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ABSTRACT

We present a scheme by means of which one can infer the charge and magnetization structures of any quantum system from its electromagnetic form factors. For spin 1/2 and spin 0 particles the new results here obtained lead us to a correct non relativistic limit as well as to a consistent description of the mean charge radii. Our analysis of the existing data on elastic electron-proton scattering suggest that the charge density should be more concentrated than the magnetization density within the proton. The hadronic matter density of the proton is obtained, from the experimental data on pp elastic collisions at high energy, by assuming the Chou-Yang model. It is found that as the energy increases the hadronic matter distribution becomes more similar to the magnetization than to the charge density inside the proton. The same seems to be true for the neutron.

I - INTRODUCTION

There is a growing amount of experimental evidence that hadrons, like nuclei, are composite systems. Since the constituents are charged objects, one can visualize a particle as possessing a structure in which charge and magnetic currents are somehow distributed in space.

Intuitively, one should be able to infer from elastic scattering experiments (with appropriate test bodies) what these distributions are. Within the one photon exchange approximation, elastic scattering cross sections of electrons (the test body) by nucleons are parametrized in terms of form-factors. What is then sought is a consistent connection between the form-factors and charge or magnetization distributions.

This problem is an old one. As far as the electric charge distribution is concerned, there exists a variety of papers in the literature[1-6] searching for the desired connection. Some derivations[3] make use of tipically non relativistic approximations hence making them trustworthy only in such a domain. Some connections[4] lead to difficulties[5] concerning the prediction of the mean square radius of spin 1/2 particles.

Within the realm of phenomenological models of hadron structure, such connections might play an important role. This is indeed the case for the Chou-Yang[7] model. Although what is actually required in such a model is the hadronic matter distribution, the pragmatic assumption that "the matter distribution is similar to charge (or magnetization) distribution" makes the determination of the charge and magnetization densities a very relevant task in this context. The importance of this is enhanced by the probable failure[8] of the Chou-Yang model in describing more recent data on high energy pp elastic collisions.

In this paper we will derive new relations between the Fourier transform of the charge and magnetization distributions and the form-factors. Besides an interesting implication for the Chou-Yang model, these connections reduce to the correct ones in the non relativistic limit and lead us to a consistent description of mean charge radii of spin 0 and spin 1/2 particles.

The plan of presentation is the following. In the next section we develop a method by means of which one can relate the charge density of any quantum system to its electromagnetic form-factors. Explicit results are presented for spin 0 and spin 1/2 particles.

In section III we extend the method in order to relate the form-factors to the intrinsic current distribution which generates the dipole magnetic moment of spin 1/2 particles. In the case of the proton we suggest that the charge density is more concentrated than its magnetization density, both of them being spherically symmetric with a radius of approximately 0.9 fm. Under certain assumptions on the short distance behavior of the densities, one can derive the assymptotic behavior of the form-factors.

In section IV, we extract the proton hadronic matter distribution out of recent experimental data on pp elastic collisions at high energy, by assuming the Chou-Yang model. We compare the matter density thus obtained with the proton charge and magnetization distributions obtained by us in the previous sections. We have verified that, under the hypothesis that the imaginary part of the elastic pp amplitude be always positive, the proton matter density becomes more similar to its magnetization density when the energy increases. The implications of this phenomenological finding to neutron-proton elastic collisions at high energies are discussed.

Section V is dedicated to conclusions.

II - CHARGE DENSITY AND FORM FACTORS

Information on the charge distribution inside a single hadron is obtained by means of experiments in which momentum is transfered to a test body which is usualy the electron. Within the one-photon exchange approximation the elastic scattering cross sections of electrons by protons are parametrized in terms of the so called Dirac (F_1) and Pauli (F_2) form-factors[3]. These are defined by means of the matrix elements of the electromagnetic current operator $\hat{J}_\mu(x)$ between one particle states as[9]:

$$\langle \vec{p}\,',s'|\,\hat{J}_\mu(x)\,|\vec{p},s\rangle = \exp\left[i(p_0'-p_0)x_0 - i(\vec{p}\,'-\vec{p}\,)\cdot\vec{x}\,\right]\cdot$$

$$\cdot\ \frac{e\,m}{V\sqrt{p_0 p_0'}}\,\bar{u}(p',s')\left[F_1(t)\gamma_\mu + i\sigma_{\mu\nu}(p'-p)^\nu\frac{F_2(t)}{2m}\right]u(p,s) \qquad (2.1)$$

Here $|\vec{p},s\rangle$ represents the state of a proton with 4-momentum $p_\mu=(p_o,\vec{p})$ and spin s, $t=(p'-p)^2$ is the momentum transfer, e is the proton charge and V is the normalization volume.

The basic question to be answered is: which form-factor (or combinations thereof) is associated with the Fourier transform of the charge distribution in the proton rest frame? We do not know how to get information on the charge density by looking directly to (2.1). This can be understood on the grounds that, if $\vec{x}$ represents an arbitrary point of the extended particle, one cannot get any information on how charge is distributed around the "charge center" without first specifying its position (which is difficult due to the uncertainty principle). One should then develop a scheme which somehow eliminates the uncertainty of the object as a whole.

This can be achieved if one follows a suggestion first proposed by Weisskopf[1]. The idea is to introduce a correlation function, which we will denote by $W_{ch}(\vec{\xi})$, which depends upon the distance $\vec{\xi}$ separating two points inside the extended object. If $\rho_{ch}(\vec{x})$ is the charge density in the proton rest frame, $W_{ch}(\vec{\xi})$ is defined by[1]:

$$W_{ch}(\vec{\xi}) = \int d^3x \; \rho_{ch}(\vec{x}+\frac{\vec{\xi}}{2}) \, \rho_{ch}(\vec{x}-\frac{\vec{\xi}}{2}) \tag{2.2}$$

This classical correlation function has the obvious property that $\int d^3\xi \; W_{ch}(\vec{\xi})=e^2$ and its physical meaning[1] should be that of "probability of finding charge simultaneously at two points separated by an arbitrary distance $|\vec{\xi}|$ ".

Within the quantum field context, Weisskopf[1] proposed that (aside from subtraction terms) the correlation function which possesses the above meaning is:

$$W_{ch}(\vec{\xi}) = \int d^3x \langle \vec{p}=0,s | \hat{J}_0(\vec{x}+\frac{\vec{\xi}}{2},x_0) \, \hat{J}_0(\vec{x}-\frac{\vec{\xi}}{2},x_0) | \vec{p}=0,s \rangle \tag{2.3}$$

where $\hat{J}_o(\vec{x},x_o)$ is the charge density operator and x_o is an arbitrary instant.

The correlation function (2.3) contains much more information about the proton than we are looking for. That can be realized by inserting, between the charge density operators in (2.3), a complete set of particle states. Since, as can be understood intuitively, only elastic experiments can be associated with the measurement of the charge density of a single proton, we select just these states in the sum of the completeness relation. Our proposal is then to associate $W_{ch}(\vec{\xi})$ in (2.2) to the expression:

$$W_{ch}(\vec{\xi}) = \int d^3x \langle 0,s| \hat{J}_0(\vec{x}+\tfrac{\vec{\xi}}{2},x_0) \sum_{\vec{q}} \sum_{s'} |\vec{q},s'\rangle\langle\vec{q},s'| \hat{J}_0(\vec{x}-\tfrac{\vec{\xi}}{2},x_0)|0,s\rangle \tag{2.4}$$

If one uses the decomposition (2.1) in (2.4), then, after a straightfoward calculation, we get the following result:

$$W_{ch}(\vec{\xi}) = e^2 \int \frac{d^3q}{(2\pi)^3} \left(\frac{1 - \frac{t}{4m^2}}{1 - \frac{t}{2m^2}} \right) G_E^2(t) \exp(-i\vec{q}\cdot\vec{\xi}) \tag{2.5}$$

where $G_E(t) = F_1(t) + \frac{t}{4m^2} F_2$ is the electric form-factor and t is given by:

$$t = 2m\left(m - \sqrt{q^2 + m^2}\right) \tag{2.6}$$

It is interesting to note that the same result, namely expressions (2.5) and (2.6), can be obtained from (2.4) if we keep the proton with a fixed polarization in its own rest frame since $\langle\vec{q},s'|\hat{J}_0(0)|0,s\rangle = \frac{e}{V}\left[\left(1-\frac{t}{4m^2}\right)/\left(1-\frac{t}{2m^2}\right)\right]^{1/2} G_E(t)\,\delta_{s,s'}$.

Another observation that can be made by looking at (2.5) is that the replacement $G_E(t) \to 1$ does not lead to the point particle limit (characterized by the property $W_{ch}(\vec{\xi}) = e^2\delta(\vec{\xi})$). This can be achieved if we also take $m \to \infty$. This is expected because a point charged particle would have an infinite electrostatic energy.

For what follows it is convenient to introduce the Fourier transform $f_{ch}(\vec{q})$ of the charge density:

$$\rho_{ch}(\vec{x}) = e \int \frac{d^3q}{(2\pi)^3} f_{ch}(\vec{q}) \exp(i\vec{q}\cdot\vec{x}) \tag{2.7}$$

With this, (2.2) can be written as:

$$W_{ch}(\vec{\xi}) = e^2 \int \frac{d^3q}{(2\pi)^3} f_{ch}(\vec{q}) f_{ch}(-\vec{q}) \exp(-i\vec{\xi}\cdot\vec{q}) \tag{2.8}$$

The connection betwwen form-factor and the charge distribution can be achieved, within our approach, by just indetifying (2.8) with (2.5). Doing so, we get:

$$f_{ch}(q^2) = \left(\frac{1 - \frac{t}{4m^2}}{1 - \frac{t}{2m^2}} \right)^{1/2} G_E(t) \tag{2.9}$$,

where t is defined in (2.6).

