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by

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ABSTRACT

We calculate the quantized non-local charge of the CP^{n-1} model in the framework of renormalized 1/n perturbation theory, and prove that it is not conserved.

1. INTRODUCTION

Classically the CP^{n-1} model is known to posses an infinite number of conservation laws and to be classically integrable[1]. At the quantum level, one would naively suspect the same behavior as in the 0(n) non-linear sigma[2] and Gross-Neveu[3] models, in which the amplitude of pair production is suppressed as a consequence of the infinite number of conservation laws. In the 1/n expansion, however, this model allows pair production, and the S-matrix does not belong to the class II of ref. (4). In this paper, we show that in spite of some hints from the coupling constant perturbation theory at high energy[5], the infrared phase, governed by the 1/n expansion, has anomalies in the conservation of the quantum non-local charge, destroying the usual constraints on the S-matrix elements. The model and some of its basic properties are reviewed in section 2. In section 3 we discuss the short distance behavior of the product of two currents, a necessary step for the construction of the quantum analog of the classical non-local charge. We show thereafter that due to the presence of anomalous terms this quantum (non-local) charge is no longer conserved.

2. DEFINITION OF THE MODEL

The CP^{n-1} model is defined by the Lagrangian

$$\mathcal{L} = \overline{D_\mu z}\, D_\mu z \tag{2.1}$$

with $D_\mu z = \partial_\mu z + \frac{2f}{n} A_\mu z$ (2.1a)

and the constraint $\bar{z} z = \frac{n}{2f}$ (2.1b)

where z is a complex n component field $z = (z_1, \dots, z_n)$. If the index i does not appear, it is summed over.

This model is known to possess instanton solutions, to be asymptotically free and $1/n$ expandable[6]. In the framework of the $1/n$ expansion, the model describes partons, and its S-matrix does not factorize, in spite of the classical integrability of the model. It was recently shown that for a model to be classically integrable it is necessary[13] and sufficient[7] to be defined on a symmetric space. In this case there is a Noether current j_μ^{ij} whose conservation is equivalent to the equations of motion and which satisfies:

$$\partial_\mu j_\nu^{ij} - \partial_\nu j_\mu^{ij} + 2.\frac{2f}{n}\left[j_\mu, j_\nu\right]^{ij} = 0 \tag{2.2}$$

Using (2.2) it is easily verified that the non-local charge

$$Q = \int dy_1 dy_2\, \varepsilon(y_1 - y_2)\, j_0(t,y_1)\, j_0(t,y_2) - \frac{n}{2f}\int dy\, j_1(t,y) \tag{2.3}$$

is conserved.

In the CP^{n-1} model, the current j_μ^{ij} is given by:

$$j_\mu^{ij} = z_i \overleftrightarrow{\partial_\mu} \bar{z}_j + 2 A_\mu z_i \bar{z}_j \qquad (2.4)$$

At the quantum level the charge (2.3) is not well-defined since it involves a product of currents at the same point. The (non integrable) singularity of this product must be analyzed in order to obtain a renormalized charge. For the 0(n) non-linear sigma model this was done in ref. (9), where it was shown that finiteness and conservation of the charge can be achieved by just changing the coefficient of the second term in (2.3).

Conservation of this charge has far-reaching consequences for the theory. Because of its non-local character, the dynamics will be much constrained. Putting it in terms of asymptotic fields, Lüscher[9] showed that in the 0(n) non-linear sigma model, it forbids pair production. Furthermore, this charge is only compatible with a non-trivial S-matrix. However, in that case, renormalization is trivial because of the reduced number of composed operators compatible with the symmetry and dimension, which are the current j_μ itself and its derivative $\partial_\mu j_\nu$, whereas in the CP^{n-1} case, we have many other composite operators for the Wilson expansion, i.e., $z_i \bar{z}_j F_{\mu\nu}$, $z_i \bar{z}_j D_\mu z \overline{D_\nu z}$, $\partial_\mu (z_i \bar{z}_k) j_\nu^{kj}$ etc. As we will see, one of these terms gives rise to an anomaly, destroying conservation of the would-be charge.

