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ABSTRACT

Equations of motion for correlated nucleon pairs are derived and then applied to some even lead isotopes. The connexion with the interacting boson model is pointed out.

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The interacting boson model (IBM) has been very successful in providing a phenomenological description for a large variety of even-even nuclei in terms of a system of interacting bosons[1]. In its usual, restricted form, the IBM makes the further assumption that, for the states of interest, only the monopole (s) and quadrupole (d) bosons need be taken into account. A natural extension, however, would be the inclusion of bosons with other multipolarities, as done in the multipole-pairing model[2], which is known to work well at least in the lead region. The similarity in the microscopic foundations of the two models has been made evident by the identification of the bosons in IBM as approximations to correlated pairs of identical valence nucleons[3]. The widespread validity of the IBM suggests, then, that it might be possible to view such pairs as general building blocks for low-lying nuclear states. The purpose of this letter is to test this hypothesis in a realistic calculation. To this end, we derive general equations of motion for excitation operators of particle-particle type. To avoid all arbitrariness of linearization procedures, we follow the double-commutator formalism of Rowe[4], which has been proved useful in the particle-hole description of collective states[5]. However, for the nuclear particle-particle excitations corresponding to the bosons in IBM, the formalism has to be extended by relaxing the condition of annihilation of the parent state, since such condition is no longer valid in the present case[6].

Guided by the (restricted) IBM, one might try the

following form for the states of a system of 2N identical valence nucleons:

$$|N,JM\rangle=(\beta S^{+}+\sum_{\mu=-2}^{2}\alpha_{\mu}D_{\mu}^{+})^{N}_{JM}|0\rangle, \tag{1}$$

where $|0\rangle$ is the doubly-closed-shell state, S^{+} and D^{+} are monopole and quadrupole fermion pair creation operators and the lower indices on the parentheses indicate that the whole expression is to be coupled to angular momentum JM. A more flexible alternative, though, is to write

$$|N,JM\rangle=O^{+}_{JM}|N-\nu\rangle, \tag{2}$$

so that the excitation operator

$$O^{+}_{JM}=(\beta S^{+}+\sum_{\mu=-2}^{2}\alpha_{\mu}D_{\mu}^{+})^{\nu}_{JM} \tag{3}$$

creates ν pairs on the parent state $|N-\nu\rangle$, chosen here as the ground state of the system of $2(N-\nu)$ nucleons.

Taking (2) as a variational wave function and making a convenient choice for the zero of energy, we obtain the following equations of motion for the excitation operators:

$$\langle N-\nu|[\,\delta O_{JM},H,O^{+}_{JM}\,]|N-\nu\rangle - \langle N-\nu|O^{+}_{JM}H\delta O_{JM}|N-\nu\rangle =$$

$$W_{J}(\langle N-\nu|[\,\delta O_{JM},O^{+}_{JM}\,]|N-\nu\rangle + \langle N-\nu|O^{+}_{JM}\delta O_{JM}|N-\nu\rangle), \tag{4}$$

where the double-commutator is defined[4] by

$$2[\,\delta O_{JM},H,O^{+}_{JM}\,] = [\,\delta O_{JM},[\,H,O^{+}_{JM}\,]\,] + [\,[\,\delta O_{JM},H\,],O^{+}_{JM}\,] \quad (5)$$

It should be noted that our equations of motion (4) would be reduced to those of Rowe[4] if one insisted on the condition $\delta O_{JM}|N-\nu>=0$, which is generally violated in the present case.

The procedure of reaching a state, starting from the doubly-closed shell, by successive 2ν-nucleon transfers through (2) with $\nu=1$ is expected to be reasonable in the beginning of major shells. On the other hand, for nuclei in the middle of a shell one should give ν a value equal to or higher than two, in order to incorporate the collective effects due to the strong coupling between the monopole and quadrupole pairs.

In the present work, we illustrate our method in the most simple case, namely that of the states supposedly mapped onto those in the vibrationsl limit[7] of IBM, labeled by $|[N](n_d)\nu n_\Delta JM>$ with $n_d=0$ or 1, $\nu=0$ or 1, $n_\Delta=0$ and $J=0$ or 2. We also extend our calculation to the states that could be roughly described as a mixture of N-1 s-bosons plus one g-boson or one i-boson. This is done by using the same equations as above, except that the D-operators are replaced by the appropriate multipole-pair operators. We thus take the ground state of the doubly magic nucleus ^{208}Pb as our vacuum and reach, by the iterative procedure (2) with $\nu=1$, the low-lying states of the even lead isotopes.