The scheme presented here can be extended in a straightfoward manner to other spin 1/2 quantum systems (like ^{3}He nucleus for instance). For spin ≥ 1 objects, which can have a quadrupole deformation in the direction $\vec{s}$ of the spin (like the deuteron[10] for instance), the correlation function W_{ch} should depend on both $\vec{\xi}$ and $\vec{s}$.

For the case of spin 0 objects the electromagnetic current matrix elements are usually written as[11]:

$$\langle \vec{p}' | \hat{J}_\mu(x) | \vec{p} \rangle = \frac{e(p'+p)_\mu}{2V\sqrt{p_0 p_0'}} F(t) \exp[i(p'-p)\cdot x] \tag{2.10}$$,

where $F(t)$ is the electromagnetic form-factor.

If we interpret (2.10) by using the same method explained above we get the following expression for the Fourier transform of the charge density:

$$f_{ch}^{(spin\ 0)}(q^2) = \frac{\left(1-\frac{t}{4m^2}\right)}{\left(1-\frac{t}{2m^2}\right)^{1/2}} F(t) \qquad (2.11),$$

where t is given by (2.6) and m is the mass of the spin 0 particle.

The main results of this section are contained in equations (2.9) and (2.11). The lessons that can be learned from them, are the following:

a) The usual Sachs' relation[4], namely

$$f_{ch}(q^2) \simeq G_E(-q^2) \qquad (2.12),$$

is obtained as the non relativistic limit ($q^2 \ll m^2$) of ours. The well-known expression $f_{ch}^{(spin\ 0)}(q^2) \simeq F(-q^2)$ is also obtained as the non relativistic limit of (2.11).

b) By looking at (2.9) and (2.11) one sees that our results differ from previous ones in the litterature (besides a different dependence of G_E and F on $\vec{q}^{\,2}$) by the presence of spin dependent factors multiplying the form-factors. The possible existence of such contributions has been realized by Yennie et al.[3] and by Gourdin[5]. These factors affect the expression of the mean square charge radius of the particle when expressed in terms of the form-factors. In the case of spin 1/2 particles, like the proton, we get from (2.9):

$$\langle r^2\rangle_{ch} \equiv \frac{1}{e}\int d^3x\, r^2 \rho_{ch}(r) = \frac{3}{4m^2} + 6\, G_E'(0) \qquad (2.13)$$

which is the expected result[3,5]. It is worth mentioning that this expression was obtained in Ref. 3 by using a completely different line of reasoning. The usual relation (2.12) does not lead to the above result (as was pointed out by Gourdin[5] the neglect of $3/4m^2$ was not properly justified in Ref. 4).

c) It is interesting to note that in the case of a Dirac particle, for which $G_E=1$, one gets from (2.13) that $\langle r^2\rangle_{ch}=3/4m^2$. This result is equal to that obtained from Darwin's non relativistic correction to the electrostatic interaction (of an extended particle) with an external electric field[2] and is observed experimentally for the electron in the hydrogen atom[12]. This $3/4m^2$ contribution to $\langle r^2\rangle_{ch}$ is usually[12] atributed to the zitterbewegung of a Dirac particle. In our opinion this is an incorrect belief since the above mentioned contribution can be traced back[3] to the overlap of the two spinors which are present in (2.1). The analysis by Huang[13] and by Lock[14] confirms our statement. They have found that zitterbewegung produces a growth in the size of the particle many orders of magnitude smaller than the particle's Compton wavelength.

d) The mean square charge radius for a spin 0 particle, as obtained from (2.11), is:

$$\langle r^2\rangle_{ch}^{(spin\,0)} = 6\,F'(0) \tag{2.14}$$

If one puts $F(t)=1$ in (2.11), then it is easy to check that:

$$\langle r^2\rangle_{ch}^{(spin\,0)} = 0 \tag{2.15a}$$

and

$$\langle r^4 \rangle_{ch}^{(\text{spin}\,0)} = \frac{15}{4m^4} \qquad (2.15b),$$

both of which are in agreement with Darwin's non relativistic corrections to the interaction of a Klein-Gordon particle in an electrostatic field[15]. To our knowledge no other previous connection leads to such a consistent description (summarized by equations (2.13) to (2.15)) of the mean charge radii.

e) The asymptotic form of $G_E(t)$ is fixed by the short distance behavior of $\rho_{ch}(r)$. More explicitly, one can write[16]:

$$G_E(t \to -\infty) \sim \sum_{\ell=1}^{\infty} \left(\frac{-4m^2}{t^2}\right)^{\ell} \left(\frac{d}{dr}\right)^{2(\ell-1)} \left[r\,\rho_{ch}(r)\right]_{r=0} \qquad (2.16).$$

From (2.16) it follows that if $\rho_{ch}(r)$ is sufficiently smooth at the origin ($[r\,\rho_{ch}(r)]_{r=0}=0$), then $G_E(t)$ should fall off at least as t^{-4}. The asymptotic form $G_E \sim t^{-2}$ is compatible, within our approach, with a singular behavior of $\rho_{ch}(r)$ at the origin, namely $\rho_{ch}(r) \underset{r\to 0}{\sim} r^{-1}$.

III - MAGNETIZATION DENSITY

In a recent paper Jackson[17] pointed out that there is experimental evidence that all known intrinsic magnetic moments (of electron, muon, proton, neutron and nuclei) are caused by circulating electric currents rather than by bound pairs of magnetic charges. Therefore it will be interesting to express the intrinsic currents (in the rest frame of a polarized spin 1/2 particle or nucleus) in terms of the measured Dirac and Pauli form-factors. With this purpose we will extend the method presented in the preceeding section. Before doing so, two comments are needed.

First of all one notes that in order to probe the magnetization current distribution one needs to make experiments in which one measures the polarization of the particle. If the particle has a spin orientation given by $\vec{s}$ in the initial configuration (for instance at rest in the laboratory frame), then one should select those experiments in which its final spin orientation (in its own rest frame) is the same as before the interaction with the test body.

Second, one notes that in elastic scattering experiments one gets information about the intrinsic current ($\hat{J}_\mu^{spin}$) as well as on the convective piece ($\hat{J}_\mu^{conv}$) of the current. The total current is just a sum of these contributions:

$$\hat{J}_\mu(x) = \hat{J}_\mu^{spin}(x) + \hat{J}_\mu^{conv}(x) \tag{3.1}$$

One can separate these contributions, at the level of matrix elements of the current operator between one particle states, by making use of Barnes' prescription[18]. Following Barnes we write:

$$\langle \vec{p}', s' | \hat{J}_\mu^{conv}(0) | \vec{p}, s \rangle = \frac{e\, G_E(t)}{\left(1 - \frac{t}{4m^2}\right)} \frac{(p'+p)_\mu}{2V\sqrt{p_0 p_0'}} \bar{u}(p', s')\, u(p, s) \tag{3.2}$$

Besides Barnes' arguments for this identification of the convective current, there is another way of checking the consistency of his prescription: to determine the charge density by using just the convective part. What ones has to do is to substitute the 0-th component of the total current matrix elements in (2.4) by the 0-th component of the convective part as given by (3.2). One gets, as one should, the some result. No other separation has this property.

By using (3.1), (3.2) and (2.1) one can check easily that the intrinsic part of the current operator has matrix elements:

$$\langle \vec{p}', s' | \hat{J}^{spin}_{\mu}(0) | \vec{p}, s \rangle =$$

$$= \frac{e m}{\sqrt{p_0 p_0'}} G_M(t) \bar{u}(p', s') \left[\gamma_\mu - \frac{(p'+p)_\mu}{2m(1-t/4m^2)} \right] u(p, s) \tag{3.3}$$

where

$$G_M(t) = F_1(t) + F_2(t) \tag{3.4}$$

Following the same line of reasoning, as in the last section, we define a correlation function $W_{mg}(\vec{\xi}, \vec{s})$, depending now on the relative coordinate $\vec{\xi}$ and upon the spin orientation $\vec{s}$, by:

$$W_{mg}(\vec{\xi}, \vec{s}) = \int d^3x \; \vec{j}_{spin}\left(\vec{x} + \frac{\vec{\xi}}{2}\right) \cdot \vec{j}_{spin}\left(\vec{x} - \frac{\vec{\xi}}{2}\right) \tag{3.5}$$

Here $\vec{j}_{spin}$ is the intrinsic stationary current which generates the magnetic dipole moment of the particle. In terms of the magnetization density $\rho_{mg}(\vec{x})$, in the rest frame of the particle, we have:

$$\vec{j}_{spin}(\vec{x}) = \vec{\nabla} \times \left[\rho_{mg}(\vec{x}) \frac{e\vec{S}}{2m} \right] \tag{3.6}$$

where $\vec{s}$ is the expectation value of the Pauli spin operator $\hat{\vec{\sigma}}$, namely $\vec{s} = \langle\hat{\vec{\sigma}}\rangle = (0, 0, 1)$ if the spin is "up".

