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by

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SPONTANEOUS BREAKDOWN OF SYMMETRY IN COSMOLOGICAL MODELS

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1. Recently Frolov, Grib and Mostepanenko[1] studied the spontaneous breakdown of the global gauge symmetry of the $\lambda(\phi^*\phi)^2$ theory defined on a curved space-time, precisely that of the open Friedmann model.

In this note, based upon their work, we show that the problem of the stability (in the energetic sense) of the solutions of the differential equation satisfied by the vacuum expectation value of the bosonic field, can be completely solved, showing not only that the solutions of Ref.(1) correspond to the minimum value of the energy density, but that there are no local minima or false vacua, in the sense of Coleman[2]. We repeat the analysis for the closed Friedmann model, where no breakdown of symmetry occurs, and also discuss the role of the term $\frac{R}{6}\phi^*\phi$ present in the lagrangian.

The open Friedmann model is described by the metric

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2 - \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{1}$$

where the notation of Landau, Lifshitz[3] is used. The bosonic lagrangian is

$$L = \sqrt{-g}\left[g^{ik}\frac{\partial\phi^*}{\partial x^i}\frac{\partial\phi}{\partial x^k} + \frac{R}{6}\phi^*\phi - \frac{\lambda}{6}(\phi^*\phi)^2\right] \tag{2}$$

giving the Euler-Lagrange equations

$$\Box\phi(x) - \frac{R}{6}\phi(x) + \frac{\lambda}{3}\phi^*(x)\,\phi^2(x) = 0 \tag{3}$$

Use of spatial homogeneity and C invariance allows one to write for

$$g(\eta) = \langle 0|\phi(x)|0\rangle$$

the equation (tree approximation)

$$\ddot{g} + 2\,\frac{\dot{a}}{a}\,\dot{g} + \left(\frac{\ddot{a}}{a} - 1\right) g + \frac{\lambda a^2}{3}\, g^3 = 0 \tag{4}$$

where a dot stands for differentiation with respect to η.

Putting

$$g(\eta) = \sqrt{\frac{3}{\lambda}}\,\frac{f(\eta)}{a(\eta)}$$

equation (4) transforms into

$$\ddot{f} - f + f^3 = 0 \tag{5}$$

which can be solved exactly. In fact, from the first integral

$$\dot{f}^2 - f^2 + \frac{f^4}{4} = C \tag{6}$$

one gets the general solution

$$\eta = \int \left(f^2 - \frac{f^4}{2} + C\right)^{-1/2} df \tag{7}$$

which, for

$$\{1 + (1 + 2C)^{1/2}\}^{1/2} > f \geq 0 \tag{8}$$

gives

$$\{1 + (1+2C)^{1/2}\}\, f^2(\eta) = 1 - \mathrm{sn}^2 \left\{ (1 + 2C)^{1/4}\,\eta,\ \frac{\{1+(1+2C)^{1/2}\}^{1/2}}{\sqrt{2}\,(1+2C)^{1/4}} \right\} \tag{9}$$

where sn is the Jacobian elliptic function called sine-amplitude. This is the general solution of (5). There is, of course, another (trivial) additive constant which was omitted, as it only redefines the origin of η.

The energy density is given by

$$\varepsilon(\eta) = \langle 0 | T^0{}_0 | 0 \rangle \tag{10}$$

where

$$T_{ik} = \frac{\partial\phi^*}{\partial x^i}\frac{\partial\phi}{\partial x^k} + \frac{\partial\phi^*}{\partial x^k}\frac{\partial\phi}{\partial x^i} - g_{ik}\frac{L}{\sqrt{-g}} - \frac{1}{3}\left[-R_{ik} + \nabla_i\nabla_k - g_{ik}\Box\right]\phi^*\phi \tag{11}$$

It is straightforward to show that

$$\varepsilon(\eta) = \frac{3C}{\lambda a^4} \tag{12}$$

which is our most important relation. It gives $\varepsilon(\eta)$ directly in terms of C, which characterizes every solution. We thus see that the solutions corresponding to the smallest value of the energy are those corresponding to the smallest value of C allowed by equation (8), that is $C = -\frac{1}{2}$. The solutions are $f = \pm 1$, in agreement with Ref.(1). We can also see that no local minima exist, so that false vacua cannot be present.

