HYDRAULIC SHOCK

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Abstract. We perform a rough estimate of the "hydraulic shock" or "water hammer" due to a compressible liquid moving along an horizontal tube. This effect is also analyzed in vertical fall of water columns in glass tubes.
***key words**: compressible liquid; long horizontal tube; water hammer.*

(I) Introduction.

In order to estimate the ""hydraulic shock" in a long horizontal tube we first present, in **Section 1,** elastic properties of a compressible liquid, like *Bulk modulus* and *sound velocity*. In **Section 2** are estimated the "shock" force F* and pressure P*. In **Section 3** are shown the shock effects in vertical fall of water columns in glass tubes.

(1) Elastic Properties of Liquids.

Let us consider a liquid that at a pressure P has a volume V and when it is submitted to an hydrostatic pressure P = P + dP its volume becomes V = V + dV. In these conditions we define its ***volumetric elasticity modulus*** K or ***bulk modulus*** B, by

$$K = B = - dp/(dV/V) = -V(dp/dV) \qquad \textbf{(1.1)},$$

where the signal - means that when the pressure increases the body volume decreases. The ***elastic compressibility modulus*** **κ** of a liquid would be defined by

$$\kappa = - (dV/dp)/V = 1/K = 1/B \qquad \textbf{(1.2)}.$$

When a liquid in a tube is submitted to longitudinal compressions, are created oscillations of the liquid particles, named *compression waves*. These are also named ***sound waves*** that are transmitted with velocity c or u,

$$c = u = (B/\rho)^{1/2} \qquad \textbf{(1.3)},$$

where ρ is the liquid *density*.

For water(see **Section (3)),** K = B = 2.2 Gpa = 0.32 Mpsi and the sound velocity u = c ≈ 1.5 10^3 m/s.

(2) Hydraulic Shock in Horizontal Tube.

Figure (1) shows a compressible liquid in motion inside a long horizontal tube with a transversal section area A and closed at the right end.

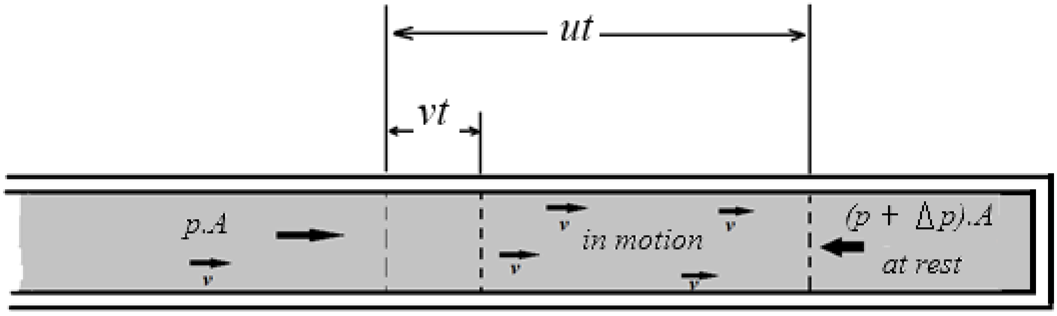

Figure (1). Compressible fluid in motion at time t≠ 0 in a closed tube at right.

In **Fig.(1)** the fluid is seen at $t \neq 0$, initially with incident velocity v and at a pressure P, entering in contact with the closed right end of the tube. Near to the end the fluid is at rest and its pressure becomes P + ΔP. Due to this compression, a *sound wave*[1] propagates inside the liquid. After a time t, while the liquid propagates a distance vt the sound propagates a distance ut (**see Fig.(1)**), remembering that $u = c >> v$.[1]

Inside the volume (u - v)t, all portions of the fluid move with velocity v while the other ones at right remain at rest. Thus, during the time t, we have a relative variation of volume Aut/Avt.

In this way, according to **Eq.(1.2)** we can write

$$B = - \Delta P/(\Delta V/V) = \Delta P/\{Aut/Avt\} \quad \textbf{(2.1),}$$

that is, $$\Delta P = B(v/u) \quad \textbf{(2.2).}$$

So, if A is tube section area, according to **Eq.(1.2)**, the "shock" force $F^* = A\,\Delta P$ on the closed end of the tube would be given by

$$F^* = A\,\rho\,v\,u \quad \textbf{(2.3),}$$

taking into account that $u = (B/\rho)^{1/2}$, that is, $B = \rho u^2$. This force F* is also named "hydraulic force" or "hammer force". The shock pressure P* on the right end of tube would be P* = F*A.

