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STATISTICAL FLUCTUATIONS AND DOORWAYS

by

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ABSTRACT

An important type of statistical preequilibrium reaction is one that has recently been described as "multistep compound", because it proceeds through successive classes of doorways. That is, a reaction that proceeds through resonances which overlap, and whose partial widths are enhanced by doorways which themselves overlap, with the partial widths of the doorways themselves enhanced by even broader overlapping doorways, etc.

A particularly concise theoretical description of such a reaction can be achieved by the use of a nested sequence of energy-averaging intervals $I_1 > I_2 \ldots > I_N$, where each I_n is intermediate between the widths of two successive classes of doorways. The present Report provides the full details of this approach and employs it to obtain the multidoorway generalizations, first, of the Hauser-Feshbach expression for $\sigma^{f\ell}$ and, more importantly, of the Ericson expression for the autocorrelation function, which is found to exhibit several correlation widths, one for each class of doorways. The results are derived in the Feshbach projection-operator formalism.

A comparison with approaches of Agassi, Weidenmüller and Mantzouranis, and of Feshbach, Kerman and Koonin, shows that all three share the features of being (1) probability-conserving and (2) Markovian in their description of the percolation of flux through the doorway classes.

1. INTRODUCTION

A preequilibrium reaction* is, by definition, one which occurs with a delay time or reaction time which is intermediate between that of a direct reaction and that of a compound-nucleus reaction. Unfortunately, it appears highly unlikely that it will be possible to measure such times directly with any reliability in the near future. Instead, preequilibrium reactions have generally been identified by indirect criteria which are more readily measured, such as angular distributions and energy spectra of reaction products.

However, the most commonly assumed model for preequilibrium phenomena is the exciton model [1], which contains definite, identifiable delay times, the lifetimes t_n for the various classes of exciton states. These times can be measured more or less directly as $t_n = \hbar/\Gamma_n$, by measuring the energy widths of the states of the various classes, provided these are isolated, $\Gamma_n \ll D_n$. In the inverse situation that the states in each class overlap, the excitation function for reactions proceeding through them will exhibit statistical or Ericson fluctuations. It is this case which is the subject of the present study. In this situation the time or energy structure of the reaction is accessible only through a study of the autocorrelation function

$$C(\epsilon) = \langle \sigma(E)\, \sigma(E+\epsilon) \rangle - \langle \sigma(E) \rangle \langle \sigma(E+\epsilon) \rangle ,$$

which will contain as parameters more than one correlation width, $\tilde{\Gamma}$, which provide the characteristic energy variations produced by the classes. When

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*In the present paper we shall not discuss the multistep direct component of the preequilibrium reaction cross section; we concentrate our analysis on the multistep compound processes.

there is reason to believe that preequilibrium states of this sort play an important role in a reaction, it is clear that neither the customary Ericson expression $C(\epsilon) \sim (\tilde{\Gamma}^2 + \epsilon^2)^{-1}$ for the autocorrelation function, nor the Hauser-Feshbach expression $\sigma^{fl}_{cc'} = T_c T_{c'} / \sum_{c''} T_{c''}$ for the fluctuation cross section, is valid.

A certain amount of experimental evidence has already been obtained for the existence of more than one correlation width in autocorrelation studies [2]. The earlier studies [2a] were directed toward an investigation of isotopic spin in nuclear reactions, and an attempt to identify two correlation widths, $\tilde{\Gamma}_>$ and $\tilde{\Gamma}_<$ in the same reaction. This included a theoretical study which obtained a 2-width generalization of Ericson's function $(\tilde{\Gamma}^2 + \epsilon^2)^{-1}$, which is very analogous to some of the results we obtain below, even though the widths involved were quite comparable to each other.

Another example is the compound nucleus ^{30}P, which has recently been investigated by Bonetti et al. [2b] in $^{27}Al(^3He,p)^{29}Si$ at bombarding energies near 11 MeV. The essential feature of an autocorrelation function for a compound nucleus which has only one type (e.g., fine structure states) of resonances is that it must exhibit the same correlation width for all reactions proceeding through these states. As Fig.1 shows, however, the autocorrelation functions for the above reaction proceeding through ^{30}P to two different final states of ^{29}Si exhibit the two very different correlation widths of 55 keV and 230 keV. Even more interesting is Fig. 2, for a third channel, which is fit by Eq. (2.17) below including <u>both</u> these widths in the same channel. The interpretation appears to be that the reaction to the channel shown in Fig. 1a couples primarily to the fine-structure (55 keV) states, that to the channel in Fig. 1b couples primarily

to the doorways (230 keV), and that to the channel in Fig. 2 couples to both.

Our purpose in this report is to obtain the generalization of the Ericson expression for the autocorrelation function and the Hauser-Feshbach formula for the fluctuation cross section when: a) more than one class of states is present; b) states in each class are overlapping and c) the correlation widths of the various classes are substantially different.

Assumption c) will be the crucial feature of our approach, for we have discovered that a simple method of analysis exists in that limit. In essence our result is equivalent to the lowest-order term in an expansion of σ^{fl} or $C(\epsilon)$ in powers of the ratios of the correlation widths. The resulting formulas are especially simple and transparent, and we feel that the result for $C(\epsilon)$ is particularly worthy of experimental investigation, for it appears to be the most sensitive probe available of the existence of more than one class, as well as the only practicable source of information on the time scales involved in the various stages of the reaction.

Although our primary emphasis is on a "nested average" approach to the multiclass model of preequilibrium reactions, we first briefly discuss and compare the four distinct approaches which are under active investigation at present. These are the approaches of (1) Feshbach, Kerman and Koonin (FKK) [3] (chaining); (2) Agassi, Weidenmuller and Mantzouranis (AWM) [4] (random matrix elements in the particle-hole model); (3) Hussein and McVoy (HM) [5] (nested energy averages); and (4) Friedman (F) [6] (probability flow approach). Although not immediately evident in all cases, all four approaches yield a fluctuation cross section which, in the absence of direct reactions, has the form

$$\sigma_{cc'}^{f\ell} = \sum_n X_{n,cc} X_{n,c'c'} \tag{1.1}$$

and, where calculated [4,7], an S-matrix autocorrelation function of the form

$$C_{cc'}^{S}(\epsilon) = \left\langle S_{cc'}^{f\ell}(E) S_{cc'}^{f\ell *}(E+\epsilon) \right\rangle = \sum_n \frac{X_{n,cc} X_{n,c'c'}}{1+i\epsilon/\tilde{\Gamma}_n} \tag{1.2}$$

from which the cross section autocorrelation function in a single partial wave and in the absence of direct reactions is obtained as (see Appendix B)

$$C_{cc'}(\epsilon) = \left| C_{cc'}^{S}(\epsilon) \right|^2 . \tag{1.3}$$

The first two approaches (FKK and AWM) explicitly employ multiclass models, writing $H = H_o + V$ and sorting the levels associated with H_o into "classes" (generally by their degree of coupling to a particular entrance channel, which may be determined by a quantum number such as in Griffin's original exciton model [1]) which then define the various states of the reaction. Although these approaches differ in calculational techniques, they both rely on the following assumptions:

a) The many-body matrix elements of the Hamiltonian governing the reaction are random in phase (i.e., $\sigma_{cc'}^{f\ell}$ depends only on probabilities without coherent phase information, just as in a one-stage Hauser-Feshbach calculation).

b) Overlapping resonances for each class.

c) The number of open channels is large.

Employing these approximations, these first two approaches produce detailed derivations of $\sigma_{cc'}^{f\ell}$, which (as has been discussed elsewhere [7a]) can be written in the form of Eq. (1.1), with explicit recipes for X's.

The last two approaches (HM and F) are closer to the original Hauser-Feshbach point of view. They obtain $\sigma_{cc'}^{f\ell}$, in the Eq. (1.1) form, but they do not employ a specific model (e.g., particle-hole) to define the classes. The HM approach uses assumptions (b) and (c). In addition, it assumes that the classes of states have very different correlation widths, $\tilde{\Gamma}_n \gg \tilde{\Gamma}_{n+1}$, and that the phases of partial-width amplitudes (for all classes) are random.* Rather than giving a model-based recipe for the X's, this approach primarily provides their relation to "optical-model" transmission coefficients, which are to be obtained from a nested sequence of energy averages of S-matrices. The FKK and HM approaches arrive at a result which describes a Markovian process, presumably as a consequence of the statistical assumptions that were made; AWM obtains a similar result in the limit in which their parameter y is small. Finally, the F approach rests only on the Markovian assumption. It does not assume the existence of resonances, but merely assumes the existence of coupled stages and traces the flow of (conserved) probability among them, assuming the exit route from each stage to be statistically independent of its entrance route. It also leads to a cross section from channel c to c' of the Eq. (1.1) form, in this case with no restrictions on the lifetimes or widths of the stages, or the number of open channels. Since this approach does not explicitly involve the energy-dependence of the S-matrix, it cannot be employed to obtain the auto-correlation function.

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*As will be seen below, this produces a substantial algebraic simplification over the method employed in AWM, merely because $\sigma_{cc'}^{f\ell}$ is a simple, explicit function of the partial-width amplitudes, but a very complicated function of the Hamiltonian matrix elements.

This report is organized as follows. In Section 2 we give a simple heuristic derivation of the fluctuation cross section and the cross section autocorrelation function for multistep compound processes using the nested-average method. In Section 3 we supply the complete quantum mechanical foundation of the nested average model using Feshbach's projection operator techniques and generalization of the optical-background representation of Kawai, Kerman and McVoy [8]. In Section 4 we present a simple physical interpretation of the results of the nested-average model using the probability flow approach of [6], and discuss the results of other formulations [3,4]. In Section 5 we discuss possible applications of our results and indicate the range of their validity, and finally, we present several concluding remarks in Section 6.

2. DERIVATION OF $\sigma^{fl}_{cc'}$ AND THE AUTOCORRELATION FUNCTION IN THE NESTED-AVERAGE APPROACH

2.1. A Simple Derivation of a Generalized Hauser-Feshbach Formula

Before deriving the autocorrelation function, we first provide a very simple derivation of the fluctuation cross section, using the nested-average method as a generalization of the Hauser Feshbach (HF) approach.

The conventional (HF) theory defines as "direct" those processes that are governed by the energy average of the S-matrix, while it defines as "compound" those processes governed by the remainder of the S-matrix energy dependence. The energy averaged cross section for any reaction $c \rightarrow c'$ has the form $\langle \sigma_{cc'} \rangle = \sigma^{dir}_{cc'} + \sigma^{fl}_{cc'}$, i.e., an incoherent sum of contributions from these two processes. Furthermore, under the following

three assumptions:

a) that the S-matrix is unitary; b) the S-matrix elements are invariant under time reversal; and c) that the decay of the compound nucleus is independent of its formation,* the Hauser-Feshbach formalism permits the evaluation of the compound contribution to the averaged cross section using factors which depend only on the direct, or energy averaged, S-matrix.

We shall extend HF formalism by generalizing the concept of the "direct" processes. We shall also define them as those processes governed by an energy averaged S-matrix with the additional provision that we may take arbitrary energy averaging intervals, I, and by so doing associate different "direct" and "compound" processes with different energy averaging intervals I. Whereas in the original HF approach the terms "direct" and "compound" had a relatively unique meaning, in our approach these terms require the further specification of the averaging interval, I.

Let us consider the following generalization of the HF representation of the S-matrix [9], [5],

$$\underset{\sim}{S} = \overline{\underset{\sim}{S}}_1 + (\overline{\underset{\sim}{S}}_2 - \overline{\underset{\sim}{S}}_1) + (\overline{\underset{\sim}{S}}_3 - \overline{\underset{\sim}{S}}_2) + \ldots + (\underset{\sim}{S} - \overline{\underset{\sim}{S}}_N) \ , \tag{2.1}$$

where $\overline{\underset{\sim}{S}}_n \equiv \langle \underset{\sim}{S} \rangle_{I_n}$ represents an average of $\underset{\sim}{S}$ over an energy interval I_n, and $I_n \gg I_{n+1}$.

If we interpret each of the above differences as a fluctuation contribution relative to the appropriate energy average,

$$\underset{\sim}{S}^{fl}_n = \overline{\underset{\sim}{S}}_{n+1} - \overline{\underset{\sim}{S}}_n \ , \tag{2.2}$$

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*This implies, of course, the absence of direct reactions. This restriction will be lifted in the following section.

we have

$$\underset{\sim}{S} = \overline{\underset{\sim}{S}}_1 + \sum_{n=1}^{N} \underset{\sim}{S}_n^{f\ell} \tag{2.3}$$

In the original HF formalism, N=1, so that $\underset{\sim}{S}$ involves only $\overline{\underset{\sim}{S}}_1$ (the optical S-matrix) and one fluctuating contribution. If n=N in (2.2), we define $\underset{\sim}{S}_{N+1} = \underset{\sim}{S}$.

Clearly, the definition in Eq. (2.2) provides that $\left\langle \underset{\sim}{S}_n^{f\ell} \right\rangle_{I_n} = \left\langle \underset{\sim}{S}_n^{f\ell} \right\rangle_{I_1} = 0$ by construction. To proceed we make one more assumption, namely, $\left\langle \underset{\sim}{S}_n^{f\ell} \underset{\sim}{S}_{n'}^{f\ell *} \right\rangle_{I_1} = 0$ for $n \neq n'$. This is valid when the characteristic energy variations of the successive $\underset{\sim}{S}_n^{f\ell}$, i.e., their correlation widths $\tilde{\Gamma}_n$, are distinctly different

$$\tilde{\Gamma}_1 \gg \tilde{\Gamma}_2 \gg \cdots \gg \tilde{\Gamma}_N \tag{2.4}$$

and the averaging intervals I_n are chosen in the nested fashion.

$$\tilde{\Gamma}_{n-1} \gg I_n \gg \tilde{\Gamma}_n \tag{2.5}$$

Implied in the representation of $\underset{\sim}{S}$ given in Eq. (2.1) is a set of "compound" and "direct" processes characterized by I_n: direct processes from c to c' are associated with $\overline{S}_{n,cc'}$; transmission coefficients $P_{n,c}$ for "getting into" the corresponding "compound" nucleus are given by $P_{n,c} = 1 - \sum_{c'} |\overline{S}_{n,cc'}|^2$.

Using Eq. (2.3) we have

$$\left\langle \sigma_{cc'} \right\rangle_{I_1} = \left\langle |\delta_{cc'} - S_{cc'}|^2 \right\rangle_{I_1} = |\delta_{cc'} - \overline{S}_{1,cc'}|^2 + \sum_{n=1}^{N} \left\langle |S_{n,cc'}^{f\ell}|^2 \right\rangle_{I_1} . \tag{2.6}$$

The first term is the conventional direct or optical contribution, whereas the remainder involves an incoherent sum over contributions from classes n, which we label $\left\langle \sigma^{f\ell}_{n,cc'} \right\rangle_{I_1} = \left\langle |S^{f\ell}_{n,cc'}|^2 \right\rangle_{I_1} \equiv \sigma^{f\ell}_{n,cc'}$.

Next we use standard HF techniques to find and to interpret these contributions. From Eq. (2.2) we have

$$\sigma^{f\ell}_{n,cc'} = \left\langle |\bar{S}_{n+1,cc'}|^2 - |\bar{S}_{n,cc'}|^2 \right\rangle_{I_1} , \tag{2.7}$$

the generalization of the familiar $\langle |S|^2 \rangle - |\langle S \rangle|^2$.

We now introduce the assumption that each partial cross section $\sigma^{f\ell}_{n,cc'}$ is the product of a factor, σ^{n}_{c}, giving the probability for going from c into the "structure" associated with class n and a factor for the decay from this structure into channel c'. The former is found by summing $\sigma^{f\ell}_{n,cc'}$ over c' to obtain

$$\sigma^{n}_{c} = \sum_{c'} \sigma^{f\ell}_{n,cc'} = \left\langle \sum_{c'} |\bar{S}_{n+1,cc'}|^2 - \sum_{c'} |\bar{S}_{n,cc'}|^2 \right\rangle_{I_1}$$

$$= \left\langle P_{n,c} - P_{n+1,c} \right\rangle_{I_1} , \tag{2.8}$$

where the transmission factors $P_{n,c}$ represent the probability of entering the "compound" nucleus associated with the respective averaging intervals. Just as in the customary HF argument, this immediately implies that

$$\sigma^{f\ell}_{n,cc'} = \begin{cases} \dfrac{\left\langle P_{n,c} - P_{n+1,c} \right\rangle_{I_1} \left\langle P_{n,c'} - P_{n+1,c'} \right\rangle_{I_1}}{\sum_{c''} \left\langle P_{n,c''} - P_{n+1,c''} \right\rangle_{I_1}} , & n < N \\ \dfrac{\left\langle P_{N,c} \right\rangle_{I_1} \left\langle P_{N,c'} \right\rangle_{I_1}}{\left\langle P_{N,c''} \right\rangle_{I_1}} , & n = N \end{cases} \tag{2.9}$$

which expresses $\sigma_n^{f\ell}$ entirely in terms of energy-averaged quantities.

2.2. The Nested-Energy-Average Derivation

The fundamental assumption of the nested-average approach is that the compound (A_1+A_2) system through which the reaction proceeds exhibits intermediate structure and that this structure clearly separates into "classes", each identified by a width $\tilde{\Gamma}_n$ and a lifetime $\hbar/\tilde{\Gamma}_n$. The exciton model [1], e.g., is one which similarly employs the concept of classes of states distinguished by the number of particle-hole pairs; these, however, are "model" classes, identified by configurations of a model Hamiltonian H_o. The class states we refer to, in contrast, are obtained by performing energy averages over the exact S-matrix, and hence over the exact resonant eigenstates of the full Hamiltonian H_o+V. For class n, these "states" correspond to the resonant structure of $\underset{\sim}{\bar{S}}_{n+1}$ and possess a correlation width $\tilde{\Gamma}_n$. These correlation widths can, in principle, be directly obtained experimentally by fitting Eq. (2.17) below to autocorrelation data, without reference to any model Hamiltonian H_o. As mentioned above, an essential assumption of the nested-average approach is that the widths $\tilde{\Gamma}_n$ and $\tilde{\Gamma}_{n+1}$ are sufficiently different that classes can be unambiguously assigned on the basis of these widths. In practice a factor of, say, 5 between successive widths may be adequate.