2. The case of the closed Friedmann model is treated similarly. The line element is

$$ds^2 = a^2(\eta)\{d\eta^2 - d\chi^2 - \sin^2\chi\,(d\theta^2 + \sin^2\theta\, d\phi^2)\} \tag{13}$$

and the lagrangian is the same. Instead of equation (5) one gets

$$\ddot{f} + f + f^3 = 0 \tag{14}$$

and

$$\dot{f}^2 + f^2 + \frac{f^4}{2} = C \tag{15}$$

replaces equation (6). The energy density now is

$$\varepsilon(\eta) = \frac{3C}{\lambda a^4} \tag{16}$$

From equation (15) it is clear that $C \geq 0$. The lowest value of $\varepsilon(\eta)$ is zero, and it corresponds to $f=0$. Hence, no spontaneous breakdown of symmetry occurs.

3. Consider lagrangians with a different dependence on R, e.g.,

$$L = \sqrt{-g} \left[g^{ik} \frac{\partial \phi^*}{\partial x^i} \frac{\partial \phi}{\partial x^k} + p\, R\, \phi^* \phi - \frac{\lambda}{6} (\phi^* \phi) \right] \tag{17}$$

where p is a constant number. The corresponding energy-momentum tensor is

$$T_{ik} = \frac{\partial \phi^*}{\partial x^i} \frac{\partial \phi}{\partial x^k} + \frac{\partial \phi^*}{\partial x^k} \frac{\partial \phi}{\partial x^i} - g_{ik} \frac{L}{\sqrt{-g}} - 2p \left[-R_{ik} + \nabla_i \nabla_k - g_{ik} \Box \right] {}^* \phi \tag{18}$$

while the equation for the vacuum expectation value is

$$\ddot{g} + 2 \frac{\dot{a}}{a} \dot{g} - 6p \left(1 - \frac{\ddot{a}}{a}\right) g + \frac{\lambda a^2}{3} g^3 = 0 \tag{19}$$

The energy density is

$$\varepsilon(\eta) = \frac{3f^2}{\lambda a^4} \left[\frac{\dot{f}^2}{f^2} + 2(6p - 1) \frac{\dot{f}\dot{a}}{fa} + (1 - 6p) \frac{\dot{a}^2}{a^2} + \frac{f^2}{2} - 6p \right] \tag{20}$$

where, as before, we put $g = \sqrt{3/\lambda}\, \frac{f}{a}$.

To simplify things, let us choose an equation of state for the matter that generates the (external) gravitational field. Let it be

$$p_M = \frac{\varepsilon_M}{3} \tag{21}$$

where p_M and ε_M are, respectively, the pressure and energy density of the matter. In this case, R=0, and

$$\ddot{a} = a \tag{22}$$

and

$$\dot{a}^2 - a^2 = K \tag{23}$$

K being a constant. Using (15), which turns out to be true also in this case, and (23), the energy density is written

$$\varepsilon(\eta) = \frac{3}{\lambda a^4}\left\{C + (1-6p)\left[(2 + \frac{K}{a^2})f^2 - 2\sqrt{1 + \frac{K}{a^2}}\,\dot{f}\,f\right]\right\} . \qquad (24)$$

Now, the smallest value of C is -1/2 and, in this case, f equals 1, while $f\dot{f}$ vanishes. Therefore,

$$\varepsilon(\eta) = \frac{3}{\lambda a^4}\left[-\frac{1}{2} + (1-6p)(2 + \frac{K}{a^2})\right] . \qquad (25)$$

Einstein's equations give

$$8\pi k\ \varepsilon_M = \frac{3}{a^4}(\dot{a}^2 - a^2) = \frac{3}{a^4}K \qquad (26)$$

so that K > 0. In order to have spontaneous breakdown of symmetry one must have $\varepsilon(\eta) < 0$, that is

$$1 - 6p < \frac{1}{2(2 + \frac{K}{a^2})} \qquad (27)$$

which, by using (26), gives the following sufficient condition for the existence of spontaneous breakdown of symmetry:

$$p > \frac{1}{6} - \frac{1}{12(2 + \frac{8\pi k\varepsilon_M a^2}{3})} . \qquad (28)$$

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