Instabilities In Vertical Cylindrical Water Jets

C.H.Furukawa, F.D.Saad and M.Cattani(*)
Instituto de Física da Universidade de São Paulo
(*)mcattani@if.usp.br

Abstract. Using Stroboscopic lamp,[1] we have investigated instabilities, with droplets formation, in vertical water jets in air in a gravitational field.
Key words: *water jets*; *hydrodynamic equations; flow instabilities; droplets formation.*

(I) Introduction.

We perform a simple analysis of instabilities that appear in a vertical water jet submitted to a gravitational field **g** in the air. As a first approach, in **Section 1,** both water and air are assumed as ideal fluids. In these conditions, water flow is **laminar.**[2-6,8] In **Section 2** will be analyzed vertical jets taking water and air as real fluids. It will be shown that instabilities are created by forces acting on the air-water interface. These are due to liquid *superficial tension* γ, *viscous forces* and *difference of pressure* between air and water.[2 -6] In these conditions the flow will become **turbulent**[2-6,8] and, finally, water **droplets** are created.

(1)Ideal Fluids.

Let us analyze water ejected in air along a vertical line submitted to a gravitational acceleration **g**. Assuming that they are *ideal fluids* we put their densities ρ = constants and viscosities $\eta = 0$. It will be also neglected water superficial elastic effects (**Appendix A).**

In these conditions, we can assume, in a first approximation, that water mass elements would fall along a vertical **z-axis** submitted only to the gravitational force. In this way, their velocities **U(z)** along the z-axis would be given by,

$$\partial \mathbf{U}/\partial t = - g\, \mathbf{k} \qquad \textbf{(1.1).}$$

Integrating **Eq.(1.1)**, neglecting the initial speed**,** we verify that fluid moves in a **free fall** along the **z-axis**, with **U**(t) = gt **k,** getting,

$$\mathbf{U}(z) = (2gz)^{1/2}\mathbf{k}. \qquad \textbf{(1.2).}$$

As water is incompressible, the mass flux $\Phi = U(z)A(z) = Q$ will be constant along the flow tube. Being $A(z) = \pi\, r^2(z)$ the tube cross section area and r(z) its radius we get, taking into account **Eq.(1.2)**:

$$r(z) = \{Q/\pi\sqrt{2gz}\}^{1/2} \qquad \textbf{(1.3),}$$

showing that the tube radius decreases as z increases.

In **Figure (1.a)** is shown the case of ideal water ejected in ideal air by a circular orifice with radius **a.** The tube flow would have a cylindrical symmetry.

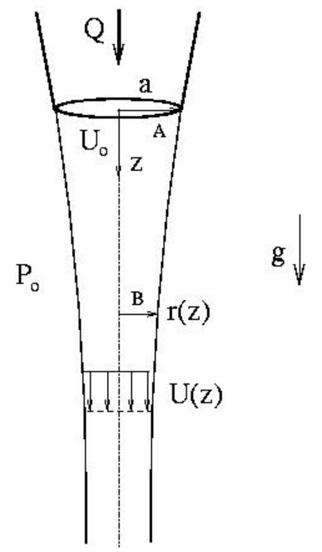

Figure (1.a) Jet flow of an **ideal water** extruded from an orifice of radius **a** accelerated under the influence of gravity. Its **shape** is influenced by the gravitational acceleration and also by the and surface tension γ.

(2)Real Fluids.

When water emerges from the tap, at the very beginning of the vertical flow, we see that there are no instabilities. The trajectories of mass elements δm along the flow tube would be "*parallel"* lines**.** It is known as **laminar flow**.[2-6](see **Figure(2.1)).**

Figure (2.1). Initial laminar flow of mass elements during the free vertical fall.

Far from a tap we always observe that it will *break up into droplets*, no matter how smoothly the stream is emitted from the tap. In **Figure (2.2)** is seen the stroboscopic photo of the water flow. Are clearly seen *instabilities* (turbulence) in the flow tube, that is, sometimes it becomes twisted, gets thinner and, finally, it breaks and droplets are created.

Figure (2.2). Instabilities of the falling water thread in air with formation of droplets.

The mathematical description of **real fluids** instabilities in air is an extremely difficult task. To do this it would be necessary to use the **Navier-Stokes** equation given by,[3,5,7]

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\bullet\mathbf{grad})\mathbf{v} = -\,\mathrm{grad}(p)/\rho - \mathrm{grad}(\varphi)/\rho + \nu\,\mathrm{lapl}(\mathbf{v}) \qquad \mathbf{(2.1),}$$

where φ(x,y,z) is the (gravitational) external potential, p(x,y,z) fluids pressures, ρ(x,y,z) fluids densities, η(x,y,z) fluids viscosities and $\nu = \eta/\rho$ their cinematic viscosities. Note that **Eq.(1.1)** is obtained from **Eq.(2.1)** neglecting **(v•grad)v,** grad(p)/ρ, ν lapl(**v**) and putting grad(φ)/ρ = **g**/ρ.

