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ABSENCE OF PAIR PRODUCTION IN THE CP^{n-1} MODEL

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ABSTRACT

We prove that CP^{n-1} model doesn't accommodate pair formation up to third order perturbation theory.

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The CP^{n-1} model is known to possess, at classical level, an infinite number of conservations laws, in such a way that it probably shares the known properties of non-linear σ and Gross-Neveu models. However, no one has ever succeed in building the S-matrix of the model. It was even suspected that the model, at quantum level didn't present factorization. In this paper, we take advantage of the assymptotic freedom of the model, and construct perturbation theory, (which is valid at high energies) to show that pair formation doesn't occur up to third order.

The lagrangian of the model is given by[7]

$$\mathcal{L} = \partial_\mu \bar{z} \partial^\mu z + \frac{g}{2} (\bar{z} \overleftrightarrow{\partial_\mu} z)^2 \tag{1}$$

where $z = (z_1 \cdots z_n)$ (2)

$$\bar{z} z = \sum_{i=1}^{n} \bar{z}_i z_i = \frac{1}{2g} \tag{3}$$

$$\bar{z} \overleftrightarrow{\partial_\mu} z = \sum_i (\bar{z}_i \partial_\mu z - (\partial_\mu \bar{z}_i) z_i) \tag{4}$$

We have, at our disposal a gauge symmetry, which can be used in order to choose z_n real. In such a case we have:

$$z_n = \sqrt{\frac{1}{2g} - \bar{z}z} \tag{5}$$

where $\bar{z}z = \sum_{i=1}^{n-1} \bar{z}_i z_i$ (6)

Substituting into the lagrangian density, and noting that

$$\bar{z} \overleftrightarrow{\partial_\mu} z = \bar{z} \overleftrightarrow{\partial_\mu} z \tag{7}$$

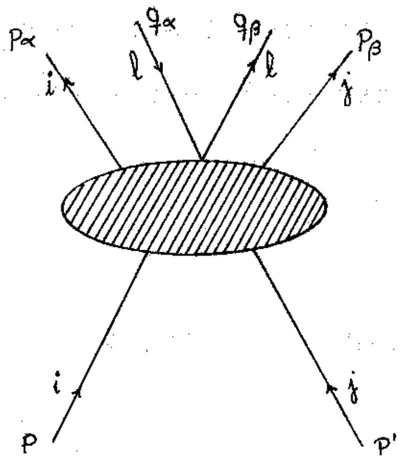
we have

$$\mathcal{L} = \partial_\mu \bar{z}\, \partial^\mu z + \frac{g}{2}\left(\bar{z} \overleftrightarrow{\partial_\mu} \dot{z}\right)^2 + \frac{1}{4}\,\frac{\left[\partial_\mu (\bar{z}z)\right]^2}{\frac{1}{2g} - \bar{z}z} \tag{8}$$

Which is appropriate to perturbation theory; we can expande in g:

$$\mathcal{L} = \partial_\mu \bar{z}\, \partial^\mu z + \frac{g}{2}\left[\left(\bar{z} \overleftrightarrow{\partial_\mu} z\right)^2 + \left(\partial_\mu (\bar{z}z)\right)^2\right] + g^2 \bar{z}z \left[\partial_\mu (\bar{z}z)\right]^2 + \mathcal{O}(g^3) \tag{9}$$

We handle with the following amplitude



We introduce the renormalization mass μ , such that:

$$p^2 = p'^2 = \cdots = \mu^2 \tag{10}$$

For $\mu = 0$, a general on-mass-shell momentum r_μ must be of the form

$$r = (r_0\,, \pm r_0) \tag{10a}$$

Without loss of generality we choose p_1', p_{α_1}, $q_{\alpha_1} < 0$ and p_1, p_{β_1}, $q_{\beta_1} > 0$, in such a way that, for $\mu = 0$

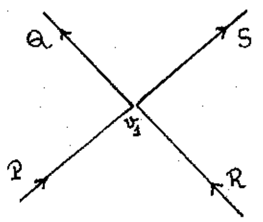
$$p' \cdot p_\alpha = p' \cdot q_\alpha = p_\alpha \cdot q_\alpha = p \cdot p_\beta = p \cdot q_\beta = p_\beta \cdot q_\beta = 0 \tag{11}$$

$$p' p = 2\, p_0 p_0' \quad \text{etc.} \tag{12}$$

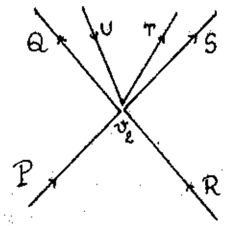
$$p = p_\beta + q_\beta \tag{13}$$

$$p' = q_\alpha + p_\alpha \tag{14}$$

As we are seeking for high energy behavior, which is the same as zero mass limit, we can freely use the above relations, where no divergence appear, in order to facilitate calculations . The Feynman rules for the vertices used in our third order calculation are given as follows:

