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I. INTRODUCTION

The growing interest in the study of magnetic confinement of plasmas in Tokamaks is mainly due to the fact that the Tokamak design appears to be the most promising for future fusion reactors [1]. In order to reach a detailed understanding of the physical processes which occur in Tokamaks, a large effort has been devoted to the development of theoretical descriptions and the collection of experimental data. Numerical techniques have been used to model the behaviour of the plasma, reproducing experimental results and making predictions for new situations. Computer codes which treat the problems of collisional transport, diffusion, turbulence, particle injection, impurities, etc. have been developed [2].

Research directly related to controlled thermonuclear fusion is carried out in large Tokamaks, where plasmas of high density and temperature are produced and confined [1]. Nevertheless there exist a large number of smaller devices, which are simpler to construct and less expensive, where physical processes in the plasma and diagnostic methods are studied in detail [3]. To investigate the physical phenomena which occur in these smaller devices, it is necessary to develop a theoretical model which takes into account the most important transport processes. A lower limit for the energy and pa ticle fluxes lost from the Tokamak can be determined from collisional transport theory. In practice higher fluxes arise due to anomalous transport associated with plasma microinstabilities [4]. In Tokamaks, the appropriate collisional transport model which accounts for the geometry of the confining magnetic field is called Neoclassical theory [5]. Although this theory does not include anomalous transport, it serves as a valid starting point for comparison with experimental data.

In the present work, a simple transport model for a small ohmically-heated Tokamak is presented. More elaborate computer programs than the one described here have been written to model Tokamak transport [2]. However these programs generally require a computer with large memory capacity and need a substantial time for computation. The present numerical simulation is suitable for use with a small computer and requires very little computing time. Thus it is useful for immediate comparison with experimental, or, in the design phase, for investigating the effect of varying the external parameters on the plasma temperature, lifetime, etc..

In a Tokamak, the plasma is confined by a helical magnetic field with a toroidal component, B_ϕ , generated by external coils, and a poloidal component, B_p , generated by the current, I_p , which flows in the toroidal direction in the plasma [6]. The plasma current, which is responsible for the ohmic heating of the plasma, is induced by a changing magnetic flux in a coil situated at the centre of the Tokamak. This coil is the primary winding of the so-called ohmic heating transformer, with the toroidal plasma column forming the secondary winding. The value of the plasma current, I_p , depends on the plasma resistivity, which is proportional to $T_e^{-3/2}$ [7], where T_e is the electron temperature. Since T_e itself depends on I_p through the ohmic heating, the coupled external circuit-plasma system is non-linear. The numerical model presented in this work allows a rapid solution to this problem, furnishing values of the plasma current, I_p , the mean electron temperature, $\langle T_e \rangle$, and the mean ion temperature, $\langle T_i \rangle$, as functions of time.

In the following section the transport equations are presented; in Section III, the ohmic-heating circuit is described; in Section IV, the numerical technique employed is discussed; in Section V, some applications of the model are presented; and Section VI contains concluding remarks.

II. TRANSPORT MODEL

The transport model used in this work is based on Neoclassical theory. The model is described by the equations of continuity, energy balance for ions and electrons, and Maxwell's equations which become* [8].

$$\frac{\partial n}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\, D\, \frac{\partial n}{\partial r}\right)$$

$$\frac{3}{2}\frac{\partial}{\partial t}(nT_e) = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(K_e \frac{\partial T_e}{\partial r} + \frac{3}{2} D\, T_e \frac{\partial n}{\partial r}\right)\right] - \frac{3m_e}{m_i}\frac{n}{\tau_e}(T_e - T_i) +$$

$$+\, \eta \frac{c^2}{(4\pi)^2}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\, B_p)\right]^2 - P_{br}$$

$$\frac{3}{2}\frac{\partial}{\partial t}(nT_i) = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(K_i \frac{\partial T_i}{\partial r} + \frac{3}{2} D\, T_i \frac{\partial n}{\partial r}\right)\right] + \frac{3m_e}{m_i}\frac{n}{\tau_e}(T_e - T_i)$$

$$\frac{\partial B_p}{\partial t} = \frac{c^2}{4\pi}\frac{\partial}{\partial r}\left[\eta \frac{1}{r}\frac{\partial}{\partial r}(r\, B_p)\right] \qquad (1)$$

Where n is the electron number density (equal to the ion density); T_e and T_i are the electron and ion temperatures (expressed in energy units) respectively; B_p is the poloidal magnetic field; P_{br} is the power loss associated with Bremsstrahlung; m_e and m_i are the electronic and ionic mass respectively; c is the speed of light; τ_e is the electron collision time [7]; K_e

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* Equations expressed in c.g.s. units.

and K_i are the electron and ion thermal conductivities; and η is the plasma resistivity.

