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COUPLED-CHANNELS EXTENSION OF THE CLOSED FORMALISM FOR HEAVY-ION COLLISIONS (I). ELASTIC SCATTERING

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Coupled-channels extension of the closed formalism for heavy-ion collisions (I). Elastic scattering*

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Abstract: We present an extension of the closed formalism for elastic and quasielastic heavy-ion collisions to account for channel coupling effects on these processes. Starting from coupled-channels equations, we use suitable approximations to calculate directly the corrections to the elastic partial-wave S-matrix that arise from the feedback of certain strongly coupled channels on elastic scattering, without having to determine effective potentials as an intermediate step. The S-matrix corrections are completely determined by the characteristics of the transitions to the intermediary channels (spectroscopic and form factors) and by the uncoupled elastic S-matrix. The corresponding contributions to the scattering amplitude are evaluated in closed form. As examples we derive explicit expressions for coupling to inelastic collective channels, by both Coulomb and nuclear excitation, and to transfer reaction channels.

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1. Introduction

It is well known that in heavy-ion collisions channel coupling often has important effects on elastic scattering, inelastic scattering, and transfer reactions. These effects manifest themselves in deviations from the "normal" shapes of angular distributions, in anomalous large-angle scattering and unusual structures of excitation functions. Since customary optical model and DWBA calculations and simple S-matrix descriptions are found to be inadequate in these cases, the use of much more elaborate coupled-channels computations appears to be inevitable. Because these can be prohibitively expensive and time-consuming, it is highly desirable to devise methods by which channel coupling effects may be represented in a form, albeit very approximate, that can be used in straightforward extensions of the conventional calculations.

One such method is to derive effective potentials which can be added to the normal optical potential to simulate the feedback of strongly coupled channels on the elastic scattering. Since the effective interaction is nonlocal, it is usually approximated by an "equivalent" local potential which then, in general, depends on the orbital angular momentum ℓ. In the case of coupling to low-lying collective (2^+) states via Coulomb excitation, such potentials have been derived by several authors[1-3]. As a by-product of the method described in this paper, the present authors have derived more general effective potentials which represent the coupling to inelastic channels by both Coulomb and nuclear excitation, and to transfer channels[4].

An alternative to the optical model and DWBA description of heavy-ion elastic scattering and direct reactions has been developed by one of us, in which the simple general properties of the partial-wave elastic S-matrix (due to the strong absorption and strong Coulomb interaction in heavy-ion collisions) are utilized to derive closed-form asymptotic expressions for the amplitudes for elastic scattering[5]), inelastic scattering[6]) and transfer reactions[7]). Aside from an extension that accounts for the effect on elastic scattering of dynamic polarization by Coulomb excitation[8]), this theory - called the Closed Formalism (CF) for short - does not incorporate channel coupling effects.

In the present paper we develop a procedure which enables us, starting from a set of coupled-channels equations and using suitable approximations, to calculate in explicit form the modifications of the elastic S-matrix that arise from the coupling to certain types of strongly excited channels. These modifications are determined solely by the characteristics of the transitions to the intermediate states (i.e., the spectroscopic factors and the form factors) and by the uncoupled elastic S-matrix. Their analytic forms are such that the corresponding contributions to the scattering amplitude can be evaluated in closed form by the same methods as developed in refs.[5-7]). In this way we achieve a natural and straightforward extension of the CF to account for channel coupling effects.

We consider it a significant feature of our approach that the modifications of the uncoupled S-matrix, and hence of the scattering amplitude, are calculated directly, without the need to determine explicitly the effective interactions which appear only in an intermediary role in our derivation. Thereby we avoid, in contrast to refs.[1-4]), the two-step procedure of first calculating an effective potential and then solving the Schrödinger equation to calculate the scattering amplitude. Moreover, whereas the latter can only be done numerically, the analytic form of our results displays explicitly the physical nature and dynamical origin of the various contributions to the scattering cross section.

For simplicity we consider only individual channels and treat the coupling in lowest order, but we show for some cases how the method can be extended to higher-order, multi-step processes by iteration procedures. As specific examples we consider the most important channels: inelastic scattering (by Coulomb and nuclear excitation), and particle transfer (including one-step and two-step elastic transfer processes).

In the present paper (Part I) we deal with elastic scattering; in a subsequent paper (Part II) the method is extended to inelastic scattering by modification of the DWBA amplitude. Similar extensions to transfer reactions, and applications of our results, will be described in further papers.

A brief summary account of our method and results has been presented previously[9].

2. Generic formulation

We start from the coupled-channels equations for the radial wave functions $\chi_{(\ell I)J}(k,r)$ for orbital angular momentum ℓ, spin I and total angular momentum J in channels denoted by n, m:

$$\left[\frac{d^2}{dr^2} + k_n^2 - \frac{\ell_n(\ell_n+1)}{r^2} - \frac{2\mu_n}{\hbar^2}\overset{\circ}{U}(r)\right]\chi_{(\ell_n I_n)J}(k_n,r)\,e^{i\sigma_{\ell_n}(k_n)}$$

$$= \sum_{\ell_m I_m} \frac{2\mu_n}{\hbar^2} V^J_{\ell_n I_n,\,\ell_m I_m}(r)\,\chi_{(\ell_m I_m)J}(k_m,r)\,e^{i\sigma_{\ell_m}(k_m)}, \tag{2.1}$$

where μ is the reduced mass and $\sigma_\ell(k)$ are Rutherford phase shifts. For simplicity we consider spin-zero nuclei in the entrance channel which we denote by n, i.e., $I_n = 0$. This fixes the value of J to be ℓ_n. The interaction in the elastic channel in the absence of coupling to the channels m is assumed to be described by a local optical potential $U_N(r)$ and the Coulomb potential $V_C(r)$,

$$\overset{\circ}{U}(r) = V_c(r) + U_N(r), \qquad U_N(r) = V(r) + iW(r). \tag{2.2}$$

We write the coupling interaction as

$$V^{\ell_n}_{\ell_n 0,\,\ell_m L}(r) \equiv V^L_{\ell_n \ell_m}(r) = a^{(m)}_{\ell_n \ell_m}\, F^{(m)}_L(r), \tag{2.3}$$

where $F_L^{(m)}(r)$ is the form factor for the transition $n \rightarrow m$ with multipolarity L, and all spectroscopic and geometric factors are lumped together in $a_{\ell_n \ell_m}^{(m)}$.

Since the nonelastic channels $m \neq n$ have outgoing waves only, the wave functions $\chi_{\ell_m}(k_m, r)$ (in simplified notation) satisfy the integral equations

$$\chi_{\ell_m}(k_m, r)\, e^{i\sigma_{\ell_m}} = \frac{2\mu_n}{\hbar^2} \int_0^\infty dr'\, \overset{\circ}{G}{}_{\ell_m}^{(+)}(r, r') \sum_{\ell_s} V^L_{\ell_m \ell_s}(r')\, \chi_{\ell_s}(k_s, r')\, e^{i\sigma_{\ell_s}}, \tag{2.4}$$

where $\overset{\circ}{G}{}_{\ell_s}^{(+)}(r, r')$ is the outgoing-wave Green function for the left-hand side of eq.(2.1).

Our first approximation is to neglect all couplings except to the elastic channel,

$$\chi_{\ell_m}(k_m, r)\, e^{i\sigma_{\ell_m}} \approx \frac{2\mu_n}{\hbar^2} \int_0^\infty dr'\, \overset{\circ}{G}{}_{\ell_m}^{(+)}(r, r')\, V^L_{\ell_m \ell_n}(r')\, \chi_{\ell_n}(k_n, r')\, e^{i\sigma_{\ell_n}}. \tag{2.5}$$

Insertion of (2.5) in (2.1) results in a Schrödinger equation for $\chi_{\ell_n}(k_n, r)$ with an effective interaction

$$U_{\ell_n}(r) = \overset{\circ}{U}(r) + \tilde{U}_{\ell_n}(r), \tag{2.6}$$

where

$$\tilde{U}_{\ell_n}(r)\,\chi_{\ell_n}(k_n,r) = \frac{2\mu_n}{\hbar^2}\sum_{\ell_m} V^L_{\ell_n\ell_m}(r)\int_0^\infty dr'\,\overset{\circ}{G}{}^{(+)}_{\ell_m}(r,r')\,V^L_{\ell_m\ell_n}(r')\,\chi_{\ell_n}(k_n,r'). \tag{2.7}$$

We bypass an explicit determination of the (non-local) interaction operator $\tilde{U}_{\ell_n}(r)$ by <u>calculating directly the partial-wave S-matrix elements.</u> From the "radial Gell-Mann-Goldberger relation" (see appendix A) we obtain for the nuclear part of the total S-matrix $S_\ell = S_\ell^{(N)} \exp(i2\sigma_\ell)$,

$$S^{(N)}_{\ell_n}(k_n) = \overset{\circ}{S}{}^{(N)}_{\ell_n}(k_n) + \tilde{S}^{(N)}_{\ell_n}(k_n), \tag{2.8}$$

where $\overset{0}{S}_\ell$ is the uncoupled elastic S-matrix and

$$\tilde{S}^{(N)}_{\ell_n}(k_n) = -i\,\frac{4\mu_n}{\hbar^2 k_n}\int_0^\infty dr\,\overset{\circ}{\chi}_{\ell_n}(k_n,r)\,\tilde{U}_{\ell_n}(r)\,\chi_{\ell_n}(k_n,r) \tag{2.9}$$

gives the channel-coupling contribution. Insertion of (2.7) yields

$$\tilde{S}^{(N)}_{\ell_n}(k_n) = -i\,\frac{8\mu_n^2}{\hbar^4 k_n}\sum_{\ell_m}\int_0^\infty dr\,\overset{\circ}{\chi}_{\ell_n}(k_n,r)\,V^L_{\ell_n\ell_m}(r) \times \int_0^\infty dr'\,\overset{\circ}{G}{}^{(+)}_{\ell_m}(r,r')\,V^L_{\ell_m\ell_n}(r')\,\chi_{\ell_n}(k_n,r'). \tag{2.10}$$

Our next approximation is to replace χ_{ℓ_n} on the right-hand side of (2.10) by the uncoupled wave function $\overset{0}{\chi}_{\ell_n}$. This amounts to treating the coupling to lowest order; higher-order contributions can be obtained by iteration. Our third step is to approximate the

Green function in channel m by its on-energy-shell part (see appendix B)

$$\overset{\circ}{G}{}^{(+)}_{\ell_m}(r,r') \approx -\frac{i}{k_m}\, \overset{\circ}{\chi}_{\ell_m}(k_m,r)\, \overset{\circ}{\chi}_{\ell_m}(k_m,r') \left[\overset{\circ}{S}{}^{(N)}_{\ell_m}(k_m)\right]^{-1} . \quad (2.11)$$

Using (2.3) we can then write eq. (2.10) as

$$\tilde{S}^{(N)}_{\ell_n}(k_n) = -\frac{\mu_n^2 k_m k_n}{2\pi^2 \hbar^4} \quad (2.12)$$

$$\times \sum_{\ell_m} a^{(n)}_{\ell_m \ell_n}\, a^{(m)}_{\ell_n \ell_m}\, R^{(n)L}_{\ell_n \ell_m}(k_n,k_m)\, R^{(m)L}_{\ell_m \ell_n}(k_m,k_n) \left[\overset{\circ}{S}{}^{(N)}_{\ell_m}(k_m)\right]^{-1} ,$$

where

$$R^{(m)L}_{\ell_m \ell_n}(k_m,k_n) = \frac{4\pi}{k_m k_n} \int_0^\infty dr\, \overset{\circ}{\chi}_{\ell_m}(k_m,r)\, F^{(m)}_L(r)\, \overset{\circ}{\chi}_{\ell_n}(k_n,r)$$

(2.13)

are the DWBA radial integrals for the transition n → m with multipolarity L.

The total elastic S-matrix can now be written in the form

$$S_{\ell_n}(k_n) = \left[1 - t^{(n,m)}_{\ell_n}(k_m,k_n)\right] \overset{\circ}{S}_{\ell_n}(k_n) \quad (2.14)$$

with

$$t^{(n,m)}_{\ell_n}(k_m,k_n) = \frac{\mu_n^2 k_m k_n}{2\pi^2 \hbar^4} \sum_{\ell_m} a^{(n)}_{\ell_m \ell_n}\, a^{(m)}_{\ell_n \ell_m}\, T_{\ell_m \ell_n}(k_m,k_n) ,$$

(2.15)

where we have defined

$$T^{(n,m)}_{\ell_m \ell_n}(k_m,k_n) = \frac{R^{(n)L}_{\ell_n\ell_m}(k_n,k_m)\,R^{(m)L}_{\ell_m\ell_n}(k_m,k_n)}{\overset{\circ}{S}{}^{(N)}_{\ell_m}(k_m)\,\overset{\circ}{S}{}^{(N)}_{\ell_n}(k_n)} \quad . \tag{2.16}$$

In fact, the use of the on-energy-shell approximation of the Green function, eq.(2.11), allows us to perform a summation over terms of all orders in $t^{(n,m)}_{\ell_n}(k_m,k_n)$ in the form of a geometrical (Born-type) series, with the result

$$S_{\ell_n}(k_n) = \frac{1-\frac{1}{2}\,t^{(n,m)}_{\ell_n}(k_m,k_n)}{1+\frac{1}{2}\,t^{(n,m)}_{\ell_n}(k_m,k_n)}\;\overset{\circ}{S}_{\ell_n}(k_n)\;. \tag{2.17}$$

Note that because of the coupling $\underset{\sim}{\ell}_m + \underset{\sim}{L} = \underset{\sim}{\ell}_n$ only a few (at most 2L + 1) terms contribute to the sum in (2.15).