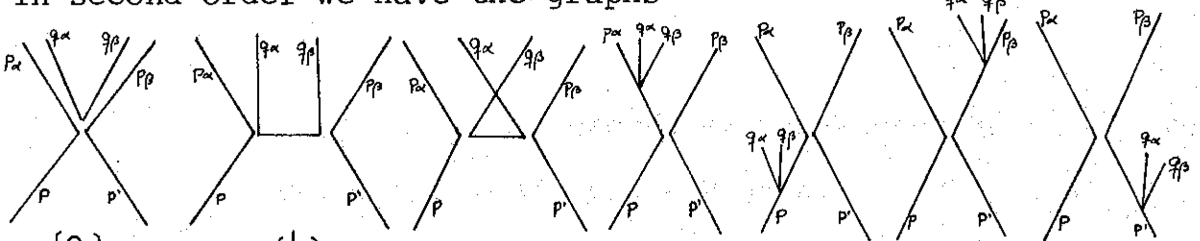


$$v_1 = \left[(Q-P)(S-R) + (Q+P)(S+R)\right] \tag{15}$$



$$v_2 = \left[(Q-P)(T+U) + (Q-P)(S-R) + (S-R)(T+U)\right] \tag{16}$$

In second order we have the graphs



(a) (b) (c) (d) (e) (f) (g)

and they sum up to zero. ($T^{(2)}=0$)

In third order the calculation is much more involved. We have a total of 111 Graphs. We use the famous Källen-Toll rule to handle with them, and achieve several group cancelations, in which the tree amplitude $T^{(2)}$ factorizes. Only one term survives, but it is zero by explicit calculations.

Along the calculations we use frequently the following functions:

$$A=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k-q_\alpha-q_\beta)^2-\mu^2}\,[-2\,q_\beta\,q_\alpha] \tag{18}$$

$$B=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k+p'-p_\beta)^2-\mu^2}\,[2\,p_\beta\,p'] \tag{19}$$

$$C=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k+p-p_\alpha)^2-\mu^2}\,[2\,p\,p_\alpha] \tag{20}$$

$$D=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k-p-p')^2-\mu^2}\,[2\,p\,p'] \tag{21}$$

$$E=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k-p_\alpha-q_\beta)^2-\mu^2}\,[2\,q_\beta\,p_\alpha] \tag{22}$$

$$F=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k+q_\alpha+p_\beta)^2-\mu^2}\,[-2\,q_\alpha\,p_\beta] \tag{23}$$

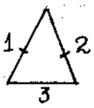
$$G=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k-p+q_\alpha)^2-\mu^2}\,[-2\,p\,q_\alpha] \tag{24}$$

$$H=\int d^2k\,\frac{1}{k^2-\mu^2}\,\frac{1}{(k+p'-q_\beta)^2-\mu^2}\,[2\,p'\,q_\beta] \tag{25}$$

$$I = \int d^2k \, \frac{1}{k^2 - \mu^2} \, \frac{1}{(k - p_\alpha - p_\beta)^2 - \mu^2} \, [2 p_\alpha p_\beta]$$

$$\gamma = 2(p + q_\beta)(p_\alpha - q_\alpha) \tag{26}$$

$$\alpha = 2(p + p_\beta)(p' + p_\alpha) \tag{27}$$

We use also a loop denoted by 

. It means the part of the graph calculated by means of the Kallen-Toll-rule by contracting lines 1 and 2.