The extension of Weisskopf's idea to this case will be implemented by extracting from the quantum correlation function:

$$\int d^3x \, \langle \vec{p}=0, s | \hat{\vec{J}}_{spin}(\vec{x}+\tfrac{\vec{\xi}}{2}, x_0) \cdot \hat{\vec{J}}_{spin}(\vec{x}-\tfrac{\vec{\xi}}{2}, x_0) | \vec{p}=0, s \rangle \tag{3.7}$$

all those contributions which cannot be associated with a measurement of the intrinsic current of a single particle and then identifying the remaining part with $W_{mg}(\vec{\xi}, \vec{s})$ as defined by (3.5).

As explained previously, this amounts to: a) eliminating the convective current; b) suppressing the contribution of all multi-particle states and c) for the one particle states, to supress the sum over polarizations. All that can be summarized by means of the expression:

$$W_{mg}(\vec{\xi}, s) =$$

$$= \int d^3x \, \langle 0, s | \hat{\vec{J}}_{spin}(\vec{x}+\tfrac{\vec{\xi}}{2}, x_0) \sum_{\vec{q}} | \vec{q}, s \rangle \cdot \langle \vec{q}, s | \hat{\vec{J}}_{spin}(\vec{x}-\tfrac{\vec{\xi}}{2}, x_0) | 0, s \rangle \tag{3.8}$$

For what follows it is convenient to introduce the Fourier transform $f_{mg}(\vec{q})$ of magnetization density:

$$\rho_{mg}(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} f_{mg}(\vec{q}) \exp(-i\vec{q}\cdot\vec{x}) \tag{3.9}$$

With this the left hand side of (3.8) can be written in the form:

$$W_{mg}(\vec{\zeta},\vec{\zeta}) = \frac{e^2}{4M^2}\int \frac{d^3q}{(2\pi)^3} f_{mg}(\vec{q})\, f_{mg}(-\vec{q})(\vec{\zeta}\times\vec{q})^2 \exp(+i\vec{q}\cdot\vec{\zeta}) \quad (3.10),$$

where we have used (3.5) , (3.6) and (3.9).

The calculation of the right hand side of (3.8) can be done by using (3.3). The resulting expression is:

$$W_{mg}(\vec{\zeta},\vec{\zeta}) = \frac{e^2}{4M^2}\int \frac{d^3q}{(2\pi)^3} \frac{G_M^2(t)(\vec{\zeta}\times\vec{q})^2}{\left(1-\frac{t}{4M^2}\right)\left(1-\frac{t}{2M^2}\right)} \exp(-i\vec{q}\cdot\vec{\zeta}) \quad (3.11)$$

where $t = 2m\,(m - \sqrt{m^2+\vec{q}^{\,2}})$.

By the identification of (3.10) with (3.11) one gets the following connection between the Fourier transform of the magnetization density and the form-factors:

$$f_{mg}(q^2) = \frac{G_M(t)}{\left[\left(1-\frac{t}{4M^2}\right)\left(1-\frac{t}{2M^2}\right)\right]^{1/2}} \quad (3.12)$$

The above expression is the main result of this section. The conclusions which can be drawn are the following:

a) The well know relation[3,4]:

$$f_{mg}(q^2) \simeq G_M(-q^2) \quad (3.13),$$

is obtained from ours by just taking the non relativistic limit

$(\vec{q}^{\,2} << m^2)$.

b) The magnetization density is spherically symmetric and the mean square magnetization radius, obtained by using (3.9) and (3.12), is:

$$\langle r^2 \rangle_{mg} = \frac{9}{4M^2} + \frac{6}{G_M(0)} G_M'(0) \qquad (3.14)$$

c) By using the recent experimental data on proton electromagnetic form-factors (19):

$$\left[6\, G_E'(0)\right]^{1/2} = 0.88 \pm 0.03 \text{ fm}$$

$$\left[\frac{6\, G_M'(0)}{G_M(0)}\right]^{1/2} = 0.84 \pm 0.03 \text{ fm} \qquad (3.15)$$

we get from (3.14) and (2.13):

$$\langle r^2 \rangle_{ch}^{1/2} = 0.90 \pm 0.03 \text{ fm}$$

$$\langle r^2 \rangle_{mg}^{1/2} = 0.93 \pm 0.03 \text{ fm} \qquad (3.16).$$

One can see from (3.16) that, within the present experimental errors, the magnetization and charge radii of the proton are about the same. Thus one should look somewhere else in order to find if there is any evidence for a different behavior of the charge and magnetization densities.

d) From (3.12) and (2.9) one gets

$$\frac{G_E(t)}{G_M(t)} = \frac{f_{ch}/f_{mg}}{1-\frac{t}{4m^2}} \qquad (3.17).$$

In FIG.1 we plot the experimental data[20] for the ratio $\mu^2 G_E^2/G_M^2$ (where $\mu=G_M(0)=f_{mg}(0)=2.79$ for the proton). The continuous curve corresponds to the hypothesis $\mu f_{ch}= f_{mg}$ or equivalently $\mu\,\rho_{ch}(r)=e\,\rho_{mg}(r)$ as suggested by (3.16). For $|t|<2\ \mathrm{GeV}^2$ the experimental data are above the continuous curve suggesting in this way that the charge density should be more concentrated than the magnetization density inside the proton. This fact can be atributed to the existence of a cloud of charged pairs, circulating around the charge center in such a way that it does not contribute appreciably to the charge density, but it does contribute to the magnetization density. If such a picture is correct then these pairs should affect in a different way the electric charge and hadronic[7] matter distributions. That will be seen in the next section.

e) The asymptotic form of $G_M(t)$ can be determined from the short distance behavior of $\rho_{mg}(r)$. By making use of (3.9) and (3.12) one gets[16]:

$$G_M(t\to\infty) \sim \frac{t}{m^2}\sum_{\ell=1}^{\infty}\left(\frac{-4m^2}{t^2}\right)^{\ell}\left(\frac{d}{dr}\right)^{2(\ell-1)}\left[r\,\rho_{mg}(r)\right]_{r=0} \qquad (3.18)$$

Under the hypothesis that $\rho_{mg}(r)$, as well as $\rho_{ch}(r)$, be finite at the origin, or under the weaker assumption that $\rho_{ch}(r) \underset{r\to 0}{\sim} \rho_{mg}(r)$, one can conclude, from (2.16) and (3.19), that:

$$\left(\frac{G_E^2}{G_M^2}\right)_{t\to -\infty} \dot{\sim} \; t^{-2} \qquad (3.19).$$

As can be seen from FIG.1, (3.19) is not ruled out by the experimental data.

The conclusion is that, in spite of very similar mean square radii, the charge and magnetization densities are different for the proton. The implication of this to the Chou-Yang model will be discussed in the next section.

IV - HADRONIC MATTER DENSITY

In this section we will be mainly concerned with the hadronic (or absortive) matter distribution inside the proton(7). Basically, the idea which we will pursue here, is similar to that of sections II and III: we try to extract from the experimental results on proton-proton elastic scattering what this distribution is. There are differences however which stem mainly from two sources: the model dependence of our results and the nature of the approximations used (we neglect in our analysis the real part of the elastic scattering amplitude and spin effects which are typical approximations in the high energy domain).

For the calculation of the imaginary part $a(q^2,s)$ of the amplitude in the limit $s \gg |\vec{q}|^2$ (where $\sqrt{s}$ is the center of mass energy and $\vec{q}$ is the transverse momentum transfer). We shall assume the eikonal approximation(7). Thus the elastic differential cross section will be written as:

$$\frac{d\sigma}{dq^2} \simeq \pi |a(q^2;s)|^2 = \pi |\langle 1 - \exp[-\Omega(b,s)]\rangle_q|^2$$
$$\equiv \pi \left| \int \frac{d^2b}{2\pi} \left(1 - \exp[-\Omega(b,s)]\right) \exp(i\vec{q}\cdot\vec{b}) \right|^2 \qquad (4.1),$$

where $b=|\vec{b}|$ is the impact parameter and $\Omega(b,s)$ is the opacity associated with the colliding protons[7].

Within the Chou-Yang[7] model, the opacity per unit of time $(\Delta\Omega/\Delta x_o)$ is defined, heuristically, as being proportional to the instantaneous superposition of the hadronic matter densities $\rho_h(\vec{x} \pm \frac{\vec{b}}{2}, x_o)$ of the colliding protons. Under the assumption that in the limit $s \gg q^2$ the trajectory of each proton will be a straight line, and we have:

$$\Omega(b,s) \sim \int_{-\infty}^{+\infty} dx_0 \int d^3x\, \rho_h(\vec{x}-\frac{\vec{b}}{2}, x_0)\, \rho_h(\vec{x}+\frac{\vec{b}}{2}, x_0) \qquad (4.2)$$

If one denotes by $f_h(q^2)$ the Fourier transform of the hadronic matter density $\rho_h(\vec{x})$ in the rest frame of the proton, namely

$$f_h(q^2) = \int d^3x\, \rho_h(\vec{x}) \exp(-i\vec{q}\cdot\vec{x}) \qquad (4.3),$$

and by $\gamma = (1-v^2)^{-1/2} = \sqrt{s}/2m$ the Lorentz factor of each proton in the C.M. system, then one can write the following expression for the hadronic matter densities of the moving protons:

$$\rho_h(\vec{x} \pm \frac{\vec{b}}{2}, x_0) = \gamma \int \frac{d^3q}{(2\pi)^3} f_h(q^2) \exp\left[i\vec{q}_\perp\cdot(\vec{x}_\perp \pm \frac{\vec{b}}{2}) + i\gamma q_\parallel (x_\parallel \pm v x_0)\right] \qquad (4.4)$$

Here $\vec{q}_\perp$ and $\vec{x}_\perp$ are vectors which are normal to the collision axis and $q_\parallel$ and $x_\parallel$ are the components parallel to this direction.

