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preprint

IFUSP/P-199

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March, 1980

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ABSTRACT

We show that the fluctuation cross section for the generalized-exciton or nested-doorway model can be obtained explicitly and exactly in the limit that doorways of successive classes have very different widths, $\Gamma_n \gg \Gamma_{n+1}$, and that the doorways of a given class are overlapping, $\Gamma_n > D_n$. The result is given in terms of experimentally observable quantities, and explicitly separates the compound and pre-compound contributions. It contains the results of previous, more specialized, models as limiting cases.

Invited paper presented at the 3rd Annual Symposium on Nuclear Physics, Oaxtepec, Mexico, January, 1980

†Supported in part by the Tinker Foundation, the U.S.-N.S.F., the CNPq-Brazil and FEPESP-Brazil.

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A pre-equilibrium reaction is one which by definition is more complex than a 1-step or direct reaction, but less complex than one which passes through a compound-nucleus or thermally equilibrated intermediate stage. The doorway picture[1] is the one most generally applied to understand such reactions, and the exciton model[2] is its most thoroughly studied example. Feshbach, Kerman, and Koonin,[3] in an important recent contribution, divide pre-equilibrium reactions in general into multi-step-direct and multi-step-compound components, primarily on the basis of angular distributions, the former being forward-peaked and the latter symmetric about 90^{o}.

Employing this terminology, we wish to discuss here a derivation of the multi-step compound component of the pre-equilibrium cross section in the generalized exciton doorway model. It is a derivation which employs a nested set of energy averages, and its principal merits are algebraic simplicity, a close connection to experimental data, and a division of the cross section into additive contributions characterized by different time delays.

We are aware of four distinct derivations (including the present one) of this multi-step compound cross section, all very different and all interesting because they appear to be based on distinct assumptions. Our purpose in the present contribution is to provide the full details of our approach (a condensed version of which has been published elsewhere[4]), and to compare its assumptions with those employed by other authors.

Stated very briefly, the approach of Agassi et al.[5] employs the original exciton model, using shell-model particle-hole states as its excitons. It expands the fluctuation amplitude in a Born series, which it energy averages term-by-term, employing the ergodic theorem to replace incident-energy averages by averages over ensembles of random matrix elements.

The assumption of random phases for all matrix elements (potential and S-matrix elements) is central to all approaches as is the assumption that $\Gamma >> D$ for each class of doorways. One of the further key assumptions of Agassi et al. seems to be that of a large number M of open channels, since their result seems to be the leading term in an expansion in powers of M^{-1}.

The second approach, by Feshbach et al.,[3] employs the Feshbach coupled-channels formalism to obtain a formal expression for the fluctuation amplitude, in the basic $\langle f|VGV|i\rangle$ form. G(E) supplies the fine-structure resonance poles, whose residues are assumed to be modulated by doorway states; these residues are thus found to have the same VGV form, with the new G(E) containing the doorway poles. Their residues in turn are modulated by a class of wider doorways and so have their own poles, thus developing a hierachy of doorways, all assumed overlapping. These authors employ several rather special assumptions, such as their "chaining" hypothesis, (that doorway states of class n couple only to those of classes $n \pm 1$) but one of their key assumptions seems to be that the reaction entrance channel couples only to class 1 (broadest doorways), which never couples directly to the exit channel.

The third approach is one under development by Friedman, which dispenses with phase information from the outset, and treats the flow of probability between the various classes of doorways as the flow of a classical, incompressible fluid. Remarkably, this approach finds that the geometrical series describing any number of "round trips" between pairs of classes can be summed to provide just the Hauser-Feshbach type of denominators which appear in the other approaches. Because there is no explicit reference to resonances or even energy-dependence, the question of energy averaging never arises in this approach.

II. THE GENERALIZED KKM APPROACH

A. $\sigma^{f\ell}$ and the P-matrices

The present approach is algebraically simpler than that of Agassi et al.,[5] because they begin with the assumption that the matrix elements of the scattering potential are random in phase, and so are lead to average over products of an arbitrary number of them in the Born series for the scattering amplitude; in contrast, we start from the assumption that this same randomness produces random phases in the residues of the resonance poles, which occur only quadratically in the cross section. Furthermore, the algebra is simplified relative to that of Feshbach et al.[3] because we are able to make use of algebraic results previously obtained by Kawai, Kerman and McVoy[7] (KKM) in treating fluctuation cross sections in the absence of doorways. These results follow directly from the fact that the S-matrix can be written

