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# THE STABILITY OF MACROSCOPIC QUANTUM WAVES

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## ABSTRACT

We show that, in the $|\phi|^4$ theory, the macroscopic quantum waves are stable against small perturbations.

In two previous papers[1,2], we proposed a new strategy to understand Helium II superfluidity. Our ideas came from the observation that Bogoliubov's superfluidity theory[3] - in the $|\phi|^4$ approximation - describes some topological macroscopic quantum waves (MQWs), which cut the condensate into sectors of phase, analogous to magnetic domains. This provides a nice insight into the origin of the $\lambda$-transition. We noticed also that quasi-particles bound to MQWs have an spectrum similar to that of liquid Helium[1,2].

More recently, we showed that MQWs occur in a large class of local[4] and non local[5] theories - coming these results to reinforce our original proposal.

The topological charge these waves carries is itself an strong evidence of their stability[(1,2,4)]. Nevertheless, it is also very instructive to understand this fact in terms of stability angles. By using the language of Dàshen, Hasslacher and Neveu[6] (DHN), we confirm here that, at least in the realm of the $|\phi|^4$ theory, the MQWs are indeed stable objects.

The equation of motion in the $|\phi|^4$ theory is:

$$i\partial_t\phi = -\frac{1}{2m}\nabla^2\phi + \lambda\phi^*\phi^2 \quad . \tag{1}$$

$\phi$ is the $^4$He field, and $m$ its mass.

In a system of density $\rho$, the velocity of phonons with large wave length (which we call the sound velocity) is[1-3] $c = \sqrt{\lambda\ \rho/m}$ .

Consider the real members $V$ such that $|V| \leq 1$ , and define $\gamma = \sqrt{1-V^2}$ . If $x$ is a particular coordinate and $\xi = mc\,(x - cVt)$ , the MQWs are the following solutions of (1):

$$W_V = (V - i\gamma\,\mathrm{tgh}\,\gamma\xi)\,\sqrt{\rho}\,\exp\,(-\,i\lambda\rho t) \quad . \tag{2}$$

$W_V$ describes a sheet of low density moving in the condensate with velocity $cV$. Its topological charge is[1,2] $2 \arccos V$ — the difference between the phases of $W(+\infty)$ and $W(-\infty)$.

To study the stability of $W_V$ in the manner of DHN we add to it a small fluctuation $\eta \exp\{-i\lambda\rho t\}$, defining the field:

$$\{(V - i\gamma \operatorname{tgh} \gamma\xi)\sqrt{\rho} + \eta\} \exp(-i\lambda\rho t) . \tag{3}$$

Plugging this expression in Eq. (1), and retaining only terms of first order in $\eta$, one gets the equation of motion of the fluid elementary excitations in the presence of a MQW:

$$(i/mc^2)\partial_t \eta = (-\frac{1}{2m^2c^2} \nabla^2_{\vec{s}} - \frac{1}{2} \partial^2_\xi + iV\partial_\xi + 1 - 2\gamma^2 \operatorname{sech}^2 \gamma\xi)\eta +$$

$$+ (V^2 - \gamma^2 + \gamma^2 \operatorname{sech}^2 \gamma\xi - 2iV\gamma \operatorname{tgh} \gamma\xi)\eta^* , \tag{4}$$

where $\vec{s} = (y,z)$ is made up by the coördinates transverse to the MQW's motion; $\nabla^2_{\vec{s}}$ is the transverse Laplacian; and $\eta = \eta(\xi,\vec{s},t)$.

Since there is invariance under translations on the transverse directions, and since Eq. (4) contains $\eta$ as well as $\eta^*$, it is convenient to represent the eigensolutions of (4) in the following way[1,2]:

$$\eta_{\vec{k}} = u(\xi) \exp i(\vec{k}\vec{s} - \omega t) + v(\xi) \exp i(-\vec{k}\vec{s} + \omega t) \tag{5}$$

$\vec{k}$ is the transverse wave number and $\omega$ is the fluctuation frequency.

To verify the MQW's stability it must be shown that $\omega$ is always a real number. In refs. (1) and (2), when quantizing $\eta$ by means of the Bohr-Sommerfeld rule, we have implicitly assumed $\omega$ to be real, otherwise the fluctuation should not have periodicity to make the rule useful. Of course, such an assumption was legitimated by the existence of topological charges, but it is still deserving an explicit proof. In what follows we shall show that $\omega$ is in fact always real.

Inserting $\eta_{\vec{k}}$, given by Eq. (5), in Eq. (4), we obtain a system of coupled equation for $u(\xi)$ and $v(\xi)$:

$$\hat{\Sigma}\,(V,k)\psi \;=\; \frac{\omega}{mc^2}\,\sigma_3\,\psi\,. \tag{6a}$$

Here $\psi$ is an "spinor" defined in terms of $u(\xi)$ and $v^*(\xi)$:

$$\psi \;=\; \begin{pmatrix} u(\xi) \\ v^*(\xi) \end{pmatrix}\,, \tag{6b}$$

and the operator $\hat{\Sigma}$ is given by

$$\hat{\Sigma}(V,k) \;=\; (-\tfrac{1}{2}\,\partial_\xi^2 - 2\,\gamma^2\,\mathrm{sech}^2\,\gamma\xi + 1 + \frac{k^2}{2m^2c^2})I \;+$$

$$+\;(iV\partial_\xi)\sigma_3 \;+\; (V^2-\gamma^2+\gamma^2\,\mathrm{sech}^2\,\gamma\xi)\sigma_1 + (2V\gamma\;\mathrm{tgh}\;\gamma\xi)\sigma_2\,. \tag{6c}$$

$I$ is the identity matrix and $\{\sigma_i\}$ are the Pauli matrices.

To show that $\omega$ is a real number we notice that Eq.(6a) can be written as

$$T^\dagger T\,\psi \;=\; R\,\psi \tag{7}$$

where the operators $T$ and $R$ are respectively

$$T \;=\; \frac{1}{\sqrt{2}}\,(-i\partial_\xi\sigma_2 + 2\,\gamma\;\mathrm{tgh}\;\gamma\xi\sigma_3 - iV\sigma_1) \tag{8}$$

and

$$R = (\omega/mc^2)\sigma_3 + (1-2\gamma^2)\sigma_1 + \frac{1}{2}\{1-3\gamma^2+(k/mc)^2\}I \qquad (9)$$

From Eq. (7) one concludes that R is a self-adjoint operator. Inspecting then Eq. (9) one observes that this fact makes $\omega$ to be always real.

We have therefore shown that, in the $|\phi|^4$ theory, MQWs are stable against infinitesimal perturbations (as already expected).

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