With the I_n chosen according to Eq. (2.5), $\underset{\sim}{S}_n^{f\ell}(E)$ will contain only the energy-structure associated with class n. It will be shown in Section 3 that, by a procedure similar to that of Kawai, Kerman and McVoy (KKM) [8], $\underset{\sim}{S}_n^{f\ell}(E)$ can formally be written as a pure "pole sum",

$$S^{f\ell}_{n,cc'}(E) = -i \sum_i \frac{g_{ni,c}\, g_{ni,c'}}{E - \mathcal{E}_{ni}} \tag{2.10}$$

Eq. (2.10) defines the resonances in class n for the nested-average approach. The $g_{ni,c}$ have an energy dependence on the scale of classes n-1,n-2,...,1.

We employ this standard sum-over-poles form to obtain the S-matrix autocorrelation function,

$$C^S_{cc'}(\epsilon) = \left\langle S^{f\ell}_{cc'}(E)\; S^{f\,*}_{cc'}(E+\epsilon) \right\rangle_{I_1}, \tag{2.11}$$

from which $\sigma^{f\ell}_{cc'}$ can be obtained as $\sigma^{f\ell}_{cc'} = C^S_{cc'}(0)$. An important step in the argument is the recognition, from Eqs. (2.3) and (2.10) that the total $\underset{\sim}{S}^{f\ell}(E)$ can be written as a sum over the resonances associated with all classes. A key assumption, which is a slight extension of that employed by Ericson [10] and KKM, is that the partial-width amplitudes $g_{ni,c}$ of Eq. (2.10) are random in the level-indices ni, which eliminates all (ni, n'i') cross terms, either within a class or between two classes. This latter assumption requires overlapping resonances within each class, i.e.,

$$\Gamma_n \gg D_n$$

where $\Gamma_n/2 = \left\langle \mathrm{Im}\, \mathcal{E}_{ni} \right\rangle_i$. A slight generalization of the KKM argument then gives

$$C^S_{cc'}(\epsilon) = \left\langle S^{f\ell}_{cc'}(E)\; S^{f\,*}_{cc'}(E+\epsilon) \right\rangle_{I_1} \approx \sum_n \left\langle S^{f\ell}_{n,cc'}(E)\; S^{f\ell\,*}_{n,cc'}(E+\epsilon) \right\rangle_{I_1}$$

$$\approx \sum_n \left\langle \sum_i \frac{|g_{ni,c}|^2\, |g_{ni,c'}|^2}{(E-E_{ni} + i\,\Gamma_{ni}/2)(E-E_{ni}+\epsilon - i\,\Gamma_{ni}/2)} \right\rangle_{I_1}$$

$$\approx \sum_n \frac{2\pi}{D_n} \left\langle |g_{ni,c}|^2 \, |g_{ni,c'}|^2 \right\rangle_i \left\langle \frac{1}{\Gamma_{ni} + i\epsilon} \right\rangle_i$$

$$\approx \sum_n \frac{2\pi}{D_n} \left\langle |g_{ni,c}|^2 \, |g_{ni,c'}|^2 \right\rangle_i \frac{1}{\tilde{\Gamma}_n + i\epsilon} , \qquad (2.12a)$$

where we have defined

$$\left\langle \frac{1}{\Gamma_{ni}} \right\rangle_i = \frac{1}{\tilde{\Gamma}_n} \qquad (2.12b)$$

A detailed discussion of the approximations employed here can be found in Ref. [11]. We have dropped terms of order $\tilde{\Gamma}_n / I_1$ and ϵ / I_1, but have not assumed all states of class n to have the same width; this means that in general $\tilde{\Gamma}_n < \Gamma_n$.

If, following KKM, we define a matrix X_n for each class by*

$$X_{n,cc'} = \sqrt{\frac{2\pi}{\tilde{\Gamma}_n D_n}} \left\langle g_{ni,c} \, g^*_{ni,c'} \right\rangle_{E_{ni} \in I_n} , \qquad (2.13)$$

we obtain

$$C^S_{cc'}(\epsilon) = \sum_n \left\langle X_{n,cc} \, X_{n,c'c'} + X_{n,cc'} \, X_{n,c'c} \right\rangle_{I_1} \frac{\tilde{\Gamma}_n}{\tilde{\Gamma}_n + i\epsilon}$$

$$= \sum_n \sigma^{f\ell}_{n,cc'} \frac{\tilde{\Gamma}_n}{\tilde{\Gamma}_n + i\epsilon} , \qquad (2.14)$$

where we have used the KKM relation

$$\sigma^{f\ell}_{n,cc'} = \left\langle X_{n,cc} \, X_{n,c'c'} + X_{n,cc'} \, X_{n,c'c} \right\rangle_{I_1} \qquad (2.15a)$$

for $\sigma^{f\ell}_{n,cc'} = \left\langle |S^{f\ell}_{n,cc'}|^2 \right\rangle_{I_1}$ defined in Section 2.1.

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*This definition includes an additional square root factor not included in Ref. [8].

From the $\epsilon=0$ limit of the last line of Eq. (2.14), we have

$$\sigma^{f\ell}_{cc'} = \sum_n \sigma^{f\ell}_{n,cc'} \quad , \tag{2.15b}$$

an incoherent sum over classes, as also obtained by the argument in Section 2.1.

As for the cross section autocorrelation function, $C_{cc'}(\epsilon)$, itself, we use Ericson's general result [10] that, in the presence of direct reactions, it can be expressed in terms of $C^S_{cc'}(\epsilon)$ by the relation (see Appendix B)

$$\begin{aligned} C_{cc'}(\epsilon) &= \left\langle \sigma_{cc'}(E)\, \sigma_{cc'}(E+\epsilon) \right\rangle_{I_1} - \left\langle \sigma_{cc'}(E) \right\rangle_{I_1} \left\langle \sigma_{cc'}(E+\epsilon) \right\rangle_{I_1} \\ &= \left| C^S_{cc'}(\epsilon) \right|^2 + 2\sigma^{dir}_{cc'} \ \mathrm{Re}\, C^S_{cc'}(\epsilon) \ . \end{aligned} \tag{2.16}$$

This immediately yields our central result,

$$C_{cc'}(\epsilon) = \left| \sum_n \sigma^{f\ell}_{n,cc'} \frac{\tilde{\Gamma}_n}{\tilde{\Gamma}_n + i\epsilon} \right|^2 + 2\sigma^{dir}_{cc'} \sum_n \sigma^{f\ell}_{n,cc'} \frac{\tilde{\Gamma}_n^2}{\tilde{\Gamma}_n^2 + \epsilon^2} \ , \tag{2.17}$$

which is a direct generalization of Ericson's 1-class formula. It is this expression which was fitted to the experimental data of Fig. 2.

An interesting and important generalization of Eq. (2.17) can be obtained without further effort, by recognizing that what was called $\underset{\sim}{\bar{S}}_1 = \underset{\sim}{S}^{dir}$ in (2.3) need not be "purely direct" in nature. All that is required of $\underset{\sim}{S}^{dir}$ is that it be constant across the averaging interval I_1. Consequently, if, e.g., $\tilde{\Gamma}_1$ is the largest doorway width present, and $I_1 \rangle \tilde{\Gamma}_1$, Eq. (2.17) is valid as it stands, with n=1 included in the

sums. If however, we average over a smaller interval I_2, with $\Gamma_1 \gg I_2 \gg \Gamma_2$, then the n=1 term from the sum of Eq. (2.3) will be removed, and will appear instead in $\bar{\underset{\sim}{S}}_2$. Then for $\epsilon \ll I_2$, Eq. (2.17) is still valid, except that the n=1, or widest doorway, term is missing from the sum of fluctuation contributions. (In this case the region $\epsilon \gg I_2$ is uninteresting. All the fluctuation contributions to $C(\epsilon)$ for $n > 1$ will be small because the energy dependence due to the n=1 doorways is essentially unaveraged, so that $\langle \sigma\sigma \rangle - \bar{\sigma}\bar{\sigma} = 0$. Thus for $\epsilon \gg I_2$, $C(\epsilon) \approx 0$.)

This suggests that $C(\epsilon)$ be generalized to a 2-variable function, $C(\epsilon, I)$, with Γ-information obtainable from its dependence on both variables. Exactly this was done several years ago by Pappalardo, [18] who investigated experimentally the special case of $C(0,I)$, obtaining the "sedia di Pappalardo", a somewhat chair-shaped curve suggesting the presence of two steps-up, at two different values of I. This behavior follows immediately from Eq. (2.17), which we can write as

$$\begin{aligned} C(0,I) &= [\sigma^{f\ell}(I)]^2 + 2\,\sigma^{dir}(I)\,\sigma^{f\ell}(I) \\ &= (\sigma^{tot})^2 - [\sigma^{dir}(I)]^2 \quad , \end{aligned} \tag{2.18}$$

where

$$\sigma^{f\ell}(I) = \sum_{n=n_o(I)}^{N} \sigma_n^{f\ell} \quad , \tag{2.19}$$

with the lower limit n_o of the sum determined by the condition $\tilde{\Gamma}_{n_o} \ll I$. In other words, $\sigma^{tot} = \sigma^{dir}(I) + \sum_{n=n_o(I)} \sigma_n^{f\ell}$ is independent of the averaging interval I (whose choice merely determines how the S-matrix is to be divided between "direct" and "fluctuation" parts), but $C(0,I)$ assigns different weights to σ^{dir} and $\sigma^{f\ell}$, and so does depend on the division determined by I. In particular, as I increases, $\sigma^{dir}(I)$

decreases, and C(0,I) increases as Pappalardo found experimentally [18]. Although the approximations we have made do not permit us to calculate the exact shape of the curve, it is clear that it will rise more of less abruptly each time I increases past one of the $\widetilde{\Gamma}_n$, thus providing a means of obtaining the $\widetilde{\Gamma}_n$'s directly from the variance of the cross section itself, without constructing the autocorrelation function. The full function $C(\epsilon, I)$ may be worth investigating; we conjecture that its form should be something like that indicated* in Fig. 4.

The only task remaining is to express $\sigma^{f\ell}_{n,cc'}$, Eq. (2.15a), in terms of transmission matrices corresponding to the various energy-averaging intervals I_n. To this end we obtain from Eq. (2.2)

$$\left\langle \underset{\sim}{S}^{f\ell\dagger}_n \underset{\sim}{S}^{f\ell}_n \right\rangle_{I_n} = \left\langle \underset{\sim}{\bar{S}}^{\dagger}_{n+1} \underset{\sim}{\bar{S}}_{n+1} - \underset{\sim}{\bar{S}}^{\dagger}_n \underset{\sim}{\bar{S}}_n \right\rangle_{I_n} , \tag{2.20}$$

where we have used the fact that $\left\langle \underset{\sim}{S}^{f\ell\dagger}_{n+1} \underset{\sim}{S}^{f\ell}_n \right\rangle_{I_n} = 0$. Defining for any class n a transmission matrix in the channel indices (analogous to Satchler's $\underset{\sim}{P} = \underset{\sim}{I} - \underset{\sim}{\bar{S}}^{\dagger}\underset{\sim}{\bar{S}}$ for one class)

$$\underset{\sim}{P}_n = \underset{\sim}{I} - \underset{\sim}{\bar{S}}^{\dagger}_n \underset{\sim}{\bar{S}}_n \tag{2.21}$$

we have

$$\left\langle \underset{\sim}{S}^{f\ell\dagger}_n \underset{\sim}{S}^{f\ell}_n \right\rangle_{I_n} = \left\langle \underset{\sim}{P}_n - \underset{\sim}{P}_{n+1} \right\rangle_{I_n} . \tag{2.22}$$

We are defining $\underset{\sim}{\bar{S}}_{N+1} = S$, $\underset{\sim}{P}_{N+1} = 0$, so that $\left\langle \underset{\sim}{S}^{f\ell\dagger}_N \underset{\sim}{S}^{f\ell}_N \right\rangle_{I_N} = \underset{\sim}{P}_N$. However, the methods of KKM immediately allow us to evaluate this same expression in terms of the $\underset{\sim}{X}$'s,

---

*It is important to recognize that for I small, $C(\epsilon, I)$ is really $C(\epsilon, I, E_o)$, and depends also on E_o, the center of the averaging interval.

$$\left\langle \underset{\sim}{S}_n^{f\ell\dagger} \underset{\sim}{S}_n^{f\ell} \right\rangle = \underset{\sim}{X}_n \, \mathrm{Tr}\, \underset{\sim}{X}_n + \underset{\sim}{X}_n^2 \quad . \tag{2.23}$$

Consequently, our net conclusion from unitarity is that

$$\left\langle \underset{\sim}{P}_n - \underset{\sim}{P}_{n+1} \right\rangle_{I_n} = \underset{\sim}{X}_n \, \mathrm{Tr}\, \underset{\sim}{X}_n + \underset{\sim}{X}_n^2 \approx \underset{\sim}{X}_n \, \mathrm{Tr}\, \underset{\sim}{X}_n \ , \tag{2.24}$$

the latter approximation being the one customarily employed [8] when the number of open channels is large. Employing it, we have

$$\underset{\sim}{X}_n = \frac{\underset{\sim}{P}_n - \left\langle \underset{\sim}{P}_{n+1} \right\rangle_{I_n}}{\left| \mathrm{Tr}\, \left(\underset{\sim}{P}_n - \left\langle \underset{\sim}{P}_{n+1} \right\rangle_{I_n} \right) \right|^{1/2}} \tag{2.25}$$

Inserting (2.23) into (2.15) gives the fluctuation cross section in terms of the "optical" transmission matrices of the problem

$$\sigma_{cc'}^{f\ell} = \left\langle \sum_{n=1}^{N-1} \frac{\left(P_n - \langle P_{n+1} \rangle_{I_n} \right)_{cc} \left(P_n - \langle P_{n+1} \rangle_{I_n} \right)_{c'c'} + \left(P_n - \langle P_{n+1} \rangle_{I_n} \right)_{cc'} \left((P_n - \langle P_{n+1} \rangle)_{I_n} \right)_{cc'}}{\mathrm{Tr} \left(\underset{\sim}{P}_n - \left\langle \underset{\sim}{P}_{n+1} \right\rangle_{I_n} \right)} \right.$$

$$\left. + \frac{P_{N,cc} P_{N,c'c'} + P_{N,cc'} P_{N,c'c}}{\mathrm{Tr}\, \underset{\sim}{P}_N} \right\rangle_{I_1} \tag{2.26a}$$

Using the result of Appendix A, we can factorize the above averages, so that we have

$$\sigma_{cc'}^{f\ell} = \sum_{n=1}^{N-1} \frac{\langle P_{n,cc} - P_{n+1,cc} \rangle_{I_1} \langle P_{n,c'c'} - P_{n+1,c'c'} \rangle_{I_1} + \langle P_{n,cc'} - P_{n+1,cc'} \rangle_{I_1} \langle P_{n,c'c} - P_{n+1,c'c} \rangle_{I_1}}{\mathrm{Tr} \left\langle \underset{\sim}{P}_n - \underset{\sim}{P}_{n+1} \right\rangle_{I_1}}$$

$$+ \frac{\langle P_{N,cc} \rangle_{I_1} \langle P_{N,c'c'} \rangle_{I_1} + \langle P_{N,cc'} \rangle_{I_1} \langle P_{N,c'c} \rangle_{I_1}}{\mathrm{Tr} \left\langle \underset{\sim}{P}_N \right\rangle_{I_1}} \tag{2.26b}$$

The factorization holds if the number of channels $N \gg 1$, and $\Gamma_n/D_n \gg 1$, both of which have been assumed throughout. In the absence of direct reactions, $X_{n,cc'} \cong 0$, $c \neq c'$, as is shown in Eq. (A.32), so the nth term of (2.26b) reduces to Eq. (2.9), obtained from our previous, abbreviated, argument.

Eq. (2.26) is our central result. Its essential feature, which distinguishes it from, e.g., a similar result by AWM [4], is that it is automatically separated, by the use of nested energy-averages, into contributions corresponding to different time-delays. For example, if only the energy average over the smallest interval I_N were performed, the corresponding $\langle \sigma^{fl} \rangle_{I_N}$ would contain only the last term of Eq. (2.26). The others, which correspond to time-delays less than $\hbar/I_N$, could not be distinguished from the direct-reaction components in a measurement whose energy resolution is $E = I_N$. Thus, using successively wider energy-averaging intervals moves successively more of the precompound components from σ^{dir} into σ^{fl}. The fact that σ^{fl} is given by Eq. (2.26) as a sum of generalized Hauser-Feshbach terms evidently implies that an equilibrium among the degrees of freedom of each class n is reached before the system decays back into the open channels, so that the process appears to be Markovian at each stage. We note that Eq. (2.26b) is manifestly unitary, i.e., $\sum_c \sigma^{fl}_{cc'} = P_{l,cc}$.

2.3 σ^{fl} and the τ-Matrices

Although Eq. (2.26) provides the desired generalization of the 1-class Hauser-Feshbach expression, previous authors [4] have obtained their results in a somewhat different form, expressed in terms of a set of transmission matrices $\underset{\sim}{\tau}_n$, which are the transmission matrices for the hypothetical case of an S-matrix involving only the states of the nth

class. As we shall see, the relation between our penetration matrices $\underset{\sim}{P}$'s and these $\underset{\sim}{\tau}$'s is a linear one. We shall give here a heuristic presentation of the results which will be more fully discussed in Sections 3 and 4.

Let us first interpret the quantity σ_c^n introduced in Eq. (2.8). We see that this quantity literally represents the cross section for going into those configurations which are considered part of the "compound" nucleus with regard to an averaging interval I_n, but not those states which are considered part of the compound nucleus with regard to the averaging interval I_{n+1}. To make this interpretation more concrete, we use the techniques of the Feshbach projection operators. Let us divide the Hilbert space for the total wave function into portions associated with projection operators p, $d_1, d_2, d_3, d_4, \ldots, d_N$ where p represents the "optical" open channels, and $d_1 \ldots d_N$, the sequence of classes. One example of such a division is to be found in FKK. These portions of Hilbert space are intimately related to the averaging intervals discussed above.

