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LIBERATION OF U(N) SOLITONS IN THE CP^{N-1} MODEL BY MASSLESS QUARKS

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Abstract

We show that in a model obtained by adding N minimally coupled massless fermions to the original CP^{N-1} model, one obtains assymptotic states, belonging to the fundamental representation of U(N). We view these states as liberated fundamental CP^{N-1} solitons. They have an S-matrix , whose factorizability was verified to order $1/N^2$ and whose scattering amplitudes coincide to order 1/N with a previously computed exact S-matrix without bound states. All fermionic degrees of freedom except one decouple from the physical Hilbert space.

In the last few years two-dimensional field theoretical models have acquired a reputation as a fertile ground for generating new ideas possibly relevant for four-dimensional quantum chromodynamics. Among these the CP^{N-1} model |1-3| seems to be particularly close to a realistic theory, but unfortunately its more interesting properties are still poorly understood.

Together with the nonlinear σ-model, where the existence of an infinite number of local and nonlocal conservation laws surviving quantization and a factorizable S-matrix has been proven |4,5| it belongs to a class of models expected to share these properties.(*)

The Green functions of the CP^{N-1} model may be generated by the following functional integral |1|:

$$Z = \int [dz\, dz^*][dA_\mu][d\omega] \exp \frac{i}{2\alpha} \int d^2x \left\{ (\partial_\mu - iA_\mu) z^* (\partial^\mu + iA^\mu) z + \omega (z^* z - 1) \right\} \tag{1}$$

where the normalization and source fields have been omitted for simplicity.

This generating functional may be expanded in powers of 1/N by integrating over the z field and expanding the result around the stationary point $A_\mu = 0,\ \omega = m^2$. This produces Feynman rules which contain a pole at $p^2 = 0$ in the A_μ- propagator. It was argued in Ref.1, that this dynamically generated pole is responsible for a long range force confining the fundamental particles in the CP^{N-1} model. The same pole furthermore allows for a θ-dependence of the vacuum energy density. Thus confinement and a nontivial θ-dependence are in this model tightly correlated facts.

Reasoning along these lines leads one to believe that eliminating the θ-dependence will also supress confinement.

Following current believe |7| we now supress the θ-dependence adding N massless fermions (**) minimally coupled to the "gauge" field A_μ , obtaining

$$\mathcal{L}^{CP^{N-1}} \rightarrow \mathcal{L}^{CP^{N-1}} + \bar{\Psi}\, i\,(\not\partial - i\not A_\mu)\,\Psi \tag{2}$$

From the equivalent boson representation |8| of ψ , one sees that the fermions remain massless and that the fermionic SU(N) degrees of freedom decouple, so that the fermions do not appear asymptotically at all, being confined just as in the massless Schwinger model. On the other hand the field A_μ looses its pole at $p^2 = 0$ and the particles corresponding to the z field emerge, as can be seen introducing the Lagrangian (2) into the functional integral (1) and integrating over the fermion fields. This yields the following propagators in the 1/N expansion:

$$D_z(p^2) = \frac{i}{p^2 - m^2 + i\epsilon} \tag{3a}$$

$$D_\omega = 4\pi i\, m^2 \frac{\mathrm{sh}\,\phi}{\phi} \tag{3.a}$$

$$D_A^{\mu\nu} = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) 2\pi i \frac{\mathrm{th}\frac{\phi}{2}}{\phi} \tag{3b}$$

where $p^2 = -4m^2\, \mathrm{sh}^2 \frac{\phi}{2}$ (3c)

The vertices are the same as in the usual CP^{N-1} model of Eq. (1).[1]

