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SEMICLASSICAL DERIVATION OF A LOCAL OPTICAL POTENTIAL FOR HEAVY-ION ELASTIC SCATTERING*

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A B S T R A C T

A semi-classical method to determine the contribution to the optical potential in the elastic channel due to the coupling to other processes taking place in heavy-ion collisions is developed. An application is made to the case of coulomb excitation. The lowest order term of our potential is shown to be identical to the quantum mechanical expression of Baltz et al.

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I - INTRODUCTION

In a typical heavy-ion reaction one frequently encounters the situation where several strongly coupled channels are open. The complexity of a full coupled channel calculation and the computer time needed increases rapidly with the number of channels involved. On the other hand one is often interested in only a few, if not just one, of these channels. It is thus natural to attempt reducing the number of channels appearing explicitly in the calculation to just those of direct interest. This is achieved in the optical model [1,2] where the effects of the eliminated channels are accounted for by the addition of a complex potential to the channel Hamiltonian. Such complex optical potentials are usually determined trough phenomenological analysis. However it is clearly desirable to base these potentials on more solid theoretical grounds. This can be accomplished by constructing a physical model for the effects of the eliminated channels on the remaining ones.

Recently such a program was carried out for the case of Coulomb excitation of a deformed nucleus by Love et al [3] and was subsequently improved upon by Baltz et al [4]. In this case one has an electric quadrupole potential coupling the different states $(0^+,2^+,4^+,\ldots)$ of the ground rotational band. In order to study the elastic (0^+) channel Baltz et al [4] employed Feshbach's formalism with the approximation of replacing the full interacting Green's function for the inelastic channel by the Coulomb Green's function. This approximation amounts to neglecting multiple Coulomb excitation and therefore the effects on elastic scattering due to channels other than the 2^+ are not considered at all. The range of applicability of such a potential is consequently limited since in most cases Coulomb excitation is very strong, thus demanding the inclusion of higher order effects as was shown recently by Doll et al [5].

The purpose of the present work is to suggest an alternative method to derive the complex optical potential based on a semi-classical description of a nuclear collision process.

In section 2 we summarize the conventional quantum mechanical treatment of the effective local potential. An alternative approach based upon classical trajectories for the colliding nuclei is developed in section 3. In section 4 an application of the formalism of the previous section is made for the case of Coulomb excitation. The results are then compared to other existing quantum mechanical expressions. Finally, in section 5 the main conclusions of the present work are presented.

II - EFFECTIVE LOCAL POTENTIAL : QUANTUM MECHANICAL TREATMENT

Let us consider two colliding nuclei. The complete Hamiltonian for the system may be written as

$$H = H_o + V \tag{1}$$

where H_o is the part of the Hamiltonian which is diagonal in channel space and V is the interaction coupling the different channels. Let P be the projection operator which slects out of the whole channel space a particular channel which we want to consider explicitly.The complementary operator,

$$Q = 1 - P \tag{2}$$

will therefore project on the subspace spanned by the remaining channels. According to Feshbach[2] the component $P|\Psi\rangle$ of the full wave function $|\Psi\rangle$ satisfies the Schrödinger-type equation

$$(H_f - E)\, P|\Psi\rangle = 0 \tag{3}$$

with the effective Hamiltonian H_f given by

$$H_f = PHP + PHQGQHP \tag{4}$$

In eq.(4) G is the Green function

$$G = (E - QHQ + i\epsilon)^{-1} \tag{5}$$

The generalized optical Hamiltonian for the channel we have isolated is given by

$$h = (\Phi_0|H_f|\Phi_0) = (\Phi_0|H|\Phi_0) + (\Phi_0|HQGQH|\Phi_0) \tag{6}$$

where $|\Phi_0)$ is that part of the channel wave function $P|\Psi>$ which describes the intrinsic state of the nuclear system. The first term in eq.(6) is the channel Hamiltonian. The second term , which accounts for the effects of the other channels on $P|\Psi>$, is in general a complex, energy dependent, non-local potential, whose matrix elements in the coordinate space representation we designate by $W(\underset{\sim}{r},\underset{\sim}{r}')$. In principle $W(\underset{\sim}{r},\underset{\sim}{r}')$ can be evaluated provided that the Green function, G, is known. However, finding an exact expression for G is a very difficult task and one has to resort to some approximate way to evalute $W(\underset{\sim}{r},\underset{\sim}{r}')$. In ref. 3) and 4) this is done by employing an approximate form for G.

