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by

Ivan Ventura
Instituto de Física - Universidade de São Paulo

UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

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Ivan Ventura

Universidade de São Paulo, Instituto de Física, C.P. 20516, São Paulo, São Paulo, Brasil.

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MACROSCOPIC QUANTUM WAVES IN NON LOCAL THEORIES

Ivan Ventura

Universidade de São Paulo, Instituto de Física.

ABSTRACT: By means of an expansion in the density, it is shown that Macroscopic Quantum Waves also appear in non local theories. This result reinforces my conjecture that these waves should exist in liquid ^{4}He.

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Some months ago, I presented a microscopic description of superfluidity[1,2], which shed new light on the λ-transition mechanism and on the nature of the liquid Helium quasi-particles. I showed that, in the $|\phi|^4$ case - i.e., when the atomic potential is a delta function - , Bogoliubov's theory of superfluidity[3] exhibits some macroscopic quantum waves (MQWs) that carry a topological charge. In states of multi - MQWs, the condensate is cut in domains of phase, analogous to magnetic domains. This fact may explain the existence of the λ-point, and a crude estimate of its temperature gave $T_\lambda \simeq 2 - 4$ K. In addition, bound to the MQWs, there is a new type of elementary excitations whose spectrum is very similar to that of liquid Helium[1,2].

In a subsequent paper[4], I proved that MQWs appear not only in the $|\phi|^4$ theory, but in a large set of local models, namely the $\lambda|\phi|^n$ models, with $\lambda>0$ and $n>2$.

Here, I will show that these topological waves exist in non local theories as well.

If $V(\vec{x})$ is the interaction potential between two ^{4}He atoms, the classical equation of motion of Bogoliubov's theory is given by[3]:

$$i\partial_t \phi(\vec{x}) = -\frac{1}{2m}\nabla^2\phi(\vec{x}) + \int d\vec{x}'\phi^*(\vec{x}')\phi(\vec{x}')V(\vec{x}-\vec{x}')\phi(\vec{x}) \tag{1}$$

Here $\phi(\vec{x})$ is the ^{4}He atom field, and m the ^{4}He mass.

Let me take $V(\vec{x})$ to be integrable,

$$\lambda = \int d\vec{x}V(\vec{x}), \tag{2}$$

considering always $\lambda>0$.

For a system of density ρ, Bogoliubov's condensate is represented by the following solutions of (1):

$$\Omega_{\theta_o} = \sqrt{\rho}\ \exp - i(\lambda\rho t-\theta_o) \tag{3}$$

where θ_o is a real constant. The arbitrariness in the choice of θ_o leads to a ground state degeneracy of infinity degree.

Phonons of large wave length travel in the condensate with a velocity given by $c = \sqrt{\lambda\rho/m}$.

If x is a particular coordinate, MQWs are solutions of Eq. (1) such that

$$\lim_{x\to-\infty} W = \Omega_{\theta_1}\ , \text{ and } \lim_{x\to+\infty} W = \Omega_{\theta_2} \tag{4}$$

$\theta_2-\theta_1$ is a topological charge which makes the MQW stable.

Let V be a real number in the interval $-1\leq V\leq 1$. In order to look for MQWs I define

$$\xi = mc(x-cVt); \quad \xi' = mc(x'-cVt), \tag{5}$$

and

$$U(\xi-\xi') = \frac{1}{mc\lambda} \int dydz\ V(\vec{x}-\vec{x}') \tag{6}$$

(This integral is done on the transverse coordinate only), and seek solutions of the type

$$W_V = \sqrt{\rho s(\xi)}\ \exp i\theta(\xi)\ \exp(-i\lambda\rho t) \tag{7}$$

The boundary conditions consistent with (5) are:

$$\lim_{|\xi|\to\infty} s(\xi) = 1, \quad \text{and} \quad \lim_{|\xi|\to\infty} \frac{d\theta}{d\xi} = 0 \tag{8}$$

Inserting W_V into the equation of motion (1), and using (8), one gets, after a simple integration:

$$\theta(\xi) = V \int_o^\xi \{1 - \frac{1}{s(\xi')}\}\ d\xi' \tag{9a}$$

and

$$\frac{1}{2}\,\partial_\xi^2 \sqrt{s} + \frac{V^2}{2}(1 - \frac{1}{s^2})\sqrt{s} + \sqrt{s} = \int d\xi'\ U(\xi-\xi')\,s(\xi')\sqrt{s(\xi)} \tag{9b}$$

This last equation is equivalent to a problem of classical mechanics where the force at any instant depends upon the position at all others.