We may regard eq.(2.14), or the iterated form (2.17), with the expressions (2.15) and (2.16) as our main result: it gives the coupling corrections to the S-matrix in the form of well-defined expressions determined by the DWBA radial integrals and the unperturbed S-matrices.

However, in order to proceed towards analytic expressions and closed-form evaluation of the scattering amplitude, we employ for the examples treated in the following sections further approximations that have been used before in the closed formalism for quasielastic heavy-ion reactions[5-7].

3. Dynamic polarization

3.1 Modification of the S-matrix

The effect of the coupling to (low-lying, collective) inelastic channels on elastic scattering is termed "dynamic polarization". For a given channel of spin $I_m = L$ this is, to lowest order, a two-step process in which the level in question is excited and de-excited by inelastic transitions of multipolarity L. This effect has been studied, for Coulomb excitation, with the effective potential method in refs.[1-3]), and with the extended CF in ref.[8]). In our present treatment we include the effects of nuclear excitation as well.

In the "extended optical model" the coupling interaction (2.3) is

$$V^{L}_{\ell\ell'}(r) = a^{L}_{\ell\ell'} F_L(r), \tag{3.1}$$

where

$$a^{L}_{\ell\ell'} = \frac{5}{2\pi^{1/2}} \left[\frac{(2\ell+1)(2\ell'+1)}{2L+1}\right]^{1/2} (-)^{\ell} \begin{Bmatrix} \ell & \ell' & L \\ L & 0 & \ell \end{Bmatrix} \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.2}$$

and the form factor is given by

$$F_L(r) = \delta_L^{(C)} \frac{3}{2L+1} Z_A Z_B e^2 \frac{R_c^{L-1}}{r^{L+1}} - \delta_L^{(N)} \frac{dU_N(r)}{dr} \tag{3.3}$$

for $r \geqq R_C$, where R_C is the charge radius. (Here and in all following expressions we write $\ell_n = \ell$ and $\ell_m = \ell'$.) The Coulomb and nuclear deformation lengths are denoted by $\delta_L^{(C)}$ and $\delta_L^{(N)}$, respectively, and Z_A, Z_B are the atomic numbers of the scattering nuclei A, B.

Now we use the same approximations for the radial integrals (2.13) as in ref.[6]) in the CF for inelastic scattering,

$$R^L_{\ell'\ell}(k',k) = \frac{4\pi}{k'k}\int_0^\infty dr\, \overset{\circ}{\chi}_{\ell'}(k',r)\, F_L(r)\, \overset{\circ}{\chi}_\ell(k,r)$$

$$\approx \frac{\pi\hbar^2}{\mu}\left\{\delta_L^{(C)}\,\hat{C}_{L-K}(\bar{\lambda})\left[\overset{\circ}{S}{}^{(N)}_{\ell'}(k')\,\overset{\circ}{S}{}^{(N)}_{\ell}(k)\right]^{1/2} - i\delta_L^{(N)}\left[\frac{d\overset{\circ}{S}{}^{(N)}_{\ell'}(k')}{d\ell'}\,\frac{d\overset{\circ}{S}{}^{(N)}_{\ell}(k)}{d\ell}\right]^{1/2}\right\}, \tag{3.4}$$

where

$$\hat{C}_{L-K}(\bar{\lambda}) = \frac{3}{2L+1}\left(\frac{\bar{k}R_c}{\bar{n}}\right)^{L-1} I_{L-K}(\vartheta,\xi)\,. \tag{3.5}$$

We have written $k_n = k$, $k_m = k'$, $\mu_n = \mu$ and defined

$$K = \ell-\ell', \qquad \bar{\lambda} = \bar{\ell}+\tfrac{1}{2}, \qquad \bar{\ell} = \tfrac{1}{2}(\ell+\ell'),$$
$$\bar{n} = \tfrac{1}{2}(n+n'), \qquad \xi = n'-n, \qquad \vartheta = 2\arctan(\bar{n}/\bar{\lambda}), \tag{3.6}$$

where n and n′ are the Sommerfeld parameters in the initial and final channels, respectively. The functions $I_{L-K}(\vartheta,\xi)$ are defined as in ref.[6]) as standard WKB radial integrals for Coulomb excitation. For details of the approximations leading to eq. (3.4) (which involve the "Sopkovich approximation" for the Coulomb part of the radial integrals, the WKB approximation for the Coulomb excitation integrals, and the Austern-Blair relation with Hahne's modification for the nuclear part of the radial integrals) we refer to ref.[6]) and the references given there.

Note that expression (3.4) is symmetrical in the initial and final channels. Thus we obtain for the quantities defined by eq. (2.16),

$$T_{\ell'\ell}(k',k) = \frac{\left[R^{L}_{\ell'\ell}(k',k)\right]^2}{\overset{\circ}{S}{}^{(N)}_{\ell'}(k')\,\overset{\circ}{S}{}^{(N)}_{\ell}(k)} = \left(\frac{\pi\hbar^2}{\mu}\right)^2 \left[\delta^{(C)}_L\,\hat{C}_{L-K}(\bar{\lambda}) - i\,\delta^{(N)}_L\,\hat{N}_K(\bar{\lambda})\right]^2 , \tag{3.7}$$

with the definition

$$\hat{N}_K(\bar{\lambda}) = \left[\frac{d\overset{\circ}{S}{}^{(N)}_{\ell'}(k')/d\ell'}{\overset{\circ}{S}{}^{(N)}_{\ell'}(k')}\cdot\frac{d\overset{\circ}{S}{}^{(N)}_{\ell}(k)/d\ell}{\overset{\circ}{S}{}^{(N)}_{\ell}(k)}\right]^{1/2} . \tag{3.8}$$

Since also the coefficients $a^{L}_{\ell\ell'}$ are symmetrical in ℓ and ℓ', our result for the total elastic S-matrix, modified by dynamic polarization, can be written in the form (2.14),

$$S_\ell(k) = \left[1 - t_\ell(k',k)\right]\overset{\circ}{S}_\ell(k) \tag{3.9}$$

with

$$t_\ell(k',k) = \frac{1}{2} kk' \sum_{K=-L}^{L} [a_{LK}(\bar\lambda)]^2 \tag{3.10}$$

$$\times \left\{ (\delta_L^{(C)})^2 [\hat C_{L-K}(\bar\lambda)]^2 - (\delta_L^{(N)})^2 [\hat N_K(\bar\lambda)]^2 - i2\delta_L^{(C)}\delta_L^{(N)} \hat C_{L-K}(\bar\lambda)\hat N_K(\bar\lambda) \right\},$$

where we have written $a^L_{\ell\ell'} \equiv a_{LK}(\bar\lambda)$ and replaced the summation over $\ell_m = \ell'$ by a sum over K.

This expression displays the contributions of dynamic polarization due to Coulomb excitation, nuclear excitation, and the <u>interference</u> between Coulomb and nuclear excitation.

The result (3.9), (3.10) gives only the lowest-order effect of the dynamic polarization (bilinear in the deformation lengths $\delta_L^{(C)}$ and $\delta_L^{(N)}$). As shown in sect.2, a Born type iteration procedure allows us to sum all higher-order contributions - within our approximations of channel coupling - in the form of a geometrical series, with the result

$$S_\ell^{(geom)}(k) = \frac{1-\frac{1}{2}t_\ell(k',k)}{1+\frac{1}{2}t_\ell(k',k)} \mathring{S}_\ell(k), \tag{3.9a}$$

where $t_\ell(k',k)$ is given by eq.(3.10). A different way of including higher-order effects (which we cannot derive within our procedure because it corresponds to a WKB-type or eikonal approximation) leads to an exponentiated form of the dynamic polarization correction,

$$S_\ell^{(exp)}(k) = \mathring{S}_\ell(k) \exp[-t_\ell(k',k)]. \tag{3.9b}$$

Obviously, both expressions (3.9a) and (3.9b) reduce to (3.9) to lowest order in the coupling constants; in fact they agree even to second order in t_ℓ.

For pure Coulomb excitation and in the adiabatic limit $k' = k$, i.e. with $t_\ell(k,k) \to t_C(\bar{\lambda})$ given by

$$t_c(\bar{\lambda}) = \frac{1}{2}\left(k\delta_L^{(c)}\right)^2 \sum_{K=-L}^{L} \left[a_{LK}(\bar{\lambda})\,\hat{C}_{L-K}(\bar{\lambda})\right]^2 , \qquad (3.10a)$$

the exponential modification of $\overset{\circ}{S}_\ell(k)$ in (3.9b) is equivalent to the one derived in ref.[8]) from the ℓ-dependent effective potential of Baltz et al.[2]), using the so-called "Coulomb-distorted eikonal approximation".

A comparison of the closed-form expressions for the differential cross sections derived from the "summed Born series" form (3.9a) and the "exponentiated" form (3.9b) using the $t_\ell^{(C)}$ given by (3.10a) for scattering below the Coulomb barrier will be shown in subsect.3.3, see fig.1.

3.2. Evaluation of the scattering amplitude

Our next aim is to derive closed-form expressions of the elastic scattering amplitude by evaluating the partial-wave series

$$\begin{aligned} f(\theta) &= \frac{i}{k}\sum_{\ell=0}^{\infty}\left(\ell+\tfrac{1}{2}\right)\left[1-S_\ell(k)\right]P_\ell(\cos\theta) \\ &= \overset{\circ}{f}(\theta) + \tilde{f}(\theta) , \end{aligned} \qquad (3.11)$$

where

$$\overset{\circ}{f}(\theta) = \frac{i}{k}\sum_{\ell=0}^{\infty}\left(\ell+\tfrac{1}{2}\right)\left[1-\overset{\circ}{S}_\ell(k)\right]P_\ell(\cos\theta) \qquad (3.12)$$

is the amplitude for no coupling (i.e., scattering by the potential $\overset{o}{U}(r)$), and

$$\tilde{f}(\theta) = -\frac{i}{k} \sum_{\ell=0}^{\infty} \left(\ell+\frac{1}{2}\right) \tilde{S}_\ell(k)\, P_\ell(\cos\theta) \tag{3.13}$$

is the channel-coupling contribution, with

$$\tilde{S}_\ell(k) = -\, t_\ell(k',k)\, \overset{o}{S}_\ell(k) \tag{3.14}$$

according to eq. (3.9).

Analytic expressions for $\overset{o}{f}(\theta)$, representing the leading terms in an asymptotic expansion for large Sommerfeld parameters n and large grazing angular momenta ℓ_g, have been derived in ref.[5]) to which we refer for details. Now we use similar methods to those developed in refs.[6,10]) to evaluate the amplitude (3.14) in closed form. The procedure consists in replacing the summation over ℓ in (3.13) by an integration over the continuous variable $\lambda = \ell + \frac{1}{2}$ (or by the exact Poisson series), using appropriate asymptotic expressions for $P_\ell(\cos\theta)$, and exploiting the general properties of $\tilde{S}_\ell(k)$. The latter is interpolated by a continuous function of λ,

$$\tilde{S}_\ell(k) \longrightarrow \tilde{S}(\lambda) = -\, t(\bar{\lambda})\, \overset{o}{S}_N(\lambda)\, \exp\left[i2\delta(\lambda)\right], \tag{3.15}$$

with $\bar{\lambda} = \lambda - \frac{1}{2}K$. The λ-integrations are carried out in different ways, depending on whether a given term in $t(\bar{\lambda})\overset{o}{S}_N(\lambda)$ is slowly or rapidly varying with λ, assuming that $|\overset{o}{S}_N(\lambda)|$ has a "normal strong-absorption profile"[10]): the slowly-varying parts are evaluated in stationary phase approximation (SPA), while for the rapidly-varying terms the Rutherford phase function $\sigma(\lambda)$ is expanded linearly about the points $\lambda = \Lambda_m$ of maximum variation,

$$2\sigma(\lambda) \approx 2\sigma(\Lambda_m) + (\lambda - \Lambda_m)\,\theta_R^{(m)} \quad , \tag{3.16}$$

where

$$\theta_R^{(m)} = \left[\frac{d\,2\sigma(\lambda)}{d\lambda}\right]_{\lambda=\Lambda_m} \cong 2\arctan\left(n/\Lambda_m\right) \tag{3.17}$$

is the Rutherford scattering angle pertaining to Λ_m. The latter procedure leads to expressions for the amplitudes that involve Fourier transforms of the rapidly-varying parts of $t(\bar{\lambda})\overset{o}{S}_N(\lambda)$.