Integrating eq.(4) over the internal degrees of freedom contained in $|\Phi_0)$ and making the usual decomposition into partial waves one obtains the Schrödinger equation for the radial wave function $u_l(r)$

$$h^0_l(r)\, u_l(r) + \int_0^\infty W_l(r,r')\, u_l(r')\, dr' = E\, u_l(r) \tag{7}$$

In eq.(7) $h^0_l(r)$ is the l-component of the channel-Hamiltonian while E is the center of mass energy.

Most optical model computer codes were developed to

specifically deal with local potentials. It is thus convenient to define for each state $u_\ell(r)$ a local potential $U_\ell(r)$,

$$U_\ell(r)\, u_\ell(r) = \int_0^\infty W_\ell(r,r')\, u_\ell(r')\, dr' \tag{8}$$

which is equivalent to $W_\ell(r,r')$. It should be clear, however, that the non-locality of $W_\ell(r,r')$ still appears in $U_\ell(r)$ through its intrinsic state dependence.

In cases where the monopole part of the Coulomb interaction is dominant, such as in low energy heavy-ion collisions, we can approximate the wave function $u_\ell(r)$ in eq. (8) by the regular Coulomb wave function $F_\ell(kr)$, where k is the asymptotic wave number in the channel considered. Such a procedure was adapted by Baltz et al[4] through which they obtained the following expression for $U_\ell(r)$

$$\bar{U}_\ell(r) = \frac{1}{F_\ell(kr)} \int_0^\infty W_\ell(r,r')\, F_\ell(kr')\, dr' \tag{9}$$

Let us define in a similar way effective local potentials for the outgoing and ingoing Coulomb waves, $H^+_\ell(kr)$ and $H^-_\ell(kr)$, respectively.

$$U^\pm_\ell(r) = \frac{1}{H^\pm_\ell(kr)} \int_0^\infty W_\ell(r,r')\, H^\pm_\ell(kr')\, dr' \tag{10}$$

where

$$H^\pm_\ell(kr) = G_\ell(kr) \pm i\, F_\ell(kr) \tag{11}$$

$G_\ell(kr)$ being the irregular Coulomb function.

Combining eqs. (9), (10) and (11) we can express $\bar{U}_\ell(r)$ in terms of $U^+_\ell(r)$ and $U^-_\ell(r)$ as

$$\bar{U}_\ell(r) = \frac{1}{2}\left\{ U^+_\ell(r) + U^-_\ell(r) \right\} + \frac{G_\ell(kr)}{2i F_\ell(kr)} \left\{ U^+_\ell(r) - U^-_\ell(r) \right\} \tag{12}$$

The advantage of this formula is that, as we shall see in the following section, it expresses the effective local potential in terms of local potentials for ingoing and outgoing waves. These waves are readily associated with classical particle trajectories and this fact will provide an alternative way to evaluate the local potential $U_\ell(r)$.

III - EFFECTIVE LOCAL POTENTIAL : SEMICLASSICAL TREATMENT

Semiclassical methods have been shown to be quite adequate in describing processes taking place in heavy ion collisions[6-10]. We shall now try to apply these ideas to the evaluation of the complex optical potential in the elastic channel resulting from the coupling to other competing channels.

The connection between the quantum mechanical description given in the previous section and the semiclassical one to be developed here is established through the association of ingoing and outgoing Coulomb wave functions with the corresponding ingoing and outgoing Rutherford trajectories of a classical charged particle (see fig. 1 for an illustration).

The local optical potential acts along these trajectories changing the amplitude for finding the system in the elastic channel. For the case of an ingoing trajectory this amplitude, $a_\ell^-(r)$, is related to the potential $U_\ell^-(r)$ through the WKB expression

$$a_\ell^-(r) = \exp\left[-\frac{i}{\hbar}\int_r^\infty \frac{U_\ell^-(r)}{\left(\frac{dr_\ell(t)}{dt}\right)} dr_\ell(t)\right] \qquad (13)$$

where $r_\ell(t)$ is the Rutherford trajectory. An equation similar to eq.(13) holds for the outgoing amplitude, $a_\ell^+(r)$, where in this case the potential $U_\ell^+(r)$ is integrated along the outgoing trajectory

(see fig.1b).