The operator p represents the space for the "direct" processes associated with the averaging interval I_1. The operator $p+d_1 = P_1$ represents the space for "direct" processes associated with I_2, and in general $p+d_1+\ldots+d_{n-1}$ represents the space for the "direct" process associated with I_n. (This symbol, P_n, should be distinguished from the penetration coefficient matrix $P_{n,cc'}$ we have used above and which is a matrix in channel space.) The space complementary to P_{n-1} is given by $d_n+d_{n+1}+\ldots+d_N$ for which we introduce the notation $Q_n = d_n + Q_{n+1}$. The reason for explicitly separating d_n from Q_n is because of the important role it plays in the interpretation of $S_n^{f\ell}$ and σ_c^n. In terms of the spaces associated with

these projection operators we see that $\sigma_c^n = \langle P_{n,c} - P_{n+1,c} \rangle_{I_1}$ measures the flux into the space d_n which does not proceed to space Q_{n+1}.

It is convenient to introduce a generalized transmission coefficient, T_{nc}, which represents the probability for getting from channel c into the space of d_n. This includes probability for direct entry and also probability for entry via the doorways d_j $(j < n)$ which are included in P_{n-1}.

To connect T_{nc} with $P_{n,c} - P_{n+1,c}$ we define η_{nm}, $n < m$, as the "downward" branching ratio for a state of d_n to decay directly or indirectly to one of d_m and we define

$$\eta_{n\downarrow} = \sum_{m(>n)} \eta_{nm} \tag{2.25}$$

as the total downward branching ratio out of n. It then follows that

$$\langle P_{n,c} - P_{n+1,c} \rangle_{I_1} = T_{n,c} - T_{n,c}\,\eta_{n\downarrow} = T_{n,c}\,(1 - \eta_{n\downarrow}) \tag{2.26}$$

gives the flux which reaches d_n but does not proceed to classes of narrower width.

Motivated by the results of KKM we generalize the transmission coefficient $T_{n,c}$ to the transmission matrices $T_{n,cc'}$, defined in terms of the Satchler penetration matrices $P_{n,cc'}$, as

$$\underset{\sim}{T}_n(1 - \eta_{n\downarrow}) = \underset{\sim}{P}_n - \underset{\sim}{P}_{n+1}\,, \tag{2.27}$$

where P_1 is the penetration matrix of the conventional optical approach. For a large number of open channels we have from Eq. (2.24),

$$(1 - \eta_{n\downarrow})\,\underset{\sim}{T}_n = \underset{\sim}{X}_n \,\mathrm{Tr}\, \underset{\sim}{X}_n\,. \tag{2.28}$$

When written in terms of the T's, Eq. (2.26) assumes the particularly transparent form:

$$\sigma_{cc'}^{f\ell} = \sum_n (1 - \eta_{n\downarrow}) \frac{T_{n,cc} T_{n,c'c'} + T_{n,cc'} T_{n,c'c}}{\mathrm{Tr}\, \underset{\sim}{T}_n} \tag{2.29}$$

which is the simplest possible generalization of the 1-class Hauser-Feshbach expression. The factors $(1 - \eta_{n\downarrow})$ clearly play the role of "depletion factors", indicating how much of the flux which entered d_n from channel c survives the "downward leakage" to decay back to c'. This factor is absent from the n=N term, as well as from the 1-class Hauser-Feshbach formula, because in these cases there are no lower classes available.

Finally, to obtain the relation of the $\underset{\sim}{T}$'s and $\underset{\sim}{P}$'s to the $\underset{\sim}{\gamma}$'s employed by previous authors [4], we define $\underset{\sim}{\gamma}_n$ as the matrix describing <u>direct</u> entry into the states of class n from the open channels. This differs from $\underset{\sim}{T}_n$, which includes both <u>direct</u> and <u>indirect</u> entry into d_n. The relationship between $\underset{\sim}{T}$ and $\underset{\sim}{\gamma}$ is then defined by the following set of equations

$$\underset{\sim}{T}_n = \underset{\sim}{\gamma}_n + \sum_{m=1}^{n-1} \underset{\sim}{T}_m \eta_{mn} \quad , \tag{2.30}$$

where the second term on the righthand side includes all of the indirect entry routes (see Fig. 3).

Consider the specific case of 3 classes. For that case we have the following set of recursion relations

$$\begin{aligned} \underset{\sim}{T}_1 &= \underset{\sim}{\gamma}_1 \\ \underset{\sim}{T}_2 &= \underset{\sim}{\gamma}_2 + \underset{\sim}{T}_1 \eta_{12} \\ \underset{\sim}{T}_3 &= \underset{\sim}{\gamma}_3 + \underset{\sim}{T}_2 \eta_{23} + \underset{\sim}{T}_1 \eta_{13} \quad , \end{aligned} \tag{2.31}$$

which can clearly be extended by induction if more classes are present. Solving them gives

$$\begin{aligned} \underset{\sim}{T}_1 &= \underset{\sim}{\tau}_1 \\ \underset{\sim}{T}_2 &= \underset{\sim}{\tau}_2 + \underset{\sim}{\tau}_1 \eta_{12} \\ \underset{\sim}{T}_3 &= \underset{\sim}{\tau}_3 + \underset{\sim}{\tau}_2 \eta_{23} + \underset{\sim}{\tau}_1 (\eta_{12}\eta_{23} + \eta_{13}) , \end{aligned} \tag{2.32}$$

indicating how all $\underset{\sim}{\tau}_k$'s, $k \leq n$, combine out to describe the flux into d_n from above.

For comparison with the work of previous authors, we restrict ourselves further to the case of only 2 classes and assume no direct reactions. Eq. (2.29) for the fluctuation cross section then becomes, in terms of the $\underset{\sim}{\tau}$'s,

$$\sigma^{f\ell}_{cc'} = (1-\eta_{12}) \frac{\tau_{1,cc}\,\tau_{1,c'c'}}{\mathrm{Tr}\,\underset{\sim}{\tau}_1} + \frac{(\underset{\sim}{\tau}_2 + \underset{\sim}{\tau}_1\eta_{12})_{cc}(\underset{\sim}{\tau}_2 + \underset{\sim}{\tau}_1\eta_{12})_{c'c'}}{\mathrm{Tr}(\underset{\sim}{\tau}_2 + \underset{\sim}{\tau}_1\eta_{12})} \tag{2.33}$$

Since, for the 2-class case, the downward branching η_{12} is nothing more than the direct coupling from d_1 to d_2 called μ in Ref. [4], we have agreement with the results of those references, especially 4d.

2.4 A Brief Comparison with the Heidelberg Approach, AWM.

Although Section 4.2 below is devoted to an extended consideration of the relation between the nested-average (NA) and other approaches to the multi-step compound problem, we include here a brief comparison with the Heidelberg work (AWM).

Eq. (2.33) for $\sigma^{f\ell}_{cc'}$ is obviously a bilinear form in the transmission matrices τ_m which couple the classes directly to the channels, and could be written

$$\sigma^{f\ell}_{cc'} = \sum_{\mu\nu} \tau_{\mu,c} \Pi^{NA}_{\mu\nu} \tau_{\nu,c'} . \tag{2.34}$$

This is exactly the form of the AWM result,

$$\sigma^{fl}_{cc'} = \sum_{mn} \tau^{c}_{m} \Pi_{mn} \tau^{c'}_{n}, \tag{2.35}$$

(except that our τ is their T), and the agreement goes deeper than just the form, because of the fundamental fact that both formalisms are unitary, i.e., probability-conserving. In the NA approach, probability-conservation is contained in the relations

$$T_{n,c} = \tau_{n,c} + \sum_{\alpha<n} T_{\alpha,c} \eta_{\alpha n}, \tag{2.30}$$

which express the transmission coefficients $T_{n,c} \equiv T_{n,cc}$ in terms of the τ's. In the AWM formalism, the unitarity relation,

$$\sum_{c'} \sigma^{fl}_{cc'} = P_{1,cc} = \sum_{n} \tau_{n,c'} \tag{2.36}$$

is most conveniently expressed in the form *

$$\sum_{n} \Pi_{mn} \sum_{c} \tau^{c}_{n} = 1, \qquad \text{(all m)} \tag{2.37}$$

where 1 is the column vector (in class space) with all elements equal to 1.

To see that these two expressions of unitarity or flux-conservation are indeed equivalent, we re-write Eq. (2.30) as a matrix transformation in class space (a transformation which diagonalizes Π_{mn}),

$$\tau_{m,c} = \sum_{\alpha} T_{\alpha} N_{\alpha m}, \tag{2.38}$$

where

$$\begin{aligned} N_{\alpha m} &= \delta_{\alpha m} - \eta_{\alpha m}, \quad \alpha \le m, \\ &= 0 \qquad\quad , \quad \alpha > m, \end{aligned} \tag{2.39}$$

i.e., the matrix

$$-N = \begin{bmatrix} -1 & \eta_{12} & \eta_{13} & \eta_{14} \cdots \eta_{1N} \\ & -1 & \eta_{23} & \eta_{24} \\ & & -1 & \eta_{34} \\ & & & -1 \\ & & & & -1 \end{bmatrix}, \tag{2.40}$$

* We thank Y.-T. Tang for pointing out this form, which is expressed as it

By inspection, the rows of N are linearly independent, so N is non-singular, and we can equivalently write (2.38) as

$$T_\alpha = \sum_m \tau_m \, (N^{-1})_{m\alpha}, \tag{2.41}$$

so that

$$\sigma^{fl}_{cc'} = \sum_\alpha \frac{T_{\alpha,c}\, T_{\alpha,c'}}{\sum_b T_{\alpha,b}} \, (1-\gamma_\alpha) \tag{2.42}$$

$$= \sum_{m,n} \tau_{m,c} \, \Pi^{NA}_{mn} \, \tau_{n,c'} \, , \tag{2.43}$$

with $\gamma_\alpha = \sum_{m>\alpha} \eta_{\alpha m}$, (2.25)

where

$$\Pi^{NA}_{mn} = \sum_\alpha \frac{(N^{-1})_{m\alpha} \, (N^{-1})_{n\alpha}}{\sum_\ell \sum_b \tau_{\ell,b} \, (N^{-1})_{\ell\alpha}} \, (1-\gamma_\alpha); \tag{2.44}$$

this expresses Π^{NA} explicitly in terms of the class-mixing coefficients $\eta_{\alpha n}$.

To check the agreement with Eq.(2.37), we construct

$$\sum_n \Pi^{NA}_{mn} \sum_b \tau_{n,b} = \sum_\alpha (N^{-1})_{m\alpha} \, (1-\gamma_\alpha). \tag{2.45}$$

If this is indeed 1, for all m, as Eq. (2.37) requires, we can multiply by N to obtain [from Eq. (2.40)]

$$1 - \gamma_\alpha = \sum_m N_{\alpha m} = 1 - \sum_{m>\alpha} \eta_{\alpha m} \, , \tag{2.46}$$

in agreement with Eq. (2.25). But since N is invertible, we can trace our steps backwards to see that Eq. (2.45) does in fact yield 1, without explicitly constructing N^{-1}. (These manipulations exactly parallel those of the Appendix of ref. 6, without requiring a complete evaluation of Π^{NA}_{mn}.)

Thus the Π_{mn}'s of AWM and NA agree in form, and both conserve flux. This implies that the (many) approximations made in both approaches, although different in detail, carry the same physical content.

Closer agreement than this cannot be expected, for two reasons. First, as

AWM carefully point out, the purpose of their approach is only to determine the form of Eq. (2.35); the specific identification of the doorway classes (and hence the numerical values of the $\tau_{n,c}$ and the Π_{mn}) is purposely left undetermined. Theirs is a $H = H_o + V$ approach, with the doorway classes identified as eigen-configurations of H_o. They are thus guaranteed to be different from our classes, whose doorway states, defined in Eq. (3.45) below, are eigenstates of $d_n H (P_{n-1}) d_n$, an energy-averaged effective (and non-hermitian) hamiltonian. A second difference in detail between our approach and theirs is that, by using optical wave functions to describe the channels, we have effectively employed a somewhat different channel definition, which includes "external mixing" in the channel states themselves. In spite of these differences in details, the way in which the cross section (and auto-correlation function) depends on the classes has exactly the same structure in the two approaches.

Finally, we remark that the restriction $\alpha < n$ on the sums in Eq. (2.37) should <u>not</u> be interpreted to imply that the flow out of α is restricted to be only "downward", towards n. Such a restriction <u>is</u> imposed in the FKK theory, by the assumption that $\rho_{n+1} >> \rho_n$. We have not made such an assumption, however; as Eq. (4.25) explains, $\eta_{\alpha n}$ includes not only the direct coupling from α to n, but also all other routes via intermediate states $m < n$.

2.5 $d\sigma^{fl}/d\Omega$ and the Multi-Class Auto-Correlation Function.

For a detailed experimental investigation of the doorway hierarchy which may be present in pre-equilibrium reactions, the most important result of the present work would seem to be Eq. (2.17) for the cross section auto-correlation function in a single partial wave,

$$C_{cc'}(\varepsilon) = \left| \sum_n \sigma^{fl}_{n,cc'} \frac{\tilde{\Gamma}_n}{\tilde{\Gamma}_n + i\xi} \right|^2 + 2\sigma^{dir}_{cc'} \sum_n \sigma^{fl}_{n,cc'} \frac{\tilde{\Gamma}^2_n}{\tilde{\Gamma}^2_n + \xi^2} \qquad (2.17)$$

In the fortunate circumstance that this partial wave dominates the full auto-correlation function, Eq. (2.17) can be employed, as it was in ref. [2b], to understand the possible occurrence of different coherence widths in different final channels. More generally, its use would seem to provide the most direct experimental test possible for the occurrence of more than one class of intermediate states, in the compound nucleus.

It cannot be expected that a single partial width will in general dominate, so for completeness we give here the full angular dependence of both the fluctuation cross section and the auto-correlation function.

The angular distribution of the multistep compound cross section for going from channel α to α' (with $\alpha \neq \alpha'$) for unpolarized particles is

$$\frac{d\sigma_{\alpha\alpha'}}{d\Omega} = \frac{1}{k_\alpha^2 (2I_1+1)(2I_2+1)} \sum_L \frac{(-)^{S-S'}}{4} \sum_{J\ell\ell'} \bar{Z}(\ell J \ell J, sL)\bar{Z}(\ell' J \ell' J, s' L) \times \left(\sigma^{f\ell,J}_{\alpha s \ell, \alpha' s' \ell'}\right) P_L(\cos\theta_\alpha) \qquad (2.47)$$

where

$$\sigma^{f\ell,J}_{\alpha s \ell, \alpha' s' \ell'} = \sum_n \sigma^{f\ell,J}_{n;\alpha s \ell, \alpha' s' \ell'} = \sum_n (1-\eta^J_n) \frac{T_{n,\alpha s \ell,\alpha s \ell} T_{n,\alpha' s' \ell',\alpha' s' \ell'} + T_{n,\alpha s \ell,\alpha' s' \ell'} T_{n,\alpha' s' \ell',\alpha s \ell}}{\sum_{\alpha_1 s_1 \ell_1} T^J_{n,\alpha_1 s_1 \ell_1,\alpha_1 s_1 \ell_1}} \qquad (2.48)$$

and I_1 and I_2 are the spins of the two particles in the incident channel α; ℓ and ℓ' are the relative angular momenta and s and s', the channel spins, in the incident and exit channel, respectively. The coefficients $\bar{Z}$ are given by

$$\bar{Z}(\ell, J, \ell_2 J_2, sL) = (2\ell_1+1)^{1/2}(2\ell_2+1)^{1/2}(2J_1+1)^{1/2}(2J_2+1)^{1/2} \times (\ell_1 \ell_2 00 | L0)\, W(\ell_1 J_1 \ell_2 J_2, sL) \qquad (2.49)$$

Since in (2.34) only $\ell_1 = \ell_2$ occur in $\bar{Z}$, the Clebsch Gordan coefficients in (2.35) insures that only even L's occur in the sum (2.47). Consequently the angular distribution is symmetric around 90°.

Similarly, the angular dependence of the cross section autocorrelation function, in the case $\alpha \neq \alpha'$ and no direct reactions, is given by:

$$\left\langle \frac{d\sigma_{\alpha\alpha'}(E)}{d\Omega} \frac{d\sigma_{\alpha\alpha'}(E+\epsilon)}{d\Omega} \right\rangle - \left\langle \frac{d\sigma_{\alpha\alpha'}(E)}{d\Omega} \right\rangle \left\langle \frac{d\sigma_{\alpha\alpha'}(\epsilon+\epsilon)}{d\Omega} \right\rangle =$$

$$\left[\frac{1}{4k_\alpha^2 (2I_1+1)(2I_2+1)} \right]^2 \sum_{L\bar{L}} \sum_{J_1 J_2 \ell_1 \ell_2 \ell_1' \ell_2'}$$

$$\bar{Z}(\ell_1 J_1 \ell_2 J_2, sL)\ \bar{Z}(\ell_1' J_1 \ell_2' J_2, s'L) \bar{Z}(\ell_2 J_2 \ell_1 J_1, sL)\ \bar{Z}(\ell_2' J_2 \ell_1' J_1, s'L) \quad x$$

$$\left[\sum_h \sigma_{n,\alpha' s' \ell_i', \alpha s \ell_1}^{f\ell, J_i} \frac{\tilde{\Gamma}_n^{J_1}}{\tilde{\Gamma}_n^{J_1} + i\epsilon} \right] \left[\sum_m \sigma_{m,\alpha' s' \ell_2', \alpha s \ell_2}^{f\ell, J_2} \frac{\tilde{\Gamma}_m^{J_2}}{\tilde{\Gamma}_m^{J_2} - i\epsilon} \right] \qquad (2.50)$$

3. THE PROJECTION OPERATOR TREATMENT OF MULTISTEP COMPOUND PROCESSES IN THE NESTED-DOORWAY MODEL

Our aim in this section is (a) to show that the fluctuation S-matrix associated with any one of the N classes of overlapping resonances can be written in the sum-over-pole form of Eq. (2.10); and (b) to derive explicit expressions for the pole positions and the residues. We accomplish both goals by using Feshbach's projection operator formalism and the KKM representation at each averaging stage. Our approach differs from that of Feshbach, Kerman and Koonin [3] primarily because we begin by isolating the fine-structure resonances of the cross section and work our way "up" through the classes of states to the smooth background associated with the channels, whereas the authors of Ref. [3] begin with the channels and work their way "down" through the doorways to the fine structure. The conditions of our model, namely $\tilde{\Gamma}_1 >> \tilde{\Gamma}_2 >> \tilde{\Gamma}_3 \ldots > \tilde{\Gamma}_N$, permit us to perform the calculation of $\underset{\sim}{S}_n^{f\ell}$ at each averaging stage I_n, in a simple way. Namely for the calculation of $\underset{\sim}{S}_n^{f\ell}$, we treat all classes, $d_{n+1}, d_{n+2}, \ldots d_N$ on the average so they act like a "sink" for the flux reaching d_n and all classes d_{n-1}, d_{n-2}, $\ldots d_1$, together with the open channel subspace p, act like an effective source of the flux that reaches d_n. Here we are using $\mathbb{1} = p + d_1 + \ldots + d_N$. This amounts to dealing with two coupled equations at each of the averaging stages. The effective Hamiltonian entering in these equations contains the doorway classes $d_{n+1}, \ldots d_N$, on the average, as will be explained fully in the next subsection (these classes thus supply d_n with a damping width). In extracting $(S_n^{f\ell})_{cc'}$ from the two coupled equations we use the KKM rep-

resentation. However, the KKM representation is further used in subsection 3 to exhibit the detailed structure of the residue factors $g_{ni,c}$ that appears in $S^{f\ell}_{n,cc'}$. This structure arises from the modulations due to those classes that are present in the "continuum" wavefunction that defines $g_{ni,c}$. It is this decomposition of the $g_{ni,c}$ into their smooth optical components and fluctuation components (arising from the abovementioned modulations), that proves to be essential for the purpose of calculating $\langle X_{n,cc'} \rangle_{I_1}$ in subsection 3.4.