3. FEYNMAN RULES AND WILSON EXPANSION IN LOWEST ORDER

The 1/n expansion of the model was treated in great detail by d'Adda et al[6]. Their Feynman rules will be used here without any more mention. All the calculations will be made in the Euclidian region.

We are interested in the short distance behavior of the product.

$$j_\mu^{ik}(x)\, j_\nu^{kj}(y) - j_\nu^{ik}(y)\, j_\mu^{kj}(x) \tag{3.1}$$

or yet, in the singular terms (as ε tends to zero) of:

$$j_\mu^{ik}(x+\varepsilon)\, j_\nu^{kj}(x-\varepsilon) - j_\nu^{ik}(x-\varepsilon)\, j_\mu^{kj}(x+\varepsilon) \tag{3.2}$$

For the product (3.2), we have a sum of the following terms

$$-\partial_\mu z_i(x+\varepsilon)\,\bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)\,\partial_\nu \bar{z}_j(x-\varepsilon) \tag{3.3a}$$

$$\partial_\mu z_i(x+\varepsilon)\,\bar{z}_k(x+\varepsilon)\,\partial_\nu z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon) \tag{3.3b}$$

$$z_i(x+\varepsilon)\,\partial_\mu \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)\,\partial_\nu \bar{z}_j(x-\varepsilon) \tag{3.3c}$$

$$-z_i(x+\varepsilon)\,\partial_\mu \bar{z}_k(x+\varepsilon)\,\partial_\nu z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon) \tag{3.3d}$$

$$2A_\mu(x+\varepsilon)\, z_i(x+\varepsilon)\,\bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)\,\partial_\nu \bar{z}_j(x-\varepsilon) \tag{3.3e}$$

$$-2A_\mu(x+\varepsilon)\, z_i(x+\varepsilon)\,\bar{z}_k(x+\varepsilon)\,\partial_\nu z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon) \tag{3.3f}$$

$$2 z_i(x+\varepsilon)\,\partial_\mu \bar{z}_k(x+\varepsilon)\, A_\nu(x-\varepsilon)\, z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon) \tag{3.3g}$$

$$-2\,\partial_\mu z_i(x+\varepsilon)\,\bar{z}_k(x+\varepsilon)\, A_\nu(x-\varepsilon)\, z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon) \tag{3.3h}$$

$$4A_\mu(x+\varepsilon)\, z_i(x+\varepsilon)\,\bar{z}_k(x+\varepsilon)\, A_\nu(x-\varepsilon)\, z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon) \tag{3.3i}$$

minus the symmetric terms (s.t) obtained from those above making the substitutions $\varepsilon \to -\varepsilon$, $\mu \leftrightarrow \nu$.

By power counting, the Green's functions which diverge in the $\varepsilon \to 0$ limit will have either two or four external z lines and zero, one or two external A lines. A term with two z's and one α (the Lagrange multiplier field which enforces $\bar{z}z$ = constant - see ref. (6)) external lines is forbidden, in this expansion, by P.T. symmetry. Also possible divergences of graphs with more than four external z lines actually cancel among themselves, as a consequence of the constraint (2.1b). Thus, up to first order in 1/n, the divergent piece receives contributions only from the following Green's functions.

$$\langle 0|T\, z_\alpha \bar{z}_\beta \left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) \right|0\rangle \tag{3.4}$$

$$\langle 0|T\, z_\alpha \bar{z}_\beta A_\gamma \left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) \right|0\rangle \tag{3.5}$$

$$\langle 0|T\, z_\alpha \bar{z}_\beta A_\gamma A_\delta \left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) \right|0\rangle \tag{3.6}$$

$$\langle 0|T\, z_\alpha \bar{z}_\beta z_\lambda \bar{z}_\sigma \left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) \right|0\rangle \tag{3.7}$$

$$\langle 0|T\, z_\alpha \bar{z}_\beta z_\lambda \bar{z}_\sigma A_\gamma \left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) \right|0\rangle \tag{3.8}$$

$$\langle 0|T\, z_\alpha \bar{z}_\beta z_\lambda \bar{z}_\sigma A_\gamma A_\delta \left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) \right|0\rangle \tag{3.9}$$