If we write the three terms of $t_\ell(k',k) \to t(\bar{\lambda})$ in eq.(3.10) in an obvious notation as

$$t(\bar{\lambda}) = t_c(\bar{\lambda}) + t_N(\bar{\lambda}) + t_{cN}(\bar{\lambda}) \quad , \tag{3.18}$$

the contribution $\tilde{f}(\theta)$ to the elastic scattering amplitude due to dynamic polarization is split into the three corresponding parts,

$$\tilde{f}(\theta) = \tilde{f}_c(\theta) + \tilde{f}_N(\theta) + \tilde{f}_{cN}(\theta) \,. \tag{3.19}$$

Using the asymptotic form of the Legendre polynomials,

$$P_\ell(\cos\theta) \cong \frac{1}{(2\pi\lambda\sin\theta)^{1/2}}\left[e^{-i(\lambda\theta-\frac{1}{4}\pi)} + e^{i(\lambda\theta-\frac{1}{4}\pi)}\right] \quad , \tag{3.20}$$

each component of (3.19) consists of two branches,

$$\tilde{f}(\theta) = \tilde{f}^{(+)}(\theta) + \tilde{f}^{(-)}(\theta) , \tag{3.21}$$

which (in a classical picture) correspond to scattering by the "near side" and the "far side" of the interaction region, respectively.

To evaluate $\tilde{f}_C(\theta)$, we must distinguish, as in the calculation of $\overset{\circ}{f}(\theta)$ according to ref.[5]), between the "illuminated" region $\theta \leqq \theta_R$ and the "shadow" region $\theta \geqq \theta_R$ of the angular distribution, where

$$\theta_R = \left[\frac{d\,2\sigma(\lambda)}{d\lambda}\right]_{\lambda=\Lambda} \cong 2\arctan(n/\Lambda) \tag{3.22}$$

is the "Rutherford grazing angle" associated with the angular momentum Λ defined by $|\overset{\circ}{S}(\Lambda)| = \frac{1}{2}$. This distinction is necessary because in evaluating the integral,

$$\tilde{f}(\theta) = \frac{i}{k}\int_0^\infty d\lambda\,\lambda\,t(\tilde{\lambda})\,\overset{\circ}{S}_N(\lambda)\,e^{i2\sigma(\lambda)}\,P_{\lambda-\frac{1}{2}}(\cos\theta) \tag{3.23}$$

with the asymptotics (3.20), there is only one contribution, namely $\tilde{f}_C^{(+)}(\theta)$ in the region $\theta \leqq \theta_R$, that arises from a point of stationary phase,

$$\lambda_\theta = n\cot\tfrac{1}{2}\theta \tag{3.24}$$

in the range $\lambda \geqq \Lambda$, as well as from the vicinity of Λ itself; all of the other leading contributions to $\tilde{f}(\theta)$ come from the neighbourhoods of the maxima Λ_m of the rapidly-varying parts of $t(\bar{\lambda})\overset{0}{S}_N(\lambda)$.

With the methods of refs. [5-8]) we obtain the following results. First, the contribution due to Coulomb excitation is

$$\tilde{f}_c^{(+)}(\theta) = \begin{cases} -f_R(\theta)\, t_c(\bar{\lambda}_\theta) + \kappa(\Lambda,\theta)\, \tilde{G}(\theta_R-\theta)\, e^{i\phi_+(\Lambda,\theta)} F_N[\Delta(\theta_R-\theta)]\, t_c(\bar{\Lambda}), \\ \qquad \theta \leqq \theta_R, \\ \kappa(\Lambda,\theta)\, \tilde{G}(\theta-\theta_R)\, e^{i\phi_+(\Lambda,\theta)} F_N[\Delta(\theta_R-\theta)]\, t_c(\bar{\Lambda}), \\ \qquad \theta \geqq \theta_R, \end{cases} \tag{3.25a}$$

$$\tilde{f}_c^{(-)}(\theta) = -\kappa(\Lambda,\theta)\, e^{i\phi_-(\Lambda,\theta)}\, \frac{F_N[\Delta(\theta_R+\theta)]}{\theta_R+\theta}\, t_c(\bar{\Lambda}), \tag{3.25b}$$

where $\bar{\lambda}_\theta = \lambda_\theta - \frac{1}{2}K$ with $\lambda_\theta = n \cot\frac{1}{2}\theta$, and $\bar{\Lambda} = \Lambda - \frac{1}{2}K$ with $\Lambda = n \cot\frac{1}{2}\theta_R$. The other quantities in (3.25) are defined as follows:

$$f_R(\theta) = -\frac{n}{2k}\, \frac{1}{(\sin\frac{1}{2}\theta)^2}\, \exp\left\{ i\left[2\sigma(0) - 2n\,\ln\left(\sin\tfrac{1}{2}\theta\right)\right]\right\} \tag{3.26}$$

is the Rutherford scattering amplitude,

$$\kappa(\Lambda,\theta) = \frac{1}{k}\left(\frac{\Lambda}{2\pi\sin\theta}\right)^{1/2}, \quad \phi_\pm(\Lambda,\theta) = 2\sigma(\Lambda) \mp (\Lambda\theta - \tfrac{1}{4}\pi), \tag{3.27}$$

and $F_N[\Delta z]$ is the Fourier transform of the derivative of $\overset{\circ}{S}_N(\lambda)$,

$$F_N[\Delta z] = \int_{-\infty}^{\infty} d\lambda\, \overset{\circ}{D}_N(\lambda)\, e^{i(\lambda-\Lambda)z} \,, \qquad \overset{\circ}{D}_N(\lambda) = \frac{d\overset{\circ}{S}_N(\lambda)}{d\lambda} \,. \tag{3.28}$$

(Specific forms of these functions are given in appendix D). The functions $\tilde{G}(\pm z)$ describe, under the conditions[11] $\Lambda \gg 1$ and $\Lambda\sin\theta_R \gtrsim 1$, the Fresnel-diffractive properties of elastic heavy-ion scattering; their analytic form is given in appendix C.

The contribution due to nuclear excitation and Coulomb-nuclear interference are

$$\tilde{f}_N^{(\pm)}(\theta) = i\kappa(\Lambda_N, \theta)\, e^{i\phi_\pm(\Lambda_N,\theta)}\, H_N(\theta_R^{(N)} \mp \theta)\, t_N(\bar{\Lambda}_N) \,, \tag{3.29}$$

and

$$\tilde{f}_{CN}^{(\pm)}(\theta) = i\kappa(\Lambda_{CN}, \theta)\, e^{i\phi_\pm(\Lambda_{CN},\theta)}\, H_{CN}(\theta_R^{(CN)} \mp \theta)\, t_{CN}(\bar{\Lambda}_{CN}) \,, \tag{3.30}$$

respectively. In these expressions, the angular momenta $\Lambda_m = (\Lambda_N, \Lambda_{CN})$ are respectively defined as the positions of the maxima of the rapidly-varying parts of $t_N(\bar{\lambda})\overset{\circ}{S}_N(\lambda)$ and $t_{CN}(\bar{\lambda})\overset{\circ}{S}_N(\lambda)$, i.e. of $|[\hat{N}_K(\bar{\lambda})]^2\overset{\circ}{S}_N(\lambda)|$ and $|\hat{N}_K(\bar{\lambda})\overset{\circ}{S}_N(\lambda)|$, and we have $\bar{\Lambda}_m = \Lambda_m - \frac{1}{2}K$. The corresponding Coulomb scattering angles are

$$\theta_R^{(N)} = 2\arctan(n/\Lambda_N) \,, \qquad \theta_R^{(CN)} = 2\arctan(n/\Lambda_{CN}) \,. \tag{3.31}$$

Similarly to (3.27) we define

$$\kappa(\Lambda_m,\theta) = \frac{1}{k}\left(\frac{\Lambda_m}{2\pi\sin\theta}\right)^{1/2}, \quad \phi_\pm(\Lambda_m,\theta) = 2\sigma(\Lambda_m) \mp (\Lambda_m\theta - \tfrac{1}{4}\pi), \tag{3.32}$$

and to (3.28) correspond the Fourier transforms

$$H_N(z) = \int_{-\infty}^{\infty} d\lambda \, [\hat{N}_K(\bar{\lambda})]^2 \, \mathring{S}_N(\lambda) \, e^{i(\lambda-\Lambda_N)z}, \tag{3.33}$$

$$H_{CN}(z) = \int_{-\infty}^{\infty} d\lambda \, \hat{N}_K(\bar{\lambda}) \, \mathring{S}_N(\lambda) \, e^{i(\lambda-\Lambda_{CN})z}. \tag{3.34}$$

Specific expressions for Λ_N, Λ_{CN} and analytic forms of the functions $H_N(z)$, $H_{CN}(z)$ are derived in appendix D.

Finally we give the CF expression[5]) for the elastic scattering amplitude without coupling,

$$\mathring{f}^{(+)}(\theta) = \begin{cases} f_R(\theta) - \kappa(\Lambda,\theta)\, G(\theta_R-\theta)\, e^{i\phi_+(\Lambda,\theta)} F_N[\Delta(\theta_R-\theta)], & \theta \leq \theta_R, \\ \\ -\kappa(\Lambda,\theta)\, G(\theta-\theta_R)\, e^{i\phi_+(\Lambda,\theta)} F_N[\Delta(\theta_R-\theta)], & \theta \geq \theta_R, \end{cases} \tag{3.35a}$$

$$\mathring{f}^{(-)}(\theta) = \kappa(\Lambda,\theta)\, e^{i\phi_-(\Lambda,\theta)} \frac{F_N[\Delta(\theta_R+\theta)]}{\theta_R+\theta}, \tag{3.35b}$$

where all quantities are the same as in eqs.(3.25), except for the functions $G(\pm z)$ which differ slightly from $\tilde{G}(\pm z)$ and are also given explicitly in appendix C.

The sum of expressions (3.35), (3.25), (3.29) and (3.30) is our closed-form result for the total elastic scattering amplitude in the presence of dynamic polarization by Coulomb and nuclear excitation.

3.3. Strong Coulomb excitation

In several cases, such as the scattering of $^{18}O + ^{184}W$ at 90 MeV[12]) analyzed in refs.[1,2,8]), the dynamic polarization by Coulomb excitation is so strong that the lowest-order approximation, represented by eqs.(3.10) and the amplitude (3.25), is insufficient. Then we must use the expressions obtained by summation over higher-order contributions, either in the geometrical (Born) series form (3.9a) or in the exponentiated (WKB) form (3.96). If we confine ourselves to pure Coulomb excitation ($\delta_L^{(N)} = 0$), the total amplitude becomes

$$f(\theta) \equiv f_c(\theta) = \overset{\circ}{f}(\theta) + \tilde{f}_c(\theta) , \tag{3.36}$$

evaluated with the S-matrix

$$S_c^{(geom)}(\lambda) = \frac{1-\frac{1}{2}t_c(\bar{\lambda})}{1+\frac{1}{2}t_c(\bar{\lambda})}\,\overset{\circ}{S}(\lambda) , \quad \text{or} \quad S_c^{(exp)}(\lambda) = \overset{\circ}{S}(\lambda)\, e^{-t_c(\bar{\lambda})} . \tag{3.37}$$

Because of the slow variation of $t_C(\bar{\lambda})$ compared with $\overset{\circ}{S}_N(\lambda)$, the corresponding amplitudes $\tilde{f}_C(\theta)$ are still of the form (3.25), but with t_C (of argument $\bar{\lambda}_\theta$ or $\bar{\Lambda}$) replaced by

$$t_c^{(geom)} = \frac{t_c}{1+\frac{1}{2}t_c} , \quad \text{or} \quad t_c^{(exp)} = 1 - e^{-t_c} . \tag{3.38}$$

As the simplest case let us consider scattering below the Coulomb barrier, where only the first terms in eqs. (3.25a) and (3.35a) remain. Then the total cross section $\sigma_C(\theta)$ divided by the Rutherford cross section $\sigma_R(\theta)$ becomes

$$\frac{\sigma_c^{(geom)}(\theta)}{\sigma_R(\theta)} = \left[\frac{1-\frac{1}{2}t_c(\bar{\lambda}_\theta)}{1+\frac{1}{2}t_c(\bar{\lambda}_\theta)}\right]^2 , \tag{3.39}$$

or

$$\frac{\sigma_c^{(exp)}(\theta)}{\sigma_R(\theta)} = \exp\left[-2\,t_c(\bar{\lambda}_\theta)\right] . \tag{3.40}$$

