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by

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ABSTRACT

In a perturbative calculation we show that no new quadrilinear counterterms are necessary to define the Federbush Model if one imposes gauge symmetry of first kind and asymptotic γ^5 invariance. A subtraction scheme satisfying these conditions is constructed and renormalization group properties of the Green functions are analysed.

1) INTRODUCTION

Historically, the Federbush Model was proposed in 1961 as a prototype of a solvable field theoretical model involving massive spinors[1]. It is a two dimensional model with a Lagrangian density given by

$$L = \sum_{j=1}^{2} \left(\frac{1}{2} i \, \bar{\psi}_j \gamma^\mu \overleftrightarrow{\partial}_\mu \psi_j - M_j \bar{\psi}_j \psi_j \right) + g\varepsilon_{\mu\nu} (\bar{\psi}_1 \gamma^\mu \psi_1)(\bar{\psi}_2 \gamma^\nu \psi_2) \qquad (1.1)$$

where the indices j=1 or 2 denote the fermion field type.

The original formulation was made more rigorous after the work of A.S.Wightman in 1963[2]. More recently there has been some controversy about the perturbative caracterization of the model[3,4,5]. The basic question is that, as graphs with four external fermion lines are superficially logarithmically divergent, counterterms having forms different of those already present in (1.1) (e.g. $(\bar{\psi}_1\psi_1)(\bar{\psi}_2\psi_2)$) could be necessary in order to define finite Green functions. However if, due to some asymptotic symmetry, there is a cancellation of the divergences of the various graphs involved then counterterms are not necessary. The existence of such cancellations, up to second order in g, has been shown in references 4 and 5.

In this communication we will show that in any order of perturbation the model (1.1) can be unambiguously caracterized by just imposing asymptotic γ^5 invariance. If this is done the only quadrilinear counterterms necessary are those needed to make both vector currents conserved.

Furthermore, we will construct a modified BPHZ subtraction scheme preserving such symmetry and show that the resulting Green's functions obey a renormalization group equation of the usual form. The paper is organized as follows:

In Section 2 we prove that no new quadrilinear counterterms are necessary if the vector currents are conserved and asymptotic γ^5 invariance is imposed. In Section 3 we construct renormalized Green's functions and derive Ward identities for the axial currents explicitly satisfying the requirements of Section 2. Finally, in Section 4, we discuss renormalization group aspects related to the approach **used.**

2) ASYMPTOTIC γ^5 INVARIANCE AND CANCELLATION OF DIVERGENCES

In this section we will prove the main result mentioned in the introduction: suppose that the basic interaction in (1.1) can be writen as $g\varepsilon_{\mu\nu}\, j_1^\mu\, j_2^\nu$ where j_1^μ and j_2^μ are renormalized versions of the field currents $\bar{\psi}_1\gamma^\mu\psi_1$ and $\bar{\psi}_2\gamma^\mu\psi_2$, having the properties

1. both j_1^μ and j_2^μ are conserved
2. their curls asymptotically vanishe, i.e.

$$\tilde{\partial}^\mu j_1^\mu \quad \text{and} \quad \tilde{\partial}^\mu j_2^\mu \;\to 0 \quad \text{as } M_1 \text{ and } M_2 \to 0$$

then no new quadrilinear counterterms are necessary.

To proof this result we introduce the graphical notation of fig.1. A generic graph contributing to the proper four point function has the structure shown in fig.2. Let q be the momentum through the indicated wavy line. Now we use the identity

$$\varepsilon^{\mu\nu} = (\tilde{q}^\mu q^\nu - q^\mu \tilde{q}^\nu)/q^2 \tag{2.1}$$

and transfer the q^μ (resp. $\tilde{q}^\mu$) factor from the line to the vertex V and apply current conservation (resp. asymptotic γ^5 invariance).

We will obtain graphs that have the same structure as before but with V replaced by a soft vertex (in the case of the rotational of the vector current) besides the usual contributions (fig.3) that come from the fact that we are considering proper time ordered functions. In the vertex V of fig.3 there is no more the momentum factor. It is therefore clear that the result is ultraviolet finite if the necessary subtractions for the two point functions have been done.