In this case, the specific particle-particle

excitation operators of multipolarity equal to J can be written, in an obvious notation, as

$$O^{+}_{JM} = \sum_{jj'} X^{(J)}_{jj'} A^{+}(jj')_{JM} \quad , \tag{6}$$

where

$$A^{+}(jj')_{JM} = -(1/\sqrt{1+\delta_{jj'}})(a^{+}_{j} a^{+}_{j'})_{JM} \quad , \tag{7}$$

and we restrict j and j' to a major shell. Inserting these expressions into our equations of motion (4), we arrive at the following eigen-equation for the coefficients $X^{(J)}_{jj'}$:

$$\sum_{j_2 j_2'} (\langle N-1| \left[A(j_1 j_1')_{JM}, H, A^{+}(j_2 j_2')_{JM} \right] |N-1\rangle -$$

$$\langle N-1| A^{+}(j_2 j_2')_{JM} H A(j_1 j_1')_{JM} |N-1\rangle) X^{(J)}_{j_2 j_2'} =$$

$$W_J \sum_{j_2 j_2'} \langle N-1| A(j_1 j_1')_{JM} A^{+}(j_2 j_2')_{JM} |N-1\rangle X^{(J)}_{j_2 j_2'} \quad , \tag{8}$$

where M can be arbitrarily chosen. If the normalization is fixed such that $\langle N-1| O_{JM} O^{+}_{JM} |N-1\rangle = 1$, the spectroscopic amplitudes for two-nucleon transfer are given by

$$S^{(J)}_{j_1 j_1'} = \sum_{j_2 j_2'} \langle N-1| A(j_1 j_1')_{JM} A^{+}(j_2 j_2')_{JM} |N-1\rangle X^{(J)}_{j_2 j_2'} \quad . \tag{9}$$

Analogous equations hold for hole-hole excitations.

We report here the main results of such a calculation, performed with a realistic effective interaction[8],

for the isotopes ^{206,204,202}Pb, for which a fair amount of spectroscopic data[9,10] is available. A more thorough account will be published soon. In Fig. 1, we show the excitation spectra for the levels with $J^{\pi}=0^{+}$, 2^{+}, 4^{+} and 6^{+}. One can see that the agreement with experiment is quite good, especially for the yrast states, drawn in heavy line in the figure. Even for the other states, however, despite the severe truncation the agreement is comparable to that obtained in a full shell model calculation[11] with the same effective interaction. (For ^{202}Pb no such calculation exists).

The natural probe for our wave functions is two-neutron stripping. We have, therefore, computed the cross sections for the (p,t) reactions between the above isotopes in a zero-range DWBA with the spectroscopic amplitudes (9). For each value of the angular momentum, the strongest transition is by far the one to the yrast state, just as found experimentally. This is an evidence for the collectivity of those states. The strengths for the transitions to the yrast states, always relative to the one of same multipolarity in the reaction ^{208}Pb(p,t)^{206}Pb, are plotted in Fig. 2, showing that our results reproduce the experimental data[9] very well.

Since our results are particularly good for the ground and yrast states, it will be interesting to take a closer look at their wave functions. A few examples are:

$$|^{206}\mathrm{Pb(g.s.)}\rangle = |N=1, J=0\rangle = O^{+}_{J=0}(1)|0\rangle, \tag{10a}$$

$$|^{204}\mathrm{Pb(g.s.)}\rangle = |N=2, J=0\rangle = O^{+}_{J=0}(2)O^{+}_{J=0}(1)|0\rangle, \tag{10b}$$

$$|^{202}\mathrm{Pb(JM)}\rangle = |N=3, JM\rangle = O^{+}_{JM}(3)O^{+}_{J=0}(2)O^{+}_{J=0}(1)|0\rangle, \tag{10c}$$

and so on, where $|0\rangle = |^{208}\mathrm{Pb(g.s.)}\rangle$ and we have used the value of N to distinguish between excitation operators with equal multipolarities. The overlaps between the operators leading to the yrast or ground states and the ones corresponding to N=1, more precisely the following quantities

$$\langle 0|O_{JM}(1)O^{+}_{JM}(N)|0\rangle / (\langle 0|O_{JM}(1)O^{+}_{JM}(1)|0\rangle\langle 0|O_{JM}(N)O^{+}_{JM}(N)|0\rangle)^{1/2}, \tag{11}$$

have values very close to unity as shown in Table 1. This means, for instance, that the three monopole-pair excitation operators are nearly equal, which helps to explain why they can all be approximated by a single s-boson in the IBM. An analogous statement can be made about the three quadrupole-pair excitations and the d-boson, as well as for the other multipolarities. Bearing this in mind, it is now clear that the wave functions (10) are indeed very similar to those in the vibrational limit of the (extended) IBM.