The graphical structure of the first 3 terms is shown in fig. 1. As we will see, the divergent parts of these graphs combine (as they should) to form gauge invariant objects. The calculation will be done in lowest order, (so that only (3.4)-(3.5) contribute) and we begin by considering contribution from the Green's function (3.4). In this case, only (3.3a) up to (3.3d) contribute, ande we have ($i \neq j$)

$$- \partial_\mu z_i(x+\varepsilon)\, \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)\, \partial_\nu \bar{z}_j(x-\varepsilon) =$$

$$= - \partial_\mu z_i(x+\varepsilon) : \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon) : \partial_\nu \bar{z}_j(x-\varepsilon) -$$

$$- \partial_\mu z_i(x+\varepsilon)\, \partial_\nu \bar{z}_j(x-\varepsilon) \langle 0|T\, \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)|0\rangle \tag{3.10}$$

The last term corresponds to a part of graph (a) of Fig. 1. Now

$$\langle 0|T\, \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)|0\rangle = \frac{n}{2\pi} K_0(2\varepsilon m) \tag{3.11}$$

K_o is a modified Bessel function of zeroth order

$$K_0(zm) \simeq -\frac{1}{2}\left[1 + \left(\frac{zm}{2}\right)^2\right] \ln \frac{m^2 z^2}{4} - \gamma + (1-\gamma)\frac{z^2 m^2}{4} \tag{3.12}$$

where γ is the Euler-Mascheroni constant, $\gamma = 0{,}577\ldots$ In order to simplify the notation, we will indicate the second (non Wick-ordered) term in (3.10) by

$$\mathrm{Div}\left(-\partial_\mu z_i(x+\varepsilon)\, \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)\, \partial_\nu \bar{z}_j(x-\varepsilon)\right) \tag{3.13}$$

We have then

$$\mathrm{Div}\left(-\partial_\mu z_i(x+\varepsilon)\, \bar{z}_k(x+\varepsilon) z_k(x-\varepsilon)\, \partial_\nu \bar{z}_j(x-\varepsilon)\right) = \frac{-n}{2\pi}\, \partial_\mu z_i(x)\, \partial_\nu \bar{z}_j(x)\, K_0 \tag{3.14}$$

where we made a Taylor expansion around x and dropped terms which tend to zero with ε. Analogously

$$\mathrm{Div}\left(\partial_\mu z_i(x+\varepsilon)\, \bar{z}_k(x+\varepsilon)\, \partial_\nu z_k(x-\varepsilon)\, \bar{z}_j(x-\varepsilon)\right) = \frac{-n}{4\pi}\, \partial_\mu z_i(x+\varepsilon)\, \bar{z}_j(x-\varepsilon) \frac{\partial}{\partial \varepsilon^\nu} K_0 \tag{3.15}$$

$$\mathcal{D}iv\left(z_i(x+\varepsilon)\,\partial_\mu \bar{z}_k(x+\varepsilon)\, z_k(x-\varepsilon)\,\partial_\nu \bar{z}_j(x-\varepsilon)\right) = \frac{n}{4\pi}\, z_i(x+\varepsilon)\,\partial_\nu \bar{z}_j(x-\varepsilon)\,\frac{\partial}{\partial \varepsilon^\mu} K_0 \qquad (3.16)$$

$$\mathcal{D}iv\left(-z_i(x+\varepsilon)\,\partial_\mu \bar{z}_k(x+\varepsilon)\,\partial_\nu z_k(x-\varepsilon)\,\bar{z}_j(x-\varepsilon)\right) = \frac{n}{8\pi}\, z_i(x+\varepsilon)\,\bar{z}_k(x-\varepsilon)\,\frac{\partial^2}{\partial \varepsilon^\mu \partial \varepsilon^\nu} K_0 \qquad (3.17)$$