$$\underline{S} = \overline{\underline{S}} + \underline{S}^{f\ell} , \tag{2.1}$$

with

$$S^{f\ell}_{cc'} = -i \sum_q \frac{g_{qc}\, g_{qc'}}{E-\mathcal{E}_q} \tag{2.2}$$

and

$$\langle S^{f\ell}_{cc'} \rangle = 0, \tag{2.3}$$

which can be achieved as indicated in Ref. 7. We then have

$$\begin{aligned} \langle \sigma^{f\ell}_{cc'} \rangle &= \langle |S^{f\ell}_{cc'}|^2 \rangle \\ &= X_{cc} X_{c'c'} + X_{cc'} X_{c'c} \end{aligned} \tag{2.4}$$

as well as energy-averaged unitarity,

$$1 = \langle \underline{S}\underline{S}^{+} \rangle = \underline{\bar{S}}\,\underline{\bar{S}}^{+} + \langle \underline{S}^{f\ell}\underline{S}^{f\ell^{+}} \rangle$$
$$= \underline{\bar{S}}\,\underline{\bar{S}}^{+} + \underline{X}\,\mathrm{Tr}(\underline{X}) + \underline{X}^{2} \ , \tag{2.5}$$

where[7]

$$X_{cc'} \equiv \left(\frac{2\pi}{\Gamma D}\right)^{1/2} \langle g_{qc}\, g^{*}_{qc'} \rangle_{q} \ . \tag{2.6}$$

Then eliminating the matrix $\underline{X}$ between these two equations expresses $\sigma^{f\ell}$ in terms of the optical S-matrix, $\underline{\bar{S}}$. Although this is complicated in general, arguments have been given[8] which indicate that the $\underline{X}^2$ term of (2.5) is generally much smaller by cancellation than its first term, if the number of open channels M is large. Neglecting it gives $\underline{X}$ and so $\sigma^{f\ell}$ directly in terms of $\underline{\bar{S}}\,\underline{\bar{S}}^{+}$, the result in the case of no channel coupling being just the Hauser-Feshbach expression.

This algebra requires the assumptions

$$\begin{aligned} &\text{a)}\ M \gg 1 \\ &\text{b)}\ \Gamma \gg D \\ &\text{c)}\ \underline{\bar{S}}\,\underline{\bar{S}}^{+} \ll 1 \end{aligned} \tag{2.7a}$$

The last condition represents the assumption of strong absorption out of the direct channels; the results stated are certainly most reliable in this limit, but the exact upper limit on the eigenvalues of $\underline{\bar{S}}\,\underline{\bar{S}}^{+}$ is not yet clear. In order to generalize these results to the case at hand, of a nested sequence of doorways, we seem to require the additional conditions

$$\begin{aligned} &\text{d)}\ \Gamma_{n} \gg \Gamma_{n+1} \ , \\ &\text{e)}\ \langle \underline{\tau}_{m}\underline{\tau}_{n} \rangle = \langle \underline{\tau}_{m} \rangle \langle \underline{\tau}_{n} \rangle \ , \end{aligned} \tag{2.7b}$$

to be explained below.

The full argument is this: The fundamental assumption of our approach (and of all others) is that the various resonances and fine-structure states of the scattering system do not have a continuous distribution of widths, but rather organize themselves clearly into a hierarchy of "classes", corresponding in time to the "stages" through which the reaction proceeds. If, following Ref. 3, we call the first (in time) stage or class number 1, our "well-nested" condition on the widths is

$$\Gamma_1 >> \Gamma_2 >> \ldots >> \Gamma_N \quad , \tag{2.8}$$

where the resonances of width = Γ_N are the actual fine-structure or compound-nucleus eigenstates of the system. The states of class n clearly contribute pre-equilibrium flux to the reaction which appears with an average time-delay of $\hbar/\Gamma_n$.