It's worth mentioning that along all of the calculation we didn't care about renormalization, but as the theory is renormalizable, and the counterterms are of the same form of the other terms in the lagrangian, we hope that the renormalization procedure does not destroy the argument. That the Källen-Toll rule can be applied to divergent graphs can be justified as follows: let it be a divergent amplitude W:

$$W = \int d^2k \; P(k) \, \frac{1}{(k+p)^2 - \mu^2} \, \frac{1}{(k+q)^2 - \mu^2} \, \frac{1}{(k+r)^2 - \mu^2} \tag{28}$$

We regularize W:

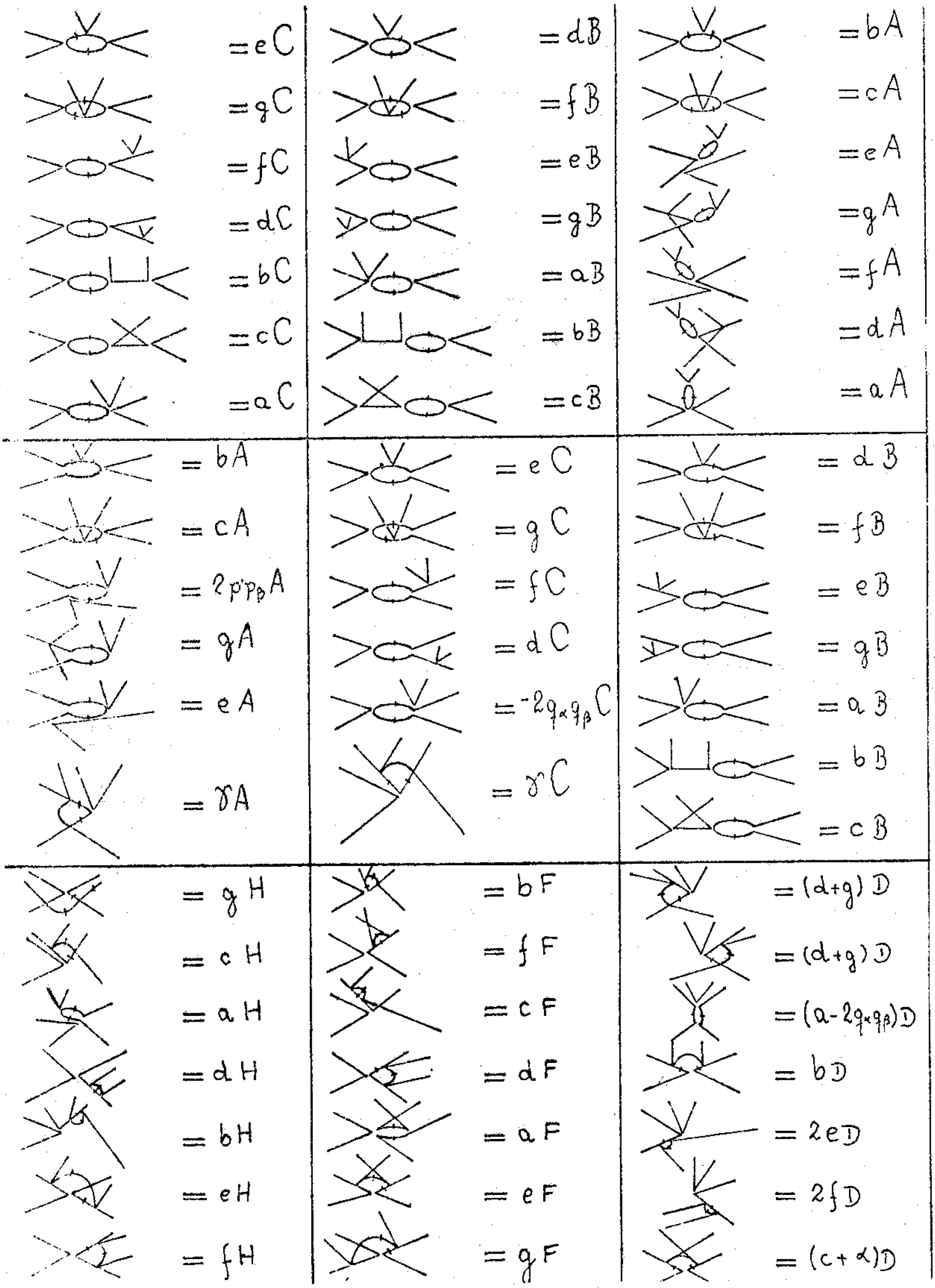
$$W_{reg} = \int d^2k \; P(k) \, \frac{1}{(k+p)^2 - \mu^2} \, \frac{1}{(k+p)^2 - \mu^2} \, \frac{1}{(k+r)^2 - \mu^2} \, \frac{M^2}{k^2 - M^2} \, \frac{N^2}{k^2 - N^2} \tag{29}$$