The above discussion shows how one should do in order to construct a graph by graph subtraction scheme without inducing quadrilinear counterterms: The subtractions for proper graphs with four external fermion lines must be done at the value zero of the masses M_1 and M_2. Possible infrared divergences of the subtraction terms can be eliminated if one replaces $\varepsilon^{\mu\nu}$ by $\varepsilon^{\mu\nu} \dfrac{q^2}{q^2-\mu^2}$. In the next section this procedure will be used to explicitly construct the desired subtraction scheme.

3) GREEN FUNCTIONS AND THE SUBTRACTION SCHEME

The observations made at the end of the previous section lead us to define the following effective Lagrangian density for the Federbush model

$$L = N_2 \{ \sum_{j=1}^{2} \left[\frac{i}{2} \bar{\psi}_j \gamma^\mu \partial_\mu \psi_j - M_j \bar{\psi}_j \psi_j \right] \} + g\varepsilon_{\mu\nu} N_2\{(\bar{\psi}_1 \gamma^\mu \psi_1)(\bar{\psi}_2 \gamma^\nu \psi_2)\} +$$

$$+ a_1 N_1(\bar{\psi}_1 \psi_1) + a_2 N_1(\bar{\psi}_2 \psi_2) = L_0 + L_{int},$$

$$L_{int} = g\varepsilon_{\mu\nu} N_2\{(\bar{\psi}_1 \gamma^\mu \psi_1)(\bar{\psi}_2 \gamma^\nu \psi_2)\} + a_1 N_1(\bar{\psi}_1 \psi_1) + a_2 N_1(\bar{\psi}_2 \psi_2) \qquad (3.1)$$

The finite counterterms a_1 and a_2 were added in order to fix the physical mass of the fields ψ_1 and ψ_2 at the values M_1 and M_2, respectively.

We adopt the graphical notation of fig. 1. By power counting the degree of superficial divergence of a proper graph is given by

$$d(\gamma) = 2 - \frac{N_\gamma}{2} - A_\gamma - B_\gamma \tag{3.2}$$

where N_γ = nº of external fermion lines of γ

B_γ = nº of external wavy lines of γ

A_γ = nº of vertices of type a_1 and a_2 in γ

Without the insertion of the counterterms (i.e. $A_\gamma=0$) the graphs to be subtracted are then

1. Fermion self-energy graphs, $\delta(\gamma)=1$
2. Vertex functions of the currents with two external fermion lines (see fig.4), $\delta(\gamma)=0$
3. Proper functions of two currents (fig.5), $\delta(\gamma)=0$
4. Four point proper functions, $\delta(\gamma)=0$

Let I_G be the unsubtracted Feynman integrand associated with the graph G. In order to construct the subtracted Feynman integrand we first substitute I_G by $\bar{I}_G$ which is obtained from I_G by replacing the $\varepsilon^{\mu\nu}$ factor associated with a wavy line in which flows the momentum q by

$$\varepsilon^{\mu\nu} \frac{q^2}{q^2 - s^2 + i\varepsilon\ (\vec{q}^{\,2}+s^2)} \tag{3.3}$$

The renormalized integrand associated with I_G is then given by the forest formula

$$R_G = S_G \sum_{U\in F_G} \prod_{\gamma} (-\tau^{d(\gamma)} S(\gamma))\, \bar{I}_G(U) \tag{3.4}$$

where $\tau^{d(\gamma)}$ is a generalized Taylor operator defined as follows:

a) $\tau^{d(\gamma)}$ is the Taylor operator of order $d(\gamma)=0$ in the external momenta p^{γ} of γ and in M^{γ} at $p^{\gamma}=0$ and $M^{\gamma}=\bar{\mu}$ if γ is the graph of fig.6.

b) For the remaining graphs $\tau^{d(\gamma)}$ is a Taylor operator of order $d(\gamma)$ in p^{γ}, M_1^{γ}, M_2^{γ} at $p^{\gamma}=0$, $M_1^{\gamma}=M_2^{\gamma}=0$ and, besides, in the last subtraction of $\tau^{d(\gamma)}$ s^{γ} is replaced by μ.

S_{γ} is a substitution operator writing the variables of $\lambda \subset \gamma$ in terms of those of γ; S_G does the additional job of setting $s=0$ and of replacing $\bar{\mu}$ either by M_1 or M_2 accordingly the type of current in fig. 6.

Notice that at $s=0$ the "propagator" of the wavy line becomes $q^2/q^2+i\varepsilon\vec{q}^{\,2}$ so that in the limit $\varepsilon\to 0$ $\bar{I}_G$ tends formally to I_G .