Because of the condition of Eq. (2.8), we can define a nested sequence of energy-averaging intervals I_n by interpolating the I_n's between the Γ_n's,

$$\Gamma_{n-1} >> I_n > \Gamma_n . \tag{2.9}$$

This permits us to define N different optical S-matrices, $\bar{\underline{S}}_n \equiv \langle \underline{S} \rangle_{I_n}$, by averaging the reaction excitation functions over these N intervals; $\bar{\underline{S}}_n(E)$ will contain the poles describing the resonances of class (n-1). E.g., $\underline{S}(E)$ really contains only the fine-structure resonances, of class N, but their partial widths are modulated by the "deepest doorway" states, of class N-1, in such a way that an average over I_N produces an $\langle \underline{S} \rangle_{I_N}$ which contains the "effective poles" of class (N-1). These poles are themselves doorway-modulated in such a way that $\langle \bar{\underline{S}}_N(E) \rangle_{I_{N-1}}$ has the effective poles of class (N-2), etc.

If the contribution of the levels of class n is written as

$$(S_{nr}^{f\ell})_{cc'} = i \sum_q \frac{g_{qc}^{(n)} \; g_{qc'}^{(n)}}{E - \tilde{\varepsilon}_q} \quad , \tag{2.10}$$

we follow KKM in defining a matrix $\underline{X}_n$ for each class, by[2]

$$(\underline{X}_n)_{cc'} = \left(\frac{2\pi}{\Gamma_n D_n}\right)^{1/2} \langle g_{qc}^{(n)} \; g_{qc'}^{(n)*} \rangle_q \quad . \tag{2.11}$$

As in KKM, averaging only over the smallest energy defines an optical S-matrix which we call $\underline{\bar{S}}_N \equiv \langle S \rangle_{I_N}$,

$$\underline{S} = \underline{\bar{S}}_N + \underline{S}_N^{f\ell} \quad , \tag{2.12}$$

with $\underline{S}_N^{f\ell}$ given by (2.10), and, via the KKM manipulation,

$$\langle \underline{S}_N^{f\ell} \rangle_{I_N} = 0 \quad . \tag{2.13}$$

But, as mentioned, any doorways present must appear in $\underline{\bar{S}}_N$, and averaging over an interval I_{N-1} separates out those doorways of width Γ_{N-1} and defines a second optical S-matrix $\underline{\bar{S}}_{N-1}$,

$$\underline{\bar{S}}_N = \underline{\bar{S}}_{N-1} + \underline{S}_{N-1}^{f\ell} \quad , \tag{2.14}$$

with $\langle \underline{S}_{N-1}^{f\ell} \rangle_{I_{N-1}} = 0$. Continuing in this way we can decompose $\underline{S}$ into contributions from all its distinct classes of doorways,

$$\underline{S} = \underline{\bar{S}}_1 + \sum_1^N \underline{S}_n^{f\ell} \quad . \tag{2.15}$$

The essential feature is that since each $\underline{S}^{f\ell}$ averages to zero over its own interval, $\langle \underline{S} \; \underline{S}^* \rangle$ will contain no cross terms, so the cross section averaged over I_1 is

$$\begin{aligned}
\langle\sigma_{cc'}\rangle_{I_1} &= (\bar{S}_1)_{cc'}\,(\bar{S}_1)^*_{cc'} + \Big\langle \sum_1^N (S_n^{f\ell})_{cc'}(S_n^{f\ell})^*_{cc'}\Big\rangle_{I_1} \\
&= \sigma^{dir}_{cc'} + \sum_1^N \langle (X_n)_{cc}(X_n)_{c'c'} + (X_n)_{cc'}(X_n)_{c'c}\rangle_{I_1} , \\
&\equiv \sigma^{dir}_{cc'} + \langle\sigma^{f\ell}_{cc'}\rangle_{I_1} ,
\end{aligned} \tag{2.16}$$

simply by applying the KKM argument to each class.

The connection with transmission matrices is obtained similarly. From unitarity and Eq. (2.12) we have

$$\begin{aligned}
1 = \langle \underline{S}\,\underline{S}^+\rangle_N &= \langle(\underline{\bar{S}}_N + \underline{S}_N^{f\ell})(\underline{\bar{S}}_N + \underline{S}_N^{f\ell})^+\rangle_N \\
&= \underline{\bar{S}}_N\,\underline{\bar{S}}_N^+ + \underline{X}_N\,\mathrm{Tr}(\underline{X}_N) + \underline{X}_N^2 ,
\end{aligned} \tag{2.17a}$$

so we can define a Satchler penetration matrix relative to averaging interval I_N,

$$\begin{aligned}
\underline{P}_N &= 1 - \underline{\bar{S}}_N\,\underline{\bar{S}}_N^+ \\
&= \underline{X}_N\,\mathrm{Tr}(\underline{X}_N) + \underline{X}_N^2 ,
\end{aligned} \tag{2.17b}$$