According to Neoclassical theory, three regimes of Tokamak transport can be distinguished: the classical, or Pfirsch-Schlüter, regime, the Plateau regime and the Banana regime [9]. The thermal conductivities in each regime are given by Düchs et al [10], and the plasma resistivity, η, is determined from Hirschmann's approximate formula [11], which is valid in all three transport regimes. The system of equations (1) is non-linear, since the coefficients depend on temperature, current density, etc. [10,11].

III. OHMIC HEATING CIRCUIT

In most Tokamaks, the plasma current is generated by means of a transformer (the ohmic heating transformer), the plasma ring forming the secondary winding. In the case of the TBR Tokamak [15], the primary current in the air-cored ohmic heating transformer is furnished by two capacitor banks which are discharged using Ignitron valves (fig. 1). The first bank which is discharged (capacitance C_1) forms the plasma and initializes the plasma current. The second bank (capacitance C_2), which is discharged when the plasma current generated by C_1 has reached a maximum, maintains the plasma current and heats the plasma. The initial voltage on the first bank, V_1, is chosen such that the resultant toroidal electric field in the Tokamak exceeds the minimum value necessary to produce breakdown [12,15]. There is also an effective upper limit on the bank voltages which is determined by the condition that, for a given toroidal field B_ϕ, I_p cannot exceed the Kruskal-Shafranov stability limit [13]. The capacitance C_2 determines the duration of the plasma current pulse; its value was fixed by economic considerations [15]. To avoid a reversed voltage on the electrolytic capacitors of the second bank, the circuit is crowbarred when the transformer voltage goes to zero (fig. 1).

The equations describing the electrical circuit are:

$$L_t \frac{dI_t}{dt} - M \frac{dI_p}{dt} + R_t \; I_t(t) + \frac{1}{C} \int I_t \, dt = 0$$

$$L_p \frac{dI_p}{dt} - M \frac{dI_t}{dt} + R_p(t) \; I_p(t) = 0 \qquad (2)$$

where $I_t(t)$ and $I_p(t)$ are the currents in the ohmic heating transformer primary and in the plasma respectively; L_t and L_p are the inductances of the primary and plasma; R_t and $R_p(t)$ are the resistances of the primary and plasma; and M is the mutual inductance between the primary and the plasma ring. Equations (2) are first solved with only the bank C , in circuit; when V_1 equals V_2 (the voltage on the second bank), the second bank is switched into the circuit and $C=C_1+C_2$, the total capacitance of the two banks. When the circuit is crowbarred, the last term on the left side of the first of equations (2) disappears.

IV. NUMERICAL MODEL

To solve the coupled non-linear set of equations (1) and (2), we have simplified the problem considerably by taking fixed spatial profiles (chosen to the parabolic) for the relevant plasma parameters. This simplification eliminates the spatial gradients in equations (1), leaving only the scale parameter a, the radius at which the electron and ion temperatures go to zero. The resultant set of first order equations in t was solved by a simple finite-difference method. Equations (2), which determine the plasma current, I_p, were solved at each step by the Laplace Transform method (for R_p fixed). The rule used to select the appropriate transport regime at a given time is described by Düchs et al [10].

V. APPLICATIONS

Three applications made of the model are described below.

a) Design parameters for the TBR Tokamak

TBR (Tokamak Brasileiro) is a small, ohmically-heated Tokamak with Major Radius 30 cm and Minor Radius 11 cm, which was constructed at the Instituto de Física, Universidade de São Paulo. The toroidal field of TBR is 5kG; the maximum plasma current is 20kA, generated by an ohmic heating transformer of resistance R_t=66mΩ and inductance L_t=1.6mH; the mutual inductance between the primary winding and the plasma is 13μH, and L_p, the plasma self-inductance, is 0.63μH. More details of the project are given by Simpson et al [15].