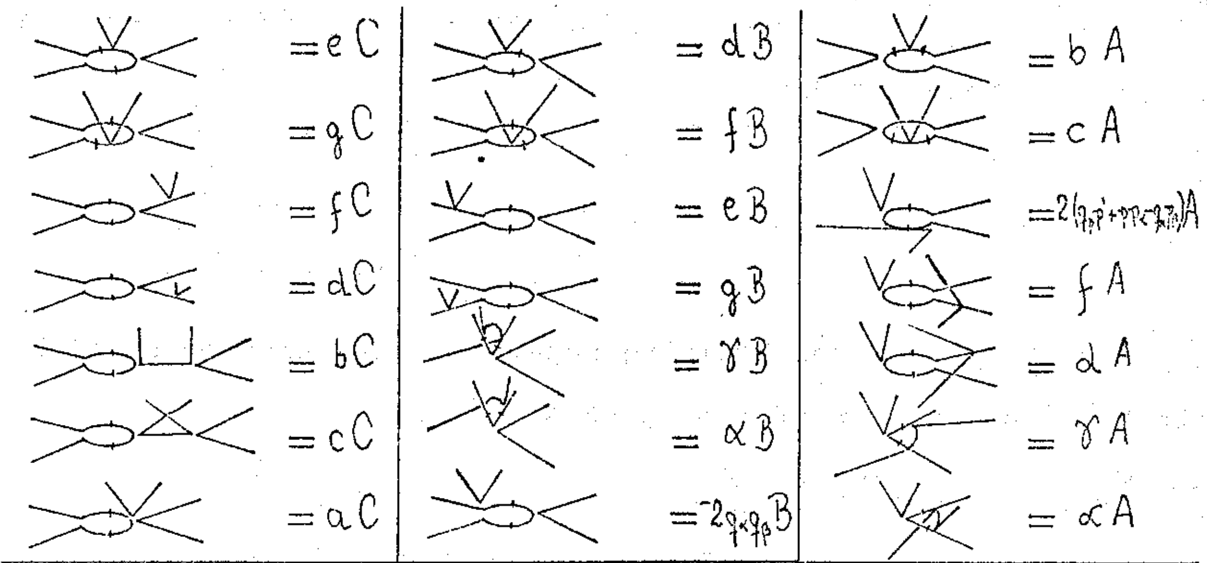
Now the rule can be applied. The contractions of the 3 inicial propagators give the usual amplitudes multiplied by finite terms in the limit M and N going to ∞. When we contract with the auxiliar propagator, we have cut-off dependent therms, which must cancel with counterterms. As this must be of the same form of the original lagrangian (renormalizability!) the whole argument follows.

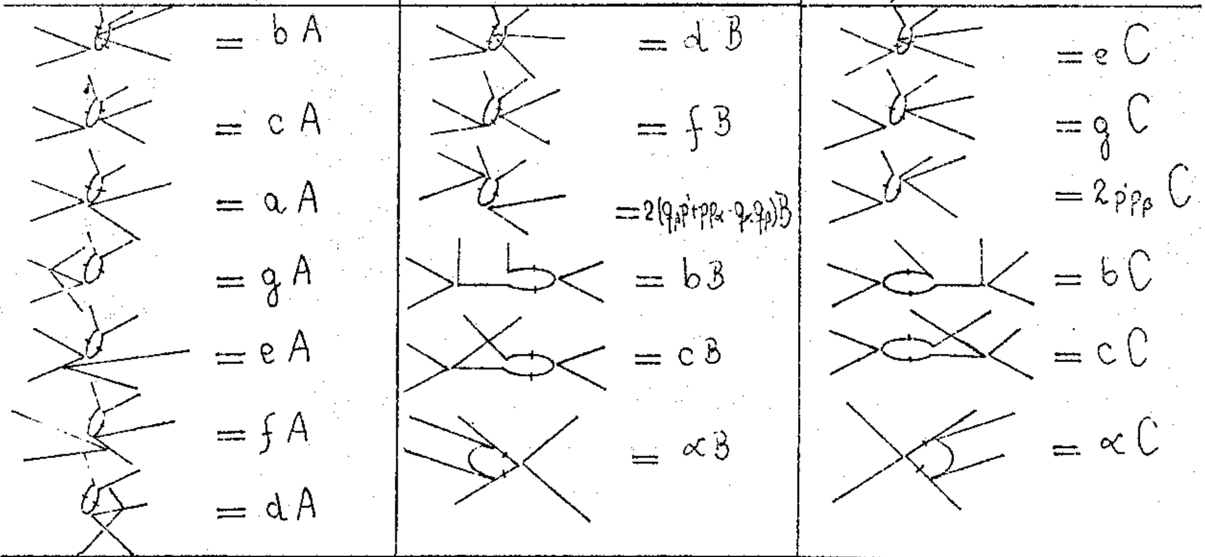
Our results are summarized in figure (1) and the surviving terms of the whole sum are:

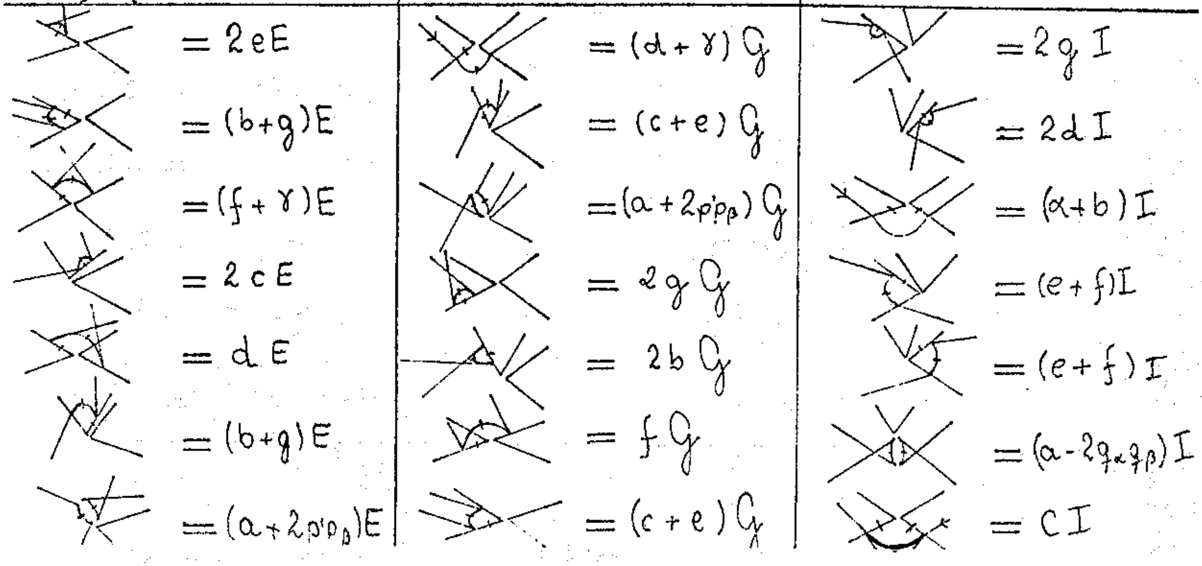
$$8\,p\,p_\alpha\,(A+B+2C+D+G+E+I) = 8\,p\,p_\alpha \ln \frac{p\cdot p' \;\; p_\alpha\cdot q_\beta \;\; p\cdot q_\alpha \;\; p_\alpha\, p_\beta}{q_\alpha\cdot q_\beta \;\; p' p_\beta \;\; p\cdot p_\alpha \;\; p\cdot p_\alpha} = 0 \qquad (30)$$

We hope in a forthcomming paper, to give arguments which permit the construction of the exact S-matrix.




= eC
= gC
= fC
= dC
= bC
= cC
= aC
= dB
= fB
= eB
= gB
= γB
= αB
= -2q_α q_β B
= bA
= cA
= 2(q_β p' + p p_γ - q_γ q_β)A
= fA
= dA
= γA
= αA


= bA
= cA
= aA
= gA
= eA
= fA
= dA
= dB
= fB
= 2(q_β p' + p p_α - q_α q_β)B
= bB
= cB
= αB
= eC
= gC
= 2p'p_β C
= bC
= cC
= αC


= 2eE
= (b+g)E
= (f+γ)E
= 2cE
= dE
= (b+g)E
= (a+2p'p_β)E
= (d+γ)G
= (c+e)G
= (a+2p'p_β)G
= 2gG
= 2bG
= fG
= (c+e)G
= 2gI
= 2dI
= (α+b)I
= (e+f)I
= (e+f)I
= (a-2q_α q_β)I
= cI

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