As a specific example we give the explicit expression for $t_C(\bar{\lambda}_\theta)$ defined by eq. (3.10a), for quadrupole excitation. With the coefficients $a_{2K}(\bar{\lambda})$ from eq. (3.2), calculated in the large-ℓ limit,

$$a_{20}(\bar{\lambda}) \cong -\frac{1}{4\pi^{1/2}} , \qquad a_{2\pm2}(\bar{\lambda}) \cong \left(\frac{3}{2}\right)^{1/2}\frac{1}{4\pi^{1/2}} , \tag{3.41}$$

we obtain

$$t_c(\bar{\lambda}_\theta) \cong \frac{1}{32\pi}\left(k\,\delta_2^{(c)}\right)^2 \left\{\left[\hat{C}_{20}(\bar{\lambda}_\theta)\right]^2 + \frac{3}{2}\left(\left[\hat{C}_{22}(\bar{\lambda}_\theta)\right]^2 + \left[\hat{C}_{2-2}(\bar{\lambda}_\theta)\right]^2\right)\right\} \tag{3.42}$$

$$\approx \frac{\pi}{50}\,\frac{k^4}{n^2}\,\frac{B(E2,\uparrow)}{Z_B^2\,e^2}\left\{\left[I_{20}(\theta,\xi)\right]^2 + \frac{3}{2}\left(\left[I_{22}(\theta,\xi)\right]^2 + \left[I_{2-2}(\theta,\xi)\right]^2\right)\right\} ,$$

and using the explicit expressions[13] for the functions $I_{2K}(\theta,0)$ in the adiabatic limit $\xi = 0$, this becomes

$$t_c(\bar{\lambda}_\theta) = \frac{2\pi}{75}\,\frac{k^4}{n^2}\,\frac{B(E2,\uparrow)}{Z_B^2\,e^2}\left\{(\sin\tfrac{1}{2}\theta)^4 + 3(\tan\tfrac{1}{2}\theta)^4\left[1-\tfrac{1}{2}(\pi-\theta)\tan\tfrac{1}{2}\theta\right]^2\right\}. \tag{3.43}$$

Expression (3.39) with (3.43) agrees with a result obtained recently by Baltz _et_ _al_.[14]). Equation (3.40) is identical to the result of ref. [2]) at $\theta = 180^0$, otherwise the explicit angular dependence is slightly different. As long as the coupling is not very strong so that $\frac{1}{2}t_C(\bar{\lambda}_\theta)$ is less than unity for all angles we expect eqs. (3.39) and (3.40) to give comparable results. To exhibit this we show in fig. 1 a comparison of the cross section ratios (3.39) and (3.40), with $t_C(\bar{\lambda}_\theta)$ given by eq. (3.43), for the scattering of $^{20}Ne + ^{152}Sm$ at $E_{lab} = 70$ MeV[15]). (Reorientation, projectile excitation, etc., are neglected in the expressions, although our formalism could be extended to include these effects as well.)

4. Coupling to transfer channels

4.1. Modification of the S-matrix

Now we turn to the coupling to rearrangement channels, and consider processes in which the intermediate channel m differs from the initial channel n by the transfer of a nucleon or a cluster c from one of the scattering nuclei (A, say) to the other (B). Here we encounter the problem of non-orthogonality of the wave functions in the initial and intermediate channels. For the two-step process $A + B \rightarrow A' + B' \rightarrow A + B$ this results in an additional term in the effective interaction potential for elastic scattering (see appendix E). Although our treatment could be extended to include such terms, we shall for simplicity neglect the non-orthogonality contributions on the expectation that their effect is of lesser importance for heavy than for light nuclei[16]).

Thus we use the results of sect.2 (writing again $\ell_n = \ell$, $\ell_m = \ell'$ and $\mu_n = \mu$) with the coupling interaction

$$V^{L}_{\ell\ell'}(r) = a_T^{(\kappa')} F_L^{(\kappa')}(r) , \tag{4.1}$$

where the coefficient $a_T^{(K')}$ includes the product of spectroscopic factors and angular momentum coupling terms, and

$$F_L^{(\kappa')}(r) = -i^L h_L^{(1)}(i\kappa' r) \cong \frac{e^{-\kappa' r}}{\kappa' r} \tag{4.2}$$

is the form factor for transfer into a bound state of imaginary wave number $\kappa' = (2\mu_c' E_B')^{1/2}/\hbar$, where μ_c' is the reduced mass and E_B' the binding energy of the transferred particle c in the nucleus B'.

Now we employ the approximations used in the closed-form description of heavy-ion transfer reactions given in ref.[7]. Thus the DWBA radial integrals (2.13) are replaced with the "Sopkovich prescription" of factoring out the square roots of the nuclear parts of the partial-wave elastic S-matrices in the initial and final channels, and the remaining transfer radial integrals with Coulomb wave functions are approximated by their WKB forms. The result is

$$R_{\ell'\ell}^{(\kappa')L}(k',k) \approx \frac{2\pi}{\zeta' k k' \kappa'} I_{L-K}^{(\kappa')}(\vartheta,\xi)\left[\mathring{S}_{\ell'}^{(N)}(k')\,\mathring{S}_{\ell}^{(N)}(k)\right]^{1/2}, \tag{4.3a}$$

where ϑ and ξ are defined in the same way as in (3.6), and ζ' is a scale factor that partly accounts for recoil effects (see refs.[7,17]). The definition and main properties of the functions $I_{L-K}^{(K)}(\vartheta,\xi)$ are given in appendix B of ref.[7]. With a similar approximation of the radial integral for the reverse transfer process,

$$R_{\ell\ell'}^{(\kappa)L}(k,k') \approx \frac{2\pi}{\zeta k' k \kappa} I_{LK}^{(\kappa)}(\vartheta,-\xi)\left[\mathring{S}_{\ell}^{(N)}(k)\,\mathring{S}_{\ell'}^{(N)}(k')\right]^{1/2}, \tag{4.3b}$$

where $\kappa = (2\mu_c E_B)^{1/2}/\hbar$ is the imaginary wave number for particle c in the initial nucleus A, and using the symmetry relation

$$I_{LK}^{(\kappa)}(\vartheta,-\xi) = I_{L-K}^{(\kappa)}(\vartheta,\xi), \tag{4.4}$$

the quantity (2.16) becomes

$$T_{\ell'\ell}^{(\kappa,\kappa')}(k',k) = \frac{4\pi^2}{\xi\xi'(kk')^2\kappa\kappa'}\, I_{L-K}^{(\kappa)}(\vartheta,\xi)\, I_{L-K}^{(\kappa')}(\vartheta,\xi) . \tag{4.5}$$

The modified S-matrix (2.14) has the form

$$S_\ell(k) = \left[1 - t_\ell^{(\kappa,\kappa')}(k',k)\right] \overset{\circ}{S}_\ell(k) \tag{4.6}$$

with

$$t_\ell^{(\kappa,\kappa')}(k',k) = \sum_{K=-L}^{L} \alpha_{LK}^{(T)}(\bar\lambda)\, I_{L-K}^{(\kappa)}(\vartheta,\xi)\, I_{L-K}^{(\kappa')}(\vartheta,\xi) , \tag{4.7}$$

where we have written

$$\alpha_{LK}^{(T)}(\bar\lambda) = \frac{2\mu^2}{\hbar^4}\, \frac{a_T^{(\kappa)}}{\xi k \kappa}\, \frac{a_T^{(\kappa')}}{\xi' k' \kappa'} \tag{4.8}$$

(since the coefficients a_T in general depend on L and K and are slowly-varying functions of $\bar\lambda$). We note especially that the contribution to the elastic S-matrix due to transfer channel coupling in eq.(4.6) is, aside from the factor $\overset{\circ}{S}_\ell(k)$, completely determined by the characteristics of the transfer process. In particular, the relative phase is fixed, and since the function $t_\ell^{(\kappa,\kappa')}$ is mainly real, the transfer coupling contribution is predominantly <u>absorptive</u>.

4.2. One-step and two-step elastic transfer processes

One-step elastic transfer is a well known contribution to elastic scattering of nuclei of the type A = (B + c), so that the transfer of particle c leads back to the elastic channel, B + A[18]), as shown schematically in fig.2. The corresponding effective interaction $\tilde{U}_\ell(r)$ in eq.(2.6) is a real exchange potential of the form

$$\tilde{U}_\ell(r) = (-)^\ell a_{ET}^{(\kappa)} F_0^{(\kappa)}(r) . \tag{4.9}$$

A simple modification of the formalism in sect.2 then leads to the following form of the total S-matrix,

$$S_\ell(k) = \left[1 - i(-)^\ell \alpha_{ET} I_{00}^{(\kappa)}(\vartheta, 0) \right] \overset{\circ}{S}_\ell(k) , \tag{4.10}$$

where

$$\alpha_{ET} = \frac{\mu}{\hbar^2} \frac{a_{ET}^{(\kappa)}}{\zeta k \kappa} . \tag{4.11}$$

Thus the contribution of one-step elastic transfer is predominantly refractive and gives rise to an "odd-even staggering" in the partial-wave amplitudes. (An expression similar to (4.10) was derived earlier, in sect.10 of the first ref.[5]).)

A two-step (sequential) elastic transfer is a possible contribution to elastic scattering of nuclei of the type A = (B + c + c), for which the intermediate channel is symmetric with nuclei A′ = (A-c) = B′ = (B+c).

This is shown schematically in fig.3(b). An example is a two-step α-transfer in $^{16}O + {}^{24}Mg$ with the symmetric intermediate channel $^{20}Ne + {}^{20}Ne$. This reaction was studied recently by Landowne and Wolter[19]) by means of a full coupled-channels calculation (with recoil and finite-range interaction but neglecting non-orthogonality contributions) including coupling to the inelastic channel with ^{24}Mg in its lowest 2^+ state. Although the two-step contributions to the elastic cross section were found to be at least 3 orders of magnitude smaller than the direct scattering cross section at 90°, an ad hoc increase of the two-step amplitude by a factor 5 would be able to account for the weak oscillatory structure observed[20]) in the elastic $^{16}O + {}^{24}Mg$ scattering at E_{lab} = 29 and 33 MeV.

The contribution of the two-step transfer process depicted in fig.3(b) corresponds again to an exchange interaction

$$\tilde{U}_{\ell}(r) = (-)^{\ell}\, \tilde{U}_{\ell}^{(T)}(r)\,, \tag{4.12}$$

where $\tilde{U}_{\ell}^{(T)}(r)$ is the effective interaction corresponding to the process shown in fig.3(a). From the formalism of sect.2, our results (4.6)-(4.8), and inspection of fig.3, it follows immediately that the S-matrix for direct elastic scattering plus the sum of the contributions from the processes of fig.3 becomes

$$S_{\ell}(k) = \left\{ 1 - [1+(-)^{\ell}]\, t_{\ell}^{(\kappa,\kappa')}(k',k) \right\} \overset{\circ}{S}_{\ell}(k)\,. \tag{4.13}$$

Thus the odd-ℓ contributions vanish while the even-ℓ amplitudes are twice as large as those given by eq.(4.7). As in eqs.(4.6), (4.7), the two-step transfer contributions are predominantly absorptive.

4.3. Evaluation of scattering amplitudes; intermediate angles

To obtain closed-form expressions for the scattering amplitudes corresponding to the S-matrices (4.6), (4.10) and (4.13), we use a similar procedure as developed in ref.[7]). We write

$$f(\theta) = \overset{\circ}{f}(\theta) + \tilde{f}_T(\theta) , \tag{4.14}$$

where $\tilde{f}_T(\theta)$ corresponds to the second term in eq.(4.6) which we replace by a continuous function of $\lambda = \ell + \frac{1}{2}$,

$$- t_\ell^{(\kappa,\kappa')}(k',k)\, \overset{\circ}{S}_\ell(k) \longrightarrow \tilde{S}_T(\lambda) = - t_T(\bar{\lambda})\, \overset{\circ}{S}(\lambda) , \tag{4.15}$$

where $t_T(\bar{\lambda})$ with $\bar{\lambda} = \lambda - \frac{1}{2}K$ is given by the right-hand side of eq.(4.7).

