \bullet \mathbf{B} \qquad \textbf{(III.4).}$$

If **B** = **B**(x) the magnetic force **F**(x) between **B**(x) and the magnetic moment **μ** would be given by

$$\mathbf{F}(x) = -\{dU(x)/dx\}\, \mathbf{i} \qquad \textbf{(III.5),}$$

according to **Eq.(II.2).**

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