From eq.(13) it is possible to solve for $U_\ell^-(r)$ in terms of the amplitude $a_\ell^-(r)$ by first taking logarithms and then differentiating both sides of the equation with respect to r. The result is

$$U_\ell^-(r) = i\hbar \left(\frac{dr_\ell}{dt}\right) \frac{d}{dr}\left[\ln\, a_\ell^-(r)\right] \tag{14}$$

Since $r_\ell(t)$ is a Rutherford trajectory $\frac{dr_\ell}{dt}$ is simply given by

$$\frac{dr_\ell}{dt} = -\left[\frac{2}{m}\left(E - \frac{Z_1 Z_2 e^2}{r} - \frac{\hbar^2 \ell^2}{2mr^2}\right)\right]^{1/2} \tag{15}$$

where E is the center of mass energy of the system, m its reduced mass and Z_1, Z_2 the atomic numbers of target and projectile, respectively. The minus sign is chosen since we are considering the ingoing branch of the Rutherford trajectory.

A similar expression holds for $U_\ell^+(r)$.

The amplitudes $a_\ell^\pm(r)$ are smoothly varying functions of r, which results in slowly varying potentials $U_\ell^\pm(r)$. If we insert them into eq.(12) we find that the potential $\bar{U}_\ell(r)$ is a sum of two terms; a smooth term given by the average of $U_\ell^+(r)$ and $U_\ell^-(r)$ and a very rapidly oscillating term which we drop since its net effect over any distance of the same order of magnitude as the diametrical dimensions of our system is negligible.

The final expression for our effective local potential is then

$$\bar{U}_\ell(r) = \frac{i\hbar}{2}\left|\frac{dr_\ell}{dt}\right|\left\{\frac{d}{dr}\left[\ln a_\ell^+(r)\right] - \frac{d}{dr}\left[\ln\, a_\ell^-(r)\right]\right\} \tag{16}$$

This general result may be applied to any specific case provided that the amplitudes $a_\ell^\pm(r)$ can be found. We would like to remark at this point that the fact that the potential is state dependent is still present

through its dependence on the Rutherford trajectories.

IV - EFFECTIVE LOCAL POTENTIAL FOR COULOMB EXCITATION

We shall consider here the elastic scattering of a spherical projectile from an even-even deformed nucleus at energies below the Coulomb barrier. Using the method developed in section 3 we evaluate the optical potential in the elastic channel resulting from the coupling of the elastic (0^+) channel to the inelastic ($2^+, 4^+, \ldots$) channels, due to the electric quadrupole interaction. In this case the amplitudes $a_\ell^\pm(r)$ may be evaluated numerically. As a matter of fact they are an optional output of the de Boer-Winther Coulomb excitation computer code[11]. It is nevertheless, interesting to obtain an analytical expression for $\bar{U}_\ell(r)$. This may be accomplished by evaluating the amplitudes $a_\ell^\pm(r)$ in the sudden limit, where the target moment of inertia is taken to be infinite. In this limit $a_\ell^\pm(r)$ are given by[7]

$$a_\ell^\pm(r) = \frac{1}{4\pi} \int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta \, d\beta \exp\left[\frac{-i}{\hbar} \int_{-\infty}^{t_\ell^\pm(r)} \frac{Z_1 Q^{(2)} e^2 \, P_2(\cos\chi_\ell^\pm(t))}{2 \, r_\ell^3(t)} dt\right] \quad (17)$$

(from now on we indicate with superscript +(-) quantities evaluated on the outgoing (ingoing) branches of the Rutherford trajectory). In the above expression α, β are the Euler angles that specify the orientation of the target's summetry axis, $Q^{(2)}$ is the intrinsic quadrupole moment of the target, $P_2(x)$ is the Legendre polynomial of order two and $\chi_\ell^\pm(t)$ are the angles subtended by the target symmetry axis and the line connecting the centers of the two nuclei. The expression for $\cos\chi_\ell^\pm(t)$ is

$$\cos\chi_\ell^\pm(t) = \cos\beta\cos\theta_\ell^\pm(t) + \sin\beta\sin\theta_\ell^\pm(t)\cos(\alpha - \varphi_\ell^\pm(t)) \quad (18)$$

where $\theta_\ell^{\pm}(t)$, $\varphi_\ell^{\pm}(t)$ are the spherical polar angles that determine the orientation of the line joining the centers of the colliding nuclei. Finally $t_\ell^-(r)$ and $t_\ell^+(r)$ are the times at which the distance between the centers is r for the ingoing and outgoing branches of the trajectory, respectively.