which is exactly as in Ref. 7 because the resonances of class N cannot decay downward. Doing the same with Eq. (2.14) gives

$$\langle \underline{\bar{S}}_N\underline{\bar{S}}_N^+\rangle_{N-1} = \underline{\bar{S}}_{N-1}\underline{\bar{S}}^+_{N-1} + \langle \underline{S}^{f\ell}_{N-1}\,\underline{S}^{f\ell^+}_{N-1}\rangle_{N-1} , \tag{2.18}$$

and subtracting this from 1 and rearranging gives

$$\underline{P}_{N-1} - \underline{\bar{P}}_N = \underline{X}_{N-1}\,\mathrm{Tr}(\underline{X}_{N-1}) + \underline{X}^2_{N-1} \simeq \underline{X}_{N-1}\,\mathrm{Tr}(\underline{X}_{N-1}) , \tag{2.19}$$

employing in the last line the above mentioned approximation, for which the condition of many open channels is necessary. Here we have defined

$$\bar{\underline{P}}_n \equiv \langle \underline{P}_n \rangle_{n-1} \quad , \tag{2.20}$$

which we shall use for general n. Physically, the extra term in (2.19) relative to (2.17) is due to the fact that the states of class N-1 can decay downward as well as upwards, while those of class N cannot.

Since the manipulation of (2.19) can be carried out for general n, it gives $\underline{X}_n$ in terms of the transmission matrices,

$$\underline{X}_n = \frac{\underline{P}_n - \bar{\underline{P}}_{n+1}}{\sqrt{\mathrm{Tr}(\underline{P}_n - \bar{\underline{P}}_{n+1})}} \, . \tag{2.21}$$

It is in this way that direct contact with experimental data is achieved, for the $\underline{P}_n$'s are in principle obtainable by fitting the "direct" part of the cross section at stage n (i.e., after averaging over I_n) with optical potentials (or coupled-channel calculations, if the channel couple "directly" at this stage) to extract $\bar{\underline{S}}_n$ and so $\underline{P}_n = 1 - \bar{\underline{S}}_n \bar{\underline{S}}_n^+$. This clearly requires knowing what the fundamental widths Γ_n are in the sequence of Eq. (2.8), in order to be able to choose the I_n intervals correctly; presumably they are most readily accessible from an analysis of Ericson fluctuations. In any case, inserting (2.21) into (2.16) gives the fluctuation cross section in terms of the "optical" transmission matrices of the problem,

$$\langle \sigma^{f\ell}_{cc'} \rangle_{I_l} = \Big\langle \sum_{1}^{N-1} \frac{(\underline{P}_n - \bar{\underline{P}}_{n+1})_{cc} (\underline{P}_n - \bar{\underline{P}}_{n+1})_{c'c'} + (\underline{P}_n - \bar{\underline{P}}_{n+1})_{cc'} (\underline{P}_n - \bar{\underline{P}}_{n+1})_{c'c}}{\mathrm{Tr}(\underline{P}_n - \bar{\underline{P}}_{n+1})}$$

$$+ \frac{(\underline{P}_N)_{cc} (\underline{P}_N)_{c'c'} + (\underline{P}_N)_{cc'} (\underline{P}_N)_{c'c}}{\mathrm{Tr}(\underline{P}_N)} \Big\rangle_{I_1} \, . \tag{2.22}$$

Eq. (2.22) is our central result. Its essential feature, which distinguishes it from, e.g., a similar result by Agassi et al.[5], is that it is automatically separated, by the use of nested energy-averages, into contributions corresponding to different time-delays. E.g., if only the energy average over the smallest interval I_N were performed, the corresponding $\langle\sigma^{f\ell}\rangle_{I_N}$ would contain only the last term of Eq.(2.21). The others, which correspond to time-delays less than $\hbar/I_N$, could not be distinguished from the direct-reaction components in a measurement whose energy-resolution is $\Delta E = I_N$. Thus, using successively wider energy-averaging intervals moves successively more of the pre-compound components from σ^{dir} into $\sigma^{f\ell}$. The fact that $\langle\sigma^{f\ell}\rangle$ is given by Eq. (2.22) as a sum of generalized Hauser-Feshbach terms evidently means that the well-nested condition, Eq. (2.8), implies that an equilibration among the degrees of freedom of each class m is reached before the system decays back into the open channels.