3.1 Projection Operator Treatment of the Nested Doorways Model

In this subsection we generalize the KKM procedure in order to construct the terms $S^{f\ell}_{n,cc'}$ that appear in Eq. (2.3). For completeness, we begin by outlining the main results of KKM. The original one-class formula for $S^{f\ell}_{cc'}$ of KKM, for a given value of the cc' total angular momentum, is given by

$$S^{f\ell}_{c'c} = -2\pi i \left\langle \varphi^{(-)c'}_{opt} \middle| V_{pq} \frac{1}{E - H_{qq} - V_{qp} \mathcal{L}^{(+)}_{opt} V_{pq}} V_{qp} \middle| \varphi^{(+)c}_{opt} \right\rangle \tag{3.1}$$

where $p+q = \mathbb{1}$, and $\varphi^{(\pm)c}_{opt}$, the optical wave function with an incident wave in channel c, is a solution of

$$(E - p\mathcal{H}(q)p)\, \varphi^{(\pm)c}_{opt} = 0 \tag{3.2}$$

$$p\mathcal{H}(q)p = pHp + pHq \frac{1}{E - qHq + iI_q} qHp \tag{3.3}$$

and $\mathcal{G}^{(+)}_{opt}$ is the open-channel space propagator

$$\mathcal{G}^{(+)}_{opt} = (E^{(+)} - p\mathcal{H}(q)\,p)^{-1} \tag{3.4}$$

The form factors V_{qp} and V_{pq} are given by

$$V_{qp} = \sqrt{\frac{iI_q/2}{E-qHq+iI_q/2}}\; qHp \tag{3.5}$$

$$V_{pq} = pHq \sqrt{\frac{iI_q/2}{E-qHq + iI_q/2}} \tag{3.6}$$

The argument q in the effective Hamiltonian $\mathcal{H}(q)$ of Eq. (3.3) stresses the fact that the q-space has been eliminated. Notice that $\mathcal{H}(q)$ does not have an energy dependence on the scale of $\mathcal{E}_{qi}$. Its energy variation is, however, only on the scale of the averaging interval specified by I_q, which is much larger than $\Gamma_{qi}/2$ (the imaginary part of $\mathcal{E}_{qi}$).

The above expression for $S^{f\ell}_{cc'}$ (Eq. (3.1)) may be written in the sum-over-poles form of Eq. (2.10) by using an expansion of the propagator

$$\mathcal{G}_q = (E-qHq - V_{qp}\mathcal{G}^{(+)}_{opt}V_{pq})^{-1} \tag{3.7}$$

in terms of biorthogonal basis states, $|\Psi_{qi}\rangle$ and $|\tilde{\Psi}_{qi}\rangle$, which satisfy the following eigenvalue equations,

$$(\mathcal{E}_{qi} - q\mathcal{H}(p)q)\,|\Psi_{qi}\rangle = 0 \quad , \tag{3.8a}$$

$$(\mathcal{E}^{*}_{qi} - q\mathcal{H}(p)^{\dagger}q)\,|\tilde{\Psi}_{qi}\rangle = 0 \tag{3.8b}$$

where $q\mathcal{H}(p)q$ is the symmetric operator

$$q\mathcal{H}(p)q \equiv qHq + V_{qp}\mathcal{G}^{(+)}_{opt}V_{pq} \tag{3.9}$$

The argument p in the effective operator $\mathcal{H}(p)$ indicates that the p-space has been eliminated. This elimination is accomplished through the introduction of the optical Green's function $\mathcal{G}^{(+)}_{opt}$ (Eq. (3.4)) into Eq. (3.9). The two set of states, $\{|\Psi_{qi}\rangle\}$ and $\{|\widetilde{\Psi}_{qi}\rangle\}$, form a biorthogonal basis characterized by the following conditions,

$$\langle\widetilde{\Psi}_{qi'}|\Psi_{qi}\rangle = \delta_{ii'} \tag{3.10}$$

Since $q\mathcal{H}(p)q$ is a symmetric operator, the dual states, $|\widetilde{\Psi}_{qi}\rangle$ are therefore also the time-reversed images of $|\Psi_{qi}\rangle$, namely $|\widetilde{\Psi}_{qi}\rangle = \theta|\Psi_{qi}\rangle$ where θ is the time reversal operator. Thus,

$$S^{f}_{c'c} = -i\sum_{i}\frac{\sqrt{2\pi}\langle\Phi^{(-)c'}_{opt}|V_{pq}|\Psi_{q_i}\rangle\sqrt{2\pi}\langle\Psi_{qi}|V_{qp}|\Phi^{(+)c}_{opt}\rangle}{E-\mathcal{E}_{qi}} \tag{3.11}$$

Since, in addition $|\Phi^{(-)c'}_{opt}\rangle$ is also the time-reversed image of $|\Phi^{(+)c'}_{opt}\rangle$, i.e., $\theta|\Phi^{(-)c'}_{opt}\rangle = |\Phi^{(+)c'}_{opt}\rangle$, we have

$$\begin{aligned} g_{qi,c'}/\sqrt{2\pi} &\equiv \langle\Phi^{(-)c'}_{opt}|V_{pq}|\Psi_{qi}\rangle \\ &= \langle\Phi^{(-)c'}_{opt}|\theta^{\dagger}\theta V_{pq}|\Psi_{qi}\rangle^{*} \\ &= \langle\Phi^{(-)c'}_{opt}|\theta^{\dagger}(\theta V_{pq}\theta^{-1})\theta|\Psi_{qi}\rangle^{*} \\ &= \langle\Phi^{(+)c'}_{opt}|(\theta V_{pq}\theta^{-1})|\widetilde{\Psi}_{qi}\rangle^{*} \end{aligned}$$

$$= \langle \tilde{\Psi}_{qi} | \, (\theta V_{pq} \theta^{-1})^{+} \, | \varphi_{opt}^{(+)c'} \rangle$$

$$= \langle \tilde{\Psi}_{qi} | \, V_{qp} \, | \varphi_{opt}^{(+)c'} \rangle \qquad (3.12)$$

where we have assumed that V_{pq} satisfies the condition

$$\theta \, V_{pq} \, \theta^{-1} = (V_{qp})^{+} \quad , \qquad (3.13)$$

which is the case for V_{pq} and V_{qp} of Eqs. (3.5) and (3.6):

$$(\theta V_{pq} \theta^{-1}) = \theta (pHq \sqrt{\frac{i\ I/2}{E-qHq + iI/2}}\ )\ \theta^{-1}$$

$$= pHq \sqrt{\frac{-i\ I/2}{E-qHq - iI/2}} \equiv (V_{qp})^{+} \qquad (3.14)$$

the above relation (Eq. (3.14)) is strictly valid only when the total Hamiltonian H is Hermitian and commutes with θ, i.e.,

$$H = H^{+}, \quad [H, \theta] = 0 \qquad (3.15)$$

As a consequence of Eq. (3.12), the following expression for $S^{f\ell}_{c'c}$ is obtained

$$S^{f\ell}_{c'c} = -i \sum_{i} \frac{g_{qi,c'}\, g_{qi,c}}{E - \mathcal{E}_{qi}} \qquad (3.16)$$

By consturction, the energy average of $S^{f\ell}_{cc'}$, $\langle S^{f\ell}_{cc'} \rangle_{I_q}$, is zero, and as a consequence one obtains the following important property of the $g_{qi,c}$'s

$$\langle g_{qi,c'}\, g_{qi,c} \rangle_{i} = 0 \qquad (3.17)$$

where by $\langle \quad \rangle_i$ we imply an average over the states i contained in the interval I_q. In KKM, the further ansatz is made that the complex $g_{qi,c}$'s are sufficiently random that their average is zero, i.e.,

$$\langle g_{qi,c} \rangle_i = 0 \tag{3.18}$$

In defining the biorthogonal states, we have dealt with only the inner product $\langle \tilde{\Psi}_{qi} | \Psi_{qj} \rangle$ (Eq. (3.10)). For later use, two other inner products may be defined namely $\langle \tilde{\Psi}_{qi} | \tilde{\Psi}_{qj} \rangle$ and $\langle \Psi_{qi} | \Psi_{qj} \rangle$, which do not satisfy the orthogonality condition. Furthermore, the diagonal elements, $\langle \tilde{\Psi}_{qi} | \tilde{\Psi}_{qi} \rangle$ and $\langle \Psi_{qi} | \Psi_{qj} \rangle$ are equal and greater than unity as a consequence of Eq. (3.10). The above inner products may be formally evaluated in terms of matrix elements of $q\mathcal{H}(p)q$ (see Eqs. (3.7) and (3.8)) as follows,

$$\langle \Psi_{qi} | \Psi_{qj} \rangle = \frac{2i \langle \Psi_{qi} | \text{Im } q\mathcal{H}(p)q | \Psi_{qj} \rangle}{\mathcal{E}_{qj} - \mathcal{E}^*_{qi}} \tag{3.19}$$

For i=j, the above relation gives

$$\langle \Psi_{qi} | \Psi_{qi} \rangle = \frac{\langle \Psi_{qi} | \text{Im } q\mathcal{H}(p)q | \Psi_{qi} \rangle}{\text{Im} \langle \tilde{\Psi}_{qi} | \quad q\mathcal{H}(p)q | \Psi_{qi} \rangle} \tag{3.20}$$

One may view Eqs. (3.19) and (3.20) as a generalization, appropriate for the KKM representation, of similar relations obtained by Bell and Steinberger[16].

The important feature of Eq. (3.1) is that optical quantities appear in a specific and clear way, namely, in the distorted waves, $\varphi^{(\pm)}_{opt}$, and in defining the average width of the q-resonances. Accordingly, the generalization of Eq. (3.1) to cases where more than one class of resonances is present is simple, as we show below. We recall that KKM start with two coupled equations, one for $p\Psi$ (where Ψ is the total

wave function for the system) and one for $q\Psi$. The operators appearing in these equations are the projections of the original Hermitian many-body nuclear Hamiltonian: pHp, the projection onto the p-space; qHq, the projection onto the q-space; and the couplings pHq and qHp.

Our aim in this subsection is to generalize the above considerations to the case of N classes of overlapping resonances and thus obtain the explicit form of $S^{f\ell}_{n,cc'}$, referred to in Section 2. We call p the projection operator associated with open channels and decompose the closed-channel part of the Hilbert space, $\mathbb{1}$ -p, into N subspaces, each associated with one of the doorway classes that we considered in the previous section. We shall call $d_1, d_2, \ldots, d_N$ the corresponding projection operators with $d_1+d_2+\ldots+d_N = \mathbb{1}-p$; d_N contains the most complicated "fine-structure" states.

As mentioned above, the essential logic of our approach is to start by separating the Hilbert space into a "resonant" part d_N, and a "smooth" part $P_{N-1} = \mathbb{1} - d_N = p+d_1+\ldots+d_N$: the terms "resonant" and "smooth" refer to energy scales determined by the averaging interval I_N, $\widetilde{\Gamma}_{N-1} \rangle I_N \rangle\rangle \widetilde{\Gamma}_N$. In this sense d_N plays precisely the same role as q in the previous (one-class) example. We then proceed iteratively by separating out the "resonant" part of $P_{N-1} = d_{N-1} + P_{N-2}$ (with reference to a scale set by an averaging interval I_{N-1} where $\widetilde{\Gamma}_{N-2} \rangle I_{N-1} \rangle\rangle \widetilde{\Gamma}_{N-1}$), then that of $P_{N-2} = d_{N-2} + P_{N-3}$, etc., until the completely non-"resonant continuum, p, is reached, $p = \mathbb{1} - (d_1+d_2+\ldots+d_N)$. It is therefore clear that in order to extract $S^{f\ell}_n$, (n=1,...N), we have to solve two coupled equations for $d_n\Psi$ and $P_{n-1}\Psi$ with non-Hermitian Hamiltonians that contain the classes $d_{n+1}, d_{n+2}, \ldots d_N$ <u>on the average</u>. This amounts to replacing the N+1

coupled-equations problem by a set of N two coupled-equations. Each of these pairs of equations associated with the corresponding averaging interval, can then be solved for the corresponding $\underset{\sim}{S}^{f\ell}$ using the KKM method. Clearly the above reduction of the problem is possible only if the experimentally accessible correlation width $\{\tilde{\Gamma}_n\}$ satisfy the condition $\tilde{\Gamma}_1 \gg \tilde{\Gamma}_2 \gg \ldots \tilde{\Gamma}_{N-1} \gg \tilde{\Gamma}_N$. The above scheme, therefore, constitutes the essential features of our model. In detail the above arguments proceed as follows.

The complete wave function of the system, Ψ, satisfies Schrodinger's equation

$$(E-H)\Psi = 0 \tag{3.21}$$

where H is the total Hermitian Hamiltonian of the system. We carry out the scheme mentioned above and start by writing $\Psi = (P_{N-1} + d_N)\Psi$. The component $P_{N-1}\Psi \equiv (p+d_1+\ldots+d_{N-1})\Psi$ is then a solution of

$$(E - P_{N-1}H P_{N-1} - P_{N-1}H d_N \frac{1}{E-d_N h d_N} d_N H P_{N-1}) P_{N-1}\Psi^{(\pm)c} = 0 \tag{3.22}$$

The "effective" Hamiltonian in the above equation has energy dependence at the level of the N^{th} class doorways (fine structure states). The wave function $P_{N-1}\Psi^{(\pm)c}$, on the other hand, has energy dependence associated with all classes . The average of $P_{N-1}\Psi^{(\pm)c}$ over an energy interval. In $\tilde{\Gamma}_{N-1} \gg I_N \gg \tilde{\Gamma}_N$, $\langle P_{N-1}\Psi^{(\pm)c}\rangle_{I_N} \equiv \varphi^{(\pm)c}_{P_{N-1}}$, is a solution of [17]

$$(E - P_{N-1}\mathcal{H}(d_N) P_{N-1})\varphi^{(\pm)c}_{P_{N-1}} = 0 \tag{3.23}$$

Eq. (3.23) is the analog of Eq. (3.2) with the "smooth", complex, effective

Hamiltonian $P_{N-1}\mathcal{H}(d_N)P_{N-1}$ given by

$$P_{N-1}\mathcal{H}(d_N)\,P_{N-1} = P_{N-1}\,H\,P_{N-1} + P_{N-1}\,H\,d_N \left\langle \frac{1}{E-d_N H\,d_N} \right\rangle_{I_N} d_N\,H\,P_{N-1}$$

$$= P_{N-1}\,H\,P_{N-1} + P_{N-1}\,H\,d_N \frac{1}{E-d_N\,H\,d_N + \frac{iI_N}{2}}\, d_N\,H\,P_{N-1}\,, \tag{3.24}$$

which is energy-independent within the interval $I_N/2$. The argument d_N in $P_{N-1}\mathcal{H}(d_N)\,P_{N-1}$ indicates, as in $p\mathcal{H}(q)p$ of Eq. (3.3), that the state in d_N appear only on the average. Clarly, $\varphi^{(\pm)c}_{P_{N-1}}$ shows an energy-dependence associated with all the doorway classes in P_{N-1}, i.e., $d_1, d_2, \ldots d_{N-1}$. The S-matrix arising from $P_{N-1}\Psi^{(+)c}$ (Eq. (3.22)) can therefore be expressed as

$$\underset{\sim}{S} = \bar{\underset{\sim}{S}}_N + \underset{\sim}{S}^{f\ell}_N \tag{3.25}$$

where $\bar{\underset{\sim}{S}}_N \equiv \langle \underset{\sim}{S} \rangle_{I_N}$ arises from $\varphi^{(\pm)c}_{P_{N-1}}$ of Eq. (3.23) and the fluctuation term is given in exactly the same form as (3.1),

$$S^{f\ell}_{N,cc'} = -2\pi i \left\langle \varphi^{(-)c'}_{P_{N-1}} \middle| V_{P_{N-1}d_N} [E-d_N H\, d_N - V_{d_N P_{N-1}} \mathcal{G}^{(+)}_{P_{N-1}} V_{P_{N-1}d_N}]^{-1} \right.$$

$$\left. V_{d_N P_{N-1}} \middle| \varphi^{(+)c}_{P_{N-1}} \right\rangle \tag{3.26}$$

where the P_{N-1}-space propagator, $\mathcal{G}^{(+)}_{P_{N-1}}$, is

$$\mathcal{G}^{(+)}_{P_{N-1}} = (E^{(+)} - P_{N-1}\mathcal{H}(d_N)P_{N-1})^{-1} \tag{3.27}$$

and the form factors $V_{P_{N-1}d_N}$ and $V_{d_N P_{N-1}}$ are

$$V_{P_{N-1}d_N} = P_{N-1}\, H\, d_N \left[\frac{iI_N/2}{E-d_N\, H\, d_N + i\, I_{N/2}} \right]^{1/2} \tag{3.28}$$

$$V_{d_N P_{N-1}} = \left[\frac{i\, I_N/2}{E-d_N\, H\, d_N + i\, I_{N/2}} \right]^{1/2} d_N H\, P_{N-1} \tag{3.29}$$

Since H above is the total Hermitian Hamiltonian, the result for $(S_N^{f\ell})_{cc'}$ above can be further reduced to exactly the same form as Eq. (3.16) for the one class formula, namely,

$$(S_N^{f\ell})_{cc'} = -i \sum_{i\epsilon\{d_N\}} \frac{g_{Ni,c}\, g_{Ni,c'}}{E - \mathcal{E}_{Ni}} \tag{3.30}$$

The major difference between Eq. (3.30) and Eq. (3.16) is (a) the $g_{Ni,c}$ in the former equation contains doorway modulations arising from the doorway classes in P_{N-1}, i.e., $d_1, d_2, \ldots d_{N-1}$, and (b) the average of the widths Γ_{Ni} that enters in the complex $\mathcal{E}_{Ni}$, s, is given by Γ_N,

$$\frac{\Gamma_N}{2} = -\mathrm{Im} \left\langle \tilde{\Psi}_{Ni} \right| \left[V_{d_N P_{N-1}} \mathcal{G}_{P_{N-1}} V_{P_{N-1} d_N} \right] \left| \Psi_{Ni} \right\rangle\!\!\rangle_i \tag{3.31}$$

Since the $\mathcal{G}_{P_{N-1}}$ in Eq. (3.31) is an (NxN) matrix propagator, Γ_N is composed in general of N^2 terms. We shall further discuss the average width at the end of this subsection. Although the calculation of $S_N^{f\ell}$ is identical with that of the one class case we have, nevertheless, given a detailed account of it in order to fix the notation as well as to present the major differences cited above.