Using eq.(17) in eq.(16) we obtain

$$\bar{U}_\ell(r) = \frac{Z_1 Q^{(2)} e^2}{4 r^3} \sum_{\pm} \left\{ \left(\int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta\, d\beta\, P_2(\cos\chi^{\pm}(t_\ell^{\pm}(r)) \times \right.\right.$$

$$\times \exp\left[-\frac{i}{\hbar} \int_{-\infty}^{t_\ell^{\pm}(r)} \frac{Z_1 Q^{(2)} e^2}{2 r_\ell^3(t)} P_2(\cos\chi^{\pm}(t))dt \right] \Bigg) \times \quad (19)$$

$$\times \left(\int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta\, d\beta \quad \times \right.$$

$$\left.\left. \times \exp\left[-\frac{i}{\hbar} \int_{-\infty}^{t_\ell^{\pm}(r)} \frac{Z_1 Q^{(2)} e^2}{2 r_\ell^3(t)} P_2(\cos\chi^{\pm}(t))\, dt \right] \right)^{-1} \right\},$$

where the summation above has two terms; one corresponding to the ingoing (-) and the other one to the outgoing(+) branch of the trajectory. It is interesting to remark that in the limit of small values of the coupling, eq.(19) leads to (see Appendix)

$$\bar{U}_\ell^{(2)}(r) = -i \frac{(Z_1 Q^{(2)} e^2)^2}{80 \hbar v} \left\{ \left[\frac{\eta^4 (3\ell^2+\eta^2)}{\ell^2(\ell^2+\eta^2)^2} - \frac{\eta^3}{\ell^3} \mathrm{Arctan}\left(\frac{\ell}{\eta}\right) \right] \frac{1}{a^2 r^3} + \right.$$

$$+ \left[\frac{4\eta^2 \ell^2}{(\ell^2+\eta^2)^2} \right] \frac{1}{a r^4} + \left[\frac{2\ell^4}{(\ell^2+\eta^2)^2} \right] \frac{1}{r^5} \Bigg\} \quad (20)$$

where $\eta = (Z_1 Z_2 e^2/\hbar v)$ is the Sommerfeld parameter and $a = \frac{Z_1 Z_2 e^2}{2E}$ is half the distance of closest approach for head-on collision. The above potential is identical to the potential of Baltz et al (eq.(8) of ref.4)). However we do not expect that for large values of $Q^{(2)}$ the potential $\bar{U}_\ell^{(2)}(r)$ to be a good approximation to $\bar{U}_\ell(r)$. Physically this means that in taking the lowest order approximation we have neglected multiple Coulomb excitation effects. This is clearly seen if we consider the type of approximation made by Baltz et al. These authors replaced the full interacting Green function, G, by the Coulomb Green function G_0.

Since G is given by the series

$$G = G_0 + G_0 \tilde{V} G_0 + G_0 \tilde{V} G_0 \tilde{V} G_0 + \cdots , \qquad (21)$$

where $\tilde{V} = QVQ$, the approximation of ref.4) amounts to neglecting $G_0 \tilde{V} G_0$, $G_0 \tilde{V} G_0 \tilde{V} G_0$, ... , which are the terms containing multiple Coulomb excitation effects.

To illustrate this point we have evaluated $\bar{U}_\ell(r)$ for $\ell = 0$ (head-on collision). In this case the radial dependence of the potential goes as r^{-3} and eq.(20) may be written as

$$\bar{U}_0^{(2)}(r) = -i \frac{8 q_2^2 a^3}{15 \eta} \frac{E}{r^3} \qquad (22)$$

where

$$q_2 = \frac{z_1 Q^{(2)} e^2}{4 \hbar v a^2} \qquad (23)$$

is the quadrupole strength parameter.