Performing a further Taylor expansion of the operators, we obtain for the graphs (3.3a) - (3.3d) of Fig. 1.

$$(3.3a) = -\frac{n}{2\pi} K_0 \left(\partial_\mu z_i\, \partial_\nu \bar{z}_j - \partial_\nu z_i\, \partial_\mu \bar{z}_j\right)$$

$$(3.3b) = -\frac{n}{4\pi}\partial_\nu K_0\, \partial_\mu z_i \bar{z}_j + \frac{n}{4\pi}\partial_\mu \partial_\nu z_i \bar{z}_j\, K_0 - \frac{n}{4\pi}\partial_\mu z_i\, \partial_\nu \bar{z}_j\, K_0 +$$

$$+ \frac{n}{8\pi}\partial_\mu \partial_\beta z_i \bar{z}_j\, \partial_\nu \partial_\beta K_1 - \frac{n}{8\pi}\partial_\mu z_i\, \partial_\beta \bar{z}_j\, \partial_\nu \partial_\beta K_1 - (s.t.)$$

$$(3.3c) = \frac{n}{4\pi}\partial_\mu K_0\, z_i\, \partial_\nu \bar{z}_j - \frac{n}{4\pi}\partial_\mu z_i\, \partial_\nu \bar{z}_j\, K_0 + \frac{n}{4\pi} z_i\, \partial_\mu \partial_\nu \bar{z}_j\, K_0 -$$

$$- \frac{n}{8\pi}\partial_\beta z_i\, \partial_\nu \bar{z}_j\, \partial_\mu \partial_\beta K_1 + \frac{n}{8\pi} z_i\, \partial_\nu \partial_\beta \bar{z}_j\, \partial_\mu \partial_\beta K_1 - (s.t)$$

$$(3.3d) = -\frac{n}{4\pi}\partial_\mu z_i \bar{z}_j\, \partial_\nu K_0 + \frac{n}{4\pi} z_i\, \partial_\mu \bar{z}_j\, \partial_\nu K_0 + \frac{n}{8\pi}\partial_\beta z_i \bar{z}_j\, \partial_\mu \partial_\nu \partial_\beta K_1$$

where $K_1(m\varepsilon) = -\frac{\partial}{\partial m^2} K_0(m\varepsilon)$

Graph b can be handled in the same way, and we have only contributions from (3.3e) and (3.3f):

$$(3.3e) = \frac{n}{2\pi}\Big[2K_0\left(A_\mu z_i\, \partial_\nu \bar{z}_j\right) - z_i \bar{z}_j\, A_\mu\, \partial_\nu K_0 + \left(-\partial_\nu z_i\, A_\mu \bar{z}_j - \partial_\nu A_\mu\, z_i \bar{z}_j + \right.$$

$$\left. + z_i A_\mu\, \partial_\nu \bar{z}_j\right) K_0 + \frac{1}{2}\left(z_i A_\mu\, \partial_\beta \bar{z}_j - \partial_\beta z_i\, A_\mu \bar{z}_j - \partial_\beta A_\mu\, z_i \bar{z}_j\right)\partial_\nu \partial_\beta K_1\Big] - (s.t.)$$

$$(3.3f) = \frac{\pi}{2\pi}\Big[2K_0\left(-A_\nu \partial_\mu z_i \bar{z}_j\right) - z_i\bar{z}_j A_\nu \partial_\mu K_0 + \left(z_i \partial_\mu A_\nu \bar{z}_j + z_i \partial_\mu \bar{z}_j A_\nu - \partial_\mu z_i A_\nu \bar{z}_j\right) K_0 + \frac{1}{2}\left(z_i \partial_\beta A_\nu \bar{z}_j + z_i \partial_\beta \bar{z}_j A_\nu - \partial_\beta z_i A_\nu \bar{z}_j\right)\partial_\mu \partial_\beta K_1\Big] - (\text{s.t.})$$