B. $\sigma^{f\ell}$ and the τ-Matrices

The transmission matrices $\langle P_{\underline{n}} \rangle$ are related as directly as possible to experimental data, but they are conceptually a bit complex because it is their differences that are related to a single class of states. Previous authors[6-9] have found it convenient to introduce a different set of transmission matrices, τ_n, defined as in Eq. (4) but for the hypothetical case of an S-matrix that contains only the nth class of resonances (which thus have no $\Gamma_n^{\downarrow}$, and decay only directly to the channels without passing through the doorways "above" them.

To obtain the (linear) relation between the $\underline{P}$'s and the τ's, we consider first the case of just two classes of resonances, fine-structure and one type of doorway. We recall from Eq. (2.10) that

$$< |g_c^{(1)}|^2 > \; = \Gamma_c^{(1)} \tag{2.23}$$

is the average partial width for the decay of the levels of class 1 (doorways) "upward" into channel c, so that

$$\begin{aligned} \text{Tr } \underline{X}_1 &= \left(\frac{2\pi}{\Gamma_1 D_1}\right)^{1/2} \sum_c \Gamma_c^{(1)} \equiv \left(\frac{2\pi}{\Gamma_1 D_1}\right)^{1/2} \Gamma_1^{\uparrow} \\ &\left(\frac{2\pi}{\Gamma_1 D_1}\right)^{1/2} (\Gamma_1 - \Gamma_1^{\downarrow}) \; . \end{aligned} \tag{2.24}$$

Then if we define

$$(\underline{\tau}_1)_{cc'} \equiv \frac{2\pi}{D_1} < g_c^{(1)} \; g_{c'}^{(1)*} > \tag{2.25}$$

as the transmission matrix to the channels from the levels of class 1 in the absence of downward coupling to class 2, and also define the branching ratio for downward decay,

$$\mu_{12} \equiv \frac{\Gamma_1^{\downarrow}}{\Gamma_1} \; , \tag{2.26}$$

we have

$$\begin{aligned} (\underline{P}_1 - \underline{\bar{P}}_2)_{cc'} &= (X_1)_{cc} \; \text{Tr}(X_1) \\ &= \frac{2\pi}{\Gamma_1 D_1} \Gamma_1^{\uparrow} < g_c^{(1)} \; g_{c'}^{(1)*} > \\ &= \frac{\Gamma_1^{\uparrow}}{\Gamma_1} (\tau_1)_{cc'} = (1-\mu_{12})(\tau_1)_{cc'} \; . \end{aligned} \tag{2.27}$$

Then,

$$\begin{aligned}\underline{\bar{P}}_2 &= \underline{P}_1 - (\underline{P}_1 - \underline{\bar{P}}_2) \\ &= \underline{P}_1 - (1-\mu_{12})\,\underline{\tau}_1 \\ &= \underline{\tau}_1\,\mu_{12} + (\underline{P}_1 - \underline{\tau}_1) \\ &\equiv \underline{\tau}_1\,\mu_{12} + \underline{\tau}_2 \quad ,\end{aligned} \tag{2.28}$$

which defines τ_2 as the limit to which $\underline{P}_2$ goes when the classes are decoupled, i.e., when $\mu_{12} = 0$. It is important to realize that $\underline{\tau}_2 = \underline{P}_1 - \underline{\tau}_1$ does not depend on μ_{12}, for $\underline{P}_1 = 1 - \underline{\bar{S}}_1 \underline{\bar{S}}_1^{+}$ only measures the upward coupling of class 1, and so is itself independent of μ_{12}. Eq. (2.28) clearly shows that the fine-structure levels of class 2 are fed both through the doorways ($\underline{\tau}_1\,\mu_{12}$) and directly from the channels ($\underline{\tau}_2$). However, note that $\underline{\tau}_2$ defined in terms of $\bar{P}_2$, not P_2, so that, unlike $\underline{P}_2(E)$, it does not have doorway-state energy dependence. We are introducing the $\underline{\tau}$'s only for comparison with the work of other authors, and it is clear that they view all τ's as being free of doorway-type energy dependence. Hence in order to agree with their usage, we shall introduce the $\underline{\tau}$'s only through equations which are always averaged over the full internal I_1. In particular, in the present case,

$$\underline{\bar{P}}_2 = \langle \underline{P}_2 \rangle_1 = \langle \underline{X}_2\, \mathrm{Tr}(\underline{X}_2) \rangle_1 = \underline{\tau}_1\,\mu_{12} + \underline{\tau}_2 \quad . \tag{2.29}$$