We have assumed, in analogy with $\rho_{ch}(\vec{x})$ and $\rho_{mg}(\vec{x})$, that $\rho_h(\vec{x})$ is spherically symmetric and we will adopt the normalization $f_h(0)=1$.

The substitution of (4.4) in (4.2) leads us to the Chou-Yang[7] expression:

$$\Omega(b,s) = C \langle f_h^2(q^2) \rangle_b \equiv \equiv C \int \frac{d^2q}{2\pi} f_h^2(q^2) \exp(i\vec{q}\cdot\vec{b}) \qquad (4.5),$$

where $q^2=q_\perp^2$ and the proportionality factor C is assumed to be independent of the impact parameter.

As formulated originally by Chou-Yang[7] the opacity $\Omega(b,s)$ is independent of s at high energies. Later on, their model was extended in such a way that one can accomodate a factorizable energy dependence[21]. In this case, the proportionality factor C should be energy dependent, $C \to C(s)$. Under the assumption that the relations (2.12) and (3.13) are valid and that the matter density ρ_h should be very similar to ρ_{ch}/e (or ρ_{mg}/μ) Chou-Yang have claimed that:

$$f_h(q^2) = G_E(-q^2) \simeq \frac{G_M(-q^2)}{\mu} \simeq \left(1 + \frac{q^2}{0.71}\right)^{-2} \qquad (4.6).$$

In this way the sole free parameter of the model $C(s)$ can be fixed by using the optical theorem and the measured pp total cross section. They have shown that this model is quite accurate[21] to describe $d\sigma/dq^2$ in the range $0<q^2<2$ GeV^2. Neverthless the model seems to fail[8] in describind the $q^2>2$ GeV^2 data. We believe that the main reason for this failure stays not in the Chou-Yang physical assumption, i.e., the similarity between the hadronic matter and the charge (or magnetization) distributions, but in the fact that the relations (2.12) and (3.13) are only approximaletty correct.

In order to discuss with more details the implications of the above hypothesis (4.5) and (4.6) it is convenient to write (4.1) in the equivalent form:

$$\langle \Omega(b,s)\rangle_q = -\langle \ln[1-\langle a(q^2,s)\rangle_b]\rangle_q \qquad (4.7),$$

where the two-dimensional Fourier transforms $\langle\rangle_q$ and $\langle\rangle_b$ are defined in (4.1) and (4.5).

By using (4.5) and (4.7) one gets the following expression for $f_h^2(q^2)$:

$$f_h^2(q^2) = \frac{\langle \Omega(b,s)\rangle_q}{\langle \Omega(b,s)\rangle_{q=0}} \qquad (4.8).$$

Expressions (4.1), (4.7) and (4.8) allow us to determine the hadronic matter distribution from the experimental data on $d\sigma/dq^2$.

Expression (4.8) can also be written[7] as:

$$f_h^2(q^2)\langle \Omega(b,s)\rangle_{q=0} = a + \frac{a\otimes a}{2} + \frac{a\otimes a\otimes a}{3} + \dots \qquad (4.9),$$

where $\otimes$ stands for the convolution integral:

$$a\otimes a = \int \frac{d^2k}{2\pi}\, a(\vec{k})\, a(\vec{q}-\vec{k}) \qquad (4.10)$$

The fact that $f_h^2(q^2)$ should be always positive for any value of q^2, and energy independent in high energy limit, can be used as a consistency[22,23] test for the Chou-Yang model. In accordance with Durand and Lipes[24], a necessary condition in order that $f_h^2(q^2)$ be always positive is that the amplitude $a(q^2,s)$ changes its signs an even number of times (as a function of q^2). As can be seen from (4.9) and (4.10), $a(q^2,s)>0$ is a sufficient condition.

Since the second minimum of $d\sigma/dq^2$ was not found up to q^2=12 GeV at $\sqrt{s}=19.4$ GeV[25] and up to $q^2=10$ GeV^2 at $\sqrt{s}=53$ GeV[26], is easy to show[22,23] that (4.8) implies that $f_h^2(q^2)$ becomes negative for $q^2 \simeq 6$ GeV^2 if $a(q^2,s)$ changes its sign at the first minimum.

This inconsistency can be avoided if we generalize the Chou-Yang model as was done by França and Hama[27]. They have analysed the high energy pp elastic scattering data for 19.4 GeV $\leq \sqrt{s} \leq$ 62 GeV by using an amplitude whose imaginary part change sign two times. Their results can be summarized as follows[27]:

a) the opacity is not fatorizable since it was found that:

$$\Omega(b,s) \simeq 0.03 \ln\left(\frac{s}{4M^2}\right) \exp(-0.029b^2) + 0.15(\lambda b)^3 K_3(\lambda b) \qquad (4.11)$$

where $\lambda \simeq 1$ GeV and K_3 is the wellknown modified Bessel function.

b) the s independent part of $\Omega(b,s)$ has a two dimensional Fourier transform which is $(1+q^2)^{-4}$ and cannot be identified with f^2_{ch} or f^2_{mg}/μ^2 except for small q^2.

c) the asymptotic $(s \to \infty)$ limit of $\Omega(b,s)$ is fatorizable but its detailed small impact parameter structure is not clear[27].

Both possibilities for $\Omega(b,s\to\infty)$ discussed in Ref. (27) does not permit us to identify $\langle\Omega(b,s\to\infty)\rangle_q$ with f_{ch}^2 or f_{mg}^2/μ^2.

In the present analysis we consider the case in which the imaginary part of the amplitude is always positive for any value of q^2. With this our analysis is similar to that of Leader et al.[28] except to the fact that we have neglected the real part of the amplitude. We shall reach results which are in qualitative agreement with those of Ref. (28).

We have fitted the experimental data on $d\sigma/dq^2$ for $\sqrt{s}=19.4$ GeV[25,29,30] and $\sqrt{s}=53$ GeV[26,31,32] by using an amplitude parametrized as:

$$a(q^2,s) = \sum_{i=1}^{4} \alpha_i \exp(-\beta_i q^2) - A\, q^{2N} \exp(-Bq^2) \tag{4.12}$$

The values of the parameters can be found in table I.

The quality of our fit can be appreciated by looking at Figs. 2, 3, 4 and 5 where our parametrization is represented by the continuous curves.

The numerical calculation of $f_h^2(q^2)$ was performed by using (4.7), (4.8) and (4.12), or more explicitelly:

$$\langle\Omega(b,s)\rangle_q = -\int_0^\infty b\,db\, J_0(bq) \ln\left[1-\langle a(q^2,s)\rangle_b\right] \tag{4.13}$$

where $J_0(bq)$ is the 0-th order Bessel function and

$$\begin{aligned} 2\langle a(q^2,s)\rangle_b = {} & \sum_{i=1}^{4} \frac{\alpha_i}{\beta_i} \exp(-b^2/4\beta_i) + \\ & - \frac{A(N!)^2}{B^{N+1}} \exp\left(\frac{-b^2}{4B}\right) \sum_{K=0}^{N} \frac{(-b^2/4B)^K}{(K!)^2\,(N-K)!} \end{aligned} \tag{4.14}$$

In Figs. 6 and 7 we have displayed, by means of discontinuous curves, the Fourier transform of the hadronic matter distribution obtained from $a(q^2, \sqrt{s}=19.4)$. The continuous curves represents $f_h(q^2)$ obtained from $a(q^2, \sqrt{s}=53)$.

We have also plotted in these figures $f_{ch}(q^2)$ and $f_{mg}(q^2)$ which are obtained from (2.9) and (3.12). For the determination of $G_E(t)$ and $G_M(t)$ we have made use of the experimental data on elastic electron-proton collisions as collected by Blatnik and Zovko[33] (for $-t>3\ GeV^2$ the data of Ref. (33) was obtained with the hypothesis $\mu G_E \simeq G_M$).

We have displayed in Fig 8 the opacity for $\sqrt{s}=19.4$ GeV and 53 GeV. From this figure we can see that these opacities are essentially the same for $b \leq 0.5$ fm (in fact we have found that $\Omega(b=0, \sqrt{s}=19.4)$ is 6% greater than $\Omega(0,53)$ but the expected uncertainty due to $a(q^2,s)$ at large q^2 can be larger than this difference). For $b \simeq 1$ fm we have $\Omega_{53} \simeq 1.15\Omega_{19}$. This expansion of the opacity as s increases is related to the shrinking observed in $d\sigma/dq^2$. All these results are in agreement with those of Ref. (28).

The most interesting results of this section can be appreciated by looking at Figs. 6 and 7.

The first conclusion we reach that factorization is not observed at these energies, i.e., $f_h = f_h(q^2,\sqrt{s})$.

Another observation is that $f_h(q^2,19.4)$ is very similar for $f_{ch}(q^2)$ up to $q^2=2.3\ GeV^2$. However the most suggestive result is the great similarity between $f_h(q^2,53)$ and $f_{mg}(q^2)/\mu$.

The conclusion that can be drawn is that the Chou-Yang model is not ruled out by the recent experimental data on pp elastic scattering at high energy since these authors have conjectured that the hadronic matter density should be similar to the charge or magnetization densities within the proton.