The calculation for the graph (c) is more involved. We formally have:

$$\langle 0|T\left(j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon)\right) A_\beta(y)|0\rangle =$$

$$= \langle 0|T N_2\left(j_\mu(x) j_\nu(x) - j_\nu(x) j_\mu(x)\right) A_\beta(y)|0\rangle + R_{\mu\nu\beta}(x,y,\varepsilon) \qquad (3.18)$$

where the symbol N_2 denotes the normal product defined[10] a la Zimmermann by making the minimum number of subtractions necessary to render the formal product of currents at the same point well-defined. $R_{\mu\nu\beta}(x,y,\varepsilon)$ are these subtraction terms which will diverge as $\varepsilon \to 0$. Now the first term does not contribute to the operator expansion since $\bar{z}z = 0$ in our subtraction procedure. Classically, if $\bar{z}z = a = \text{constant}$ we have the following equality

$$2\left(j_\mu j_\nu - j_\nu j_\mu\right) = a\left(\partial_\nu j_\mu - \partial_\mu j_\nu\right) \qquad (3.19)$$

In a subtraction scheme preserving this equality, we should have

$$2\langle 0|T N\left(j_\mu j_\nu - j_\nu j_\mu\right) X|0\rangle = a\,\langle 0|T\left(\partial_\nu j_\mu - \partial_\mu j_\nu\right) X|0\rangle + \text{delta terms} \qquad (3.20)$$

and therefore, for $a = 0$, we have the desired result. This statement can be explicitly verified in a lowest order calculation. The divergent part of the second term in (3.18) is

$$\mathcal{D}iv\,[3.3a + A_\beta] = 0 \tag{3.21a}$$

$$\mathcal{D}iv\,[3.3b + A_\beta] = -\frac{1}{2}\partial_\nu \partial_\beta K_1 \partial_\mu z_i(x)\bar{z}_j(x) A_\beta(x) + \frac{1}{2}\partial_\mu \partial_\beta K_1 \partial_\nu z_i(x)\bar{z}_j(x) A_\beta(x) \tag{3.21b}$$

$$\mathcal{D}iv\,[3.3c + A_\beta] = \frac{1}{2}\partial_\mu \partial_\beta K_1 z_i(x)\partial_\nu \bar{z}_j(x) A_\beta(x) - \frac{1}{2}\partial_\nu \partial_\beta K_1 \partial_\mu \bar{z}_j(x) z_i(x) A_\beta(x) \tag{3.21c}$$

$$\mathcal{D}iv\,[3.3c + A_\beta] = \frac{1}{2} z_i(x)\bar{z}_j(x)\partial_\mu A_\beta(x)\partial_\nu \partial_\beta K_1 - \frac{1}{2} z_i(x)\bar{z}_j(x)\partial_\nu A_\beta(x)\partial_\mu \partial_\beta K_1 \tag{3.21d}$$

The graphs (d), (e) and (f) are easily shown to vanish by symmetry. Now we collect all terms and obtain:

$$j_\mu(x+\varepsilon) j_\nu(x-\varepsilon) - j_\nu(x-\varepsilon) j_\mu(x+\varepsilon) = \frac{n}{2\pi}\left[-\frac{\delta_{\mu\nu}\varepsilon_\rho}{2\varepsilon^2} + \frac{\delta_{\mu\rho}\varepsilon_\nu}{2\varepsilon^2} + \frac{\delta_{\nu\rho}\varepsilon_\mu}{2\varepsilon^2} + \right. \tag{3.22}$$

$$\left. + \frac{\varepsilon_\mu \varepsilon_\nu \varepsilon_\rho}{(\varepsilon^2)^2}\right] j_\rho + \frac{n}{2\pi}\left[\left(\frac{\gamma}{2} + \frac{1}{4}\ln m^2\varepsilon^2\right)(\delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\nu\sigma}\delta_{\mu\rho}) + \frac{\delta_{\nu\sigma}\varepsilon_\mu\varepsilon_\rho}{2\varepsilon^2} - \frac{\delta_{\mu\sigma}\varepsilon_\nu\varepsilon_\rho}{2\varepsilon^2}\right]\partial_\sigma j_\rho$$