In conclusion, we have shown that the description of nuclear states in terms of correlated pairs of identical nucleons, as suggested by the IBM, works well in the particular example dealt with here. The agreement with experiment was especially good for the yrast and ground states. This, however, is certainly due to the fact that, for each state, we allowed for no more than one pair of non-zero multipolarity. The inclusion of two or more such pairs would improve the agreement for the remaining states. Finally, we want to point out that the present equations may be useful for the derivation of the IBM from the realistic shell model, a problem in which, despite some preliminary work[12], much still remains to be done.

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TABLE 1

Overlaps between the excitation operators for the yrast states with N=2 or 3 and those for the yrast states with N=1.

J^{π}	0^+	2^+	4^+	6^+
N=2	0.986	0.973	0.974	0.998
N=3	0.976	0.923	0.940	0.997

FIGURE CAPTIONS:

Figure 1. Levels with $J^{\pi}=0^{+}$, 2^{+}, 4^{+} and 6^{+} in $^{206,204,202}Pb$ according to experiment (EXP) and to the present calculation (EQM). All such levels below the highest one shown in each spectrum are included. The ground state energies measured from ^{208}Pb are given in MeV.

Figure 2. Experimental (EXP) and claculated (EQM) values for the strengths of the (p,t) transitions to the yrast states in ^{204}Pb and ^{202}Pb in units of those of the corresponding transitions to ^{206}Pb.

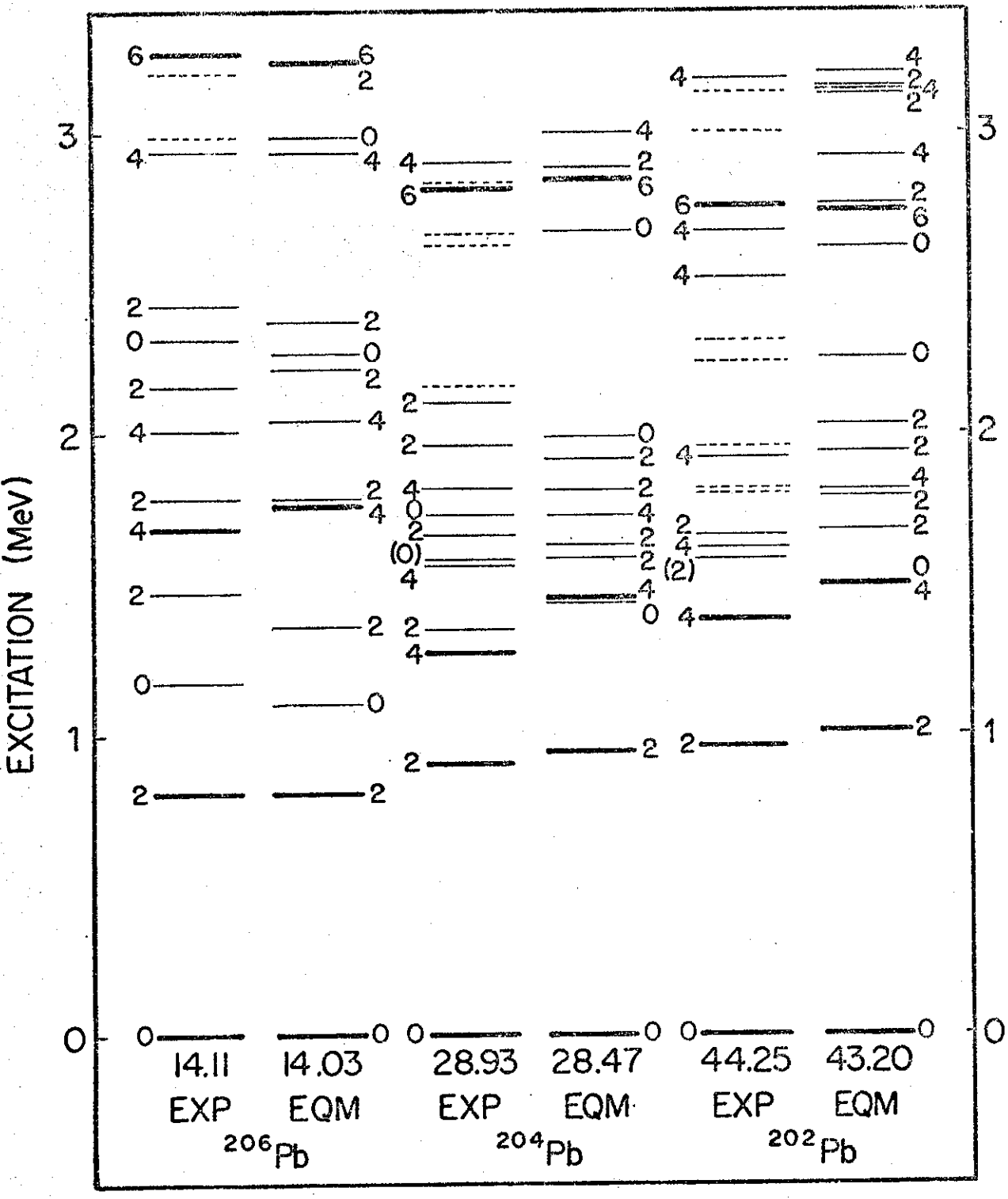
EXCITATION (MeV)
0
1
2
3
14.11
EXP
14.03
EQM
^{206}Pb
28.93
EXP
28.47
EQM
^{204}Pb
44.25
EXP
43.20
EQM
^{202}Pb