We now turn to the calculation of the other terms in $S^{f\ell}_{cc'}$. This amounts to extracting from $\bar{S}_N$ of Eq. (3.25) the contributions $\bar{S}_{N-1}$ and $S^{f\ell}_{N-1}$ with $\bar{S}_N = S_{N-1} + S^{f\ell}_{N-1}$ and then from $\bar{S}_{N-1}$ the contributions $\bar{S}_{N-2}$ and $S^{f\ell}_{N-2}$ etc.

To begin this program we start with Eq. (3.23) where the Hamiltonian is the non-Hermitian operator $P_{N-1}\mathcal{H}(d_N)P_{N-1}$ of Eq. (3.24) in which the d_N class states appear on the average. This is similar to what was done previously where we started with the full Schroedinger equation Eq. (3.21) with the total Hermitian Hamiltonian H. We then decompose $\Phi_{P_{N-1}}$ into the components $d_{N-1}\Phi_{P_{N-1}}$ and $P_{N-2}\Phi_{P_{N-1}}$ and further use the KKM method to solve the resulting two coupled equations for these components. Similarly, to extract $S^{f\ell}_{N-2}$ from $\bar{S}_{N-1}$, we start with an equation for the wave function

$$\Phi_{P_{N-2}} = \left\langle P_{N-2}\Phi_{P_{N-1}} \right\rangle_{I_{N-1}}$$

which is analogous to Eq. (3.25) but with an effective Hamiltonian $P_{N-2}\mathcal{H}(d_{N-1}+d_N)P_{N-2}$. We then decompose $\Phi_{P_{N-2}}$ into $d_{N-2}\Phi_{P_{N-2}}$ and $P_{N-3}\Phi_{P_{N-2}}$ and solve the corresponding coupled equations, following again the KKM procedure.

This demonstrates that in general, to calculate the general term (with $n=1,\ldots,N$), $S^{f\ell}_n$, we have to work with $\bar{S}_{n+1}$ which is defined in terms of the "optical" wave function Φ_{P_n} and, by writing $P_n = P_{n-1} + d_n$, we obtain the equations

$$(E - P_{n-1}\mathcal{H}(Q_{n+1})P_{n-1})\,P_{n-1}\Phi_{P_n} = (P_{n-1}\mathcal{H}(Q_{n+1})d_n)\,d_n\Phi_{P_n} \tag{3.32}$$

$$(E - d_n\mathcal{H}(Q_{n+1})d_n)\,d_n\Phi_{P_n} = (d_n\mathcal{H}(Q_{n+1})P_{n-1})\,P_{n-1}\Phi_{P_n} \tag{3.33}$$

where $Q_{n+1} = \mathbb{1} - P_n = \mathbb{1} - d_n - P_{n-1} = d_{n+1} + d_{n+2} + \cdots + d_N$

and

$$\mathcal{H}(Q_{n+1}) = \mathcal{H}(Q_{n+2}) + \mathcal{H}(Q_{n+2})\, d_{n+1} \left\langle [E - d_{n+1}\mathcal{H}(Q_{n+2})\, d_{n+1}]^{-1} \right\rangle_{I_{n+1}} d_{n+1}\, \mathcal{H}(Q_{n+2}) \tag{3.34a}$$

where the average over I_{n+1} above guarantees that the states in d_{n+1} are treated on the average and since in $\mathcal{H}(Q_{n+2})$ the states in Q_{n+2} were averaged out, all the states in Q_{n+1} appear only on the average in $\mathcal{H}(Q_{n+1})$. This implies that $\mathcal{H}(Q_{n+1})$ may also be obtained from the total Hermitian Hamiltonian H,

$$\mathcal{H}(Q_{n+1}) = H + HQ_{n+1} \left\langle \frac{1}{E - Q_{n+1} HQ_{n+1}} \right\rangle_{I_{n+1}} Q_{n+1} H \tag{3.34b}$$

With KKM we can then easily solve the above coupled equations (3.32) and (3.33) to obtain $S_n^{f\ell}$,

$$S^{f\ell}_{n,\,c'c} = -2\pi i \left\langle \varphi^{(-)\,c'}_{P_{n-1}} \middle| V_{P_{n-1}d_n} \left[E - d_n \mathcal{H}(Q_{n+1}) d_n - V_{d_n P_{n-1}} \mathcal{G}^{(+)}_{P_{n-1}\,P_{n-1}} V_{P_{n-1} d_n} \right]^{-1} \right.$$

$$= -2\pi i \left\langle \varphi^{(-)\,c'}_{P_{n-1}} \middle| V_{P_{n-1}d_n} \mathcal{G}_{d_n}\, V_{d_n P_{n-1}} \middle| \varphi^{(+)\,c}_{P_{n-1}} \right\rangle \tag{3.35}$$

where again we have defined the "optical" wave function

$$\varphi^{(\pm)c}_{P_{n-1}} = \left\langle P_{n-1} \varphi^{(\pm)c}_{P_n} \right\rangle_{I_n} \tag{3.36}$$

which is a solution of the "optical" equation

$$(E - P_{n-1}\mathcal{H}(Q_n)\, P_{n-1})\, \varphi^{(\pm)c}_{P_{n-1}} = 0 \tag{3.37}$$

The "optical" matrix propagator $\mathcal{G}^{(+)}_{P_{n-1}}$ is

$$\mathcal{G}^{(+)}_{P_{n-1}} = (E^{(+)} - P_{n-1}\mathcal{H}(Q_n)\, P_{n-1})^{-1} \tag{3.38}$$

where $Q_n = Q_{n+1} + d_n = d_n + \ldots + d_n$ and $P_{n-1}\mathcal{H}(Q_n)\, P_{n-1}$ is analogous to that of Eq. (3.4). In (3.35) we have introduced the d_n-resonance propagator, $\mathcal{G}_{d_n}$

$$\mathcal{G}_{d_n} = [E - d_n \mathcal{H}(P_{n-1}) d_n]^{-1} \tag{3.39}$$

where the effective Hamiltonian $d_n \mathcal{H}(P_{n-1}) d_n$ is defined, as

$$\begin{aligned} d_n \mathcal{H}(P_{n-1}) d_n &= d_n \mathcal{H}(Q_{n+1}) d_n \\ &+ V_{d_n P_{n-1}} \mathcal{G}^{(+)}_{P_{n-1}} V_{P_{n-1} d_n} \end{aligned} \tag{3.40}$$

which is the generalization of Eq. (3.9). The form factors $V_{d_n P_{n-1}}$ and $V_{P_{n-1} d_n}$ are

$$V_{d_n P_{n-1}} = \left[\frac{i\, I_n/2}{E - d_n \mathcal{H}(Q_{n+1}) d_n + i\, I_n/2} \right]^{1/2} d_n \mathcal{H}(Q_{n+1})\, P_{n-1} \tag{3.41}$$

and

$$V_{P_{n-1} d_n} = P_{n-1}\mathcal{H}(Q_{n+1}) d_n \left[\frac{i\, I_n/2}{E - d_n \mathcal{H}(Q_{n+1}) d_n + i\, I_n/2} \right]^{1/2} \tag{3.42}$$

Notice that the effective Hamiltonian $\mathcal{H}(Q_n)$ that appears in $\mathcal{G}^{(+)}_{P_{n-1}}$ contains all the doorway classes in $Q_n = d_N + d_{N-1} + \ldots + d_n$ on the average, whereas $\mathcal{H}(Q_{n+1})$ that appears in the form factors contains only the classes in Q_{n+1} on the average.

At this point one might ask about the formal relation between the "optical" wave function $\varphi_{P_{n-1}}$ and the total wave function of the system Ψ. This relation is simply

$$\varphi_{P_{n-1}} = \Big\langle P_{n-1} \Big\langle \cdots \Big\langle P_{N-2} \Big\langle P_{N-1} \Psi \Big\rangle_{I_N} \cdots \Big\rangle_{I_{n+1}} \Big\rangle_{I_n}$$

$$= \langle P_{n-1} \Psi \rangle_{I_n} \tag{3.43}$$

Eq. (3.43) again exhibits clearly the spirit of our model. It also indicates how the actual optical wave function, φ_{opt}, that describes the open channels in p, is formally defined in a nested average sense vis

$$\varphi^{(+)c}_{opt} = \varphi_p = \varphi_{P_o}$$

$$= \Big\langle P_o \Big\langle P_1 \Big\langle \cdots \Big\langle P_{N-1} \Big\rangle_{I_N} \cdots \Big\rangle_{I_2} \Big\rangle_{I_1}$$

$$= \langle P_o \Psi \rangle_{I_1} \tag{3.44}$$

Having derived a formal expression for S^{fl}_n in Eq. (3.35) we now cast it in the sum-over-poles form by expanding the d_n-propagator, $\mathcal{G}_{d_n}$ (Eq. (3.39)) in terms of biorthogonal basis states $|\Psi_{ni}\rangle$ and $|\tilde{\Psi}_{ni}\rangle$ which satisfy the following eigenvalue equation

$$(\mathcal{E}_{ni} - d_n \mathcal{H}(P_{n-1}) d_n) |\Psi_{ni}\rangle = 0 \tag{3.45a}$$

$$\left(\mathcal{E}^{*}_{ni} - d_n \mathcal{H}(P_{n-1})^{\dagger} d_n \right) \left| \tilde{\Psi}_{ni} \right\rangle = 0 \qquad (3.45b)$$

with the effective Hamiltonian $d_n \mathcal{H}(P_{n-1}) d_n$ given in Eq. (3.40). Eqs. (3.45a) and (3.45b) are analogous to Eqs. (3.8a), b) for the one-class case. One major difference between Eq. (3.35) and Eq. (3.26) for $\underset{\sim}{S}_N^{f\ell}$ is that in $\underset{\sim}{S}_N^{f\ell}$ the form factors are given, aside from the complex square root factor, in terms of the Hermitian operator H whereas in $\underset{\sim}{S}_n^{f\ell}$ they are given in terms of the non-Hermitian operator $\mathcal{H}(Q_{n+1})$. However, despite this difference, one can prove that $V_{P_n d_n}$ satisfies the same condition (3.13), namely,

$$\theta \, V_{P_n d_n} \, \theta^{-1} = (V_{d_n P_n})^{\dagger} \qquad (3.46)$$

The proof is as follows:

$$\theta V_{P_n d_n} \theta^{-1} = \theta P_n \, (Q_{n+1}) d_n \sqrt{\frac{i I_n/2}{E - d_n \mathcal{H}(Q_{n+1}) d_n + \frac{-i I_n}{2}}} \, \theta^{-1}$$

$$= P_n \mathcal{H}(Q_{n+1})^{\dagger} d_n \sqrt{\frac{-i I_n}{E - d_n \mathcal{H}(Q_{n+1})^{\dagger} d_n - i I_n/2}} \qquad (3.47)$$

$$= (V_{d_n P_n})^{\dagger} .$$

We have used $\theta \mathcal{H}(Q_{n+1}) = \mathcal{H}(Q_{n+1})^{\dagger} \theta$ which is a consequence of (3.34b) and (3.15). Therefore we obtain, the sum-over-poles form,

$$S^{f\ell}_{n,cc'} = -i \sum_{i \in \{d_n\}} \frac{g_{ni,c} \, g_{ni,c'}}{E - \mathcal{E}_{ni}} \qquad (3.48)$$

which justifies Eq. (2.10). The residue factors $g_{ni,c}$ are given by

$$g_{ni,c} = \sqrt{2\pi}\,\langle \varphi^{(-)c}_{P_{n-1}} \mid V_{P_{n-1}d_n} \mid \Psi_{ni} \rangle \tag{3.49}$$

and the average width, Γ_n, which one obtains from $\mathcal{E}_{ni}$ by

$$-\frac{\Gamma_n}{2} = \left\langle \mathrm{Im}\langle \tilde{\Psi}_{ni} \mid [d_n \mathcal{H}(Q_{n+1}) d_n + V_{d_n P_{n-1}} \mathcal{L}^{(+)}_{P_{n-1}} V_{P_{n-1} d_n}] \mid \Psi_{ni} \rangle \right\rangle_i \tag{3.50}$$

An interesting fact which emerges from Eq. (3.50) is that the average width Γ_n is generally composed of (N-n) + n^2 terms. This can be seen from the fact that the P_{n-1}-Green's function $\mathcal{L}^{(+)}_{P_{n-1}}$ is an (nxn) matrix

$$\mathcal{L}^{(+)}_{P_{n-1}} = \begin{pmatrix} (\mathcal{L}_{P_{n-1}})_{pp} & \cdots & (\mathcal{L}_{P_{n-1}})_{p d_{n-1}} \\ (\mathcal{L}_{P_{n-1}})_{d_1 p} & \cdots & (\mathcal{L}_{P_{n-1}})_{d_1 d_{n-1}} \\ \vdots & & \vdots \\ (\mathcal{L}_{P_{n-1}})_{d_{n-1} p} & \cdots & (\mathcal{L}_{P_{n-1}})_{d_{n-1} d_{n-1}} \end{pmatrix} \tag{3.51}$$

which when inserted into $V_{d_n P_{n-1}} \mathcal{L}^{(+)}_{P_{n-1}} V_{P_{n-1} d_n}$ generates n^2 terms. The other (N-n) terms in Γ_n come about from $d_n \mathcal{H}(Q_{n+1}) d_n$ in which the doorway classes $d_{n+1}, d_{n+2}, \ldots d_N$ appear on the average (see Eq. (3.34a)). Generally the off-diagonal elements of $\mathcal{L}^{(+)}_{P_{n-1}}$ contribute little to Γ_n as they generate terms with odd powers of the form factor V,s and thus would approximately average to zero. Therefore one may consider Γ_n to contain N terms: the N-n terms originating from $d_n \mathcal{H}(Q_{n+1}) d_n$ may loosely be referred to as the average "damping width" of the n^{th} class resonances and the remaining n terms as forming what might be called a generalized "escape width".

3.2 "Optical-Background" Representation of $g_{ni,c}$

It is clear from our previous discussion that the residue factors of Eq. (3.49) $g_{ni,c'}$ contain "modulations" due to all the doorway classes contained in P_{n-1}. To stress this point we introduce the notation $g^{P_{n-1}}_{ni,c} = g_{ni,c}$. For our later use we shall, in the following, exhibit explicitly these modulations in $g^{P_{n-1}}_{ni,c}$. To do that we resort again to KKM and analyze the "continuum" wave function $\mathcal{P}_{P_{n-1}}$ that appears in the definition of $g^{P_{n-1}}_{ni,c}$. The first step would then be to extract, from $\mathcal{P}_{P_{n-1}}$, the components $d_{n-1}\mathcal{P}_{P_{n-1}}$ and $P_{n-2}\mathcal{P}_{P_{n-1}}$. By solving the pair of coupled equations (3.32) and (3.33) with $n \to n-1$, we obtain

$$P_{n-2}\mathcal{P}^{(+)}_{P_{n-1}} = \mathcal{P}^{(+)}_{P_{n-2}} + \mathcal{G}^{(+)}_{P_{n-2}} V_{P_{n-2}d_{n-1}} \mathcal{G}_{d_{n-1}} V_{d_{n-1}P_{n-2}} \mathcal{P}^{(+)}_{P_{n-2}} \tag{3.52}$$

and

$$d_{n-1}\mathcal{P}^{(+)}_{P_{n-1}} = \left[\frac{i\, I_{n-1}/2}{E - d_{n-1}\mathcal{H}(Q_n) d_{n-1} + i I_{n-1}/2} \right]^{1/2} \mathcal{G}_{d_{n-1}} V_{d_{n-1}P_{n-2}} \mathcal{P}^{(+)}_{P_{n-2}} \tag{3.53}$$

where the "optical" Green's function $\mathcal{G}^{(+)}_{P_{n-2}}$ and the doorway Green's function $\mathcal{G}_{d_{n-1}}$ are given as in Eqs. (3.38) and (3.39), respectively, by changing n into n-1.