Writing

$$\bar{U}_0(r) = - \frac{\vartheta(q_2) + i \omega(q_2)}{\eta} \frac{a^3}{r^3} E \qquad (24)$$

we have numerically evaluated $\vartheta(q_2)$ and $\omega(q_2)$. The results are shown in fig.2, together with the approximate form $\omega^{(2)}(q_2) \equiv i \bar{U}_0^{(2)}(r) \left(\frac{\eta r^3}{a^3 E} \right)$

We notice that for small values of q_2 the potential $\bar{U}_0(r)$ is almost purely imaginary and very close to $\bar{U}_0^{(2)}(r)$. As q_2 increases the real part of $\bar{U}_0(r)$ starts developing and becomes larger in magnitude than the imaginary part. However its values will still be less than 1% of the predominant Coulomb potential for a typical heavy ion collision. The imaginary part of the potential starts departing from $\bar{U}_0^{(2)}(r)$ at $q_2 \tilde{\sim} 1.2$, which corresponds to the onset of multiple Coulomb excitation , and at large values of q_2 ($q_2 \gtrsim 3$) it is seen to be quite different.

V - CONCLUSIONS

In the present work a semiclassical method has been developed to calculate contributions to the optical potential in the elastic channel resulting from the coupling to other open channels. Our eq.(12) for the effective local petential has been shown to lead to an expression involving the semiclassical amplitudes of eq.(16). Given these amplitudes one is able, in principle, to evaluate the potential $\bar{U}_\ell(r)$ which accounts for the effects of the coupling to other channels. We have applied the formalism developed here to the specific case of Coulomb excitation. The amplitudes $a_\ell^{\pm}(t)$ were evaluated in the sudden limit and a closed expression for the potential was found for a system with small deformation. This expression was found to be identical in form to that of Baltz et al[4] . Since our method involves no other assumption than those implicit in the semiclassical limit, and it was found to work correctly in this lowest order comparison with the quantum mechanical calculation discussed above, it is therefore expected to give adequate results in the general case of strong quadrupole coupling. As we have mentioned in the text, the Coulomb excitation amplitudes needed for the calculation can easily be obtained from a widely used computer code[11].

Further applications of this semiclassical method are presently under way.

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Appendix

Weak coupling limit of Eq. (19).

By expanding Eq. (19) to lowest order in $Q^{(2)}$ we obtain

$$\bar{U}_\ell(r) = -i \frac{(Z_1 Q^{(2)} e^2)^2}{32\pi\hbar r^3} \sum_{\pm} \int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta\, d\beta \times$$

$$\times P_2(\cos \chi^{\pm}(t_\ell^{\pm}(r))) \int_{-\infty}^{t_\ell^{\pm}(r)} dt \frac{P_2(\cos(\chi^{\pm}(t))}{r^3(t)} \qquad \text{(A-1)}$$

Now we express the angles $\chi^{\pm}(t_\ell^{\pm}(r))$ and $\chi^{\pm}(t)$ in terms of α and β as indicated in eq. (18), and we choose a coordinate system such that the Rutherford trajectory lies on the XZ plane and it is bisected by the Z axis. We then obtain after integrating over α and β.

$$\bar{U}_\ell(r) = -i \frac{(Z_1 Q^{(2)} e^2)^2}{32\pi\hbar r^3} \frac{4\pi}{5} \times$$

$$\times \sum_{\pm} \int_{-\infty}^{t_\ell^{\pm}(r)} dt \frac{P_2[\cos(\theta^{\pm}(t_\ell^{\pm}(r)) - \theta^{\pm}(t))]}{r^3(t)} \qquad \text{(A-2)}$$

which may be rewritten in the simpler form

$$\bar{U}_\ell(r) = -i \frac{(Z_1 Q^{(2)} e^2)^2}{40\hbar r^3} \int_{-\infty}^{+\infty} dt \frac{P_2[\cos(\theta(r) - \theta(t)]}{r^3(t)} \qquad \text{(A-3)}$$

where $\theta(r) = \theta^+(t_\ell^+(r)) = \theta^-(t_\ell^-(r))$, and now $\theta(t)$ sweeps the full Rutherford trajectory.