By using the asymptotics (3.20), each of the two amplitudes in (4.14) splits into a "near-side" (+) and a "far-side" (-) branch, with $\overset{\circ}{f}{}^{(\pm)}(\theta)$ given by eqs.(3.35a,b), and

$$\tilde{f}_T^{(\pm)}(\theta) = \frac{i}{k} \frac{1}{(2\pi \sin\theta)^{1/2}} \int_0^\infty d\lambda\, \lambda^{1/2}\, t_T(\bar{\lambda})\, \overset{\circ}{S}_N(\lambda) \exp\left\{ i \left[2\sigma(\lambda) \mp (\lambda\theta - \tfrac{1}{4}\pi) \right] \right\} . \tag{4.16}$$

To evaluate the integrals we must identify the rapidly and slowly varying parts of the integrand. Since the main, asymptotic behaviour of the functions $I_{L-K}^{(K)}(\vartheta,\xi)$ is exponential in $\bar{\lambda}$ [7]) we write, as an auxiliary step,

$$I_{L-K}^{(\kappa)}(\vartheta,\xi) \equiv \hat{I}_{L-K}^{(\kappa)}(\vartheta,\xi)\, e^{-\gamma\bar{\lambda}} , \tag{4.17}$$

where the functions $\hat{I}^{(\kappa)}_{L-K}(\vartheta,\xi)$, defined by eq.(4.17), are slowly-varying functions of $\bar{\lambda}$. The quantity γ is mainly determined by κ and given by

$$\gamma = (\rho^2 + \xi'^2)^{1/2} / \bar{n} , \tag{4.18}$$

where

$$\rho = \frac{\kappa}{\bar{k}}\,\bar{n} , \qquad \xi' = \frac{k - \xi k'}{\bar{k}}\,\bar{n} . \tag{4.19}$$

Then we can write

$$t_T(\bar{\lambda}) \equiv \hat{t}_T(\bar{\lambda})\, e^{-2\bar{\gamma}\bar{\lambda}} , \tag{4.20}$$

where

$$\hat{t}_T(\bar{\lambda}) = \sum_{K=-L}^{L} \alpha^{(T)}_{LK}(\bar{\lambda})\, \hat{I}^{(\kappa)}_{L-K}(\vartheta,\xi)\, \hat{I}^{(\kappa')}_{L-K}(\vartheta,\xi) \tag{4.21}$$

is slowly-varying in $\bar{\lambda}$, and where $\bar{\gamma} = \frac{1}{2}(\gamma+\gamma')$, with γ' defined as in (4.18), (4.19) with κ' instead of κ.

Thus the rapidly-varying part of the integrand in (4.16) is

$$\tau_K(\lambda) = \mathring{S}_N(\lambda)\, e^{-2\bar{\gamma}\bar{\lambda}} = e^{\bar{\gamma}K}\, \mathring{S}_N(\lambda)\, e^{-2\bar{\gamma}\lambda} . \tag{4.22}$$

Since there is no contribution from any point of stationary phase to the integrals in (4.16), the main contribution comes from the vicinity of $\lambda = \Lambda_T$ where $\tau_K(\lambda)$ has its maximum. After a linear expansion of $2\sigma(\lambda)$ about this point,

$$2\sigma(\lambda) \approx 2\sigma(\Lambda_T) + (\lambda-\Lambda_T)\,\theta_R^{(T)} \quad , \tag{4.23}$$

where

$$\theta_R^{(T)} = \left[\frac{d\,2\sigma(\lambda)}{d\lambda} \right]_{\lambda=\Lambda_T} \cong 2\arctan\left(n/\Lambda_T\right) \tag{4.24}$$

is the Rutherford scattering angle pertaining to Λ_T, we obtain

$$\tilde{f}_T^{(\pm)}(\theta) = i\,\kappa\,(\Lambda_T,\theta)\,e^{i\phi_\pm(\Lambda_T,\theta)}\,H_T\left(\theta_R^{(T)} \mp \theta\right)\,t_T(\bar{\Lambda}_T)\,. \tag{4.25}$$

Here, $\bar{\Lambda}_T = \Lambda_T - \frac{1}{2}K$, and

$$\kappa(\Lambda_T,\theta) = \frac{1}{k}\left(\frac{\Lambda_T}{2\pi\sin\theta}\right)^{1/2}, \quad \phi_\pm(\Lambda_T,\theta) = 2\sigma(\Lambda_T) \mp \left(\Lambda_T\theta - \tfrac{1}{4}\pi\right), \tag{4.26}$$

while

$$H_T(z) = \int_{-\infty}^{\infty} d\lambda\; \mathring{S}_N(\lambda)\, e^{-2\bar{\gamma}(\lambda-\Lambda_T)}\, e^{i(\lambda-\Lambda_T)z} \tag{4.27}$$

is the Fourier transform of $\tau_K(\lambda)\exp(2\bar{\gamma}\bar{\Lambda}_T)$. Note that the factor $\exp(-2\bar{\gamma}\bar{\Lambda}_T)$ has been used together with $\hat{t}_T(\bar{\Lambda})$ to restore in (4.25) the original function t_T, according to definition (4.20), at the point $\bar{\Lambda}_T$. This is given by

$$t_T(\bar{\Lambda}_T) = \sum_{K=-L}^{L} \alpha_{LK}^{(T)}(\bar{\Lambda}_T) I_{L-K}^{(\kappa)}(\bar{\theta}_R^{(T)}, \xi) I_{L-K}^{(\kappa')}(\bar{\theta}_R^{(T)}, \xi), \tag{4.28}$$

where

$$\bar{\theta}_R^{(T)} = 2\arctan(\bar{n}/\bar{\Lambda}_T). \tag{4.29}$$

Specific expressions for Λ_T and $H_T(z)$ are derived in appendix D.

The sum,

$$f(\theta) = \mathring{f}^{(+)}(\theta) + \mathring{f}^{(-)}(\theta) + \tilde{f}_T^{(+)}(\theta) + \tilde{f}_T^{(-)}(\theta), \tag{4.30}$$

with the four terms given by eqs.(3.35) and (4.25), is our closed-form result for the total elastic scattering amplitude in the presence of coupling to a transfer channel.

For the amplitude for elastic scattering with a one-step transfer contribution we obtain from the S-matrix (4.10), following the same procedure as before but noting that the factor $(-)^{\ell}$ changes θ into $\pi-\theta$ and interchanges the (+) and (-) branches,

$$f(\theta) = \mathring{f}(\theta) + \tilde{f}_{ET}(\theta), \tag{4.31}$$

where

$$\tilde{f}_{ET}^{(\pm)}(\theta) = -\kappa(\Lambda_{ET}, \theta) e^{i\phi_{\mp}(\Lambda_{ET}, \pi-\theta)} H_{ET}\left[\theta_R^{(ET)} \pm (\pi-\theta)\right] \alpha_{ET} I_{00}^{(\kappa)}(\theta_R^{(ET)}, 0). \tag{4.32}$$

In this expression, Λ_{ET} is defined by the maximum of

$$\tau_{ET}(\lambda) = \overset{\circ}{S}_N(\lambda)\, e^{-\gamma\lambda} , \tag{4.33}$$

further $\theta_R^{(ET)} = 2\arctan(n/\Lambda_{ET})$, and $H_{ET}(z)$ is the Fourier transform

$$H_{ET}(z) = \int_{-\infty}^{\infty} d\lambda\, \overset{\circ}{S}_N(\lambda)\, e^{-\gamma(\lambda-\Lambda_{ET})}\, e^{i(\lambda-\Lambda_{ET})z} \tag{4.34}$$

(A specific expression is given in appendix D).

Finally, for elastic scattering with the two-step transfer contributions shown in fig. 3, the total amplitude follows immediately from the form of the S-matrix (4.13) as

$$f(\theta) = \overset{\circ}{f}(\theta) + \tilde{f}_T(\theta) + \tilde{f}_T(\pi-\theta) , \tag{4.35}$$

where $\tilde{f}_T(\pi-\theta)$ is given by eqs. (4.25) with θ replaced by $\pi-\theta$.

4.4. Evaluation of scattering amplitudes; large angles

Because of the use of the asymptotic form (3.20) for the Legendre polynomials, the expressions for the amplitudes derived in subsect. 4.3 are restricted to the angular range $\Lambda^{-1} \lesssim \theta \lesssim \pi-\Lambda^{-1}$, excluding in particular a small region near $\theta \sim \pi$. Since effects due to transfer channel coupling could be most conspicuous in large-angle scattering, it is necessary to supplement our previous results with expressions that cover the large-angle region uniformly up to and including 180°. This is possible by using the methods developed in ref. [10]).

Using instead of (3.20) the large-angle asymptotics

$$P_{\lambda-\frac{1}{2}}(\cos\theta) \cong e^{i\pi(\lambda-\frac{1}{2})}\left(\frac{\pi-\theta}{\sin\theta}\right)^{1/2} J_0\left[\lambda(\pi-\theta)\right], \qquad (4.36)$$

valid for $\lambda^{-1} \lesssim \theta \leq \pi$, and the Fourier-Bessel transform method described in Appendix A of part I of ref.[10]), the partial-wave summation in eq.(3.12) for the unmodified elastic amplitude $\overset{o}{f}(\theta) = \overset{o}{f}{}^{(+)}(\theta) + \overset{o}{f}{}^{(-)}(\theta)$ can be evaluated by means of the Poisson sum formula with the result[10])

$$\overset{o}{f}{}^{(\pm)}(\theta) = \frac{\Lambda}{2k}\, e^{i2\sigma(\Lambda)}\left(\frac{\pi-\theta}{\sin\theta}\right)^{1/2} \mathcal{G}^{(\pm)}(\theta)\left\{J_0\left[\Lambda(\pi-\theta)\right] \pm i J_1\left[\Lambda(\pi-\theta)\right]\right\}, \qquad (4.37)$$

where $J_0(x)$, $J_1(x)$ are cylindrical Bessel functions, and

$$\mathcal{G}^{(\pm)}(\theta) = \sum_{m=-\infty}^{\infty} e^{i\pi(2m+1)(\Lambda-\frac{1}{2})}\, \frac{F_N\left[\Delta\langle\theta_R+(2m+1)\pi \pm (\pi-\theta)\rangle\right]}{\theta_R+(2m+1)\pi \pm (\pi-\theta)}. \qquad (4.38)$$

(In the notation of ref.[10]) we have $\mathcal{G}^{(\pm)}(\theta) \equiv \mathcal{G}(\mp\vartheta)$ where $\vartheta = \pi-\theta$.)

Expression (4.37) is valid in the angle range

$$\theta \gg \theta_R + \left|2\Theta_R'(\Lambda)\right| = \theta_R + \frac{2}{\Lambda}\sin\theta_R, \qquad (4.39)$$

i.e., from just beyond the Rutherford grazing angle up to and including $\theta = \pi$.

By a very similar procedure the transfer channel coupling contribution $\tilde{f}_T(\theta) = \tilde{f}_T^{(+)}(\theta) + \tilde{f}_T^{(-)}(\theta)$ can be evaluated, with the result[7])

$$\tilde{f}_T^{(\pm)}(\theta) = i\,\frac{\Lambda_T}{2k}\,e^{i2\sigma(\Lambda_T)}\left(\frac{\pi-\theta}{\sin\theta}\right)^{1/2}\mathcal{H}_T^{(\pm)}(\theta)\left\{J_0\left[\Lambda_T(\pi-\theta)\right] \pm i\,J_1\left[\Lambda_T(\pi-\theta)\right]\right\} t_T(\bar{\Lambda}_T) \tag{4.40}$$

where

$$\mathcal{H}_T^{(\pm)}(\theta) = \sum_{m=-\infty}^{\infty} e^{i\pi(2m+1)(\Lambda_T-\frac{1}{2})}\, H_T\left[\theta_R^{(T)} + (2m+1)\pi \pm (\pi-\theta)\right]. \tag{4.41}$$

Similarly, the one-step elastic transfer amplitude $\tilde{f}_{ET}(\theta) = \tilde{f}_{ET}^{(+)}(\theta) + \tilde{f}_{ET}^{(-)}(\theta)$ has the form (see sect.8 of part I of ref.[10]))

$$\tilde{f}_{ET}^{(\pm)}(\theta) = -\,\frac{\Lambda_{ET}}{2k}\,e^{i2\sigma(\Lambda_{ET})}\left(\frac{\pi-\theta}{\sin\theta}\right)^{1/2}$$

$$\times\,\mathcal{H}_{ET}^{(\pm)}(\theta)\left\{J_0\left[\Lambda_{ET}(\pi-\theta)\right] \pm i\,J_1\left[\Lambda_{ET}(\pi-\theta)\right]\right\}\alpha_{ET}\,I_{00}^{(\kappa)}\left(\theta_R^{(ET)},0\right), \tag{4.42}$$

where

$$\mathcal{H}_{ET}^{(\pm)}(\theta) = \sum_{m=-\infty}^{\infty} e^{i2m\pi(\Lambda_{ET}-\frac{1}{2})}\, H_{ET}\left[\theta_R^{(ET)} + 2m\pi \pm (\pi-\theta)\right]. \tag{4.43}$$

If we use the asymptotic expressions

$$J_0(x) \pm i\, J_1(x) \cong (2/\pi x)^{1/2} \exp\left[\pm i(x - \tfrac{1}{4}\pi)\right] , \tag{4.44}$$

and retain only those terms of the Poisson series that are dominant in the intermediate-angle region (i.e., m = -1 for the (+) branches and m = 0 for the (-) branches of $\overset{\circ}{f}(\theta)$ and $\tilde{f}_T(\theta)$, but m = 0 for both branches of $\tilde{f}_{ET}(\theta)$), it is easily seen that eq.(4.37) for $\overset{\circ}{f}^{(+)}(\theta)$ reduces to eq.(3.35a) for $\theta > \theta_R$, noting that according to eq. (C.10) in appendix C the G-function has the asymptotic form $G(\theta-\theta_R) \cong (\theta-\theta_R)^{-1}$ under the condition (4.39), and eq.(4.37) for $\overset{\circ}{f}^{(-)}(\theta)$ reduces to eq.(3.35b). Similarly, eqs.(4.40) and (4.42) reduce to eqs.(4.25) and (4.32), respectively. This shows that, taken together, our closed-form expressions for $\overset{\circ}{f}(\theta)$, $\tilde{f}_T(\theta)$ and $\tilde{f}_{ET}(\theta)$ cover the whole range of scattering angles.