Substituting Eqs. (3.52) and (3.53) into $g^{P_{n-1}}_{ni,c}$ of Eq. (3.49) we finally obtain (dropping the square root factor that appears in Eqs. (3.49) and (3.42) as it has no consequence)

$$g^{P_{n-1}}_{ni,c} = g^{P_{n-2}}_{ni,c} + g^{f\ell}_{ni,c}(d_{n-1}) \tag{3.54}$$

where

$$g^{P_{n-2}}_{ni,c} = \sqrt{2\pi} \left\langle \mathcal{P}^{(-)c'}_{P_{n-2}} \middle| V_{P_{n-2}d_n} \middle| \Psi_{ni} \right\rangle \tag{3.55}$$

and

$$g^{fl}_{ni,c}(d_{n-1}) = \sqrt{2\pi}\,\langle \varphi^{(-)c'}_{P_{n-2}} | V_{P_{n-2}d_{n-1}} \mathcal{G}_{d_{n-1}} \mathbb{V}_{d_{n-1}d_n} | \psi_{ni} \rangle$$

$$= \sqrt{2\pi}\,\langle \varphi^{(-)c'}_{P_{n-2}} | V_{P_{n-2}d_{n-1}} \sum_{i\in\{d_{n-1}\}} \frac{|\psi_{(n-1)i}\rangle\langle\tilde{\psi}_{(n-1)i}|}{E - \mathcal{E}_{(n-1)i}} \mathbb{V}_{d_{n-1}d_n} | \psi_{ni} \rangle \tag{3.56}$$

In the above, we have defined an effective interaction $\mathbb{V}_{d_{n-1}d_n}$ that couples class d_n with class d_{n-1} and is given by

$$\mathbb{V}_{d_{n-1}d_n} = V_{d_{n-1}d_n} + V_{d_{n-1}P_{n-2}} \mathcal{G}^{(+)}_{P_{n-2}} V_{P_{n-2}d_n} \tag{3.57a}$$

where

$$V_{d_{n-1}d_n} = d_{n-1}(V_{P_{n-1}d_n}) \tag{3.57b}$$

$$V_{P_{n-2}d_n} = P_{n-2}(V_{P_{n-1}d_n}) \tag{3.57c}$$

and $V_{P_{n-1}d_n}$ was defined in Eq. (3.42). Notice that our decomposition of $g^{P_{n-1}}_{ni,c}(E)$ above amounts to defining an energy average of $g^{P_{n-1}}_{ni,c}(E)$, i.e., $g^{P_{n-2}}_{ni,c}(E)$, plus a fluctuation part that results from the d_{n-1} doorway modulations which are present in the "continuum" wave function $\varphi^{(\pm)c}_{P_{n-1}}$. In addition, the KKM construction results in the following property of $g^{P_{n-1}}_{ni,c}$ (see Eq. (3.18))

$$\left\langle g^{P_{n-1}}_{ni,c}(E) \right\rangle_i = 0 \tag{3.58}$$

where by the above average we imply an average over the index i which labels the states in d_n. The interpretation of $g^{fl}_{ni,c}(d_{n-1})$ is now quite clear as it corresponds to that part of $g^{P_{n-1}}_{ni,c}$ that contains explicitly the d_{n-1}-class modulation as shown in Eq. (3.56). By repeated application of

the KKM reduction procedure one can easily generate all the terms in $g^{P_{n-1}}_{ni,c}$ which contain explicit doorway modulations. A natural decomposition of $g^{P_{n-1}}_{ni,c}$ is then obtained

$$g^{P_{n-1}}_{ni,c} = g^{opt}_{ni,c} + \sum_{m=1}^{n-1} g^{f\ell}_{ni,c}(d_m) \tag{3.59}$$

where $g^{opt}_{ni,c}$ is given by

$$g^{opt}_{ni,c} = \sqrt{2\pi} \left\langle \varphi^{(-)c'}_{opt} \right| V_{pd_n} \left| \psi_{ni} \right\rangle \tag{3.60}$$

and $\varphi^{(-)c'}_{opt}$ is a solution of the optical equation in which all doorway classes have been averaged out, see Eq. (3.44). The terms $g^{f\ell}_{ni,c}(d_m)$, which are given by

$$g^{f}_{ni,c}(d_m) = \sqrt{2\pi} \left\langle \varphi^{(-)c}_{P_{m-1}} \right| V_{P_{m-1}d_m} \mathcal{G}_{d_m} V_{d_m d_n} \left| \psi_{ni} \right\rangle , \tag{3.61}$$

represent the modulations in $g^{P_{n-1}}_{ni,c}$ due to the resonances in class m ($m < n$). As can be seen from Eq. (3.61) $g^{f\ell}_{ni,c}(d_m)$ contains also modulations due to resonances in classes m-1, m-2, ...1.

For the purpose of clarity we give below the explicit form for $g^{P_{n-1}}_{ni,c}$ as given in Eq. (3.59) in the case of three classes of states, d_1, d_2 and d_3 ($\tilde{\Gamma}_1 \gg \tilde{\Gamma}_2 \gg \tilde{\Gamma}_3$). To be specific we consider first $g^{P_2}_{3i,c}$

$$g^{P_2}_{3i,c} = g^{opt}_{3i,c} + g^{f\ell}_{3i,c}(d_1) + g^{f\ell}_{3c'}(d_2) \tag{3.62}$$

with

$$g^{f\ell}_{3i,c}(d_1) = \sqrt{2\pi} \left\langle \varphi^{(-)c}_{opt} \right| V_{pd_1} \mathcal{G}_{d_1} V_{d_1 d_3} \left| \psi_{3i} \right\rangle$$

$$= \sqrt{2\pi}\,\langle \varphi^{(-)}_{opt} | V_{pd_1} \sum_{j\in\{d_1\}} \frac{|\psi_{1j}\rangle\langle\tilde{\psi}_{1j}|}{E - \mathcal{E}_{1j}} V_{d_1 d_3} | \psi_{3i}\rangle \qquad (3.63)$$

$$g^{f}_{3i,c}(d_2) = \sqrt{2\pi}\,\Big[\langle \varphi^{(-)c}_{opt} | V_{pd_2} \mathcal{G}_{d_2} V_{d_2 d_3} | \psi_{3i}\rangle$$

$$+ \langle \varphi^{(-)c}_{opt} | V_{pd_1} \mathcal{G}_{d_1} V_{d_1 d_2} \mathcal{G}_{d_2} V_{d_2 d_3} | \psi_{3i}\rangle \Big]$$

$$= \sqrt{2\pi}\Big[\langle \varphi^{(-)c}_{opt} | V_{pd_2} \sum_{j\in\{d_2\}} \frac{|\psi_{2j}\rangle\langle\tilde{\psi}_{2j}|}{E - \mathcal{E}_{2j}} V_{d_2 d_3} | \psi_{3i}\rangle$$

$$+ \langle \varphi^{(-)c}_{opt} | V_{pd_1} \sum_{k\in\{d_1\}} \frac{|\psi_{1k}\rangle\langle\tilde{\psi}_{1k}|}{E - \mathcal{E}_{1k}} V_{d_1 d_2} \sum_{j\in\{d_2\}} \frac{|\psi_{2j}\rangle\langle\tilde{\psi}_{2j}|}{E - \mathcal{E}_{2j}} V_{d_2 d_3} | \psi_{3i}\rangle \Big] \qquad (3.64)$$

Similarly, we obtain for $g^{P_1}_{2i,c}$ the following

$$g^{P_1}_{2i,c} = g^{opt}_{2i,c} + g^{f\ell}_{2i,c}(d_1) \qquad (3.65)$$

with

$$g^{f\ell}_{2i,c}(d_1) = \sqrt{2\pi}\,\langle \varphi^{(-)c}_{opt} | V_{pd_1} \mathcal{G}_{d_1} V_{d_1 d_2} | \psi_{2i}\rangle$$

$$= \sqrt{2\pi}\,\langle \varphi^{(-)c}_{opt} | V_{pd_1} \sum_{j\in\{d_1\}} \frac{|\psi_{1j}\rangle\langle\tilde{\psi}_{1j}|}{E - \mathcal{E}_{1j}} V_{d_1 d_2} | \psi_{2i}\rangle \qquad (3.66)$$

Finally,

$$g^{P_0}_{1i,c} = g^{p}_{1i,c} = g^{opt}_{1i,c} \qquad (3.67)$$

since there are no more doorways above d_1! Notice that in Eq. (3.66) the coupling $V_{d_1 d_2}$ is given by an equation similar to Eq. (3.57) except that $\mathcal{G}^{(+)}_{P_{n-2}}$ is now just $\mathcal{G}^{(+)}_{p}$, i.e., the full optical Green's function $\mathcal{G}^{(+)}_{opt}$

associated with the open channel subspace p. As for $V_{d_2 d_3}$, the Green's function that appears in the second term is G_{P_1}, with $P_1 = p+d_1$. A fuller discussion of the matrix elements of these class-class couplings will be given later.

3.3. Calculation of the X_n-Matrices

As discussed in Section 2 and in Appendix A, the fundamental quantity in our theory for $f_{cc'}$ is the average, over the largest energy averaging interval I_1, of the matrices $\underset{\sim}{X}_n$, i.e., $\langle \underset{\sim}{X}_n \rangle_{I_1}$. In this subsection we give detailed account of how the above average is calculated using the "optical-background" representation for $g^{P_{n-1}}_{ni,c}$ discussed in the previous subsection. Our final results for $\langle \underset{\sim}{X}_n \rangle_{I_1}$, which will be shown later to be related to the $\langle \underset{\sim}{X}_m \rangle_{I_1}$ with m=1,2,...n-1 in a recursive way, leads us to a very simple and appealing physical interpretation in terms of probability flow. In what follows we shall, for simplicity, assume the absence of direct reactions and thus deal with a diagonal optical S-matrix, i.e., $\langle S_{cc'} \rangle_{I_1} = \delta_{cc'} \langle \varphi^{(-)}_{opt} | \varphi^{(+)c}_{opt} \rangle$. The generalization of our results to the case where $\langle S_{cc'} \rangle_{I_1}$ is not diagonal will be given at the end of this subsection.

Let us recall the defining equation for $X_{n,cc'}$, Eq. (2.13), which gives

$$X_{n,cc} = \sqrt{\frac{2\pi}{\tilde{\Gamma}_n D_n}} \left\langle \left| g^{P_{n-1}}_{ni,c} \right|^2 \right\rangle_{i \in I_n} \tag{3.68}$$

where the i-average is over the states ni contained in I_n. Clearly the energy-dependence of $X_{n,cc'}$ due to the doorway modulations, is now an

$\mathcal{E}_{ni}$-dependence since the g 's are evaluated at the pole position namely $E = \mathcal{E}_{ni}$ (see Appendix A). This, however, does not prevent us from using the optical-background representation of the g's worked out in subsection 3.2.

Using the general result for $g^{P_{n-1}}_{ni,c}$ of Eq. (3.59) in Eq. (3.68) we immediately obtain the following simple representation for $\langle X_{n,cc} \rangle_{I_l}$ namely

$$\langle X_{n,cc} \rangle_{I_l} = X^{opt}_{n,cc} + \sum_{m=1}^{n-1} X^{f\ell}_{n,cc}(m) \qquad (3.69)$$

where cross terms have been dropped upon taking the average over I_l. In Eq. (3.69) $X^{opt}_{n,cc}$ stands for

$$X^{opt}_{n,cc} = \sqrt{\frac{2\pi}{\widetilde{\Gamma}_n D_n}} \left\langle \left\langle \left| g^{opt}_{ni,c} \right|^2 \right\rangle_{i \in I_n} \right\rangle_{I_l} \qquad (3.70)$$

$$= \sqrt{\frac{2\pi}{\widetilde{\Gamma}_n D_n}} \left\langle \left| g^{opt}_{ni,c} \right|^2 \right\rangle_{i \in I_l}$$

and $X^{f\ell}_{n,cc}(m)$ stands for

$$X^{f\ell}_{n,cc}(m) = \sqrt{\frac{2\pi}{\widetilde{\Gamma}_n D_n}} \left\langle \left\langle \left| g^{f\ell}_{ni,c}(d_m) \right|^2 \right\rangle_{i \in I_n} \right\rangle_{I_l} = \sqrt{\frac{2\pi}{\widetilde{\Gamma}_n D_n}} \left\langle \left| g^{f\ell}_{ni,c}(d_n) \right|^2 \right\rangle_{i \in I_l} \qquad (3.71)$$

where the I-average in Eq. (3.71) indicates an average over intervals I_n centered on energies within I_l. Clearly $X^{opt}_{n,cc}$ must represent the direct coupling of channel c to the n^{th} class of resonances, on the average. The terms $X^{f\ell}_{n,cc}(m)$, on the other hand, represent the non-direct coupling of c to d_n that proceeds via the classes of doorways, d_m, d_{m-1},...d_1 $(m<n)$.

The explicit form of $g^{f\ell}_{ni,c}(d_m)$, Eq. (3.61), clearly demonstrates the above fact which can be fully shown as follows. Substituting Eq. (3.61) into Eq. (3.71) we find

$$X^{f\ell}_{n,cc}(m) = \sqrt{\frac{2\pi}{\tilde{\Gamma}_{n D_n}}}\, 2\pi \left\langle\!\!\left\langle \left| \langle \varphi^{(-)c}_{P_{m-1}} | V_{P_{m-1} d_m} \mathcal{G}_{d_m} \mathbb{V}_{d_m d_n} | \psi_{ni} \rangle \right|^2 \right\rangle_{i \in I_n}\!\!\right\rangle_{I_1}$$

$$= \sqrt{\frac{2\pi}{\tilde{\Gamma}_{n D_n}}}\, 2\pi \left\langle\!\!\left\langle \left| \langle \varphi^{(-)c}_{P_{m-1}} | V_{P_{m-1} d_m} \sum_{j \in d_m} \frac{|\psi_{mj}\rangle\langle\tilde{\psi}_{mj}|}{\mathcal{E}_{ni} - \mathcal{E}_{mj}} \mathbb{V}_{d_m d_n} | \psi_{n_i} \rangle \right|^2 \right\rangle_{i \in I_n}$$

$$\approx \sqrt{\frac{2\pi}{\tilde{\Gamma}_{n D_n}}}\, 2\pi \left(\frac{2\pi}{\tilde{\Gamma}_{m D_m}}\right) \left\langle\!\!\left\langle \left| \langle \varphi^{(-)c}_{P_{m-1}} | V_{P_{m-1} d_m} | \psi_{mj} \rangle \right|^2 \right\rangle_{j \in I_m}\!\!\right\rangle_{I_1}$$

$$\left\langle\!\!\left\langle\!\!\left\langle \left| \langle \tilde{\psi}_{mj} | \mathbb{V}_{d_m d_n} | \psi_{ni} \rangle \right|^2 \right\rangle_{j \in I_m}\!\!\right\rangle_{i \in I_n}\!\!\right\rangle_{I_1}$$

$$= \sqrt{\frac{2\pi}{\tilde{\Gamma}_{n D_n}}} \sqrt{\frac{2\pi}{\tilde{\Gamma}_{m D_m}}} \left[\sqrt{\frac{2\pi}{\tilde{\Gamma}_{m D_m}}} \left\langle \left| g^{P_{m-1}}_{mj,c} \right|^2 \right\rangle_{j \in I_1} \right] \left\langle \left| \langle \psi_{mj} | \mathbb{V}_{d_m d_n} | \psi_{ni} \rangle \right|^2 \right\rangle_{\substack{i \in I_1 \\ j \in I_1}}$$

$$= \sqrt{\frac{2\pi}{\tilde{\Gamma}_{n D_n}}} \sqrt{\frac{2\pi}{\tilde{\Gamma}_{m D_m}}} \left\langle X_{m,cc} \right\rangle_{I_1} \left\langle \left| \langle \tilde{\psi}_{mj} | \mathbb{V}_{d_m d_n} | \psi_{ni} \rangle \right|^2 \right\rangle_{I_1} \qquad (3.72)$$

In the above equation $\mathbb{V}_{d_m d_n}$ is the effective coupling interaction defined in Eq. (3.57). It contains both direct coupling, $V_{d_m d_n}$ and non-direct coupling, $V_{d_m P_{m-1}} \mathcal{G}^{(+)}_{P_{m-1}} V_{P_{m-1} d_n}$, that proceeds via the classes, $d_{m-1}, \ldots, d_1$, and the open channels, all contained in P_{m-1}.

The key results of this section are contained in Eqs. (3.69) and (3.72). They show that the doorway modulation of the partial widths $g^{P_{n-1}}_{ni,c}$, given by Eq. (3.59), manifests itself in recursion relations between the X_n's of different classes. This relation is given explicitly by Eqs. (3.69) and (3.72), and is highly reminiscent of the previously-noted recursion relation between transmission coefficients, Eq. (2.30). Both relations appear to carry the same physical information, which indicates that the transmission coefficients T_n which we would like to extract from the analysis of the present section must be proportional to the X_n.

The structure of (3.72) suggests that a reasonable guess for the proportionality constants (one for each class) might be given by the definition

$$T'_{n,c} = \sqrt{\frac{2\pi\tilde{\Gamma}_n}{D_n}} \left\langle X_{n,cc} \right\rangle_{I_1} = \frac{2\pi}{D_n} \left\langle \left| g^{P_{n-1}}_{ni,c} \right|^2 \right\rangle_{I_1} \tag{3.73}$$

for the full transmission coefficients $T'_{n,cc}$, as well as

$$\tau'_{n,c} = \sqrt{\frac{2\pi\tilde{\Gamma}_n}{D_n}} X^{opt}_{n,cc} = \frac{2\pi}{D_n} \left\langle \left| g^{opt}_{ni,c} \right|^2 \right\rangle_{I_1} \tag{3.74}$$

for the optical or "direct" transmission coefficient. If we also employ what seems a reasonable definition for a branching ratio η'_{mn},

$$\eta'_{mn} = \frac{2\pi}{\tilde{\Gamma}_m D_n} \left\langle \left| \langle \Psi_{mj} | V_{d_m d_n} | \Psi_{ni} \rangle \right|^2 \right\rangle_{I_1} \tag{3.75}$$

then Eqs. (3.69) and (3.72) yield

$$T_{n,cc} = \tau'_{n,cc} + \sum_{m=1}^{n-1} T'_{m,cc} \eta'_{mn} , \tag{3.76}$$

which does indeed have the same form as Eq. (2.30), and means that the T'_n transmission coefficients are linear combinations of the τ'_n coefficients,

$$T'_{n,c} = \sum_{m=1}^{n} \tau_{m,c} M'_{mn} \tag{3.77}$$

with $M'_{mn} = 1$.