We remark that the proposed subtraction scheme does not create infrared divergences. Some of the subtractions are done at $s^{\gamma}=0$ and $M^{\gamma}=0$ but the last subtractions, which if done at $s^{\gamma}=0$ and $M^{\gamma}=0$ would lead to infrared divergences, are actually done at $s^{\gamma}=\mu$.

The scheme so constructed is a simple modification of those employed in ref.6 and both ultraviolet and infrared finiteness can be proven using similar arguments of those references.

In the same way as we did in section 2 we can prove that the subtractions for the proper four point functions are actually redundant. To verify this we write the contribution of order n as

$$R_n^{(4)} = \sum_i (1-\tau^{o}_{G_i})\bar{R}_{G_i} \qquad (3.5)$$

where, $\bar{R}_{G_i}$ is defined by the forest formula but without the subtraction associated with the graph G_i. Note that G_i has the structure shown in fig. 7. As before, we apply the identity

$$\frac{q^\mu \tilde{q}^\nu - \tilde{q}^\mu q^\nu}{q^2 + i\varepsilon \vec{q}^2} = \varepsilon^{\mu\nu} + O(\varepsilon) \qquad (3.6)$$

to the line indicated linking V_1 to V_2. We obtain a sum of graphs where $\not{q}$ in V_2 is substituted by $2M\gamma^5$ or appears, in a factorized form, the contribution of a graph which vanishes at $P_{G_i} = M_{G_i} = 0$. In both cases the application of the last subtraction operator gives zero.

Another interesting property concerns to graphs that have two closed fermion loops linked by at least one wavy line. In this case the graph will not contribute. This can be seen by the same reasoning as above. We simply use (3.6) to one of the lines linking the two closed loops and apply current conservation.

Green's functions containing normal products $N_{\delta_i}(\theta_i)$, $\delta_i \leq 2$, where δ_i = (operator dimension of θ + number of mass parameters in θ_i), are defined by (3.4) but using

$$d(\gamma) = 2 - \frac{N_\gamma}{2} - A\gamma - B\gamma - \sum_{V_i \in \gamma} (2-\delta_i) \qquad (3.7)$$

The axial currents, $N_1[\bar{\psi}_i \gamma^\mu \gamma^5 \psi_i]$ satisfy Ward identities which can be derived in the usual way. With the notation

$$X = \prod_{i=1}^{N_1} \psi_1(x_i)\bar{\psi}(y_i) \prod_{j=1}^{N_2} \psi_2(z_j)\bar{\psi}_2(w_j)$$

we have, for example

$$\partial_x^\mu \langle TN_1[\bar{\psi}_1\gamma^\mu\gamma^5\psi_1](x)X\rangle = i\langle T\{N_2[\bar{\psi}_1 \; (-i\overleftarrow{\not{\partial}}-M_1)\psi_1](x) +$$

$$+ N_2[\psi_1\gamma^5(i\not{\partial}-M_1)\psi_1](x) + 2\, N_2[M\, \bar{\psi}_1\gamma^5\psi_1](x)\}X\rangle =$$

$$= 2i \langle TN_2[M_1\bar{\psi}_1\gamma^5\psi_1](x)X\rangle - 2ia_1\langle TN_1(\bar{\psi}_1\gamma^5\psi_1)(x)X\rangle -$$

$$- \sum_{i=1}^{N_1} \{ (\delta(x-x_i)\gamma^5_{x_i} + \delta(x-y_i)\gamma^{5T}_{y_i} \} \langle TX\rangle \qquad (3.8)$$

Observing that the only difference between $N_2[M_1\bar{\psi}_1\gamma^5\psi_1]$ and $M_1N_1[\bar{\psi}_1\gamma^5\psi_1]$ comes from the subtraction for the graph of fig.8, we can write

$$2i\, N_2[M_1\bar{\psi}_1\gamma^5\psi_1] = 2iM_1N_1[\bar{\psi}_1\gamma^5\psi_1] + a\, \partial^\mu N_1(\bar{\psi}_2\gamma^\mu\psi_2)$$
$$a = -g/\pi \qquad (3.9)$$