In the case of three classes, the total width of the levels of class 1 is

$$\Gamma_1 = \Gamma_1^{\uparrow} + \Gamma_{12}^{\downarrow} + \Gamma_{13}^{\downarrow} \quad , \tag{2.30}$$

so that

$$\underline{P}_1 - \underline{\bar{P}}_2 = \underline{X}_1\, \mathrm{Tr}(\underline{X}_1) = \frac{\Gamma_1^{\uparrow}}{\Gamma_1}\,\underline{\tau}_1 \equiv (1-\mu_{12}-\mu_{13})\,\underline{\tau}_1 . \tag{2.31}$$

Similarly,

$$\langle P_2 \tilde{P}_3 \rangle_1 = \langle \underline{X}_2 \mathrm{Tr}\, \underline{X}_2 \rangle_1 = \left\langle \frac{\Gamma_2^{\uparrow}}{\Gamma_2} \; 2\pi \frac{\langle g^{(2)} g^{(2)*} \rangle_2}{D_2} \right\rangle_1 ,$$

$$\equiv \left\langle \frac{\Gamma_2^{\uparrow}}{\Gamma_2} \right\rangle_1 \left\langle 2\pi \frac{\langle g^{(2)} \; g^{(2)*} \rangle_2}{D_2} \right\rangle_1 , \tag{2.32}$$

in which, if the last step is not justified, it can be taken as a definition of $\langle \Gamma_2^{\uparrow}/\Gamma_2 \rangle_1$. Its importance is due to the fact that $g^{(2)}$ describes only the upward coupling of the states of class 2; it is thus not affected by the presence of class 3, so we can employ Eq. (2.29) to identify the second factor in Eq. (2.32) as $(\underline{\tau}_1 \mu_{12} + \underline{\tau}_2)$, giving

$$\langle P_2 \tilde{P}_3 \rangle_1 = \left\langle \frac{\Gamma_2^{\uparrow}}{\Gamma_2} \right\rangle (\underline{\tau}_1 \mu_{12} + \underline{\tau}_2) ,$$

$$\equiv (1-\mu_{23})(\underline{\tau}_1 \mu_{12} + \underline{\tau}_2). \tag{2.33}$$

These results can be extended to more classes by induction, the general result being

$$\langle \underline{P}_n \underline{P}_{n+1} \rangle_1 = \sum_{j=1}^{n} \left(1 - \sum_{m=n+1}^{N} \mu_{n,m}\right) D_{nj} \underline{\tau}_j , \tag{2.34a}$$

with

$$D_{nj} = \sum_{i=j}^{n-1} \mu_{in} D_{ij} , \tag{2.34b}$$

where $\mu_{ii} = 1$, $D_{ii} = 1$, and

$$\mu_{ij} = \frac{2\pi \overline{|\langle i|V|j\rangle|^2}}{\Gamma_i D_j} \equiv \frac{\Gamma_{ij}^{\downarrow}}{\Gamma_i} \tag{2.35}$$

is the $i \to j$ "downward" mixing parameter, given in terms of the effective interaction V that couples the classes of states. Note that

$$\Gamma_i = \Gamma_i^{\uparrow} + \sum_{j=i+1}^{N} \Gamma_{ij}^{\downarrow} \qquad (2.36a)$$

and

$$\Gamma_i^{\uparrow} = \sum_c \Gamma_{ic} = \sum \langle |g_{cq}^{(i)}|^2 \rangle_q \ . \qquad (2.36b)$$

We also note in passing that Eq. (2.11) and (2.19) (for general n) guarantee the usual unitarity sum rule, which in terms of the τ's takes the form

$$\sum \langle \sigma_{cc'}^{f\ell} \rangle_{I_1} = (\langle \underline{P}_1 \rangle)_{cc} = \sum_{n=1}^{N} (\tau_n)_{cc} \ . \qquad (2.37)$$