So far we have assumed no direct reactions between the channels. If such reactions are present, T_n and τ_n become matrices in channel space. The calculation of their non-diagonal elements exactly parallels the above calculations, employing $\langle X_{n,cc'} \rangle_{I_1}$, and produces the result

$$\begin{aligned} T'_{n,cc'} &= \sqrt{\frac{2\pi\tilde{\Gamma}_n}{D_n}} \left\langle X_{n,cc'} \right\rangle_{I_1} \\ &= \tau'_{n,cc'} + \sum_{m=1}^{n-1} T'_{m,cc'} \eta'_{mn} \end{aligned} \tag{3.78}$$

Returning to the case of no direct channel coupling for simplicity, the fluctuation cross section is, in these terms,

$$\sigma^{f\ell}_{cc'} = \sum_n X_{n,cc} X_{n,c'c'} \tag{3.79}$$

$$= \sum_n \frac{D_n}{2\pi\tilde{\Gamma}_n} T'_{n,c} T'_{n,c'} \, . \tag{3.80}$$

Since Eq. (3.79) is the standard one for $\sigma^{f\ell}_{cc'}$, it is guaranteed unitary. Eq. (3.80), which is just a re-writing of it, is equally unitary and provides an acceptable form, but one which differs from Eq. (2.29) because of the normalization of the $T'_{n,c}$ chosen in Eq. (3.73). Comparison with Eq. (2.28) shows that this was not, indeed, an appropriate normalization for this purpose and shows that, in order to write $\sigma^{f\ell}$ in the form of Eq. (2.29), we must define transmission coefficients $T_{n,c}$ which are so normalized that

$$T_{n,c} = \frac{\mathrm{Tr}\ X_n}{1 - \gamma_n} X_{n,cc} = \left(\frac{\mathrm{Tr}\ T_n}{1 - \gamma_n} \right)^{\frac{1}{2}} X_{n,cc} \, , \tag{3.81}$$

where $$\gamma_n = \sum_{m>n} \eta_{nm} , \tag{3.82}$$

in terms of quantities η_{nm} to be defined below. In order to retain the form of the recursion relations (3.76), we must similarly re-normalize the τ_l' by the definition

$$\tau_{n,c} = \frac{T_n X_n}{1 - \gamma_n} X^{opt}_{n,cc} . \tag{3.83}$$

An examination of Eqs. (3.69) and (3.72) shows that they can then be written in the described form,

$$T_{n,c} = \tau_{n,c} + \sum_{m=1}^{n-1} T_{m,c} \eta_{mn} \tag{3.84}$$

provided we define the channel-coupling coeffients by

$$\eta_{mn} = \left[\frac{2\pi}{\tilde{\Gamma}_m D_m}\right]^{\frac{1}{2}} \frac{T_r X_m}{1 - \gamma_m} \left\langle \left| \langle \tilde{\psi}_{mj} | V_{d_m d_n} | \psi_{ni} \rangle \right|^2 \right\rangle_{I_1} \frac{1 - \gamma_n}{T_r X_n} \left[\frac{2\pi}{\tilde{\Gamma}_n D_n}\right]^{\frac{1}{2}} . \tag{3.85}$$

The definition (3.81) implies that

$$\left\langle P_{n,cc} - P_{n+1,cc} \right\rangle_{I_1} = T_{n,cc} (1 - \gamma) \tag{3.86}$$

and hence that the cross section has the desired form,

$$\sigma^{f\ell}_{n,cc'} = \frac{T_{n,cc} T_{n,c'c'}}{T_r T_n} (1 - \gamma_n), \tag{3.87}$$

and satisfies the unitarity condition,

$$\sum_n \sum_{c'} \sigma^{f\ell}_{n,cc'} = P_{1,cc} = \sum_{n=1}^{N} \tau_{n,c} . \tag{3.88}$$

The definitions (3.81), (3.83) and (3.85) are distinctly not intuitively obvious, but it is clear that they are demanded by Eqs. (3.69) and (3.72)

in order to produce Eq. (3.87). In this connection we might observe, first, that in AWM [4a], the transmission coefficient $\tau_{n,c}$ is not simply $\chi_n^c \equiv \pi \rho_n |\langle n|v|c\rangle|^2$, but rather has the "re-normalized" form $\tau_{n,c} = 4\chi_n^c / (1 + \sum_k \chi_k^c)^2$, which is somewhat reminiscent of our Eq. (3.83) in that it involves <u>all</u> classes in the "normalization constant" $4/(1 + \sum_k \chi_k^c)^2$. Similarly we recall that η_{nm} is not the <u>direct</u> branching ratio from class m to class n, but involves paths through other classes as well, as Eq. (4.25), e.g., makes clear.

3.4 Calculation of the Downward Branching Ratios η'_{mn}

In this subsection we give a full account of the structure of the mixing coefficient η'_{mn} introduced in Eq. (3.75). As was already mentioned the coupling interaction $\mathcal{V}_{d_m d_n}$ that appears in η'_{mn} contains two pieces; the direct coupling interaction, $V_{d_m d_n}$, and the non-direct interaction, $V_{d_m P_{m-1}} \mathcal{G}^{(+)}_{P_{m-1}} V_{P_{m-1} d_n}$, which proceeds via the "continuum" $P_{m-1} = p+d_1+\ldots+d_{m-1}$. This non-direct coupling is a generalization of what is called external mixing in the one-class case, in which it has the form $V\mathcal{G}^{(+)}_p V$ i.e., it involves the open-channel subspace p. It would be useful, therefore, to extract the components $V\mathcal{G}^{(+)}_p V$ from $V_{d_m P_{m-1}} \mathcal{G}^{(+)}_{P_{m-1}} V_{P_{m-1} d_n}$ and then define the basic coupling interaction to be $v = V + V\mathcal{G}^{(+)}_p V$ instead of just V. This way η'_{mn} can be considered to be given in terms of the matrix elements of v which will be seen to result in a simple interpretation.

Recall the defining equation for η'_{mn}

$$\zeta'_{mn}\,\frac{\tilde{\Gamma}^{D}_{mn}}{2\pi} = \left\langle \left| \langle \tilde{\Psi}_{mj} | V_{d_m d_n} | \Psi_{in} \rangle \right|^2 \right\rangle_{I_1}$$

$$= \left\langle \left| \langle \tilde{\Psi}_{jm} | \left(V_{d_m d_n} + V_{d_m P_{m-1}} \mathcal{G}^{(+)}_{P_{m-1}} V_{P_{m-1} d_n} \right) | \Psi_{in} \rangle \right|^2 \right\rangle_{I_1} \tag{3.89}$$

where the $V_{d_m d_n}$ are given in terms of effective interactions that contain all doorway classes n+1,n+2,...,N on the average, as discussed in subsection 3.1, and $\mathcal{G}^{(+)}_{P_{m-1}}$ is an (mxm) matrix propagator associated with the $p+d_1+d_2+\ldots+d_{m-1}$ -subspace. We now introduce the projection operator $D_{m-1} = P_{m-1} - p = d_1 + d_2 + \ldots + d_{m-1}$. Clearly the term $V_{d_m P_{m-1}} \mathcal{G}^{(+)}_{P_{m-1}} V_{P_{m-1} d_n}$ can now be written as

$$V_{d_m P_{m-1}} \mathcal{G}^{(+)}_{P_{m-1}} V_{P_{m-1} d_n} = \left(V_{d_m p}, V_{d_m D_{m-1}} \right) \begin{pmatrix} \mathcal{G}^{(+)}_{pp} & \mathcal{G}^{(+)}_{pD_{m-1}} \\ \mathcal{G}^{(+)}_{D_{m-1}p} & \mathcal{G}^{(+)}_{D_{m-1}D_{m-1}} \end{pmatrix} \begin{pmatrix} V_{pd_n} \\ V_{D_{m-1}d_n} \end{pmatrix} \tag{3.90}$$

where, as before, the propagators $\mathcal{G}^{(+)}_{pp}$, etc. are defined by

$$\mathcal{G}^{(+)}_{pp} \equiv p\,\mathcal{G}^{(+)}_{P_{m-1}}\,p$$

$$\mathcal{G}^{(+)}_{pD_{m-1}} \equiv p\,\mathcal{G}^{(+)}_{P_{m-1}}\,D_{m-1}\,, \tag{3.91}$$

etc.

Now we use the KKM method to express the above Green's functions in terms of the optical Green's function $\mathcal{G}^{(+)}_{opt}$ defined below (see Appendix C for details)

$$\mathcal{G}^{(+)}_{pp} = \mathcal{G}^{(+)}_{opt} + \mathcal{G}^{(+)}_{opt} V_{pD_{m-1}} \mathcal{G}_{D_{m-1}} V_{D_{m-1}p} \mathcal{G}^{(+)}_{opt} \tag{3.92}$$

$$\mathcal{G}_{pD_{m-1}} = \mathcal{G}^{(+)}_{opt} V_{pD_{m-1}} \mathcal{G}_{D_{m-1}} \tag{3.93}$$

$$\mathcal{G}_{D_{m-1}p} = \mathcal{G}_{D_{m-1}} V_{D_{m-1}p} \mathcal{G}^{(+)}_{opt} \tag{3.94}$$

$$\mathcal{G}_{D_{m-1}D_{m-1}} = \mathcal{G}_{D_{m-1}} \tag{3.95}$$

where

$$\mathcal{G}_{D_{m-1}} = (E-D_{m-1}\mathcal{H}(Q_m)\,D_{m-1} - V_{D_{m-1}p}\mathcal{G}^{(+)}_{opt} V_{pD_{m-1}})^{-1} \tag{3.96}$$

and

$$\mathcal{G}^{(+)}_{opt} = [E^{(+)} - p\mathcal{H}(Q_m)p - p\mathcal{H}(Q_m)D_{m-1}\left\langle \frac{1}{E-D_{m-1}\mathcal{H}(Q_m)D_{m-1}} \right\rangle_{I_1} \times D_{m-1}\mathcal{H}(Q_m)p]^{-1} \tag{3.97}$$

where $Q_m = d_m+d_{m+1}+\ldots+d_N = \mathbb{1} -p-D_{m-1}$. Clearly $\mathcal{G}^{(+)}_{opt}$ can also be written as (see Eq. (3.34)).

$$\mathcal{G}^{(+)}_{opt} = (E^{(+)} - pHp - pHQ_1\left\langle \frac{1}{E-Q_1HQ_1} \right\rangle_{I} Q_1Hp)^{-1} \tag{3.98}$$

where $Q_1=D_N= d_1+d_2+\ldots d_N$.

The form factors $V_{pD_{m-1}}$ etc. are defined as usual and are given in Appendix C. Notice that $\mathcal{G}_{D_{m-1}}$ is a $(m-1)\times(m-1)$ matrix propagator with an effective interaction,

$$D_{m-1}\mathcal{H}(p)D_{m-1} = D_{m-1}\mathcal{H}(Q_m)\,D_{m-1} + V_{D_{m-1}p}\mathcal{G}^{(+)}_{opt} V_{pD_{m-1}}, \quad \text{that}$$

contains all doorway classes in Q_m on the average as well as the coupling to the open channels contained in p. Substituting the above expressions for the Green's functions given in Eqs. (3.92) - (3.95) into Eq. (3.57)

we find

$$\mathcal{V}_{d_m d_n} = V_{d_m d_n} + V_{d_m p}\,\mathcal{G}^{(+)}_{opt}\,V_{pd_n} + (V_{d_m D_{m-1}} + V_{d_m p}\,\mathcal{G}^{(+)}_{opt}\,V_{pD_{m-1}})\,\mathcal{G}_{D_{m-1}} \times (V_{D_{m-1}p}\,\mathcal{G}^{(+)}_{opt}\,V_{pd_n} + V_{D_{m-1}d_n}) \tag{3.99}$$

Defining, as was explained above, the basic coupling v by

$$v_{d_m d_n} = V_{d_m d_n} + V_{d_m p}\,\mathcal{G}^{(+)}_{opt}\,V_{pd_n}\ , \tag{3.100}$$

we thus find the following form for $\mathcal{V}_{d_m d_n}$

$$\mathcal{V}_{d_m d_n} = v_{d_m d_n} + v_{d_m D_{m-1}}\,\mathcal{G}_{D_{m-1}}\,v_{D_{m-1} d_n} \tag{3.101}$$

Notice that in Eq. (3.100) $V_{d_m d_n}$ and V_{pd_n} are related to $\mathcal{H}(Q_{n+1})$ whereas $V_{d_m p}$ is related to $\mathcal{H}(Q_{m+1})$.

With the above expression (Eq. (3.101)) for $\mathcal{V}_{d_m d_n}$ we can reexpress η'_{mn} as

$$\eta'_{mn}\,\frac{\tilde{\Gamma}_{m\,D'n}}{2\pi} = \left\langle \left| \langle \tilde{\Psi}_{mj} | v | \Psi_{ni} \rangle \right|^2 \right\rangle_{I_1} + \left\langle \left| \langle \tilde{\Psi}_{mj} | v\,\mathcal{G}_{D_{m-1}}\,v | \Psi_{ni} \rangle \right|^2 \right\rangle_{I_1} \tag{3.102}$$

which is the fundamental result of this subsection. The important advantage of (3.102) over (3.84) is that, in (3.102) we have introduced as the interaction responsible for the "direct" coupling (mixing) between d_m and d_n, the coupling potential v which contains both the pure "internal" mixing interaction, V, as well as the pure "external" mixing

interaction $V\mathcal{G}^{(+)}_{opt}V$. Notice that the doorway classes, $d_1, d_2, \ldots d_{m-1}$, which are present explicitly in $\mathcal{G}_{D_{m-1}}$, couple among themselves via exactly the same interaction v, of Eq. (3.100). The above discussion clearly demonstrates that η'_{nm} can be expressed in terms of the basic mixing parameters which are proportional to $\left\langle \left| \langle \tilde{\psi}_{n'j} | v | \psi_{ni} \rangle \right|^2 \right\rangle_{I_1}$ where $n' = m, m-1, \ldots, 1$ and $n > m$. In Section 4 we shall give a detailed account of the relationship between η'_{nm} and the basic mixing parameters referred to above. Before ending this section we give below an estimate for the external mixing part of $d_i v d_n{}^2$, i.e.,

$$\left\langle \left| \langle \tilde{\psi}_{mj} | v_{d_m p} \mathcal{G}^{(+)}_{opt} v_{p d_n} | \psi_{ni} \rangle \right|^2 \right\rangle_{i,j}$$

$$\approx \pi^2 \sum_c \left\langle \left| \langle \tilde{\psi}_{mj} | v_{d_m p} | \varphi^{(+)c}_{opt} \rangle \right|^2 \left| \langle \psi_{ni} | v_{d_n p} | \varphi^{(+)c}_{opt} \rangle \right|^2 \right\rangle_{i,j}$$

$$\approx \left(\frac{D_m D_n}{16\pi^2} \right) \sum_c \frac{4\pi^2 \left\langle \left| \langle \tilde{\psi}_{mj} | v_{d_m p} | \varphi^{(+)c}_{opt} \rangle \right|^2 \right\rangle_j}{D_m} \times \frac{4\pi^2 \left\langle \left| \langle \psi_{ni} | v_{d_n p} | \varphi^{(+)c}_{opt} \rangle \right|^2 \right\rangle_i}{D_n} \tag{3.103}$$

$$\approx \frac{D_m D_n}{16\pi^2} \sum_c \tau_{mc} \tau_{nc}$$

where we have approximated $\langle \tilde{\psi}_{mj} |$ by $\langle \psi_{ni} |$ and used the defining equations for τ' and τ, (Eq. (3.74) and (3.83)).

It is quite gratifying that after a rather lengthy algebraic manipulation we were able to express the fundamental quantity of our theory, namely $\left\langle X_{n,cc} \right\rangle_{I_1}$, in terms of the downward branching ratio, η_{nm}, and the optical transmission coefficients τ_{nc}.

4. A PROBABILITY FLOW INTERPRETATION OF THE NESTED-AVERAGE MODEL AND COMPARISON WITH OTHER FORMALISMS

In the previous section we developed expressions for the fluctuation cross section $\sigma^{f\ell}_{n,cc'}$ in the nested-average model by making extensive use of the Feshbach projection operator formalism and the generalized optical background techniques of KKM [8]. With these procedures we obtained formal expressions for all the factors necessary to construct $\sigma^{f\ell}_{n,cc'}$. In addition, since the Feshbach projection operator methods were used, comparison may be made with the multistep compound theory of FKK [3] which employs similar projection operators while proceeding along different lines.

In this section we establish a direct comparison between the nested-average model and the probability flow picture of non-direct reactions developed in Ref. [6], called here the "flow approach", (FA). In achieving a formal link between the nested-average model (NA) and the (FA), we shall provide intuitive interpretations for the factors we have formally developed in the previous section.

In Ref. [6], a formal connection was established between the flow approach and the work of AWM [4]. The model states and mixing matrices of the latter were identified with the structure obtained in the flow approach. It was further shown that a reordering of the sums appearing in AWM [4] leads to the same expressions as obtained in (FA). Thus by demonstrating, in this section, the specific relationship between the nested average model and the flow approach we will, in fact, achieve contact between the former and AWM. In short, the projection operator development of Section 3 permits comparison with FKK [3]; the work of this section, which relates the nested average model to the flow approach, allows for

comparison with AWM, since the latter is related to the flow approach in Ref.[6]. Thus by the end of this section we shall have related the current major approaches to multistep compound reactions.

In Subsection 4.1 we summarize some of the salient features of the probability flow approach developed in [6] and indicate the formal similarity between the results of [6] and those of the nested-average model of Section 3. In Subsection 4.2 we present a detailed formal comparison between the results of the different approaches to multistep compound processes. Finally, in Subsection 4.3 we indicate the kind of problems that one faces when trying to relate, in full detail, the quantities that were obtained in Section 3, where statistical assumptions were made on the S-matrix parameters, with those quantities that result when statistical assumptions are made on model states.