Let us estimate F* and P* for water taking $\rho = 1 g/cm^3 = 10^3 kg/m^3$ and $u \approx 1.5\ \ 10^3$ m/s[1], assuming that $\mathbf{A = 1,25\ 10^{-3}\ m^2}$ (diameter = 2cm) and that **v = 1.4 m/s** (10 cm high free fall velocity). In these conditions we verify that **F* ≈ 2600 N,** which is equivalent to the weight of a body with mass ~ 260 Kg, and that **P* = F*/A ≈ 2.1 10^6 N/m^2 = 2.1 MPa**.

Note that, a **copper** tube with an inner diameter = 20 mm and an outer diameter = 24 mm can withstand a maximum pressure [2] = **7.3 MP.** This pressure can be reached with a free fall of about **1.2 m in height.** This implies that a column of water falling in free fall of **1.2 meters** could collapse a copper tube with these dimensions.

(3) Free Fall Water Hammer in Glass Tubes.

In our didactical Laboratory (***LabDemo***) there is an equipment to show the hydraulic shock using water in vertical free fall, without air resistance. It is a glass tube containing water at low pressure (**Figure (2)**).

Figure (2). Glass tube with water at low pressure.

When abruptly we lift vertically the tube (**Figure 3a**), by inertia the water column is detached from the bottom of the tube. In sequence, the water column falls (**Figure 3b**) and in free fall it collides strongly with the bottom (**Figure 3c**) generating an intense "metallic sound". This experiment shows the "Water Hammer" (**Figures (3**).

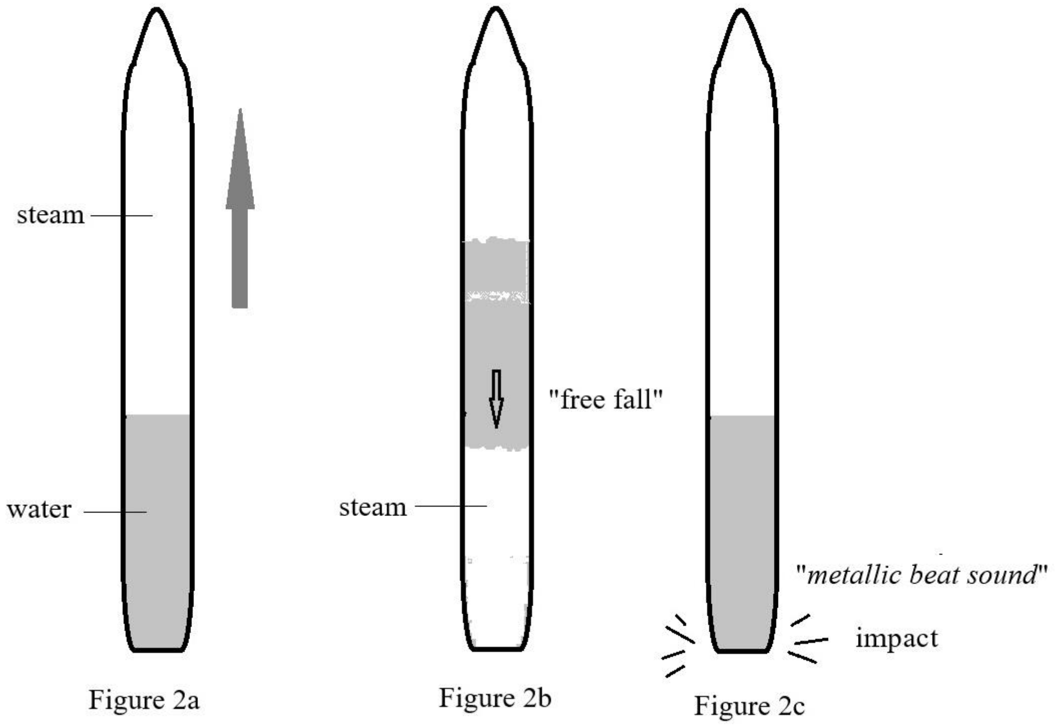

Figure (3a). Vertical lift of the glass tube .
Figure (3b). With an abrupt lift of the glass tube the water column is detached from the bottom.
Figure (3c). With a vertical free fall the water column collides with the bottom, generating a strong metallic sound.

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