As shown here, this is true as long as the imaginary part of the elastic amplitude is positive and the physical hypothesis (4.6) is replaced (according to our analyses of sections 2 and 3) by:

$$f_h(q^2, \sqrt{s} \to \infty) \to \mu^{-1} f_{mag}(q^2) = \frac{\mu^{-1}\, G_M(t)}{\left[\left(1 - \frac{t}{4M^2}\right)\left(1 - \frac{t}{2M^2}\right)\right]^{1/2}} \tag{4.15}$$

where $t = 2m\,(m - \sqrt{m^2+q^2}\,)$.

Our phenomenological finding makes the extension of the Chou-Yang model to neutron-proton (n-p) elastic scattering a very appealing one. The reason is that it is experimentally observed that $G_M^p\,(t)/2.79 \simeq G_M^n(t)/1.9$ (one notes that $G_E^p(t) \neq G_E^n(t)$). Therefore on the basis of the Chou-Yang model and in view of (4.15) one can predict that $d\sigma^{pp}/dq^2$ and $d\sigma^{np}(dq^2)$ should be very similar at $s \to \infty$ since we also have $m_p \simeq m_n$. Recent experimental data[(29)] confirms this suggestion.

V - CONCLUSIONS AND FINAL REMARKS

The characterization of the structure of an elementary particle in terms of familiar physical quantities such as charge and magnetization distributions is an old problem in physics. Such a structure is probed by means of experiments in which momentum is transfered to an appropriate test body (usually the electron). Since elastic scattering of electrons by some particle is parametrized, within the one photon exchange approximation, in terms of form-factors, the determination of the structure of the particle out of what is measured experimentally is, basically, a problem of relating the form-factors to those physical quantities.

In this paper we have proposed a new scheme, inspired by Weisskopf's ideas, for relating the charge and magnetization densities to the electromagnetic form-factors of the system. The idea is extremely simple: one considers a classical correlation function and finds its quantum analog (i.e. the correlation function which has, within the quantum context, the same meaning).

The main virtues of our method are:

a) It is possible to eliminate the uncertainty in the position of the extended system as a whole.

b) The generalization to any quantum system (particles or nuclei with arbitrary spin) is simple.

c) For spin 0 and spin 1/2 objects our explicit results reduce to those correct ones in the non relativistic limit.

d) We have obtained a consistent description (summarized by equations (2.13) to (2.15)) of the mean charge radii of particles with spin 1/2 and spin 0.

We have also made two phenomenological applications of our connection between charge and magnetization distributions and the electromagnetic form-factors. In the first example, we have shown that an analysis of the experimental data for the ratio G_E/G_M implies that the charge distribution of the proton is more concentrated than the magnetization distribution. That indicates that hadronic matter could also be distributed in a different manner inside the proton.

The matter distribution is a basic ingredient in the Chou-Yang model. Our idea was, by assuming this model, to extract the matter density of the proton from the experimental data on pp elastic scattering at high energy. We have reached the conclusion that the hadronic matter density becomes more close to its magnetization density when the energy increases (discrepancies of the order of $10 \sim 15\%$ in the Fourier transform of these densities could be atributed to the neglect of the real part of the scattering amplitude,or to spin effects, or to the fact that we have not yet reached an asymptotic energy). We just want to recall that this conclusion depends upon the hypothesis that the imaginary part of the amplitude be positive (or have a double zero at $q^2 \simeq 1.4$ GeV^2) in the interval covered by the experimental data ($q^2 \leq 10$ GeV^2) . This hypothesis deserves experimental confirmation. Polarization experiments could settle this point in the future.

The similarity pointed out by us, between the proton magnetization and matter distribution, might be relevant within the phenomenological context. As we have shown it allows us to understand, on the light of the Chou-Yang model, the similarity observed experimentally, between the proton-proton and neutron-proton elastic scattering at high energies.

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TABLE I

$\sqrt{s}$	19.4 GeV	53 GeV
α_1	1.8	1.77
α_2	6.12	6.95
α_3	0.05	0.06
α_4	0.0024	0.0002
β_1	12.0	15.0
β_2	4.61	5.3
β_3	0.95	1.02
β_4	0.42	0.16
A	150.0	172.0
B	8.3	8.64
N	8	8

FIGURE CAPTIONS

Fig. 1

Experimental data[20] for the ratio $\mu^2 G_E^2/G_M^2$ (Price , Bartel). The full curve corresponds to formula (3.17) with the hypothesis $\mu f_{ch}=f_{mg}$. For coments see section 3.

Fig. 2

Fit to pp elastic differential cross section data[31,32] at $\sqrt{s}=53$ GeV . The curve corresponds to $a(q^2)$ parametrized by (4.12). The values of the parameters can be found in table I.

Fig. 3

The same as Fig. 2 but for large q^2 . The experimental data are from Nagy et al. Ref. 26.

Fig. 4

Experimental data on pp elastic scattering at $\sqrt{s}=19.4$ GeV as given by Akerlof et al. (Ref. 29) and Carrol et al. (Ref. 30). The curve corresponds to a fit by using an amplitude parametrized by (4.12). The values of the parameters can be found in table I. For comparison we also shown the experimental data on proton-neutron (pn) elastic scattering at the same energy. (De Haven et al. Ref. 29).

Fig. 5

The same as Fig. 4 but for large q^2 . For $q^2 > 5$ GeV^2 we have used the experimental data given by Hartman et al. (Ref. 25).

Fig. 6

Comparison of the Fourier transforms of the charge (f_{ch}) , magnetization (f_{mg}) and hadronic matter (f_h) densities. For f_h we show the cases: 1) the dashed curve corresponds to $\sqrt{s}=19.4$ GeV (see also Figs. 4 and 5); 2) the full curve corresponds to $\sqrt{s}=53$ GeV (see also Figs. 2 and 3). We have used the experimental data (Ref. 33) on elastic electron-proton collisions, and our expressions (2.9) and (3.12) to construct the experimental values of f_{ch} and f_{mg} respectivelly.

Fig. 7

The same as Fig. 6 but now normalized to the dipole formula $f_d(q) = (1+q^2/0.71)^{-2}$. The continuous curve corresponds to f_h

obtained from pp data at $\sqrt{s} = 53$ GeV while the doted curve was obtained from pp data at $\sqrt{s} = 19.4$ GeV.

Fig. 8

The opacity at $\sqrt{s} = 19.4$ GeV (doted curve) and at $\sqrt{s} = 53$ GeV (full curve). The expansion with increasing energy is clearly visible for $b > 0.5\ f_m$.