The present model was used to simulate discharges in TBR. The design values of the voltages and capacitances of the fast and slow ohmic heating banks were adjusted to produce the desired temporal profiles of current and temperature in the simulated discharge.

Figure (2-A) shows the time evolution of current and plasma loop voltage for $V_1=7000V$, $C_1=30\mu F$, $V_2=600V$ and $C_2=15mF$. For these values, it can be seen that I_p reaches a maximum of about 20kA and lasts for about 4.5ms. If C_2 is doubled the plasma current lasts for 6.5ms. These calculations were carried out for a density $n=2\times 10^{13}\ cm^{-3}$ and an effective charge $Z=2.5$. Higher impurity levels increase Z, leading to lower values of the plasma current and temperatures.

Figure (2-B) shows the time dependence of mean electron and ion temperatures for the same conditions as figure (2-A). The mean electron temperature reaches a maximum of 230 eV and the mean ion temperature, 23 eV.

b) Energy confinement time

The total thermal energy of the electrons in a Tokamak with Minor Radius a and Major Radius R is given approximately by

$$U_e(t) = 2\pi R \int^a \frac{3}{2}\, n\, T_e(r,t)\, 2\pi r dr.$$

Taking inductively-stored energy into account, the ohmic heating power input to the plasma, $S(t)$, is given by

$$S(t) = \left[V_p - L_p \frac{dI_p}{dt}\right] I_p \quad . \qquad (3)$$

τ_E, the electron energy confinement time, is then defined by the energy balance,

$$\frac{dU_e}{dt} = - \frac{U_e}{\tau_E} + S \quad . \qquad (4)$$

In quasi-steady state, equations (3) and (4) give

$$\tau_E = \frac{U_e}{V_p I_p} . \qquad (5)$$

This is a commonly-employed approximation to τ_E .

In figure (3), $\tau_E(t)$ calculated numerically from expressions (3) and (4) is compared with the approximate value of equation (5). It can be seen that the approximate formula only yields useful values in a small time interval close to steady state.

c) The Fisher and Bekefi method for determining the Energy Confinement Time in a Tokamak

Using the present model, it was possible to make a numerical simulation [14] of a Tokamak discharge to which a small perturbation was applied in the steady state region. This perturbation produced an increase in the electron temperature. The temperatures in the perturbed and non-perturbed discharges were compared using the method proposed by Fisher and Bekefi [16] to estimate the electron energy confinement time. Simpson, Drozak and Galvão [17] discuss the precision of the method, the conclusion being that its accuracy depends strongly on the experimental conditions.

VI. CONCLUSIONS

This article presented a simple model to describe energy transport in a small ohmically-heated Tokamak, based on Neoclassical theory. The model offers the advantage that values of the plasma current, loop voltage, and electron and ion temperatures as functions of time are furnished rapidly without a large amount of computation. It may also be considered as a starting point for the solution of more general problems as for example the calculation of spatial profiles for current temperature and density as well as the inclusion of the effect of the neutral particle in the processes of transport.

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FIGURE CAPTIONS

Figure (1) - Ohmic heating circuit.

Figure (2-A) - The time variation of the plasma current, I_p, and loop voltage, V_p, for V_1=7000V, C_1=30μF, V_2=600V, C_2=15mF, $n=2\times 10^{13}$ cm^{-3} and $Z=2.5$.

Figure (2-B) - The time variation of the average electron temperature, T_e, and the average ion temperature, T_i, for the same conditions as figure (2-A).

Figure (3) - Comparison of the exact value of the electron confinement time, τ_E, as calculated from equation (4) with the approximate value, τ_E', given by relation (5).