Eq. (A-3) may be expressed in terms of orbital integrals as follows

$$\bar{U}_\ell(r) = -i \frac{(Z_1 Q^{(2)} e^2)^2}{40\hbar r^3} \left[\frac{1}{2} \sin^2\theta(r) \int_{-\infty}^{\infty} \frac{dt}{r^3(t)} + \right.$$

$$\left. + (2\cos^2\theta(r) - 1) \int_{-\infty}^{\infty} dt \frac{P_2(\theta(t))}{r^3(t)} \right] \qquad \text{(A-4)}$$

Since we are over a Rutherford trajectory, all quantities may be given analytically;

$$\cos\vartheta(r) = \frac{1 + \ell^2 a/\eta^2 r}{\sqrt{1+\ell^2/\eta^2}} \tag{A-5}$$

$$\int_{-\infty}^{+\infty} dt \frac{P_2(\theta(t))}{r^3(t)} = \frac{1}{2va^2}\left[\frac{1}{1+\ell^2/a^2} + \right.$$
$$\left. + \frac{\eta^2}{\ell^2}\left(1-(\eta/\ell)\,\mathrm{Arctan}(\ell/\eta)\right)\right] \tag{A-6}$$

$$\int_{-\infty}^{+\infty} \frac{dt}{r^3(t)} = \frac{1}{va^2}\frac{\eta^2}{\ell^2}\left(1-(\eta/\ell)\,\mathrm{Arctan}(\ell/\eta)\right) \tag{A-7}$$

All quantities appearing in eqs. A-5,6,7 are defined in the text. Substituting these equations into A-4 we obtain, after ordering in increasing powers of r

$$\bar{U}_\ell^{(2)}(r) = -i\frac{(Z_1 Q^{(2)} e^2)^2}{8\pi\hbar v}\left[\left(\frac{\eta^4(3\ell^2+\eta^2)}{\ell^2(\ell^2+\eta^2)^2} - \right.\right.$$
$$\left.\left. - \frac{\eta^3}{\ell^3}\,\mathrm{Arctan}\frac{\ell}{\eta}\right)\frac{1}{a^2 r^3} + \right. \tag{20}$$
$$+ \frac{4\eta^2\ell^2}{(\ell^2+\eta^2)^2}\frac{1}{a r^4} +$$
$$\left. + \frac{2\ell^4}{(\ell^2+\eta^2)^2}\frac{1}{r^5}\right]$$

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FIGURE CAPTIONS

Fig.1

Terms contributing to the optical nuclear potential: (a) ingoing wave (b) outgoing wave.

Fig.2

Exact , $\ell = 0$, optical potential of eq.(24) vs. q_2

Also shown is the lowest order potential of eq.(22) .