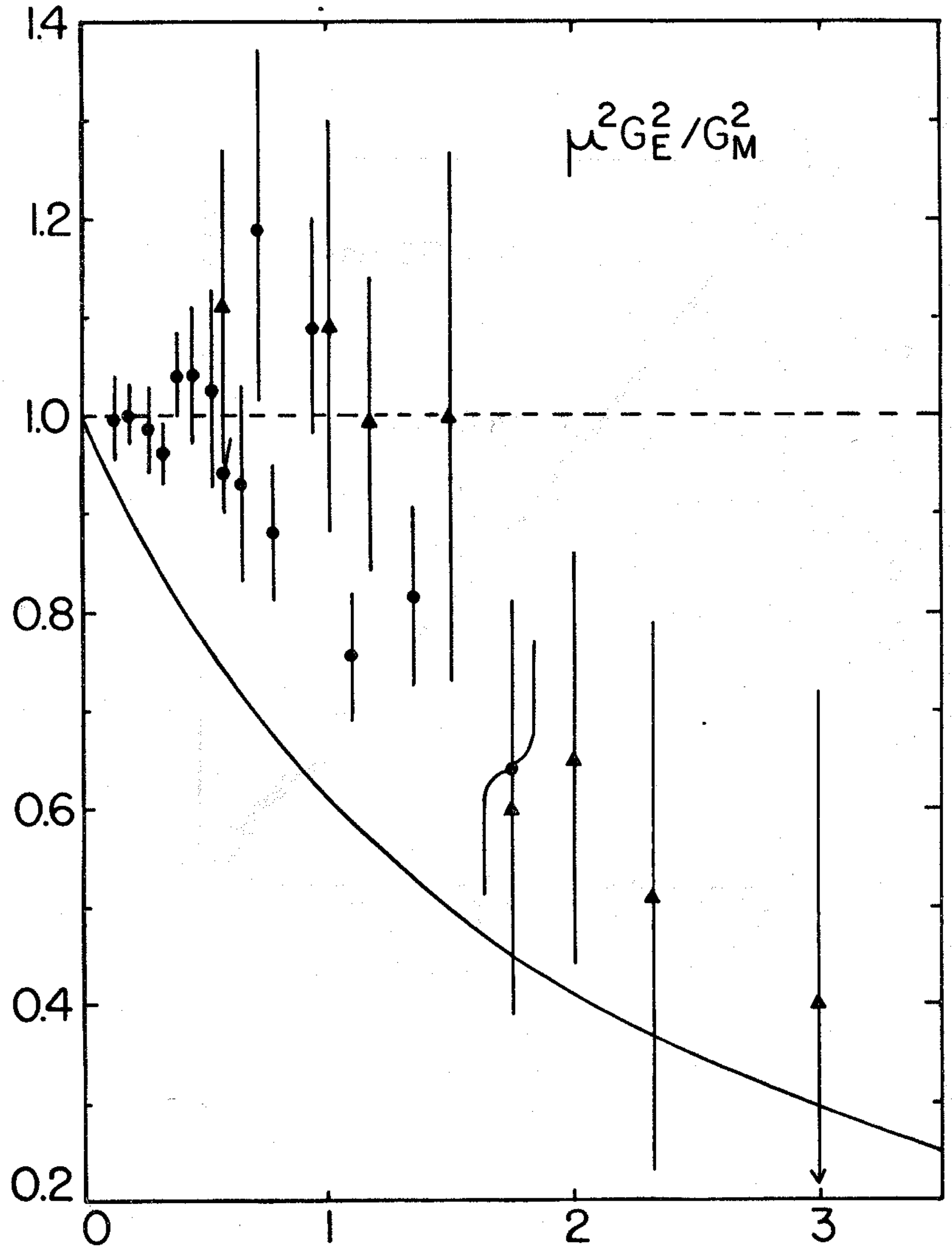


Fig. 1

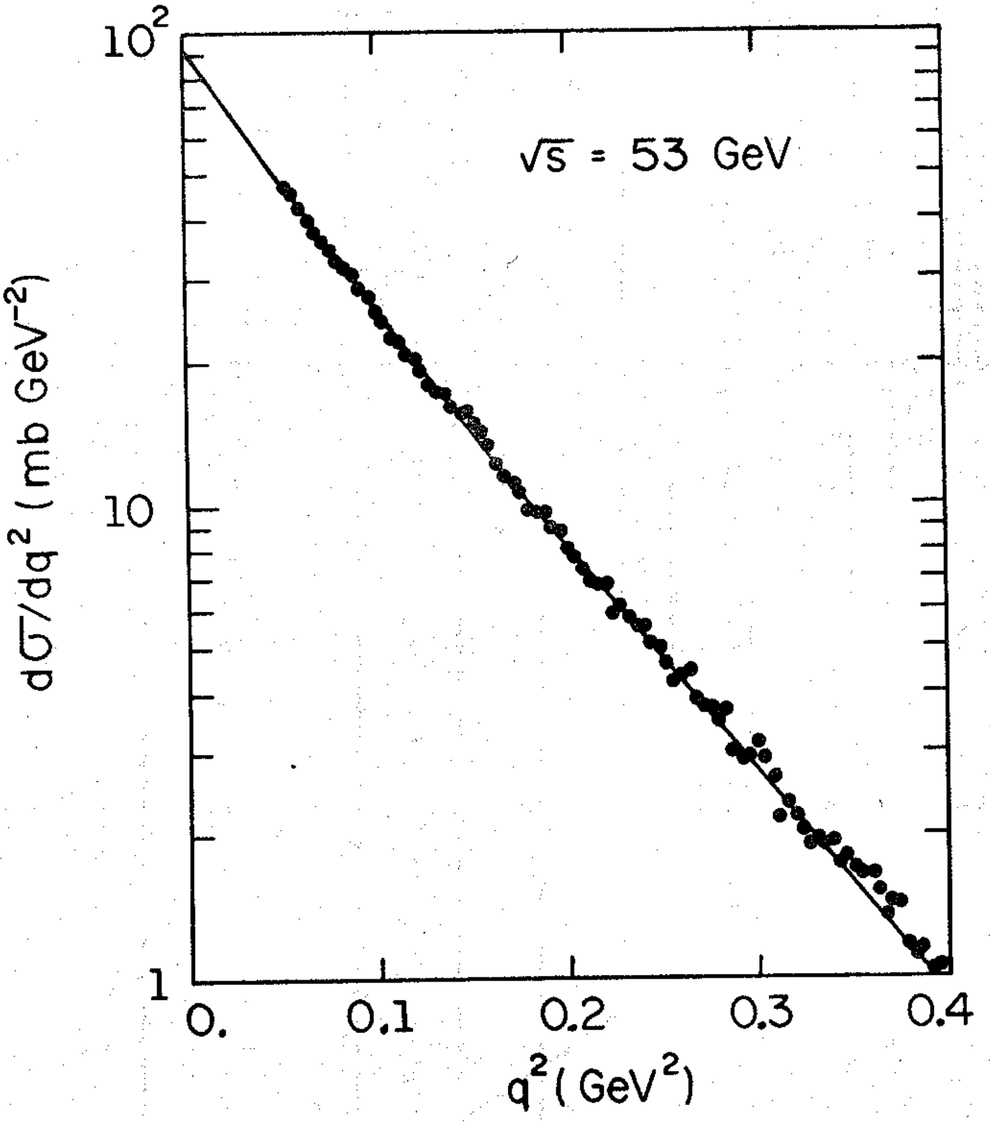
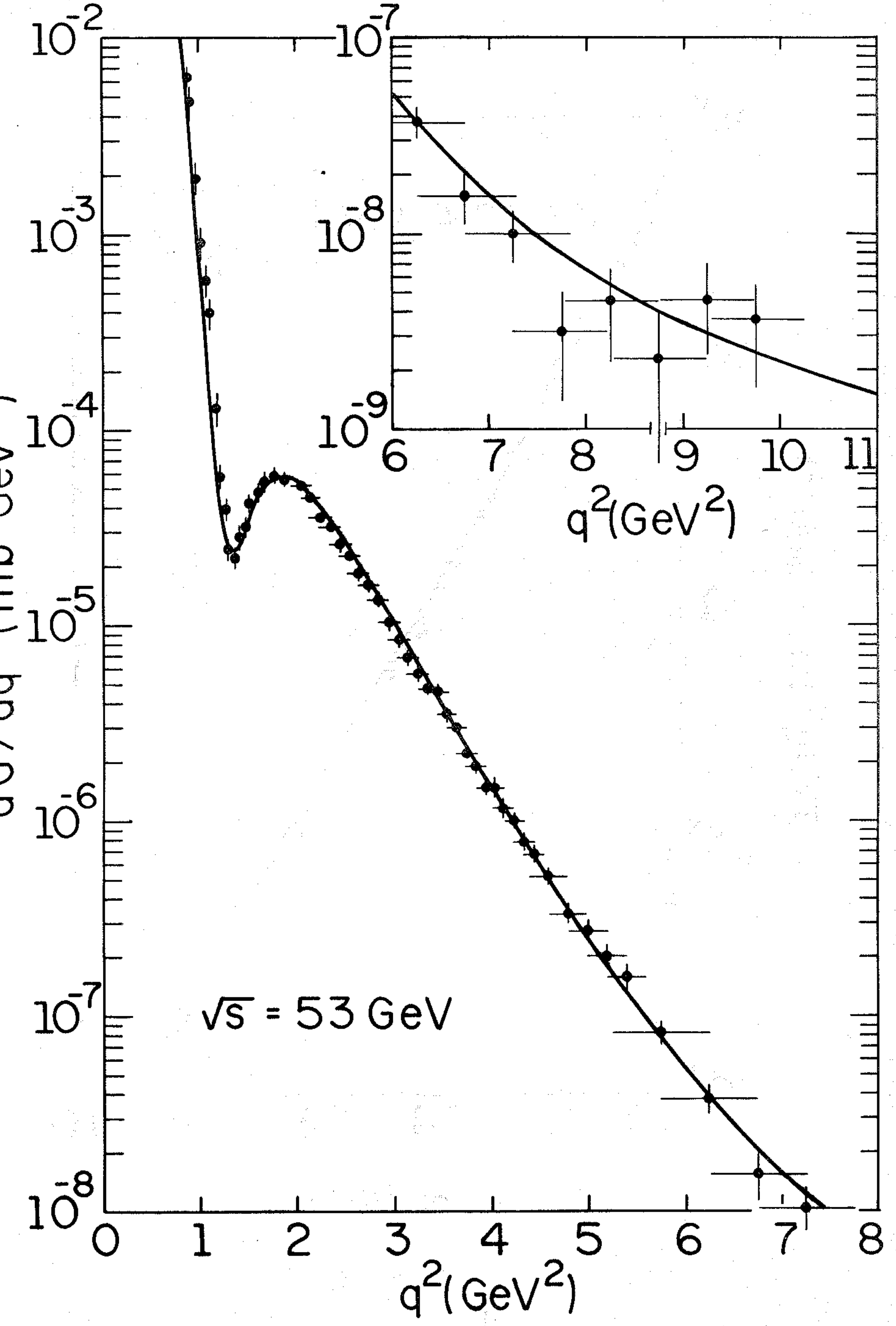
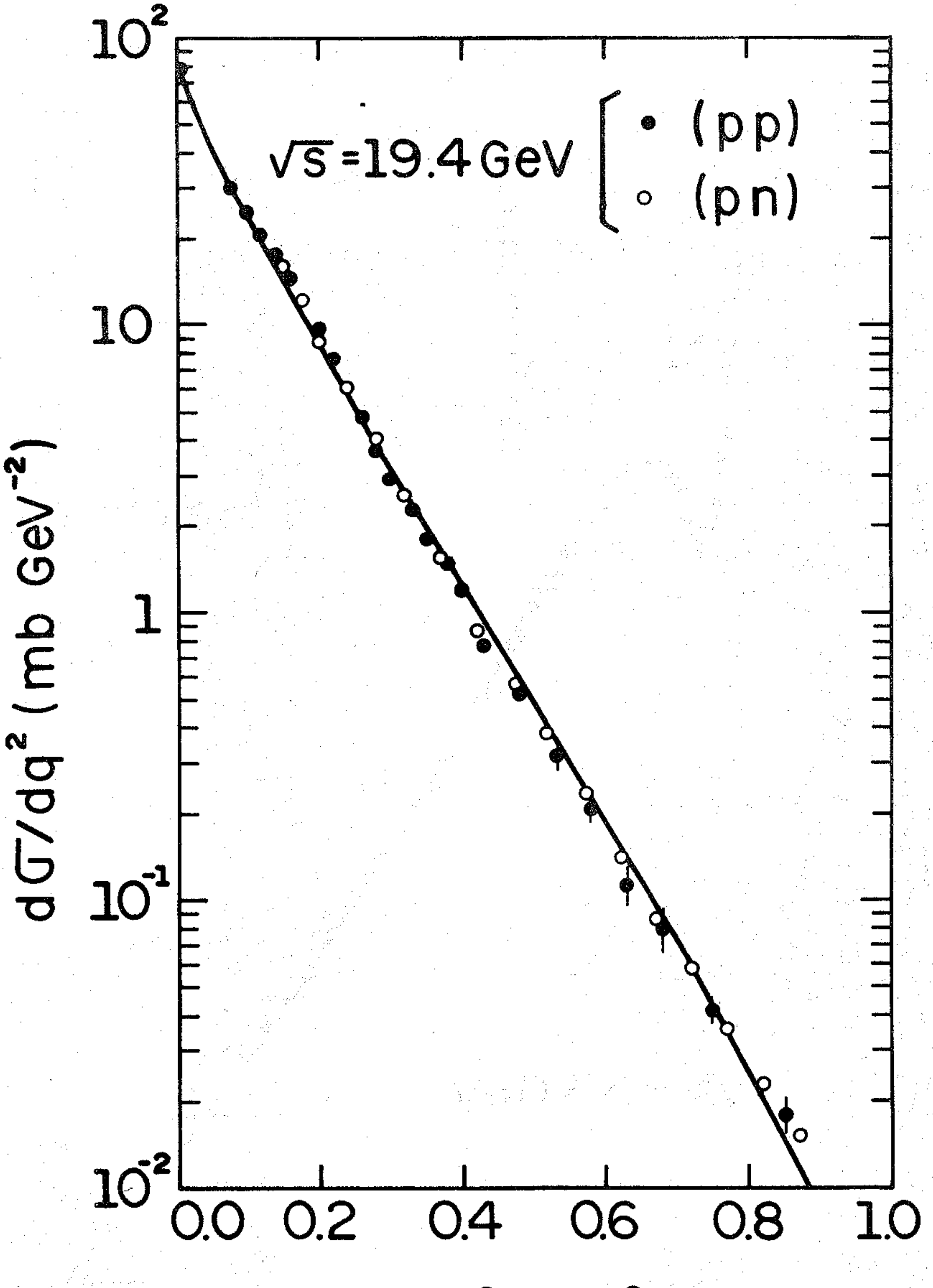