When $V(x)$ has spherical symmetry, $U(\xi)=U(|\xi|)$, so that

$$\int d\xi' \; U(\xi-\xi') s(\xi') = F(\partial_\xi^2) s(\xi) , \tag{10a}$$

where

$$F(\partial_\xi^2) = \sum_{n=0}^{\infty} \frac{a_n}{(2n)!} \partial_\xi^{2n} , \tag{10b}$$

and the coefficients a_n are:

$$a_n = \int d\xi U(\xi) \; \xi^{2n} = \frac{(m\lambda\rho)^n}{\lambda} \int d^3\vec{x} \; x^{2n} \; V(\vec{x}) \tag{10c}$$

Notice, then, that Eq.(10b) is an <u>expansion in the density</u>.

As an example of the operator $F(\partial_\xi^2)$, consider the one associated with the Yukawa potential $g \exp(-\Delta r)/r$:

$$F(\partial_\xi^2) = \{1 - (m\lambda\rho/\Delta^2)\partial_\xi^2\}^{-1} = 1 + \frac{m\lambda\rho}{\Delta^2} \partial_\xi^2 + 0(\frac{\rho^2}{\Delta^4}) \tag{11}$$

When the density is small one can solve Eq.(9b) discarding terms of order ρ^2 or larger. This procedure provides the first non local corrections to the problem.

$$\frac{1}{2}\partial_\xi^2 \sqrt{s} + \frac{V^2}{2}(1-\frac{1}{s^2})\sqrt{s} + \sqrt{s} = s \sqrt{s} + \frac{a_1}{2} \partial_\xi^2 s \sqrt{s} \tag{12}$$

Multiplying this equation by $8 \sqrt{s}\, \partial_\xi s$, and using conditions (8), one gets

$$(\partial_\xi s)^2 + \frac{4.\,(1-s)^2 (V^2-s)}{(1-4a_1 s)} = 0 \tag{13}$$

Now, take $|a_1| \ll 1$ (condition compatible with low densities). Integrating Eq. (13) is analogous to studying the frictionless motion of a particle in one dimension.

When $|V|<1$, the potential producing this motion is shown in fig. 1. In this subsonic case there are solutions $s(\xi)$ such that: $s(-\infty)=1=s(+\infty)$, and $1 \geq s(\xi) \geq a$. These solutions obey the boundary condition (8) and hence describe macroscopic quantum waves.

In fig. 2 one sees the potential of a supersonic situation ($|V|>1$). In this case the only solution consistent with (8) is $s(\xi)=1$. Therefore, as happens in local theories, there is no supersonic MQWs.

The results presented here and those of ref. (4) show that MQWs are a common phenomenon of nonrelativistic bosonic theories. This conclusion legitimates and reinforces my proposal that they should exist in liquid Helium.

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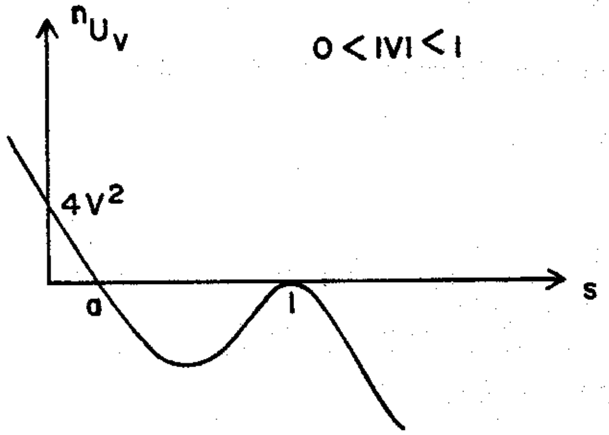

fig. 1

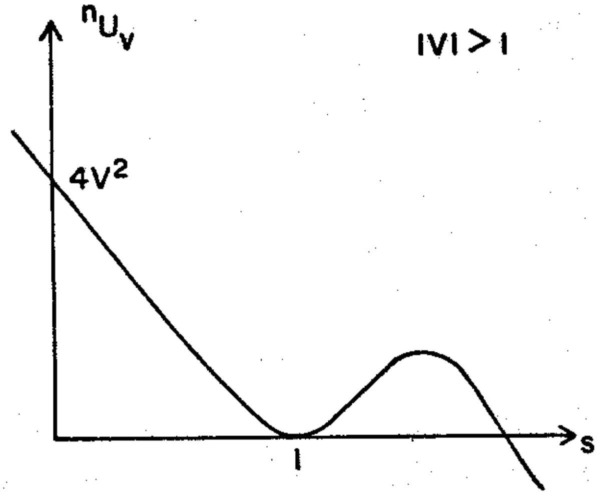

fig. 2