We believe that these Feynman rules generate a field theory with a factorized S-matrix. Our arguments are the usual ones; i) absence of particle production to order $1/N^2$ and ii) agreement to order 1/N with an exact S matrix previously computed assuming factorization [9]. The reader may check i) using the standard procedure.[10] As for ii) let's write down the proposed exact amplitudes corresponding to the $2 \to 2$ scattering of particles $\alpha(p)$ and antiparticles $\bar{\alpha}(p)$. Defining the rapidity variables $p_0^i = m\,\mathrm{ch}\,\theta^i$, $p_1^i = m\,\mathrm{sh}\,\theta^i$ and $p_1 \cdot p_2 = m\,\mathrm{ch}(\theta_1 - \theta_2)$ we write the exact amplitudes

$$_{out}\langle \beta(\theta_1)\bar{\delta}(\theta_2) | \alpha(\theta_1)\bar{\gamma}(\theta_2)\rangle_{in} = t_1(\theta_1-\theta_2)\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} + t_2(\theta_1-\theta_2)\,\delta_{\alpha\gamma}\,\delta_{\beta\delta}$$

$$_{out}\langle \beta(\theta_1)\delta(\theta_2) | \alpha(\theta_1)\gamma(\theta_2)\rangle_{in} = t_1[i\pi - (\theta_1-\theta_2)]\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} + t_2[i\pi - (\theta_1-\theta_2)]\,\delta_{\alpha\delta}\,\delta_{\gamma\beta}$$

where

$$t_1(\theta) = \frac{\Gamma\left(\frac{1}{2} + \frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2} + \frac{1}{N} - \frac{\theta}{2\pi i}\right)}{\Gamma\left(\frac{1}{2} - \frac{\theta}{2\pi i}\right)\Gamma\left(\frac{1}{2} + \frac{1}{N} + \frac{\theta}{2\pi i}\right)} \tag{4.a}$$

$$t_2(\theta) = \frac{-2\pi i}{N(i\pi-\theta)}\, t_1(\theta). \qquad (4.b)$$

and the reflection amplitudes vanishe.

These amplitudes were computed in Ref. 9, where to we refer the reader for details. Expanding (4.a, b) to first order in 1/N one gets agreemant with those computed with the Feynman rules of Eq. (3).

Another support for the above proposal comes from an analogy operating between the Gross-Neveu and the non-linear σ-model on one hand and the chiral Gross-Neveu and the CP^{N-1} model on the other hand. The S-matrices of the first two models,[5,11] each one depending on a parameter λ', are related by the substitution $\lambda' \to -\lambda'$. Upon supersymmetrizing them one obtains a fermion-boson transmission amplitude invariant under the above substitution.[12] Analogously supersymmetry couples the chiral Gross-Neveu and the CP^{N-1} model[2,13] and again the supersymmetric fermion-boson transmission amplitude is invariant under a substitution $\lambda \to -\lambda$.(***)

Thus one could suspect that the natural candidate for the S-matrix obtained from the one of the chiral Gross-Neveu model[15] via the substitution $\lambda \to -\lambda$, would be the one of the usual CP^{N-1} model. But this cannot be true if one believes that the fundamental particles of this model are confined. It is the model of Eq. (2), which has the above S-matrix and one starts to doubt wether the usual CP^{N-1} model is factorizable at all.

Thus the following picture seems to emerge. The massless fermions shield the source of the longe range force and apparently this is their only job. This allows the confined particles to emerge and they seem to interact via repulsive forces, since they do not form bound states. The implications of this picture for the usual CP^{N-1} model remain to be worked out. At any rate it would be interesting to prove the existence of bound states, as was announced in Ref.1, since this is a prerequisite for nontrivial physical content of the usual CP^{N-1} model.

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Footnotes

(*) The situation with respect to local conservation laws, is not clear at present even for the classical CP^{N-1} model [6].

(**) Only one fermion would be sufficient for our purpose, but is somewhat less symmetric.

(***) The supersymmetric CP^{N-1} model, together with a supersymmetric generalization of a factorizable Z(N) model, are exactly soluble and their S-matrices will be presented in a forthcoming paper by the present authors.[7]

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