Fig. 2



10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$d\sigma/dq^2$ (mb GeV^{-2})
$\sqrt{s}$ = 53 GeV
0 1 2 3 4 5 6 7 8
q^2(GeV^2)
10^{-7}
10^{-8}
10^{-9}
6 7 8 9 10 11
q^2(GeV^2)

Fig. 3



$\sqrt{s}$ = 19.4 GeV
• (pp)
○ (pn)
10^2
10
1
10^{-1}
10^{-2}
0.0
0.2
0.4
0.6
0.8
1.0
$d\sigma/dq^2$ (mb GeV^{-2})
q^2 (GeV^2)

Fig. 4

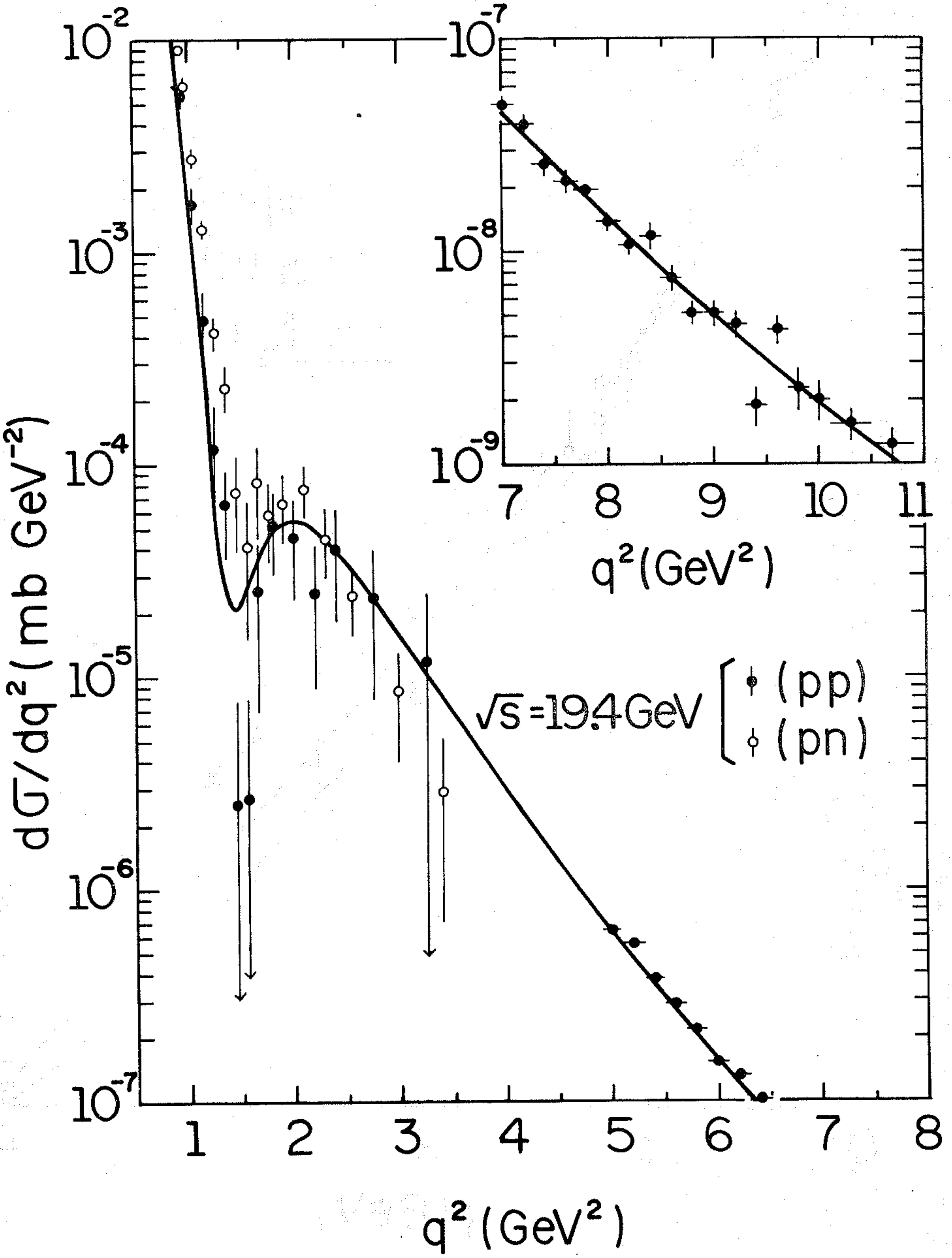

10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$d\sigma/dq^2$ (mb GeV^{-2})
1
2
3
4
5
6
7
8
q^2 (GeV^2)
10^{-7}
10^{-8}
10^{-9}
7
8
9
10
11
q^2 (GeV^2)
$\sqrt{s}$ = 19.4 GeV
(pp)
(pn)