Now, by using

$$\partial^\mu \langle TN_1[\bar{\psi}_2\gamma_\mu\psi_2](x)X\rangle = \sum_{j=1}^{N_2} \{(\delta(x-w_j) - \delta(x-z_j)\} \langle TX\rangle \qquad (3.10)$$

and (3.9) in (3.8) we obtain

$$\partial^\mu_x \langle TN_1[\bar{\psi}_1\gamma_\mu\gamma^5\psi_1](x)X\rangle = 2i(M_1-a_1)\langle TN_1[\bar{\psi}_1\gamma^5\psi_1](x)X\rangle +$$

$$+ a \sum_{j=1}^{N_2} \{(\delta(x-w_j) - \delta(x-z_j)\} \langle TX\rangle -$$

$$- \sum_{i=1}^{N_1} \{(\delta(x-x_i)\gamma^5_{x_i} + \delta(x-y_i)\gamma^{5T}_{y_i}\} \langle TX\rangle \qquad (3.11)$$

Note the mild form (proportional to the divergence of the vector current of the ψ_2 field) of the anomalous term. If, instead of the proposed scheme, we had employed the usual BPHZ procedure[7], we would get a hard breaking of the axial currents conservation.

4) THE RENORMALIZATION GROUP EQUATION

The Green's functions defined in the previous section satisfy a renormalization group equation which can be derived by using the following differential vertex operations (D.V.O.s)[8]:

$$\Delta_{1j} = i \int d^2xN_1 [\bar{\psi}_j\psi_j] \quad , \quad \bar{\Delta}_{1j} = i\int d^2xN_2 [M_j\bar{\psi}_j\psi_j](x)$$

$$\Delta_{2j} = -\frac{1}{2}\int d^2xN_2 [\bar{\psi}_j \not\partial \psi_j](x) \quad , \quad j = 1, 2 \tag{4.1}$$

$$\Delta_3 = i\int d^2xN_2 [\varepsilon_{\mu\nu} (\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_2\gamma^\nu\psi_2)](x)$$

The operators Δ_{1i} and $\bar{\Delta}_{1i}$ are not independent. Their only difference is due to the subtractions for the subgraphs of fig. 9. However a straightforward calculation shows that the sum of these subtractions is actually zero, and we have:

$$M_i \ \Delta_{1i} \ \Gamma^{(N_1,N_2)} = \bar{\Delta}_{1i} \ \Gamma^{(N_1,N_2)} \tag{4.2}$$

Using (4.1) and (4.2) we can derive, in the usual way, the relations

$$N_i \ \Gamma^{(N_1,N_2)} = (\Delta_{2i} - (M_1-a_i) \ \Delta_{1i} + g \ \Delta_3) \ \Gamma^{(N_1,N_2)} \tag{4.3}$$

$$\frac{\partial}{\partial g} \Gamma^{(N_1,N_2)} = (\frac{\partial a_1}{\partial g} \Delta_{11} + \frac{\partial a_2}{\partial g} \Delta_{12} + \Delta_3)\Gamma^{(N_1,N_2)} \tag{4.4}$$

Besides (4.3) and (4.4) we also have

$$\frac{\partial \Gamma^{(N_1,N_2)}}{\partial \mu} = \sum_{i=1}^{2} (\lambda_i \Delta_{1i} + \delta_i \Delta_{2i}) \Gamma^{(N_1,N_2)} + \gamma \Delta_3 \Gamma^{(N_1,N_2)} \tag{4.5}$$

where λ_i, δ_i and γ are power series in the coupling constant g. Equation (4.5) is obtained by noting that $\frac{\partial}{\partial\mu}\Gamma^{(N_1,N_2)}$ receives contributions only of the subtraction terms and that :

i) Renormalization parts corresponding to contributions from the proper function of two currents do not contribute. This follows because the graph of fig. 6 does not depend on μ and graphs of this type of higher order will have closed fermion loops linked by at least one wavy line and, as argued before, they identically vanish.

ii) The subtractions for proper graphs with four external fermion lines cancel among themselves.