FIGURE 1

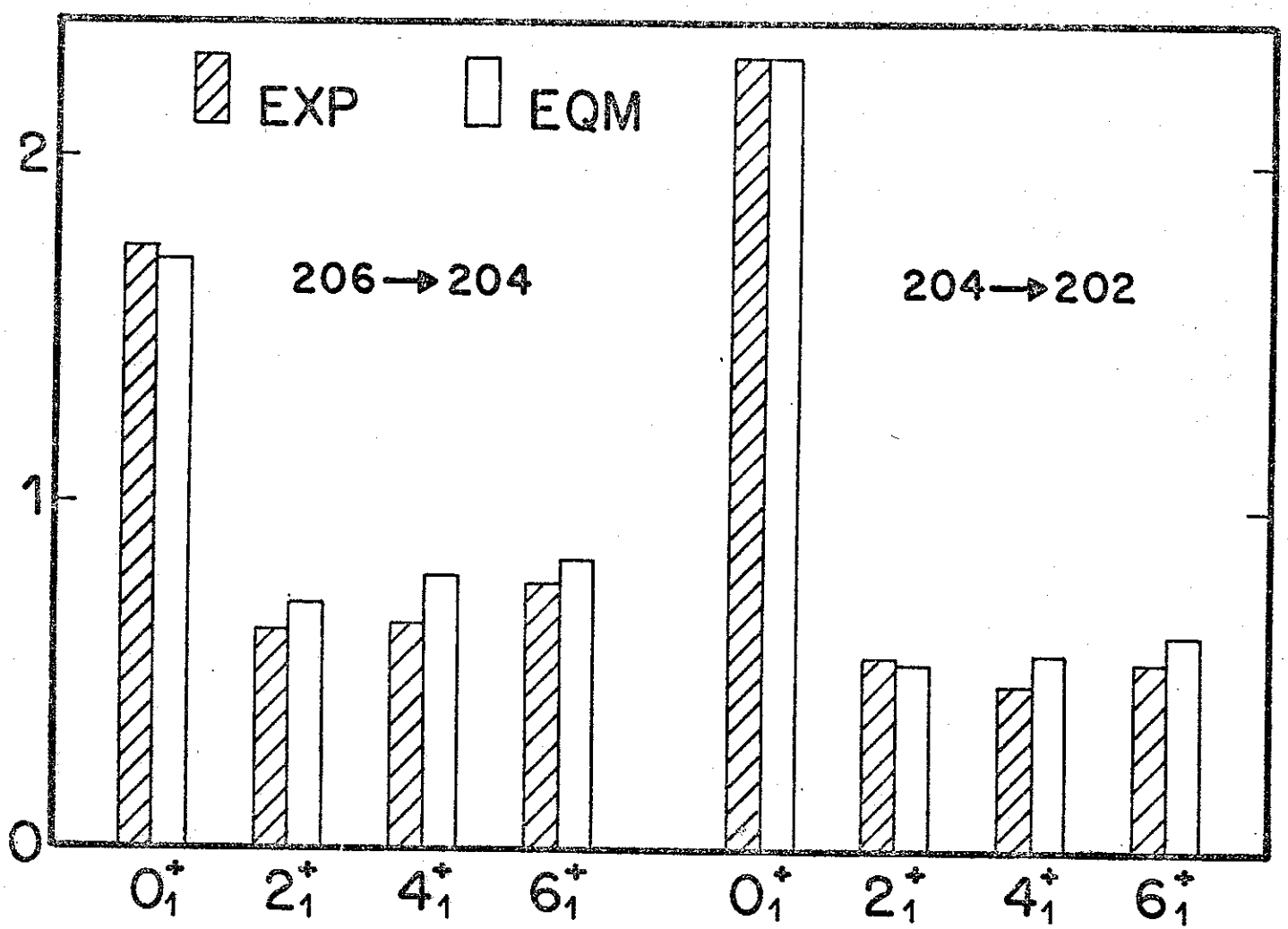

FIGURE 2