4.5. Backward-angle excitation functions

At $\theta = \pi$, eqs.(4.37), (4.40) and (4.42) reduce to

$$\overset{\circ}{f}(\pi) = \frac{\Lambda}{k}\, e^{i 2\sigma(\Lambda)}\, \mathcal{G}(\pi) , \tag{4.45}$$

$$\tilde{f}_T(\pi) = i\, \frac{\Lambda_T}{k}\, e^{i 2\sigma(\Lambda_T)}\, \mathcal{H}_T(\pi)\, t_T(\bar{\Lambda}_T) , \tag{4.46}$$

$$\tilde{f}_{ET}(\pi) = -\, \frac{\Lambda_{ET}}{k}\, e^{i 2\sigma(\Lambda_{ET})}\, \mathcal{H}_{ET}(\pi)\, \alpha_{ET}\, I_{00}^{(*)}(\theta_R^{(ET)}, 0) , \tag{4.47}$$

where the superscripts $(\pm)$ on the functions $\mathcal{G}(\pi)$, $\mathcal{H}_T(\pi)$ and $\mathcal{H}_{ET}(\pi)$ have been dropped because the two branches coincide in the backward direction. If we retain only the leading terms of the Poisson series for angles near $\theta = \pi$, the total backward-angle excitation function, divided by the Rutherford cross section $\sigma_R(\pi) = (n/2k)^2$, becomes

$$\frac{\sigma_T(\pi)}{\sigma_R(\pi)} = \frac{|\mathring{f}(\pi) + \tilde{f}_T(\pi)|^2}{|f_R(\pi)|^2}$$

$$\approx \left(\frac{2\Lambda}{n}\right)^2 \Bigg| \frac{F_N[\Delta(\theta_R - \pi)]}{\pi - \theta_R} + \frac{F_N[\Delta(\theta_R + \pi)]}{\pi + \theta_R} e^{i2\pi\Lambda} \tag{4.48}$$

$$- i\, t_T(\bar{\Lambda}_T)\left(1 + \frac{d_T}{\Lambda}\right) e^{-i d_T(\pi - \theta_R)} \left[H_T(\theta_R^{(T)} - \pi) - H_T(\theta_R^{(T)} + \pi) e^{i2\pi(\Lambda + d_T)} \right] \Bigg|^2$$

for the direct scattering plus transfer coupling contribution. In eq. (4.48) we have used $\Lambda_T = \Lambda + d_T$ as defined in appendix D, and the expansion $2\sigma(\Lambda_T) \approx 2\sigma(\Lambda) + d_T\theta_R$.

Similarly, in the presence of one-step elastic transfer we obtain for the total backangle excitation function

$$\frac{\sigma_{ET}(\pi)}{\sigma_R(\pi)} = \frac{|\mathring{f}(\pi) + \tilde{f}_{ET}(\pi)|^2}{|f_R(\pi)|^2}$$

$$\approx \left(\frac{2\Lambda}{n}\right)^2 \Bigg| \frac{F_N[\Delta(\theta_R - \pi)]}{\pi - \theta_R} + \frac{F_N[\Delta(\theta_R + \pi)]}{\pi + \theta_R} e^{i2\pi\Lambda} \tag{4.49}$$

$$- i\, \alpha_{ET}\, I_{oo}^{(\kappa)}(\theta_R^{(ET)}, 0)\left(1 + \frac{d_{ET}}{\Lambda}\right) e^{i d_{ET}\theta_R}\, H_{ET}(\theta_R^{(ET)})\, e^{i\pi\Lambda} \Bigg|^2 ,$$

where we have used $\Lambda_{ET} = \Lambda + d_{ET}$ as defined in appendix D, and the expansion $2\sigma(\Lambda_{ET}) \approx 2\sigma(\Lambda) + d_{ET}\theta_R$. Note that because of its exchange character, the elastic transfer contribution oscillates in Λ with the period $\delta\Lambda = 2$, i.e., with twice the normal period $\delta\Lambda = 1$ of the oscillations in the direct scattering term.

The energy dependence of the quantities appearing in eqs.(4.48), (4.49), and the oscillations that arise from the interference of the different terms in these expressions (which may account for some of the gross structure observed in 180° excitation functions), have been discussed in refs.[10,7]) to which we refer for details.

5. Conclusion

In this paper we have developed an extension of the closed formalism for elastic heavy-ion scattering [refs.[5,10])] which includes the effects of coupling to inelastic and transfer channels. The corresponding contributions to the S-matrix for normal scattering without coupling, $\overset{\circ}{S}_{\ell}$, have been calculated in explicit form, using appropriate approximations. These contributions depend, aside from $\overset{\circ}{S}_{\ell}$, only on the properties of the coupling transitions, and thus, if the latter are known, contain no additional parameters. We have further calculated closed-form expressions for the corresponding contributions to the total elastic scattering amplitude which cover the whole range of scattering angles up to and including $\theta = \pi$. Thus, in particular, we have obtained analytic formulae for the 180° excitation functions.

It should be emphasized that our results, like those of refs.[5,10]), do not depend on any specific form of the function $\overset{\circ}{S}_{\ell} \to \overset{\circ}{S}(\lambda)$, provided only that the reflection function $\overset{\circ}{\eta}(\lambda) = |\overset{\circ}{S}(\lambda)|$ has a normal strong-absorption profile. If one doesn't want to rely on parameterized forms of $\overset{\circ}{S}(\lambda)$, (such as the simple example given in appendix D), this function may be generated by phenomenological or semi-microscopic (e.g. folding) potentials, and the required Fourier transforms calculated numerically.

Extension of our method to inelastic heavy-ion scattering and transfer reactions to account for higher-order, multi-step contributions to the first-order DWBA amplitudes, are fairly straightforward and will be presented in subsequent papers[21]. Applications of our theory to experimental data are in progress.

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Appendix A. Radial Gell-Mann-Goldberger relation

The Gell-Mann-Goldberger relation relates the amplitude for elastic scattering by a potential $U_\ell(r) = \overset{\circ}{U}(r) + \tilde{U}_\ell(r)$ to a sum of two amplitudes for the constituent potentials,

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2}\left\langle e^{i\underset{\sim}{k}'\underset{\sim}{r}}\,\middle|\,U_\ell(r)\,\middle|\,\psi^{(+)}(\underset{\sim}{k},\underset{\sim}{r})\right\rangle \tag{A.1}$$

$$= -\frac{\mu}{2\pi\hbar^2}\left\{\left\langle e^{i\underset{\sim}{k}'\underset{\sim}{r}}\,\middle|\,\overset{\circ}{U}(r)\,\middle|\,\overset{\circ}{\psi}^{(+)}(\underset{\sim}{k},\underset{\sim}{r})\right\rangle + \left\langle\overset{\circ}{\psi}^{(-)}(\underset{\sim}{k}',\underset{\sim}{r})\,\middle|\,\tilde{U}_\ell(r)\,\middle|\,\psi^{(+)}(\underset{\sim}{k},\underset{\sim}{r})\right\rangle\right\},$$

where $\psi(\underset{\sim}{k},\underset{\sim}{r})$ is the total wave function for scattering by $U_\ell(r)$ and $\overset{\circ}{\psi}(\underset{\sim}{k},\underset{\sim}{r})$ the wave function for scattering by $\overset{\circ}{U}(r)$. Expansion in partial waves,

$$\psi(\underset{\sim}{k},\underset{\sim}{r}) = \frac{4\pi}{kr}\sum_{\ell m} i^\ell \chi_\ell(k,r)\, e^{i\sigma_\ell(k)}\, Y_{\ell m}(\hat{\underset{\sim}{k}})\, Y^*_{\ell m}(\hat{\underset{\sim}{r}}), \tag{A.2}$$

etc., and comparison with the partial-wave series for $f(\theta)$,

$$f(\theta) = i\,\frac{2\pi}{k}\sum_{\ell m}\left[1 - S_\ell(k)\right] Y^*_{\ell m}(\hat{\underset{\sim}{k}})\, Y_{\ell m}(\hat{\underset{\sim}{r}}), \tag{A.3}$$

yields, from the first part of eq.(A.1), an exact expression for the S-matrix pertaining to $U_\ell(r)$,

$$S_\ell(k) = 1 - i\,\frac{4\mu}{\hbar^2}\int_0^\infty dr\, r\, j_\ell(kr)\, U_\ell(r)\, \chi_\ell(k,r)\,. \tag{A.4}$$

From the second part of (A.1) we obtain in a similar way,

$$S_\ell(k) = -i\,\frac{4\mu}{\hbar^2}\int_0^\infty dr\, r\, j_\ell(kr)\,\mathring{U}(r)\,\mathring{\chi}_\ell(k,r)\,e^{i\sigma_\ell(k)}$$

$$-i\,\frac{4\mu}{\hbar^2 k}\int_0^\infty dr\,\mathring{\chi}_\ell(k,r)\,\widetilde{U}_\ell(r)\,\chi_\ell(k,r)\,e^{i\sigma_\ell(k)} \tag{A.5}$$

$$\equiv \mathring{S}_\ell(k) + \widetilde{S}_\ell(k)\,.$$

We call the equality of the right-hand sides of (A.4) and (A.5) the "radial Gell-Mann-Goldberger relation".

Appendix B. Green function approximation

The Green function of the left-hand side of the eq.(2.1), for outgoing-wave boundary condition, is given by (omitting the channel index n)

$$\mathring{G}_\ell^{(+)}(r,r') = -\frac{1}{k}\,\mathring{\chi}_\ell(k,r_<)\,O_\ell(k,r_>) \tag{B.1}$$

with

$$\mathring{\chi}_\ell(k,r) = \frac{i}{2}\left[\,I_\ell(k,r) - \mathring{S}_\ell^{(N)}(k)\,O_\ell(k,r)\,\right], \tag{B.2}$$

where $O_\ell(k,r)$ and $I_\ell(k,r)$ are solutions of the left-hand side of eq.(2.1) that behave asymptotically as

$$O_\ell(k,r) \rightarrow e^{i\omega_\ell},\quad I_\ell(k,r) \rightarrow e^{-i\omega_\ell},\qquad \omega_\ell = kr - n\,\ln(2kr) - \ell\frac{\pi}{2} + \sigma_\ell\,. \tag{B.3}$$

In (B.1) , $r_{>(<)}$ denotes the greater (lesser) of r, r'.

On the other hand we have the integral representation (derived from an expansion in biorthonormal basis functions)

$$\overset{\circ}{G}{}_\ell^{(+)}(r,r') = \lim_{\varepsilon\to 0} \frac{2}{\pi} \int_0^\infty d\kappa \, \frac{\overset{\circ}{\chi}_\ell(\kappa,r)\, \overset{\circ}{\chi}_\ell(\kappa,r')}{k^2-\kappa^2+i\varepsilon} \left[\overset{\circ}{S}{}_\ell^{(N)}(\kappa)\right]^{-1} \tag{B.4}$$

with $\varepsilon > 0$. By separating the contribution from the singularity at $\kappa = k$ and the remaining principal value of the integral we obtain

$$\begin{aligned}\overset{\circ}{G}{}_\ell^{(+)}(r,r') &= -\frac{i}{k}\, \overset{\circ}{\chi}_\ell(k,r)\, \overset{\circ}{\chi}_\ell(k,r') \left[\overset{\circ}{S}{}_\ell^{(N)}(k)\right]^{-1} \\ &\quad + \frac{2}{\pi}\, \mathcal{P}\int_0^\infty d\kappa \, \frac{\overset{\circ}{\chi}_\ell(\kappa,r)\, \overset{\circ}{\chi}_\ell(\kappa,r')}{k^2-\kappa^2} \left[\overset{\circ}{S}{}_\ell^{(N)}(\kappa)\right]^{-1}\end{aligned} \tag{B.5}$$

Our approximation is to neglect the second term in (B.5) compared to the first, "on-energy-shell" contribution,

$$\overset{\circ}{G}{}_\ell^{(+)}(r,r') \approx -\frac{i}{k}\, \overset{\circ}{\chi}_\ell(k,r)\, \overset{\circ}{\chi}_\ell(k,r') \left[\overset{\circ}{S}{}_\ell^{(N)}(k)\right]^{-1} \tag{B.6}$$

A criticism has been raised[22]) that with this approximation the coupled-channels effects would be overestimated for low partial waves since the factor $[\overset{\circ}{S}{}_\ell^{(N)}(k)]^{-1}$ is large for small ℓ. In our formalism, however, the reciprocal of the nuclear S-matrix in the intermediate channel, $[\overset{\circ}{S}{}_{\ell_m}^{(N)}(k_m)]^{-1}$ in eq. (2.11), is cancelled by the factor $\overset{\circ}{S}{}_{\ell_m}^{(N)}(k_m)$ arising from the "Sopkovich approximation" for the radial integrals (2.13).