Thus the only contributions for $\frac{\partial\Gamma}{\partial\mu}$ come from subtractions for the vertex and fermion's self energy graphs. Concerning the self energy graphs observe that they do not give contributions to DVO's of the type $\int d^2x N_1(\bar\psi\gamma^5\psi)$ or $\int d^2x N_2[\bar\psi\gamma^\mu\gamma^5\partial^\mu\psi](x)$. To see this consider, for example, possible contributions to $\int d^2x N_1[\bar\psi\gamma^5\psi](x)$ coming from graphs of the type shown in fig.(10) (in this case the diagram must be of order odd in g). These arise from the subtraction term $M\frac{\partial}{\partial\mu}\frac{\partial}{\partial M} I_G\Big|_{\substack{p=M=0\\ s^2=\mu^2}}$. Now if $\frac{\partial}{\partial M}$ acts on the propagators outside the fermion loop we obtain zero (as result of vector and axial vector current conservation applied to any of the vertices in the loop). On the other hand if $\frac{\partial}{\partial M}$ is applied in the lines of the fermion loop the result is also zero since now the loop will have an odd number of gamma matrices and the trace gives zero (graphs of this type with insertion of mass counterterms will not contribute by the same argument).

That $\int N[\bar{\psi}\gamma^{\mu}\gamma^{5}\partial_{\mu}\psi]d^2x$ do not give contribution to $\frac{\partial\Gamma}{\partial\mu}$ can be seen most easily by choosing the routing of the external momentum so that it does not flow throughout the lines of the fermion loop and applying the same reasoning as before.

Using (4.3), (4.4) and (4.5) we can write the renormalization group equation:

$$\left[\mu \frac{\partial}{\partial\mu} + \sigma \frac{\partial}{\partial g} - N_1\tau_1 - N_2\tau_2\right] \Gamma^{(N_1,N_2)} = 0 \tag{4.6}$$

The proof of (4.6) is standard[8]. We follow the usual argument: by substituting (4.3), (4.4) and (4.5) in (4.6) and equating to zero the coefficient of each DVO we get

$$\mu\lambda_1 + \sigma \frac{\partial a_1}{\partial g} + (M_1-a_1)\,\tau_1 = 0$$

$$\mu\lambda_2 + \sigma \frac{\partial a_2}{\partial g} + (M_2-a_2)\tau_2 = 0$$

$$\mu\delta_1 - \tau_1 = 0$$

$$\mu\delta_2 - \tau_2 = 0$$

$$\mu\gamma + \sigma + g\,(\tau_1 + \tau_2) = 0 \tag{4.7}$$

The last three equations can be used to determine τ_1, τ_2 and σ. To show that the first two equations are then identically satisfied we use the fact that $\Gamma^{(2,0)}\Big|_{\not p=M_1} = 0$ and $\Gamma^{(0,2)}\Big|_{\not p=M_2} = 0$

so that

$$\left[\mu\frac{\partial}{\partial\mu} + \sigma\frac{\partial}{\partial g} - 2\tau_1\right] \Gamma^{(2,0)}\Big|_{\not p=M_1} = 0$$

$$\left[\mu\frac{\partial}{\partial\mu} + \sigma\frac{\partial}{\partial g} - 2\tau_2\right] \Gamma^{(0,2)}\Big|_{\not p=M_2} = 0 \qquad (4.8)$$

and therefore

$$C_1 \; \Delta_{11} \; \Gamma^{(2,0)}\Big|_{\not p=M_1} + C_2 \; \Delta_{12} \; \Gamma^{(2,0)}\Big|_{\not p=M_1} = 0$$

$$C_1 \; \Delta_{11} \; \Gamma^{(0,2)}\Big|_{\not p=M_2} + C_2 \; \Delta_{12} \; \Gamma^{(0,2)}\Big|_{\not p=M_2} = 0 \qquad (4.9)$$

where C_1 and C_2 are the left hand side of the first two equations in (4.7), respectively. As the determinant of the coefficients in (4.9) is different from zero, then necessarily $C_1=C_2=0$.

The renormalization group equation (4.6) shows that, although our scheme employs an auxialiarily mass μ, the value taken by μ is irrelevant in the sense that changes in this parameter can be absorbed in coupling constant and wave function renormalizations.

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REFERENCES

1) P.Federbush; Phys.Rev.121, 1247 (1961).

2) A.S.Wightman; Cargese Lectures in Theor.Phys., 1964, ed. by Levy Gordon and Breach, N.Y. (1966).

3) M.L.Tapper; Nuovo Cimento 25A, 165 (1975).

4) B.Schroer, T.T.Truong e P.Weisz; Ann.Phys. 102, 156 (1976).

5) P.Hählen; Nucl.Phys. B116, 491 (1976).

6) E.Abdalla, M.Gomes and R.Koberle; Phys.Rev. D 18, 3634 (1978). M.El Afioni, M.Gomes and R.Koberle; Phys.Rev. D 19, 1791 (1979).