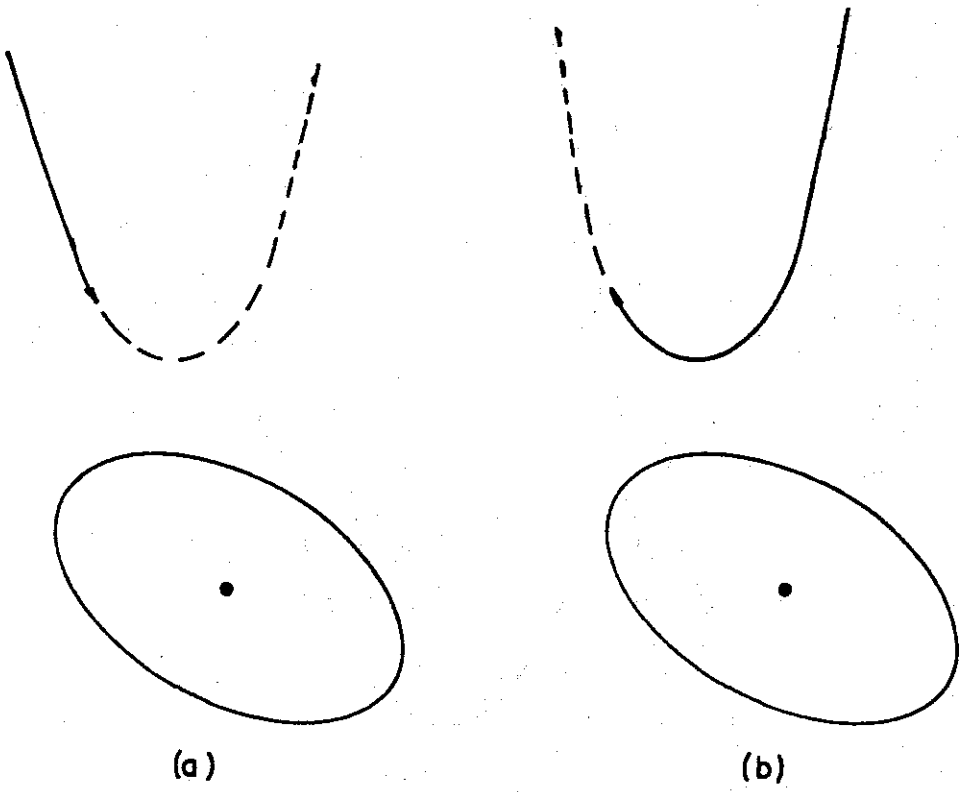

FIG.1

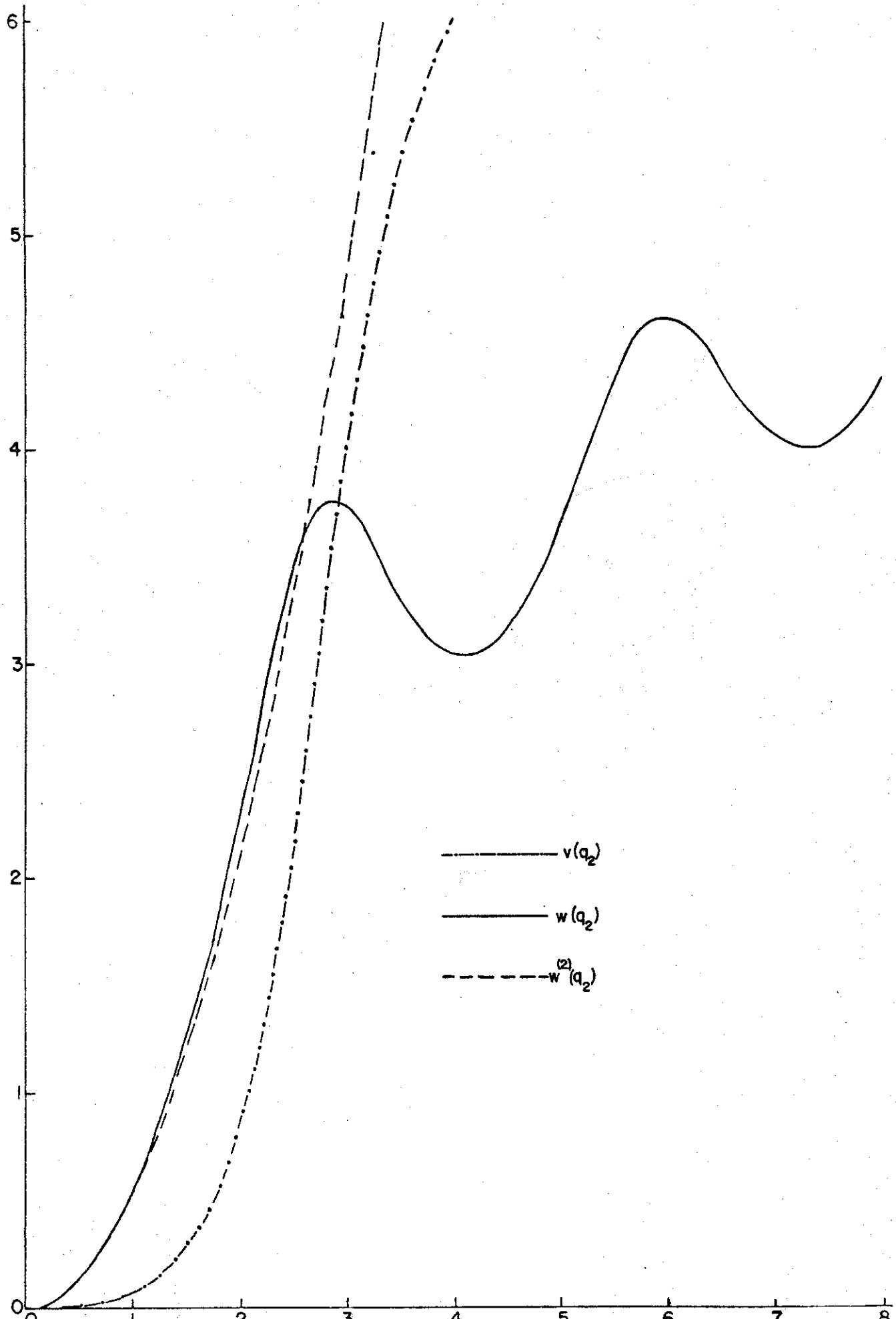

FIG. 2