Fig. 5

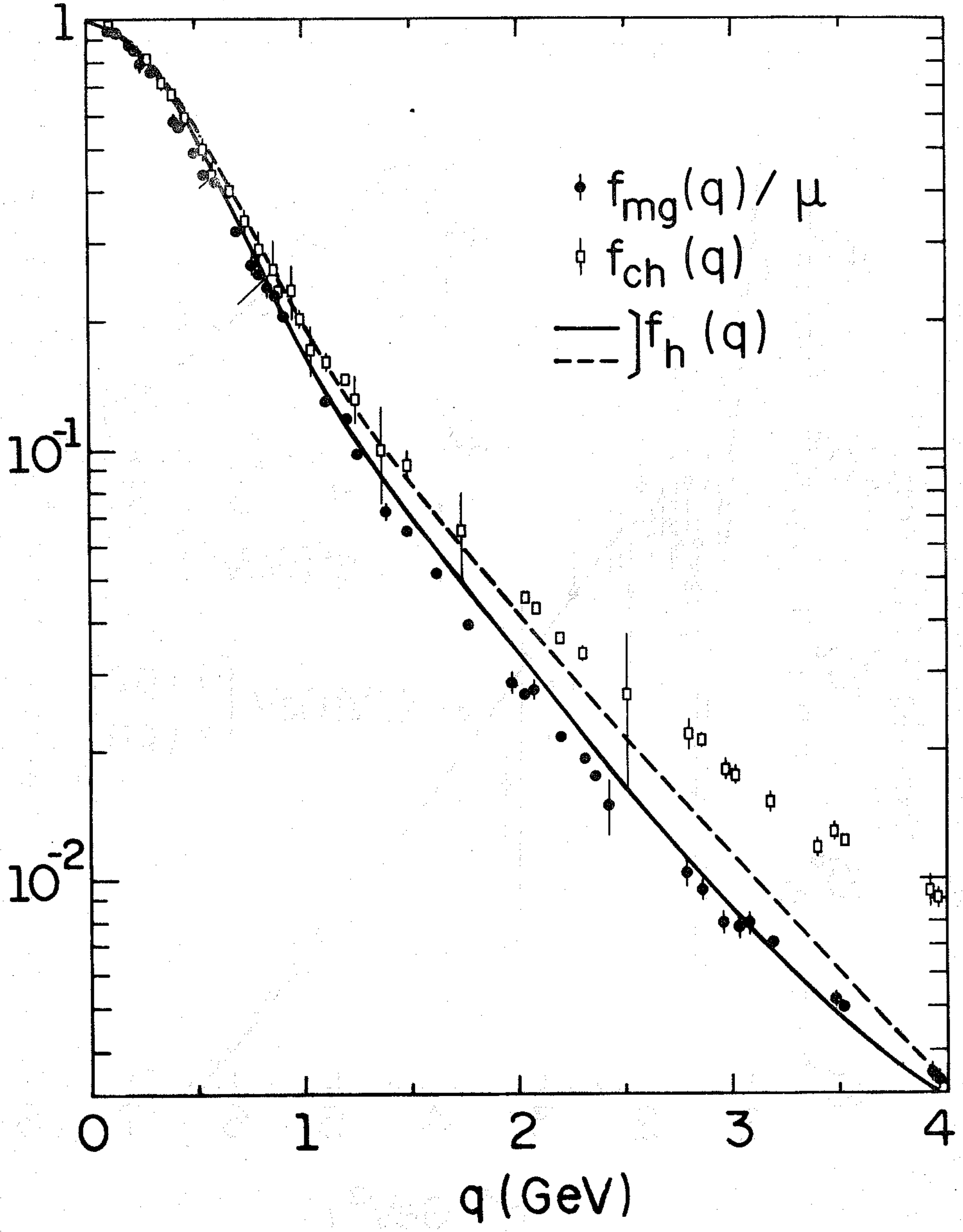


Fig. 6

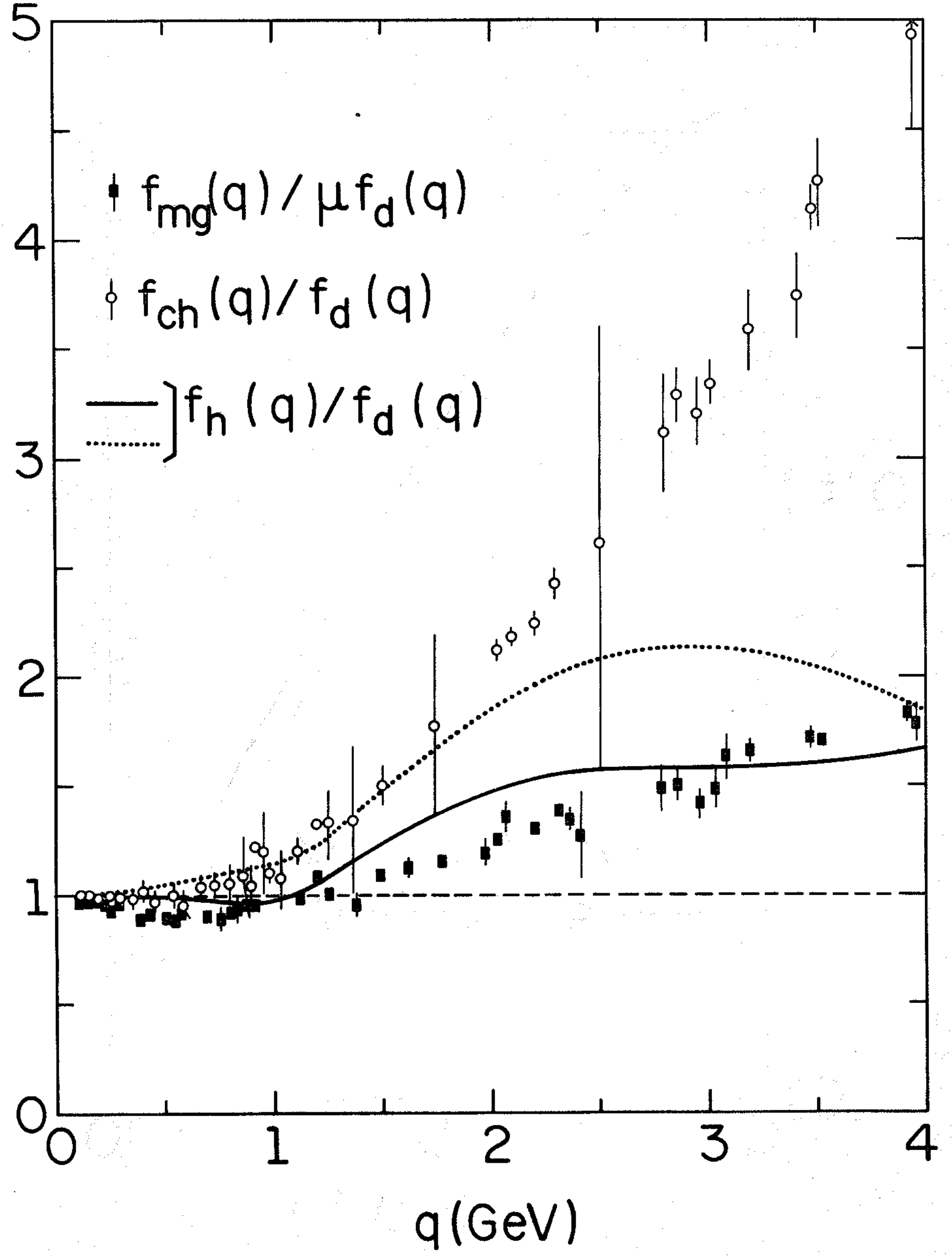


Fig. 7

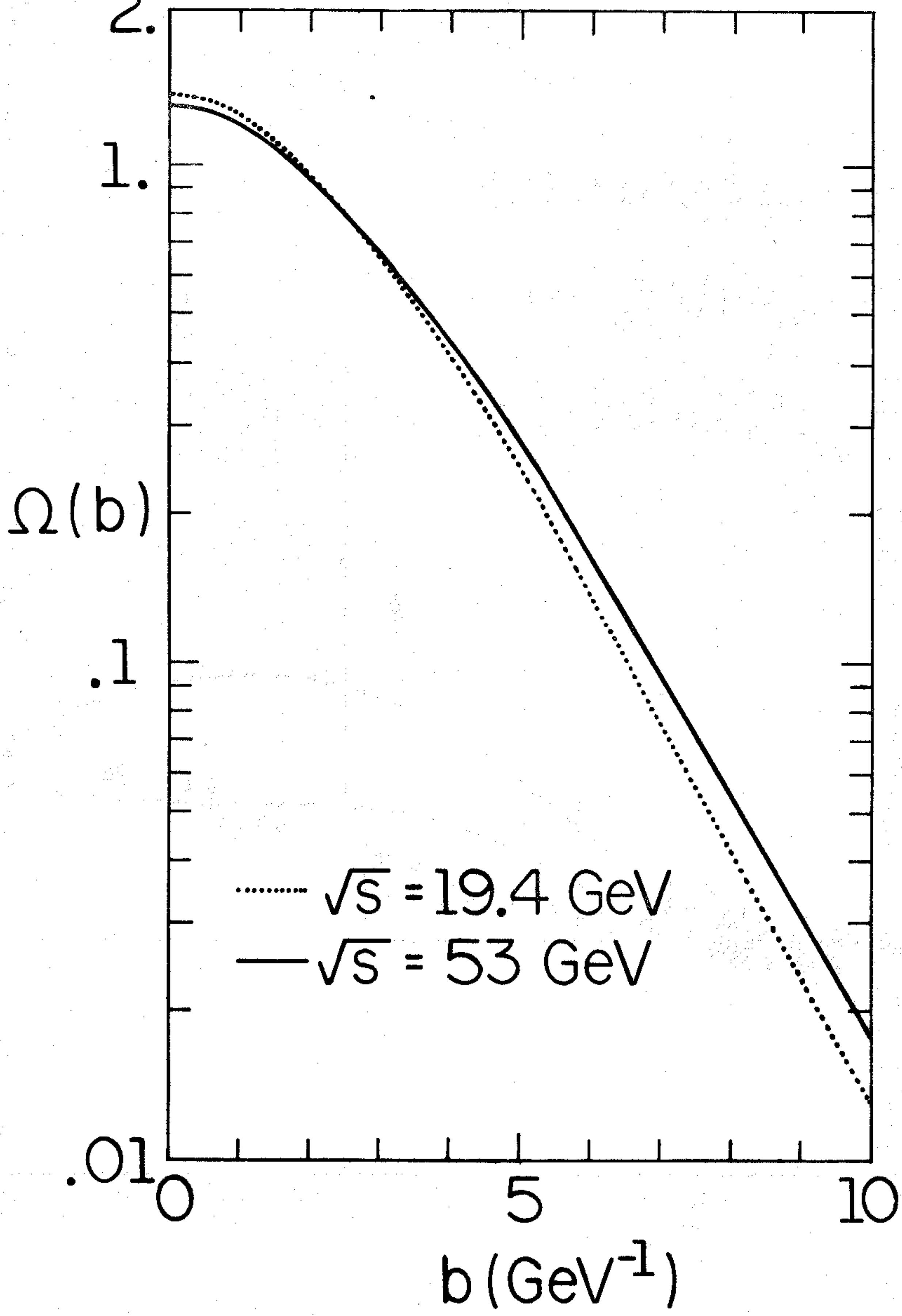


Fig. 8