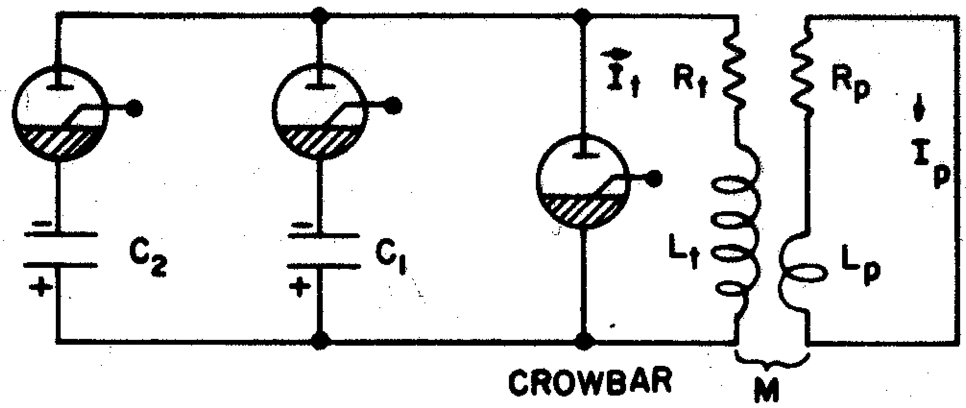

Figure 1

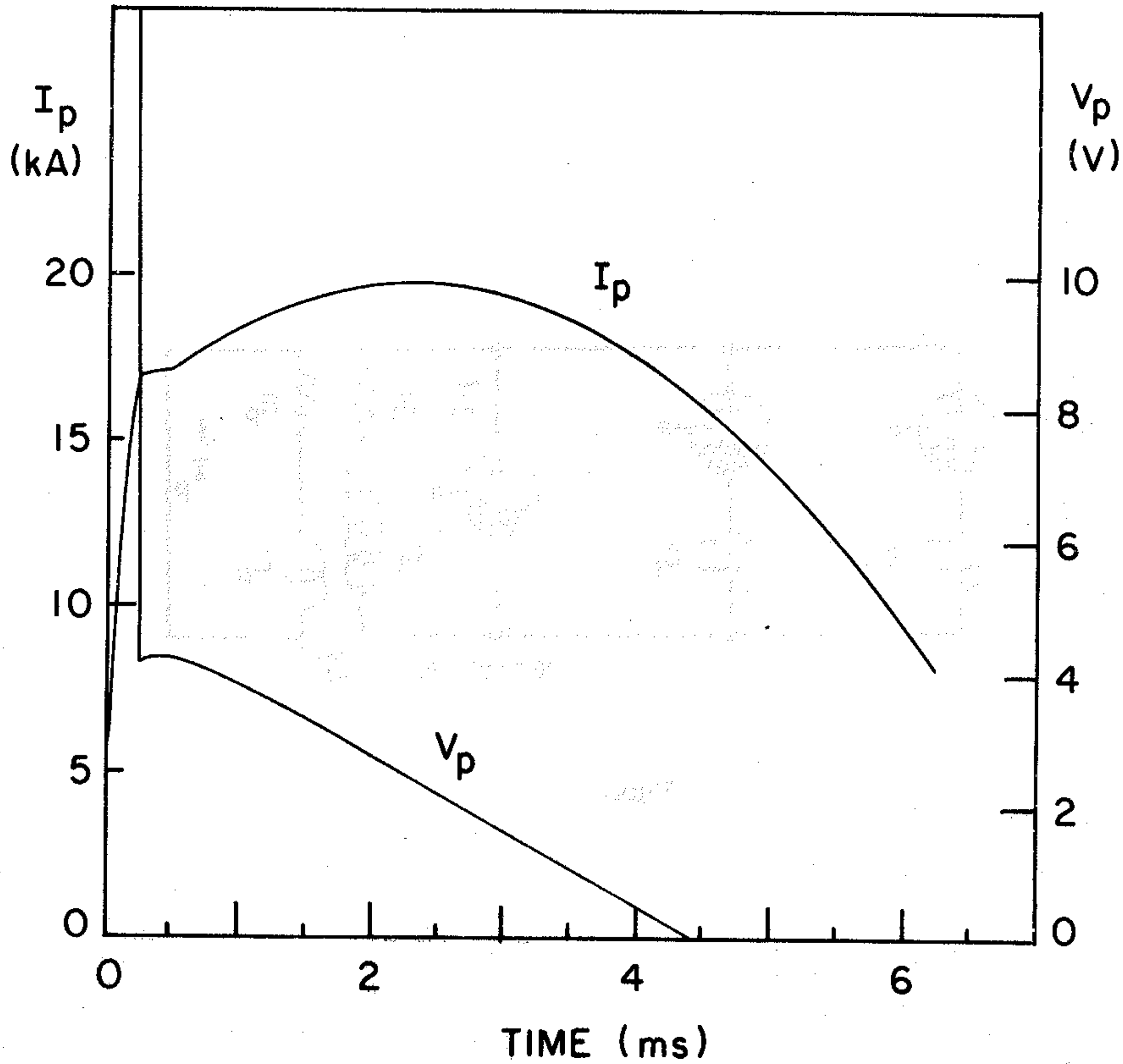

Figure (2-A)