Substituting Eq. (2.35) into Eq. (2.22) re-expresses $\langle \sigma^{f\ell} \rangle$ in terms of the $\underline{\tau}$'s, giving it a form which we can adequately illustrate by the case of only two classes. Assuming, for simplicity of notation, that the $\underline{\tau}$'s have no $c \neq c'$ terms (absence of direct-reaction channel coupling), we have in this case

$$\langle \sigma_{cc'}^{f\ell} \rangle_{I_1} = \left\langle (\underline{\tau}_1)_{cc} \frac{(1-\mu_{12})(\underline{\tau}_1)_{cc'}}{\mathrm{Tr}(\underline{\tau}_1)} + \frac{(\mu_{12}\underline{\tau}_1+\underline{\tau}_2)_{cc}(\mu_{12}\underline{\tau}_1+\underline{\tau}_2)_{c'c'}}{\mathrm{Tr}(\mu_{12}\underline{\tau}_1+\underline{\tau}_2)} \right\rangle_{I_1} \ . \qquad (2.38)$$

The second term of this equation, symmetric in c and c', is the Hauser-Feshbach term for the long-lived compound-nuclear states (2), which are entered both directly from the channels ($\underline{\tau}_2$) and through the class-1 doorways ($\underline{\tau}_1\mu_{12} = \underline{\tau}_1\Gamma_1^{\downarrow}/\Gamma_1$). The first term of Eq. (2.38) describes that part of the reaction which occurs with the shorter time-delay $\hbar/\Gamma_1$, and so passes only through the doorway states without reaching class 2. It is not symmetric in c and c' if we interpret $(\underline{\tau}_1)_{cc}$ as the transmission coefficient into the doorways from channel c and $(1-\mu_{12})(\underline{\tau}_1)_{c'c'}/\mathrm{Tr}(\underline{\tau}_1)$ as the branching ratio for decay back to c'. This latter interpretation follows from Eqs. (2.34) and (2.35) which give

$$(1-\mu_{12})\,\frac{(\underline{\tau}_1)_{c'c'}}{\mathrm{Tr}(\underline{\tau}_1)} = \frac{\Gamma_1^{\uparrow}}{\Gamma_1}\,\frac{\Gamma_{1c'}}{\Gamma_1^{\uparrow}} = \frac{\Gamma_{1c'}}{\Gamma_1} \quad ; \qquad (2.39)$$

summed over the exit channels c' this gives $\sum_{c'} (\Gamma_{1c'}/\Gamma_1) = 1 - \Gamma_1^{\downarrow}/\Gamma_1$, which is less than unity because the doorways <u>also</u> lose flux downward to class 2. This loss of flux is in fact irreversible in the first (doorway only) term of Eq.(2.38) it does not return to class 1 from class 2 in this term, and so destroys time-reversibility and hence the symmetry in c and c'. The physical reason that this term contains no "up-feeding" is that it describes only the <u>prompt</u> contribution to $\langle\sigma^{f\ell}\rangle$; keeping only lowest order in $\Gamma_2/\Gamma_1 \ll 1$ essentially gives class 2 an infinite time-delay relative to the doorways, and so includes $2 \to 1$ feeding only in the compound-nucleus term. This lack of up-feeding is an essential physical feature of the general expression Eq.(2.22), that is automatically introduced by the use of well-nested energy averages: the term contributed by class n, although it is fed from and decays to all classes above it, contains no feeding from the longer-lived states below it. We also note in passing that this term (for $n < N$) contains the factor $\Gamma_n^{\uparrow}/\Gamma_n$, and so vanishes if $\Gamma_n^{\uparrow}/\Gamma_n^{\downarrow} \to 0$. This is as it should be, for $\Gamma_n^{\uparrow}/\Gamma_n^{\downarrow} \to 0$ only if either that class of doorways is decoupled from the channels altogether ($\Gamma_n^{\uparrow} = 0$), or if their lifetime becomes so short that they cease to function as doorways at all ($\Gamma_n^{\downarrow} \to \infty$).

The above results solve the nested-doorway problem in general, within the constraints imposed by Eq. (2.7) but by considering limiting cases we can make contact with previous approaches to the problem.

a) Isospin mixing through a single class of doorways (isobaric analog states) has been discussed by Grimes et al.[8] and by Harney et al.[9] It was subsequently discussed by Lane,[10] who showed that their expressions

for $\langle\sigma^{f\ell}\rangle$ are equivalent, and that both can be written in exactly the form of our Eq.(2.38) for the N=2 case, provided that one assumes $\langle\underline{\tau}_i\underline{\tau}_j\rangle = \langle\underline{\tau}_i\rangle\langle\underline{\tau}_j\rangle$. These authors have obviously dropped the $\tau_{cc'}$ terms ($c \neq c'$) for simplicity, but they are readily restored if needed.