Instead of an intricate mathematical analysis[7] we only intend to describe, in a simple way, the main mechanisms responsible by the flow instabilities and the droplets formation. These occur due to surface tension perturbation effects that are *always* present in the interface of the two fluids which are in relative motion. Instabilities are created by forces acting on the air-fluid interface due to liquid *superficial tension γ*, *viscous forces* and *difference of pressure* between air and liquid and the flow becomes **turbulent**.[2-6] In **Figure (2.3)** these perturbations are represented by circles R(z). As z increases r(z) decreases and R(z) increases, finally breaking the jet tube into droplets (**Figure (2.2)).** It can be shown that when r(z) → **0** we have v(z) → ∞.[7]

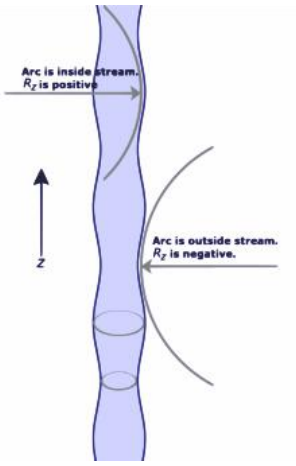

Figure (2.2). Surface perturbations are represented by circles with radius R(z) . When these R(z) increase the stream tube radius r(z) decrease and the water jet breaks into droplets.

Appendix A - Surface Physics.

As seen in basic physics courses,[2-5] a paperclip, an insect or a needle can float on water. Drops of mercury or water do not spread on a surface. When the clean glass with a small diameter are immersed in water, the water rises in the tube, however, when the liquid is mercury it falls into the tube. This phenomenon is known as *capillarity*. Bubbles of water with soap can created and float freely in the air. Thin liquid films of water with soap can be created in wire frames. To stretch these films is necessary to apply a force F.[2] In this stretching if the film area is modified by dA is realized a work dW defined by $dW = \gamma\, dA$, where γ is named *surface tension coefficient* or, simply, *surface tension effects* of the film. It has the dimension of *energy per unit of area* [J/m^2] or *force per unit of length* [N/m]. The thin film behaves as an elastic membrane.

In materials science, *surface tension* is used for either *surface stress* or *surface free energy* and, usually, instead of "force" F we take into account "*tension*" T which is "*force per unit of length*". These phenomena and many others are observed in *interfaces* of fluids in contact. A clear understanding of the *interface physics* can be only obtained taking into account the molecular structure of the fluids and their mutual interactions.

Usually, in undergraduate physics courses[1,3] are studied only liquid-air interfaces since in this case is easier to understand basic effects derived from surface tension. At liquid-air interfaces, surface tension results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air. Thus, the surface becomes under tension from the imbalanced forces. The net effect is an inward force at its surface that causes the liquid to behave as if its surface were covered with a stretched elastic membrane. Surface tension[6] is the elastic tendency of a fluid surface which makes it acquire the least surface area possible. An illustrative example is the water droplet free in the air: the liquid assumes a spherical shape which is the least surface area possible, that is, an area with a minimum of elastic energy.

Appendix B. Turbulent or Chaotic flows.

In **Section 1** was seen that ideal water jets in the air are **laminar,**[2-6] that is, water would travel smoothly or in regular paths; the flow lines are "parallel" according to **Figure (B.1).** In **turbulent** or **chaotic** case[7] there are no continuous flow lines. Water travels along small mixing paths submitted to irregular fluctuations. These cases are shown in **Figure (B.1).**

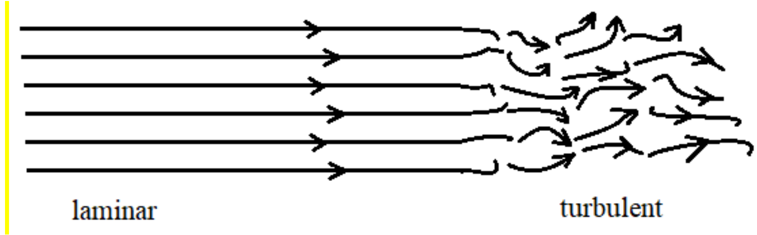

Figure (B.1). At the left side, fluid is moving following a laminar flow and at right it is moving following a turbulent flow.

In **Figure (B.2)** is shown a candle flame flow: the lower part is laminar and the upper one is turbulent.

Figure (B.2). Candle flame flow. The low part is laminar and the upper is turbulent.

In fluid mechanics is very difficult to exactly determine the necessary conditions to have laminar or turbulent flows.[2-6] There is, however, a dimensionless quantity called **Reynold number**(Re) that can help us to see when we can have a turbulent or laminar flow.[4,8] It is defined by

Re =

where L = characteristic linear flow dimension, v = flow velocity,ρ = fluid density and η = fluid viscosity. For a tube with diameter D we put L = D.

(1)**Laminar flow** occurs at **low Re**, where viscous forces are dominant and is characterized by smooth, constant fluid motion.[4,8]

(2)**Turbulent flow** occurs at **high Re.** It is dominated by viscous and inertial forces that tend to produce chaotic eddies, vortices and other flow instabilities.[4,8]

Acknowledgements. *The author thanks the librarian Maria de Fatima A. Souza and the administrative technician Tiago B. Alonso for their invaluable assistances in the publication of this paper.*

REFERENCES

[1]https://en.wikipedia.org/wiki/Strobe_light
[2]R.Resnick, D.Halliday e K.S.Krane. "Física 2". LTC(2003).
[3]M.Cattani. "Elementos de Mecânica dos Fluidos". Editora Edgard Blücher (2005).
[4]F.W.Sears e M.W.Semansky. "Física" vol.1. LTC (1975).
[5]L.Landau et E.Lifchitz."Mécanique des Fluides". Éditions Mir(1971).
[6] https://en.wikipedia.org/wiki/Surface_tension
[7]M.Cattani and M.C.Salvadori "Water Threads Instabilities in Cylindrical Jets" http://publica-sbi.if.usp.br/PDFs/pd1695
[8]https://en.wikipedia.org/wiki/Reynolds_number