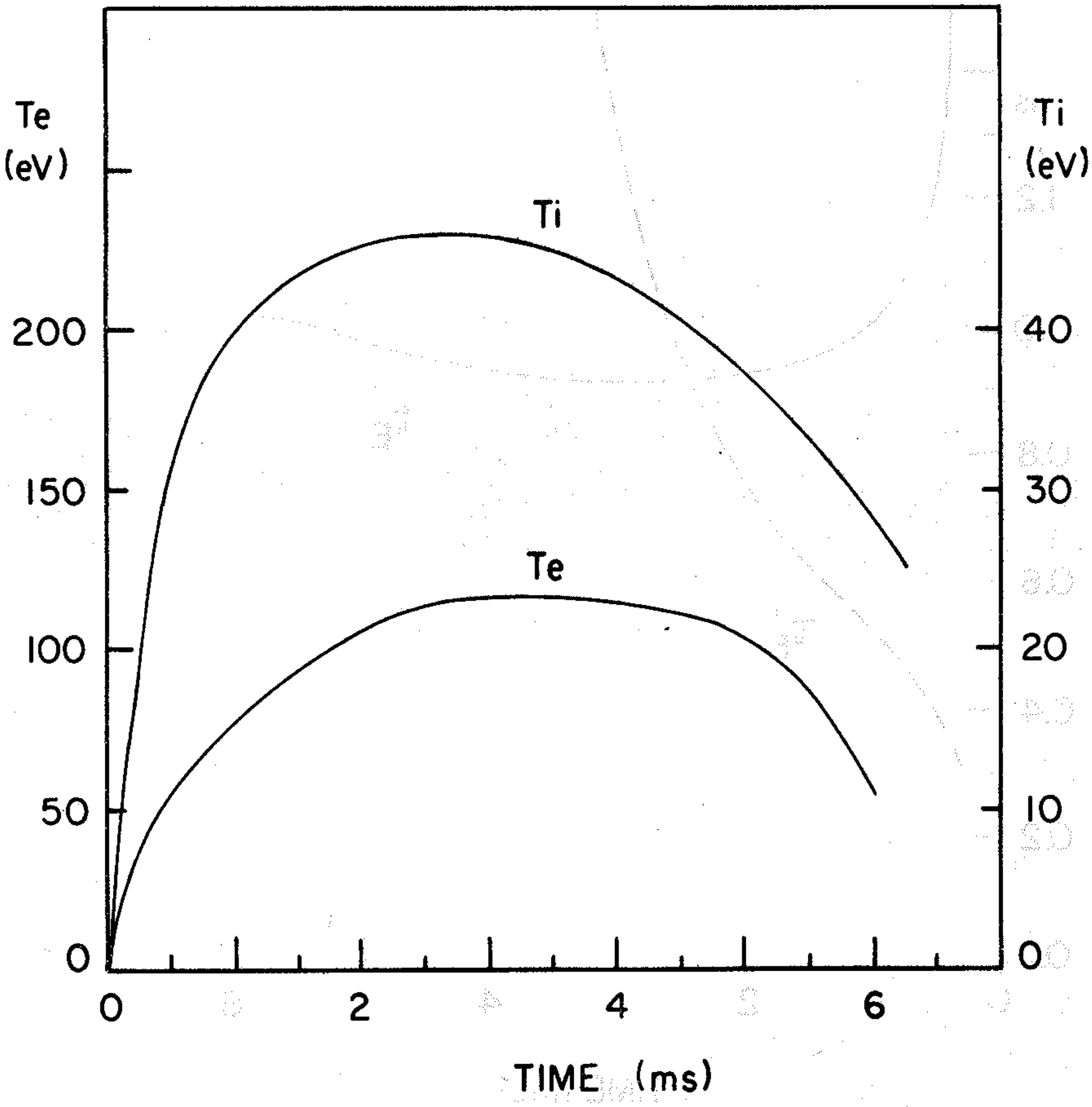

Figure (2-B)