b) The "chaining approximation" of Feshbach et al.[3] assumes that (1) only nearest-neighbor classes couple with each other, i.e., $\mu_{mn} = 0$ unless $|m-n| \leq 1$; (2) the incident channel c couples only to class 1; (3) class n decays directly to the exit channel c', without returning through classes $m < n$; and (4) direct-reaction channel coupling is neglected. In this case the matrix D_{nm} of Eq. (2.34)is $D_{nm} = \prod_{k=n}^{m-1} \mu_{k,k+1}$, and, again assuming $\langle\underline{\tau}_m\,\underline{\tau}_n\rangle = \langle\underline{\tau}_m\rangle\langle\underline{\tau}_n\rangle$,

$$\langle\sigma^{f\ell}_{cc'}\rangle_1 = (\underline{\tau}_1)_{cc} \sum_{n=1}^{N} \Big(\prod_{k=1}^{n-1} \mu_{k,k+1}\Big)(1-\mu_{n',n+1}) \frac{(\underline{\tau}_n)_{c'c'}}{\mathrm{Tr}(\underline{\tau}_n)}$$

$$= (\underline{\tau}_1)_{cc} \sum_{n=1}^{N} \Big(\frac{\Gamma^{\downarrow}_{12}}{\Gamma_1}\frac{\Gamma^{\downarrow}_{23}}{\Gamma_2}\cdots\frac{\Gamma^{\downarrow}_{n-1,n}}{\Gamma_{n-1}}\Big)\frac{\Gamma_{nc'}}{\Gamma_n}\,, \tag{2.40}$$

which is the result given in Ref.3; we have used Eq.(2.34) and assumed, with Feshbach et al.,[3] that $(\mathrm{Tr}\,\underline{X}_n)^2 \simeq \mathrm{Tr}\,\underline{\tau}_n$, implying that direct decay to the channels dominates for all classes and most channels.

c) In the "strong mixing" limit, defined by $\Gamma^{\downarrow}_n >> \Gamma^{\uparrow}_n$ for all $n < N$ (equivalent to $\underline{P}_n - \underline{P}_{n+1} = \underline{X}_n\,\mathrm{Tr}\,\underline{X}_n = 0$, we have $\underline{X}_N\,\mathrm{Tr}\,\underline{X}_N = \sum_1 \underline{\tau}_n$, and Eq.(2.22) reduces to

$$\langle\sigma^{f\ell}_{cc'}\rangle_1 = \frac{(\underline{P}_N)_{cc}\,(\underline{P}_N)_{c'c'} + (\underline{P}_N)_{cc'}\,(\underline{P}_N)_{c'c}}{\mathrm{Tr}\,(\underline{P}_N)} \tag{2.41}$$

$$= \frac{(\sum \underline{\tau}_n)_{cc}(\sum \underline{\tau}_n)_{c'c'} + (\sum \underline{\tau}_n)_{cc'}(\sum \underline{\tau}_n)_{c'c}}{\mathrm{Tr}\,(\sum \underline{\tau}_n)}\,,$$

which is just the Hauser-Feshbach formula for the fine-structure states alone, all doorway effects having vanished. In the opposite limit of zero mixing between the classes, $\mu_{ij} = 0$, each class couples only to the channels and we have

$$\langle \sigma^{f\ell}_{cc'} \rangle_I = \left\langle \sum_n \frac{(\underline{\tau}_n)_{cc} (\underline{\tau}_n)_{c'c'} + (\underline{\tau}_n)_{cc'} (\underline{\tau}_n)_{c'c}}{\mathrm{Tr}\ \underline{\tau}_n} \right\rangle_{I_1} \tag{2.42}$$

d) In the limit that only class 1 couples to the channels ($\underline{\tau}_m = 0$, $m \neq 1$), all the terms of Eq. (10) contribute, but combine to give

$$\langle \sigma^{f\ell}_{cc'} \rangle_I = \frac{(\underline{\tau}_1)_{cc} (\underline{\tau}_1)_{c'c'} + (\underline{\tau}_1)_{cc'} (\underline{\tau}_1)_{c'c}}{\mathrm{Tr}\ (\underline{\tau}_1)} . \tag{2.43}$$

All flux to the "internal" classes must enter and exit through class 1, and the net effect is the same cross section as though the other classes were not present at all.

We wish to thank A. K. Kerman and W. A. Friedman for stimulating discussions.

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