$$+ \frac{n}{2\pi}\left[2\delta_{\mu\rho}\frac{\varepsilon_\nu\varepsilon_\sigma}{\varepsilon^2} + 2\delta_{\nu\sigma}\frac{\varepsilon_\mu\varepsilon_\rho}{\varepsilon^2}\right] z_i\bar{z}_j F_{\rho\sigma}$$

or yet:

$$j_\mu(x+\varepsilon) j_\nu(x) - j_\nu(x) j_\mu(x+\varepsilon) = \left[-\frac{\delta_{\mu\nu}\varepsilon_\rho}{\varepsilon^2} + \frac{\delta_{\mu\rho}\varepsilon_\nu}{\varepsilon^2} + \frac{\delta_{\nu\rho}\varepsilon_\mu}{\varepsilon^2} + \frac{2\varepsilon_\mu\varepsilon_\nu\varepsilon_\rho}{(\varepsilon^2)^2}\right] j_\rho$$

$$+ \left[\left(\frac{\gamma}{2} + \frac{1}{4}\ln\frac{m^2\varepsilon^2}{4}\right)(\delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\nu\sigma}\delta_{\mu\rho}) + \frac{\delta_{\nu\sigma}\varepsilon_\mu\varepsilon_\rho}{2\varepsilon^2} - \frac{\delta_{\mu\sigma}\varepsilon_\nu\varepsilon_\rho}{2\varepsilon^2} - \right. \tag{3.23}$$

$$\left. - \frac{\delta_{\mu\nu}\varepsilon_\rho\varepsilon_\sigma}{2\varepsilon^2} + \frac{\delta_{\mu\rho}\varepsilon_\nu\varepsilon_\sigma}{2\varepsilon^2} + \frac{\delta_{\nu\rho}\varepsilon_\mu\varepsilon_\sigma}{2\varepsilon^2} + \frac{\varepsilon_\mu\varepsilon_\nu\varepsilon_\rho\varepsilon_\sigma}{(\varepsilon^2)^2}\right]\partial_\sigma j_\rho +$$

$$+ \left[+ 2\delta_{\mu\rho}\frac{\varepsilon_\nu\varepsilon_\sigma}{\varepsilon^2} + 2\delta_{\nu\sigma}\frac{\varepsilon_\mu\varepsilon_\rho}{\varepsilon^2}\right] z_i\bar{z}_j F_{\rho\sigma}$$

4. DEFINITION OF THE QUANTUM NON LOCAL CHARGE

We define the quantum version of the non-local charge as[9]

$$Q_\delta = \int_{|y_1-y_2|\geq\delta} dy_1\, dy_2\, \epsilon(y_1-y_2)\, j_0(t,y_1)\, j_0(t,y_2) - Z\int dy\, j_1(t,y) \tag{4.1}$$

where Z depends on the cut-off δ. Because of the linearly divergent term in eq. (3.23), it is easy to see that the coefficient Z must be equal to $\frac{n}{2\pi}\ln\mu\delta$ in order that we have a well-defined finite charge Q in eq. (4.1). This charge is the only candidate to be the quantum non-local charge corresponding to (2.3).

However, as mentioned previously, this charge is no longer time-independent. This is verified as follows: Current conservation and partial integration give

$$\frac{dQ_\delta}{dt} = \frac{-1}{n}\int_{-\infty}^{\infty} dy \Big\{ \left(j_1^{ik}(t,y+\delta) + j_1^{ik}(t,y-\delta)\right) j_0^{kj}(t,y) - \left(j_1^{kj}(t,y+\delta) + j_1^{kj}(t,y-\delta)\right) j_0^{ik}(t,y) - \frac{n}{2\pi}\ln\mu\delta\, \partial_0\, j_1^{ij}(t,y)\Big\}, \text{ where } \mu = e^{\gamma-1}\frac{m}{2} \tag{4.2}$$