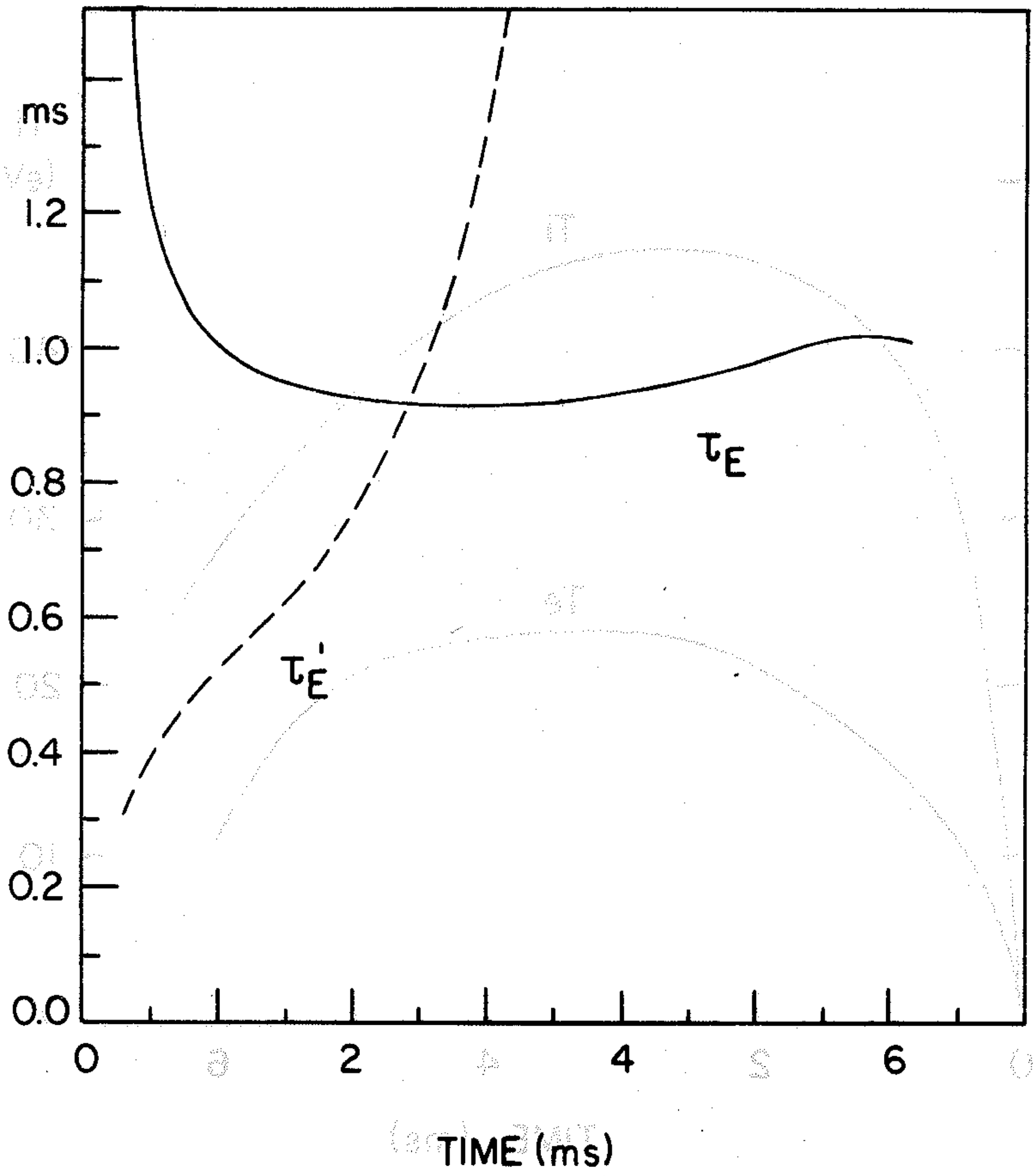

Figure 3