7) W.Zimmermann, 1970 Brandeis Lectures, Lectures on Elementary Particles and Quantum Field Theory, Vol. 1 (Cambridge, Mass., 1970), p.397.

8) J.H.Lowenstein; Comm.Math.Phys. 24, 1,(1971).

FIGURE CAPTIONS

1) Feynman rules: continuous line represents the fermion propagator of type j , the wavy line represents the "propagator" $\varepsilon^{\mu\nu}$.

2) General structure for the four point function.

3) Here the indicated wavy line has a momentum factor $\frac{q^{\nu}}{q^2}$ or $\frac{\tilde{q}^{\nu}}{q^2}$.

4) Generic graph for the vertex function of the current with two external fermion line.

5) Proper functions of two currents.

6) In this graph the fermion mass is not modified by the action of the subtraction operator.

7) The bubble represents possible contributions to the wavy line "propagator" (the two fermion lines at V_2 do not give further contributions to the propagator).

8) Only contribution to the anomaly of the axial current.

9) Fermion loop with a mass insertion.

10) A contribution to the fermion self energy.

FIGURES

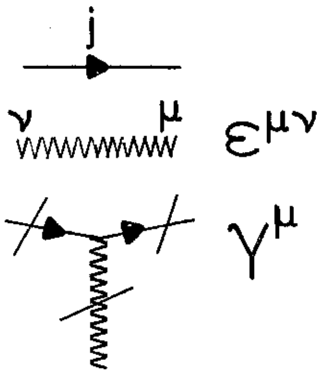

FIG. 1

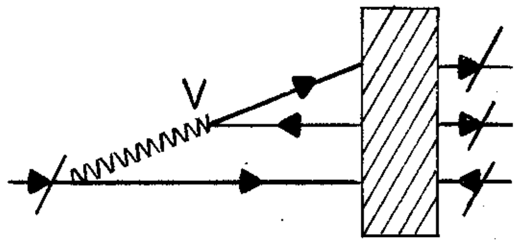

FIG. 2

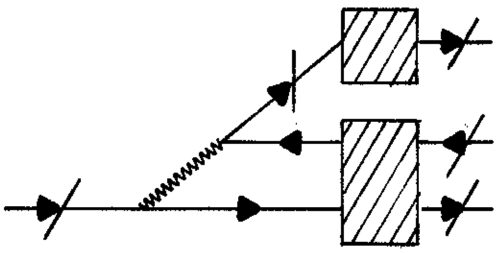

FIG. 3

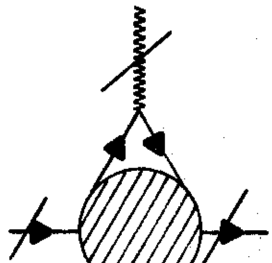

FIG. 4

FIG. 5

FIG. 6

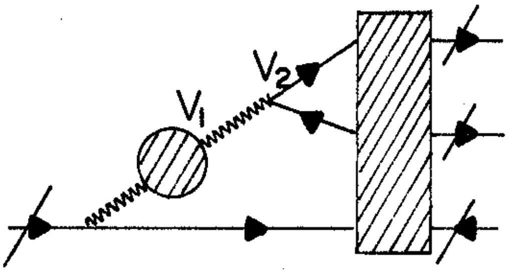

FIG. 7

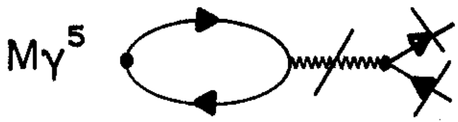

FIG. 8

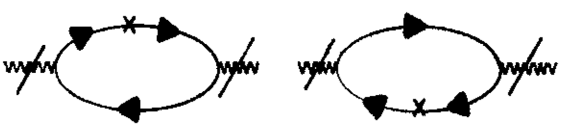

FIG. 9

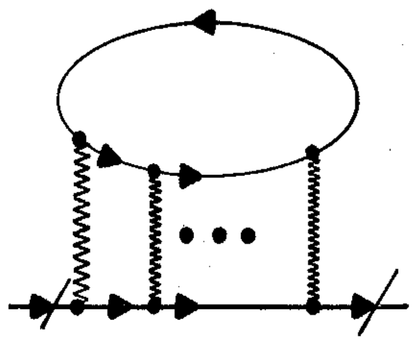

FIG. 10