As δ goes to zero we use the operator expansion (3.23) to obtain:

$$\left(j_1^{ik}(t,y+\delta) + j_1^{ik}(t,y-\delta)\right) j_0^{kj}(t,y) - \left(j_1^{kj}(t,y+\delta) + j_1^{kj}(y-\delta)\right) j_0^{ik}(t,y) =$$

$$= \left(\gamma + \frac{1}{2}\ln\frac{m^2\delta^2}{4}\right)\left(\partial_1 j_0 - \partial_0 j_1\right) + \partial_0 j_1 - 4 z_i \bar{z}_j F_{10}$$

and we have

$$\frac{dQ_\delta^{ij}}{dt} = -\frac{2}{\pi}\int_{-\infty}^{\infty} z_i \bar{z}_j F_{10}\, dy .$$

5. CONCLUSION

The CP^{n-1} model, as is well-known, using the 1/n expansion, does allow production of pairs. This can now be traced back to an anomaly in the quantum non-local charge, in contraposition to the case of the 0(n) non-linear σ-model , in which Lüscher quantized the analogous non-local charge and this turned out to be conserved. In that case, also Pohlmeyer's local conservation laws(11) provided an alternative explanation for the absence of pair production. We presume that the quantum local charges in the CP^{n-1} model must also have anomalies which prevent the model from having a factorizable S-matrix and from showing the usual soliton behavior. The supersymmetric extension of the CP^{n-1} model has already been studied and proven to factorize(12). This could in principle be traced back to a cancellation of the anomalies studied here with those coming from the coupling of the chiral model to the CP^{n-1} model.

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FIGURE CAPTION

Fig. 1 - Lowest order graphs contributing to the short distance expansion of the product of the currents.

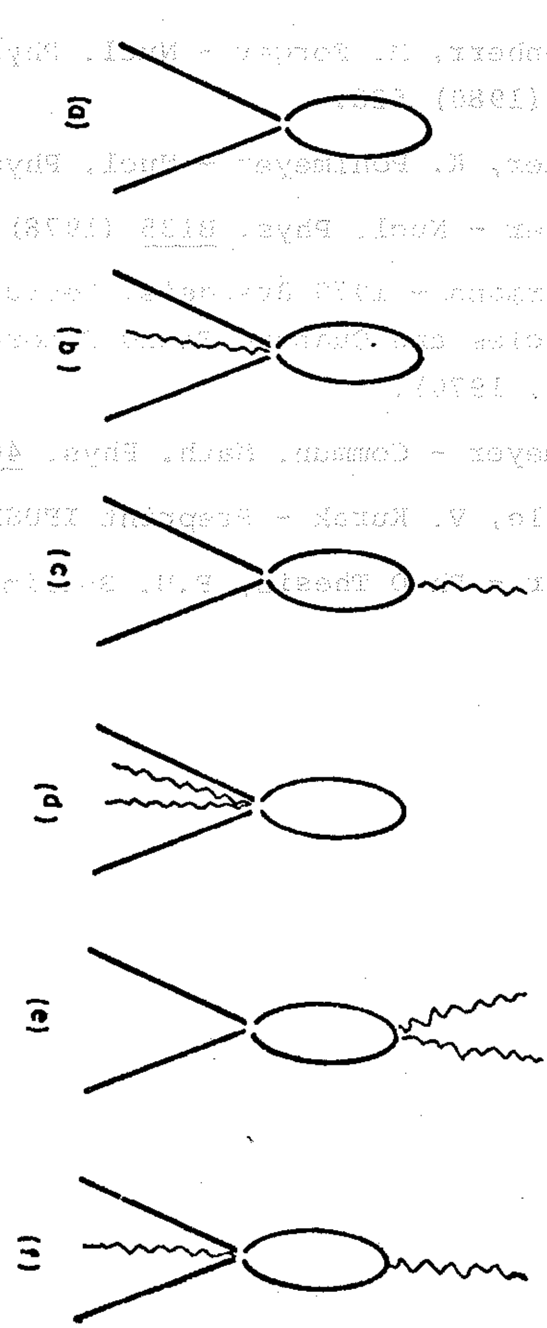

Fig.1