Appendix C. The functions $G(\pm z)$ and $\tilde{G}(\pm z)$

The functions $G(\pm z)$ that appear in eq.(3.35a) are defined as

$$G(\pm z) = \pm\,\gamma(\pm u)\left[1 - z\,a_1(u)\right] + a_0(u)\,, \tag{C.1}$$

where

$$z = \theta - \theta_R\,, \qquad u = \left(\frac{\Lambda}{2\sin\theta_R}\right)^{1/2}(\theta-\theta_R) \tag{C.2}$$

and

$$\gamma(u) = \left(\frac{2\pi\Lambda}{\sin\theta_R}\right)^{1/2} \exp\left[i\left(u^2+\tfrac{1}{4}\pi\right)\right]\frac{1}{2}\,\mathrm{erfc}\left(e^{i\frac{1}{4}\pi}u\right), \tag{C.3}$$

with the complementary error function defined by

$$\mathrm{erfc}\,(w) = \frac{2}{\pi^{1/2}}\int_w^\infty d\tau\, e^{-\tau^2}\,. \tag{C.4}$$

Further

$$a_0(u) = \frac{1}{2\sin\theta_R}\left[1 + \frac{4}{3}\left(1+iu^2\right)\left(\cos\tfrac{1}{2}\theta_R\right)^2\right], \tag{C.5}$$

$$a_1(u) = \frac{1}{2\sin\theta_R}\left[1 + 2\left(1+i\,\tfrac{2}{3}u^2\right)\left(\cos\tfrac{1}{2}\theta_R\right)^2\right]. \tag{C.6}$$

The functions $\tilde{G}(\pm z)$ in eq.(3.25a) are defined as

$$\tilde{G}(\pm z) = \pm\gamma(\pm u)\left[1 - z\tilde{a}_1(u)\right] + \tilde{a}_0(u) \quad , \tag{C.7}$$

where

$$\tilde{a}_0(u) = a_0(u) + b \; , \qquad \tilde{a}_1(u) = a_1(u) + b \tag{C.8}$$

with

$$b = \frac{\Lambda}{\sin\theta_R}\,\frac{t_c'(\bar{\Lambda})}{t_c(\bar{\Lambda})} \quad . \tag{C.9}$$

Here, t_C' denotes the derivative with respect to $\bar{\lambda}$ of the function defined by eq.(3.10a).

Finally we note that the functions $G(\pm z)$ and $\tilde{G}(\pm z)$ have very simple asymptotic forms: for $|u| \gg 1$, i.e. $|\theta-\theta_R| \gg (2\sin\theta_R)/\Lambda$,

$$G(\pm z) \approx \frac{1}{z} + O\left(\frac{1}{\Lambda z^2}\right) \cdot \tag{C.10}$$

Appendix D. Evaluation of Fourier transforms

In this appendix we derive analytic expressions for the various Fourier transforms that appear in our results for the scattering amplitudes. For this we need a convenient parameterized form of the zero-order S-matrix, and choose the often-used Ericson form[23]

$$\overset{\circ}{S}_N(\lambda) = \left[1 + \exp\left(\frac{\Lambda-\lambda}{\Delta} - i\alpha\right)\right]^{-1} , \tag{D.1}$$

where the real phase parameter α is restricted to the range $0 \leqq \alpha < \frac{1}{2}\pi$.

First, the Fourier transform (3.28) of the derivative $\overset{o}{D}_N(\lambda) = d\overset{o}{S}_N(\lambda)/d\lambda$, where $|\overset{o}{D}_N(\lambda)|$ has its maximum at $\lambda = \Lambda$, is given by[24]

$$F_N[\Delta z] = F[\Delta z]\, e^{\alpha \Delta z} \;, \qquad F[\Delta z] = \frac{\pi \Delta z}{\sinh(\pi \Delta z)} \;. \tag{D.2}$$

Next we derive the Fourier transforms $H_N(z)$ and $H_{CN}(z)$ defined by eqs.(3.33) and (3.34), respectively. These involve the functions $\hat{N}_K(\bar{\lambda})$ defined by (3.8). Clearly, the square root of the product of logarithmic derivatives of $\overset{o}{S}_N(\lambda)$ is very awkward to deal with; however, in appendix A of ref.[7]) a method has been devised by which such square roots can be replaced in good approximation by a single function of the same form as one of the product functions. This method yields

$$\hat{N}_K(\bar{\lambda}) \approx \frac{d\overset{o}{S}_N(\bar{\lambda})/d\bar{\lambda}}{\overset{o}{S}_N(\bar{\lambda})} \;, \tag{D.3}$$

with $\overset{o}{S}_N(\bar{\lambda})$ depending on $(\bar{\lambda}-\bar{\Lambda})/\Delta$, where $\bar{\lambda} = \lambda - \frac{1}{2}K$ and $\bar{\Lambda} = \frac{1}{2}(\Lambda+\Lambda')$, but we may set $\bar{\Lambda} \approx \Lambda$ in the adiabatic approximation.

Before evaluating $H_N(z)$ and $H_{CN}(z)$ we have to determine the values of Λ_N and Λ_{CN} where the functions $|[\hat{N}_K(\bar{\lambda})]^2\overset{o}{S}_N(\lambda)|$ and $|\hat{N}_K(\bar{\lambda})\overset{o}{S}_N(\lambda)|$ have their respective maxima. For simplicity we neglect the K-dependence of $\hat{N}_K(\bar{\lambda})$ (or set $K = 0$), so that from (D.1) and (D.3) we have

$$[\hat{N}_0(\lambda)]^2\, \overset{o}{S}_N(\lambda) = \frac{1}{\Delta^2}\, \frac{\zeta}{(1+\zeta)^3} \tag{D.4}$$

$$\hat{N}_0(\lambda)\, \overset{o}{S}_N(\lambda) = \frac{1}{\Delta}\, \frac{\zeta}{(1+\zeta)^2} \;, \tag{D.5}$$

where $\zeta = \exp[(\lambda-\Lambda)/\Delta+i\alpha]$. Hence we obtain

$$\Lambda_N = \Lambda - d_N \;, \qquad d_N = -\Delta \ln \left\{ \left[\frac{1}{2} + \left(\frac{1}{4}\cos\alpha\right)^2 \right]^{1/2} - \frac{1}{4}\cos\alpha \right\}, \tag{D.6}$$

(thus $d_N = \Delta \ln 2$ for $\alpha = 0$), and

$$\Lambda_{cN} = \Lambda \,. \tag{D.7}$$

To evaluate the Fourier transforms we use a contour integration method described in appendices B, C of ref.[25]). The results are

$$H_N(z) = \frac{1}{2\Delta}\left(1 + i\Delta z\right) F[\Delta z]\, e^{-\alpha\Delta z}\, e^{i d_N z} \,, \tag{D.8}$$

$$H_{cN}(z) = F[\Delta z]\, e^{-\alpha\Delta z} \,, \tag{D.9}$$

where $F[\Delta z]$ is given in eq. (D.2) and d_N in eq. (D.6).

To evaluate the Fourier transform for transfer coupling, $H_T(z)$ defined by eq.(4.27), we must first determine the maximum of $|\overset{0}{S}_N(\lambda)|\exp(-2\overline{\gamma}\lambda)$, again with the choice (D.1) for $\overset{0}{S}_N(\lambda)$. A simple analytic expression for Λ_T can be obtained only for $\alpha = 0$,

$$\Lambda_T = \Lambda + d_T \;, \qquad d_T = \Delta \ln \left(\frac{1}{2\overline{\gamma}\Delta} - 1 \right) \,, \quad (\alpha = 0) \,. \tag{D.10}$$

Then we calculate

$$H_T(z) = \int_{-\infty}^{\infty} d\lambda\, \overset{\circ}{S}_N(\lambda)\, e^{i(\lambda-\Lambda_T)(z+i2\bar{\gamma})}$$

$$= e^{i(\Lambda-\Lambda_T)(z+i2\bar{\gamma})} \int_{-\infty}^{\infty} d\lambda\, \overset{\circ}{S}_N(\lambda)\, e^{i(\lambda-\Lambda)(z+i2\bar{\gamma})}$$

$$= i\, \frac{e^{-id_T(z+i2\bar{\gamma})}}{z+i2\bar{\gamma}}\, F_N\left[\Delta(z+i2\bar{\gamma})\right] , \qquad \text{(D.11)}$$

with the function F_N given by (D.2).

For the one-step elastic transfer we obtain similarly,

$$\Lambda_{ET} = \Lambda + d_{ET} , \qquad d_{ET} = \Delta \ln\left(\frac{1}{\gamma\Delta} - 1\right) , \quad (\alpha = 0) , \qquad \text{(D.12)}$$

and the Fourier transform (4.34) has the form

$$H_{ET}(z) = i\, \frac{e^{-id_{ET}(z+i\gamma)}}{z+i\gamma}\, F_N\left[\Delta(z+i\gamma)\right] . \qquad \text{(D.13)}$$

Appendix E. Nonorthogonality effects

In this appendix we give a formal discussion of how the non-orthogonality of the channels in the case of transfer coupling manifests itself in our theory. For brevity we confine ourselves here to deriving a formal expression for the effective potential in the elastic channel due to coupling to a transfer channel, an expression that contains the nonorthogonality effects to all orders. The actual evaluation of the nonorthogonality integrals for different partial waves, and their incorporation in our explicit formulae for the modified elastic S-matrix, will be described elsewhere[26]. In our formal development we follow closely the work of Udagawa *et al.*[27].

Consider the decomposition of the total wave function $|\Psi\rangle$ into two components corresponding to the elastic (o) and transfer (t) "channel" wave functions,

$$|\Psi\rangle = \chi_o(\underset{\sim}{r}_o)|o,\underset{\sim}{r}_o\rangle + \chi_t(\underset{\sim}{r}_t)|t,\underset{\sim}{r}_t\rangle \quad . \tag{E.1}$$

Equation (E.1) serves as a definition of the distorted wave functions $\chi_o(\underset{\sim}{r}_o)$ and $\chi_t(\underset{\sim}{r}_t)$. The vectors $\underset{\sim}{r}_o$ and $\underset{\sim}{r}_t$ specify the relative distances between the centres of the heavy ions in the elastic and the transfer channel, respectively.

Now we project out of $|\Psi\rangle$ the elastic amplitude $\langle o|\Psi\rangle$ and the transfer amplitude $\langle t|\Psi\rangle$. It is these amplitudes that asymptotically would give us the scattering amplitudes. These amplitudes satisfy the usual coupled-channels equations,

$$(E_o - H_o)\langle o|\Psi\rangle = \langle o|V_o - U_o|\Psi\rangle , \tag{E.2}$$

$$(E_t - H_t)\langle t|\Psi\rangle = \langle t|V_t - U_t|\Psi\rangle\,, \tag{E.3}$$

where $H_o = T_o + U_o$ and $H_t = T_t + U_t$ are the optical Hamiltonians appropriate for the elastic and transfer channels, respectively (T is the kinetic energy operator and U the optical potential). The total Hamiltonian, H, is invariant under the different compositions, i.e., $H = H_o + V_o - U_o = H_t + V_t - U_t$.

In order to cast eqs. (E.2) and (E.3) into a form that shows explicitly the coupling between $\langle o|\Psi\rangle$ and $\langle t|\Psi\rangle$, we need to know the relation between these amplitudes and the distorted waves $X_o(\underset{\sim}{r}_o)$ and $X_t(\underset{\sim}{r}_t)$ defined in eq. (E.1). Since $\langle o|\Psi\rangle$ and $\langle t|\Psi\rangle$ depend on $\underset{\sim}{r}_o$ and $\underset{\sim}{r}_t$, respectively, we shall for clarity denote them by $\chi_o(\underset{\sim}{r}_o)$ and $\chi_t(\underset{\sim}{r}_t)$. Using (E.1) we immediately obtain for $\chi_o(\underset{\sim}{r}_o)$ and $\chi_t(\underset{\sim}{r}_t)$ the following equations,

$$\chi_o(\underset{\sim}{r}_o) = X_o(\underset{\sim}{r}_o) + \int d\underset{\sim}{r}_t\, \mathcal{N}_{ot}(\underset{\sim}{r}_o, \underset{\sim}{r}_t)\, X_t(\underset{\sim}{r}_t)\,, \tag{E.4}$$

$$\chi_t(\underset{\sim}{r}_t) = X_t(\underset{\sim}{r}_t) + \int d\underset{\sim}{r}_o\, \mathcal{N}_{to}(\underset{\sim}{r}_t, \underset{\sim}{r}_o)\, X_o(\underset{\sim}{r}_o)\,, \tag{E.5}$$

where we have introduced the nonorthogonality overlap integrals given by

$$\mathcal{N}_{ot}(\underset{\sim}{r}_o, \underset{\sim}{r}_t) \equiv \langle o, \underset{\sim}{r}_o | t, \underset{\sim}{r}_t\rangle\,, \tag{E.6}$$

$$\mathcal{N}_{to}(\underset{\sim}{r}_t, \underset{\sim}{r}_o) \equiv \langle t, \underset{\sim}{r}_t | o, \underset{\sim}{r}_o\rangle\,. \tag{E.7}$$

It is understood that these overlap integrals vanish asymptotically, thus rendering χ and X equal in that region.

Equations (E.4) and (E.5) may be regarded as two coupled integral equations for $X_o(\underset{\sim}{r}_o)$ and $X_t(\underset{\sim}{r}_t)$. A formal solution of these equations can easily be obtained and is given by

$$X_o = (1-\mathcal{N}_{ot}\mathcal{N}_{to})^{-1}(\chi_o-\mathcal{N}_{ot}\chi_t), \tag{E.8}$$

$$X_t = (1-\mathcal{N}_{to}\mathcal{N}_{ot})^{-1}(\chi_t-\mathcal{N}_{to}\chi_o). \tag{E.9}$$

By means of (E.8) and (E.9) we can then write down the form of the coupled equations (E.2) and (E.3) after denoting the coupling potential V-U by $\overline{V}$,

$$(E_o-H_o)\chi_o = \overline{V}_{ot}(1-\mathcal{N}_{to}\mathcal{N}_{ot})^{-1}(\chi_t-\mathcal{N}_{to}\chi_o), \tag{E.10}$$

$$(E_t-H_t)\chi_t = \overline{V}_{to}(1-\mathcal{N}_{ot}\mathcal{N}_{to})^{-1}(\chi_o-\mathcal{N}_{ot}\chi_t). \tag{E.11}$$

Equations (E.10) and (E.11) are the correct coupled-channels equations appropriate for transfer coupling.

Since χ_t contains only outgoing waves, its formal structure is obtained directly from (E.11) as

$$\chi_t = \mathcal{G}_t^{(+)}(E_t)\overline{V}_{to}(1-\mathcal{N}_{ot}\mathcal{N}_{to})^{-1}\chi_o, \tag{E.12}$$

where $\mathcal{G}_t^{(+)}(E_t)$ is the transfer channel Green function formally given by

$$\mathcal{G}_t^{(+)}(E_t) = \left[E_t - H_t + \overline{V}_{to} \left(1 - \mathcal{N}_{ot}\mathcal{N}_{to}\right)^{-1} \mathcal{N}_{ot} \right]^{-1} . \tag{E.13}$$

The presence of the nonorthogonality term in $\mathcal{G}_t^{(+)}(E_t)$ implies a modification of the optical potential U_t that appears in $H_t = T_t + U_t$. Since in this paper we are concerned with elastic scattering, we derive the effective equation for $\chi_o(\underset{\sim}{r}_o)$ by means of (E.13) and obtain

$$\begin{aligned}(E_o - H_o)\chi_o &= \overline{V}_{ot} \left(1 - \mathcal{N}_{to}\mathcal{N}_{ot}\right)^{-1} \\ &\quad \times \left[\mathcal{G}_t^{(+)}(E_t)\, \overline{V}_{to} \left(1 - \mathcal{N}_{ot}\mathcal{N}_{to}\right)^{-1} - \mathcal{N}_{to} \right] \chi_o .\end{aligned} \tag{E.14}$$

Equation (E.14) is the principal result of this appendix. It shows how the elastic channel is modified due to its coupling to a transfer channel. By setting $\mathcal{N}_{to}$ equal to zero we obtain the result that was used in deriving the modified elastic amplitude in sect. 4. Thus eq. (E.14) should serve as a starting point for generating the corrections to our expression (4.6) due to the nonorthogonality of the channels.

Furthermore an exact, albeit formal, expression for the effective potential, $U_o^{(T)}$, in the elastic channel in the presence of coupling to a transfer channel, can immediately be extracted from (E.14) and is given by

$$U_o^{(T)} = \overline{V}_{ot} \left(1 - \mathcal{N}_{to}\mathcal{N}_{ot}\right)^{-1} \left[\mathcal{G}_t^{(+)}(E_t)\, \overline{V}_{to} \left(1 - \mathcal{N}_{ot}\mathcal{N}_{to}\right)^{-1} - \mathcal{N}_{to} \right] . \tag{E.15}$$

In the limit of small $\mathcal{N}_{to}$, eq. (E.15) reduces to

$$U_o^{(T)} \approx \overline{V}_{ot} \left(E_t - H_t + i\eta\right)^{-1} \overline{V}_{to} - \overline{V}_{to}\mathcal{N}_{to} . \tag{E.16}$$

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Figure captions

Fig. 1. Cross section ratio $\sigma(\theta)/\sigma_R(\theta)$ for the elastic scattering $^{20}Ne + {}^{152}Sm$ at $E_{lab} = 70$ MeV, calculated from the WKB expression (3.40) (solid curve) and the Born series summation (3.39) (dashed curve) with $t_C(\overline{\lambda}_\theta)$ given by eq. (3.43). Reorientation and projectile excitation are neglected.

Fig. 2. One-step elastic transfer.

Fig. 3. Two-step elastic transfer processes.

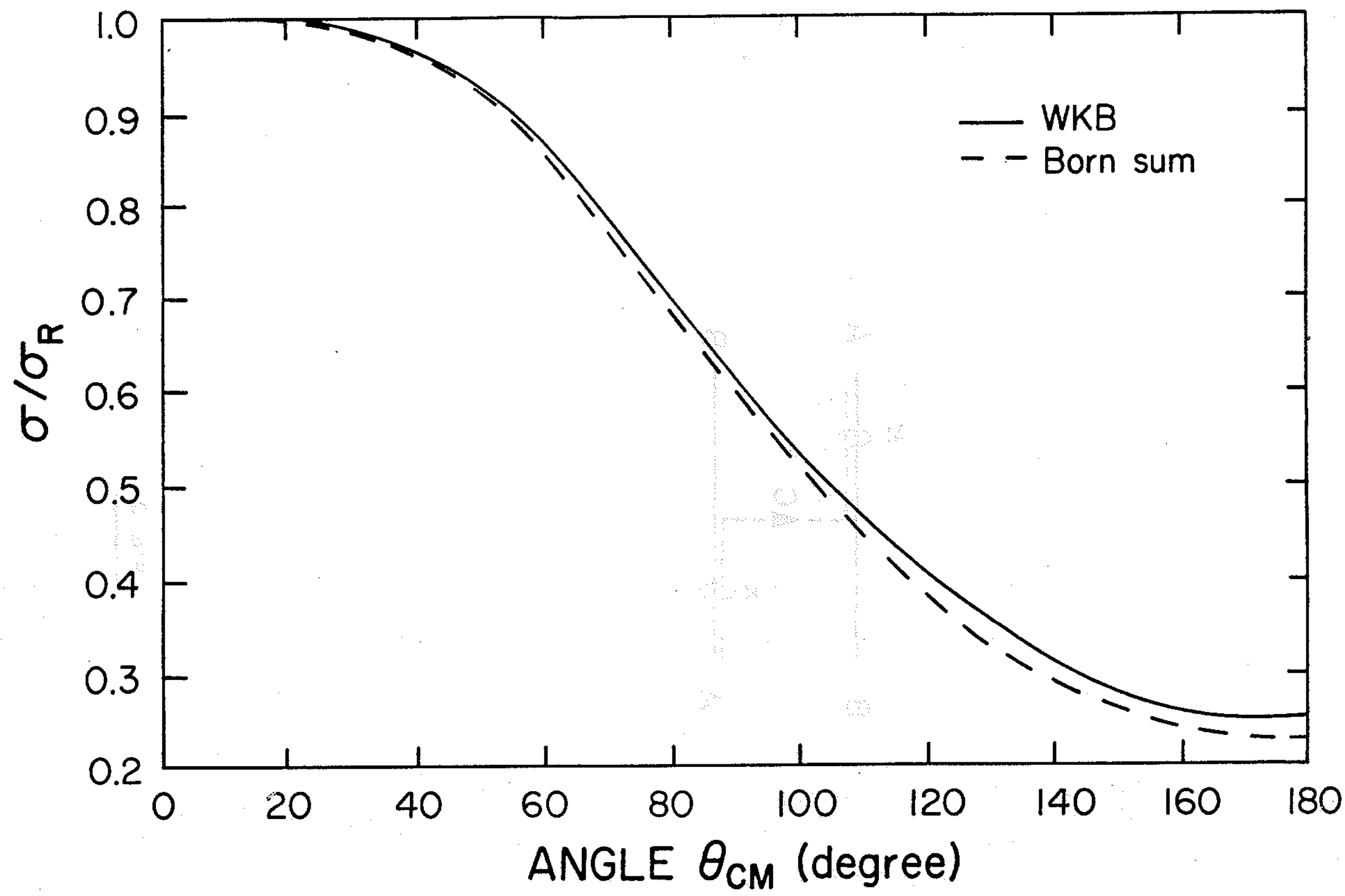
WKB
Born sum
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
σ/σR
0
20
40
60
80
100
120
140
160
180
ANGLE θCM (degree)

FIG. 1

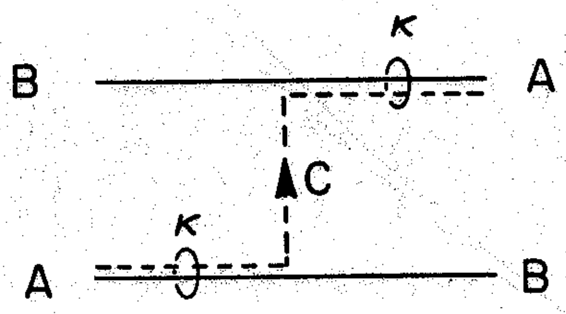

FIG. 2

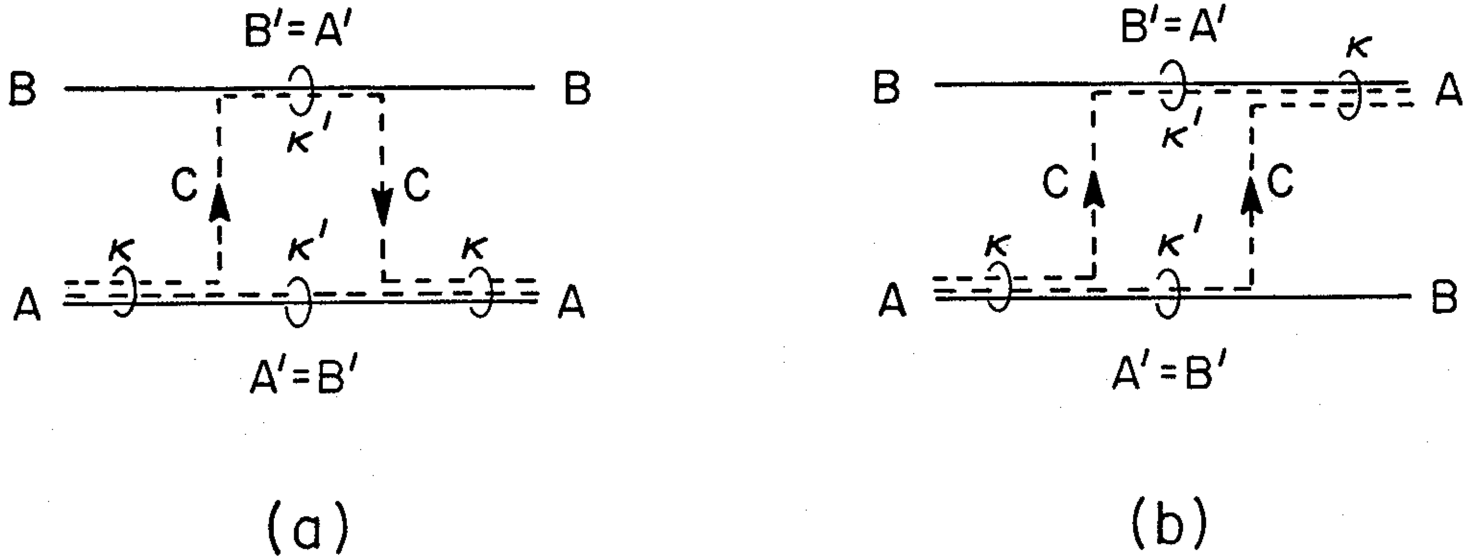

FIG. 3