4.1 The Probability Flow Approach to Multistep Compound Processes

In the flow approach, the probability for a non-direct reaction leading from channel c to channel c' is determined by summing the individual probabilities for all processes which begin with c and end with c' where a succession of classes is visited along the way as the system evolves along various routes. The sequence of events leading from c to c' is assumed to be governed by probability conserving branching ratios, independent of the previous history (Markovian), which connects the classes of levels. To facilitate the link between the nested average model and the flow approach we have already introduced, in Section 3, several symbols-- Υ, T, η --which we shall later connect with their counterparts in the FA. We

next summarize some of the major results of Ref. [6], in preparation for making these connections.

In the general flow approach one considers any set of classes of states. For the discussion here we let those states be orthonormal model states which span the spaces of the respective projection operators $d_1, \ldots d_N$, introduced in Section 3. For example, we take the set $\{ | \xi_{n\alpha} \rangle \}$ to span the space of d_n such that

$$d_n = \sum_{\alpha} | \xi_{n\alpha} \rangle \langle \xi_{n\alpha} | \tag{4.1}$$

The branching ratios governing the flow at each stage involve: the average probability for passing from the states of class n to the states of the continuum (channels c); and the average probability for passing from one class of states to another. The former is provided by transmission coefficients, $\tau^{(F)}_{n,c}$; the latter is provided by $\mu_{nm} = \frac{\Gamma^{(F)}_{n\to m}}{\Gamma^{(F)}_{n}}$ the branching ratio for going from class n to class m. We assume that each class is characterized by a width (inverse lifetime), $\Gamma^{(F)}_n$ given by

$$\Gamma^{(F)}_n = \sum_c \Gamma^{(F)\uparrow}_{n,c} + \sum_m \Gamma^{(F)}_n \mu_{nm} \tag{4.2}$$

where

$$\Gamma^{(F)\uparrow}_{n,c} = \left(\frac{D_n}{2\pi} \right) \tau^{(F)}_{n,c} \quad . \tag{4.3}$$

with D_n providing the average separation between the states of class n. We take $\Gamma^{(F)}_{n\to m} = \Gamma^{(F)}_n \mu_{nm}$, as giving the transition rate for passing from class n to m. We further take $\Gamma^{(F)}_{n\to m}$ to include the effect of direct coupling and also coupling through the continuum ("external mixing") both of which are treated in AWM [4]. Note that the width $\Gamma^{(F)}_n$ here does not correspond to the correlation width $\widetilde{\Gamma}_n$ characterizing the

fluctuations of the $S_{\sim n}^{f\ell}$. Notice also that the above $\Gamma_n^{(F)}$'s (Eq. (4.2)) calculated with the model states (Eq. (4.1)) may be different from the Γ_n's of Eq. (3.50), which were calculated with the eigenstates of the non-Hermitian Hamiltonian $d_n \mathcal{H}(P_{n-1}) d_n$ of Eq. (3.40). (A fuller discussion will be made below.)

It was shown in Ref. 6 that a partial compound cross section, $\sigma_{cc'}^{(n)}$ (here, $\sigma_{n,cc'}^{f\ell}$) includes the contributions from all routes leading from channel c to channel c' that reach states of class n at least once but which reach no states of classes m, $(m > n)$. The above definition of $\sigma_{n,cc'}^{f\ell}$ obviously requires an ordering of the classes from 1 to N (see Fig. 3b). In the general flow approach this ordering is arbitrary. For the specific study of the nested-average model, however, the ordering will be based on the widths as presented in previous sections.

It was shown in Ref. [6], that $\sigma_{n,cc'}^{f\ell}$ can be obtained in the form

$$\sigma_{n,cc'}^{f\ell} = \frac{T_{n,c}^{(F)} T_{n,c'}^{(F)}}{\sum_{c''} T_{n,c''}^{(F)}} \left(1 - \sum_{\nu > n} \eta_{n\nu}^{(F)}\right) \tag{4.4}$$

where

$$T_{n,c}^{(F)} = \tau_{n,c}^{(F)} + \sum_{m < n} \gamma_{m,c}^{(F)} M_{mn}^{(F)} \tag{4.5}$$

The symbol $M_{mn}^{(F)}$ can be most easily interpreted in terms of the product $\Gamma_m^{(F)} M_{mn}^{(F)}$ which gives the rate for going from class m to n $(n > m)$ directly and also by passing through states of class n', $(n' < n)$. The factor $M_{mn}^{(F)}$ is obtained directly from the branching ratios, μ_{mn}, by the following iterative scheme,

$$M_{mn}^{(F)} = \mu_{mn} + \sum_{n' < n} \mu_{mn'} M_{n'n}^{(F)} \quad . \tag{4.6}$$

The quantity $\eta^{(F)}_{n\nu}$ appearing in Eq. (4.4) gives the total probability for going from class n to class ν, ($\nu > n$) directly and also by first passing through the states of class n' with $n' \leq n$. We call this quantity the "downward" branching ratio. In Ref. [6] $\eta^{(F)}_{n\nu}$ was also related to the normal branching ratios, $\mu_{mm'}$, by

$$\eta^{(F)}_{n\nu} = \frac{\mu_{n\nu} + \sum_{n'<n} M^{(F)T}_{nn'} \mu_{n'\nu}}{(1 - \sum_{n''<n} \mu_{nn''} M^{(F)}_{n''n})} \tag{4.7}$$

where $M^{(F)T}_{nn'}$ (the time-reversed version of $M^{(F)}_{n'n}$) represents the probability for going from class n to n', (n' < n), directly and also through the classes n", (n" < n),

$$M^{(F)T}_{nn'} = \frac{\Gamma^{(F)}_{n'}}{\Gamma^{(F)}_{n}} \frac{D_n}{D_{n'}} M^{(F)}_{n'n} \tag{4.8}$$

It can be shown that, with Eqs. (4.6) and (4.7), the expression for $T^{(F)}_{n,c}$, given in Eq. (4.5) can also be written in terms of $\eta^{(F)}_{n'n}$ as follows

$$T^{(F)}_{n,c} = \tau^{(F)}_{n,c} + \sum_{n'<n} T^{(F)}_{n',c} \eta^{(F)}_{n'n} \tag{4.9}$$

As a means of elucidating the structural similarity between the flow approach and the nested average approach, we devote the remainder of this sub-section to the study of a particular quantal realization of the flow model. It is defined in terms of matrix elements $\gamma_{n\alpha,c}$ connecting the model states $|\xi_{n\alpha}\rangle$ with the channels, which, using the projection-operator techniques of Section 3, are shown to obey the recursion relations of the flow pattern. Thus in analogy to the amplitude $g_{ni,c}$ of Section 3, we define [recalling the definition of $\psi^{(+)c}_{P_{n-1}}$ in Eq. (3.23)]

$$\gamma_{n\alpha,c} = \sqrt{2\pi} \langle \xi_{n\alpha} | V | \varphi^{(+)c}_{P_{n-1}} \rangle \tag{4.10}$$

The sole difference between $\gamma_{n\alpha,c}$ and $\vartheta_{ni,c}$ of Section 3 is the appearance of a model state $\langle \xi_{n\alpha} |$ (a member of an orthonormal basis) as a substitution for $\langle \tilde{\Psi}_{ni} |$ (a member of a biorthogonal basis). In both amplitudes the same channel state $\varphi^{(+)c}_{P_{n-1}}$ is used.

We develop an expression for $\gamma_{n\alpha,c}$ analogous to the one given in Eq. (3.59) for $\vartheta_{ni,c}$ but having a form more suitable for finding the quantities $M^{(F)}_{nm}$ and μ_{nm} defined in Eq. (4.6). We begin by partitioning $\varphi^{(+)c}_{P_{n-1}}$ into $p\varphi^{(+)c}_{P_{n-1}}$ and $D_{n-1}\varphi^{(+)c}_{P_{n-1}}$ as was done in subsection 3.4. We then solve the coupled equations for these components, using as the effective Hamiltonian $P_{n-1}\mathcal{H}(Q_n)P_{n-1}$ given in Eq. (3.34). To simplify the expressions we use h to represent the effective Hamiltonian $P_{n-1}\mathcal{H}(Q_n)P_{n-1}$, and find,

$$\varphi^{(+)c}_{P_{n-1}} = p\varphi^{(+)c}_{P_{n-1}} + D_{n-1}\varphi^{(+)c}_{P_{n-1}} = \left(1 - \frac{1}{E - D_{n-1}\,h\,D_{n-1}} D_{n-1}hp\right) p\varphi^{(+)c}_{P_{n-1}} \tag{4.11}$$

The full optical wave function $\varphi^{(+)c}_{opt}$, Eq. (3.44) differs from $p\varphi^{(+)c}_{P_{n-1}}$ and satisfies the following equation,

$$\left(E - php - phD_{n-1}\left\langle \frac{1}{E - D_{n-1}h\,D_{n-1}} \right\rangle_{I_1} D_{n-1}hp\right)\varphi^{(+)c}_{opt} = 0 \tag{4.12}$$

The corresponding optical Green's function, $\mathcal{G}^{(+)}_{opt}$, Eq. (3.19), is the reciprocal of the operator multiplying $\varphi^{(+)c}_{opt}$ in Eq. (4.12). We next solve for $p\varphi^{(+)c}_{P_{n-1}}$ in terms of $\varphi^{(+)c}_{opt}$, using the steps leading to Eq. (3.52) and (3.53), and find

$$p\varphi^{(+)c}_{P_{n-1}} = \varphi^{(+)c}_{opt} + \mathcal{G}^{(+)}_{opt}(ph\,D_{n-1})\mathcal{G}_{D_{n-1}}\,D_{n-1}\,hp\,\varphi^{(+)c}_{opt} \tag{4.13}$$

where $\mathcal{G}_{D_{n-1}}$ is defined in Eq. (3.88). The approximation indicated in Eq. (4.13) is that of dropping the ubiquitous factor, $\sqrt{\frac{i I_1/2}{E - D_{n-1}{}^h D_{n-1}^+ + iI_1/2}}$, when it would appear in the effective interactions. This type of approximation has been consistently employed in earlier sections when the interval I_1 was taken to be much larger than the level spacing. With this approximation we obtain

$$|\mathcal{P}^{(+)c}_{P_{n-1}}\rangle = |\mathcal{P}^{(+)c}_{opt}\rangle + (1 + \mathcal{G}^{opt} V)\mathcal{G}_{D_{n-1}} {}^{D_{n-1}}V|\mathcal{P}^{(+)c}_{opt}\rangle \qquad (4.14)$$

where V is used to represent ph D_{n-1} or D_{n-1}hp as needed. (In this section we are more concerned with the structure of expressions than formal rigor.) The representation of $|\mathcal{P}^{(+)c}_{P_{n-1}}\rangle$ given above, Eq. (4.14) provides for a convenient separation of $\gamma_{n\alpha,c}$ into two terms as follows,

$$\gamma_{n\alpha,c} = \sqrt{2\pi}\langle\xi_{n\alpha}|V|\mathcal{P}^{(+)c}_{opt}\rangle \qquad (4.15)$$
$$+ \sqrt{2\pi}\langle\xi_{n\alpha}|(V + V\mathcal{G}^{(+)}_{opt}V)\mathcal{G}_{D_{n-1}}V|\mathcal{P}^{(+)c}_{opt}\rangle$$

The first term is clearly analogous to $g^{opt}_{ni,c}$ of Eq. (3.60). As in Section 3, we use here the basic interaction, $v = V + V\mathcal{G}^{(+)}_{opt}V$, given in Eq. (3.100) which includes external coupling, and we obtain,

$$\gamma_{n\alpha,c} = \sqrt{2\pi}\langle\xi_{n\alpha}|V|\mathcal{P}^{(+)c}_{opt}\rangle \qquad (4.16)$$
$$+ \sqrt{2\pi}\sum_{\beta,m\rangle n}\langle\xi_{n\alpha}|V\mathcal{G}_{D_{n-1}}|\xi_{m\beta}\rangle\langle\xi_{m\beta}|V|\mathcal{P}^{(+)c}_{opt}\rangle$$

when a complete set of states $\{|\xi_{m\beta}\rangle\}$ is inserted into Eq. (4.15).

We next consider the average $\langle|\gamma_{n\alpha,c}|^2\rangle_\alpha$, which we obtain from

$\gamma_{n\alpha,c}$, Eq. (4.16), retaining only explicitly positive definite terms. The resulting expression is,

$$\left\langle \left| \gamma_{n\alpha,c} \right|^2 \right\rangle = 2\pi \left\langle \left| \langle \xi_{n\alpha} | V | \varphi_{opt}^{(+)c} \rangle \right|^2 \right\rangle_\alpha \qquad (4.17)$$

$$+ \sum_m 2\pi \left\langle \left| \langle \xi_{m\alpha} | V | \varphi_{opt}^{(+)c} \rangle \right|^2 \right\rangle_\alpha \left\langle \left| \langle \xi_{m\beta} | \mathcal{G}_{D_{n-1}} V | \xi_{n\gamma} \rangle \right|^2 \right\rangle_{\beta,\gamma}$$

With Eq. (4.17) we now make the following definitions to provide quantal expressions for the FA quantities T, γ, M:

$$T_{n,c}^{(F)} = \frac{2\pi}{D_n} \left\langle \left| \gamma_{n\alpha,c} \right|^2 \right\rangle_\alpha \qquad (4.18)$$

$$\gamma_{n,c}^{(F)} = \frac{2\pi}{D_n} \left\langle 2\pi \left| \langle \xi_{n\alpha} | V | \varphi_{opt}^{(+)c} \rangle \right|^2 \right\rangle_\alpha \qquad (4.19)$$

$$M_{mn}^{(F)} = \frac{D_m}{D_n} \left\langle \left| \langle \xi_{m\alpha} | \mathcal{G}_{D_{n-1}} V | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} \qquad (4.20)$$

where D_n is the average level spacing in class n.

With these definitions Eq. (4.17) is equivalent to

$$T_{n,c}^{(F)} = \gamma_{n,c}^{(F)} + \sum_m \gamma_{m,c}^{(F)} M_{mn}^{(F)} \qquad (4.21)$$

Notice that the quantities $T_{n,c}^{(F)}$, $\gamma_{n,c}^{(F)}$, and $M_{mn}^{(F)}$ are expressed in terms of the model states $|\xi_{n\alpha}\rangle$ rather than the eigenstates* $|\psi_{n\ell}\rangle$ which in Section 3 are used to define T', γ', etc. It is convenient to define $\Gamma_{n,c}^{(F)\uparrow}$ as in Eq. (4.3)

$$\Gamma_{n,c}^{(F)\uparrow} = \frac{D_n}{2\pi} \gamma_{n,c}^{(F)} \qquad (4.22)$$

which with Eq. (4.19) gives

$$\Gamma_{n,c}^{(F)\uparrow} = 2\pi \left\langle \left| \langle \xi_{n\alpha} | V | \varphi_{opt}^{(+)c} \rangle \right|^2 \right\rangle_\alpha \qquad (4.23)$$

(Note, this quantity differs from that in Eq. (3.79) in that model states appear, and the coupling is directly to the channels.) The above expression gives the average coupling of the model states of class n to the optical continuum. This definition also seems consistent with the model-state γ's of Ref. [4a,b]. As we demonstrate in Appendix D the above definition of $M_{nm}^{(F)}$, Eq. (4.20) does satisfy the flow approach recursion relation Eq. (4.6) provided we defined

$$\mu_{mn} = \frac{2\pi}{\hat{\Gamma}_m D_n} \left| \langle \xi_{m\alpha} | v | \xi_{n\beta} \rangle \right|^2 \tag{4.24}$$

where $\hat{\Gamma}_m$ is given in Eq. (D.29).

We next consider the factor $\eta_{nm}^{(F)}$ which can be constructed directly from the coupling matrices through the identification with the FA relationship

$$\eta_{mn}^{(F)}(m > n) = \frac{\mu_{nm} + \sum_{k>n} M_{nk}^{T(F)} \mu_{k\nu}}{1 - \sum \mu_{ni} M_{in}^{(F)} - C_{nn}} \tag{4.25}$$

This expression differs from the one presented in Ref. 6 , Eq. (4.7), solely by the addition of the factor C_{nn} which provides for the coupling among the states of class n via the continuum channels (external mixing within the same class). This was not included in Ref. 6 since external mixing was not discussed in that paper. Using Eq. (4.8), (4.20), and (4.24) in Eq.(4.25) we obtain

$$\eta_{nm}^{(F)} = \frac{2\pi \left\langle \left| \langle \xi_{n\alpha} | v | \xi_{m\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} + 2\pi \left\langle \left| \langle \xi_{n\alpha} | v \mathcal{G}_{Dn-1} | \xi_{s\delta} \rangle \right|^2 \right\rangle_{\alpha,\delta} \left\langle \left| \langle \xi_{s\delta} | v | \xi_{m\beta} \rangle \right|^2 \right\rangle_{\delta,\beta}}{\hat{\Gamma}_n (1 - \sum_{k<n} \mu_{nk} M_{kn} - C_{kn}) D_m} \tag{4.26}$$

This expression could have also been obtained from the following equation by retaining only positive definite terms,

$$\eta^{(F)}_{nm} = \frac{2\pi \left\langle \left| \langle \xi_{n\alpha} | v + v \mathcal{G}_{D_{n-1}} v | \xi_{m\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta}}{\hat{\Gamma}_n (1 - \sum \mu_{ni} M_{in} - C_{nn}) D_m} \tag{4.27}$$

The expression in Eq. (4.28) is similar to the η'_{nm} defined in Eq. (3.75). The terms $\langle \xi_{n\alpha} | v | \xi_{m\beta} \rangle$ describes the coupling (internal and external) between class n and class m ($m > n$), whereas $\langle \xi_{n\alpha} | v \mathcal{G}_{D_{n-1}} v | \xi_{m\beta} \rangle$ describes the coupling which proceeds through the classes $d_1 \ldots d_{n-1}$ by means of the Green's function $\mathcal{G}_{D_{n-1}}$. (See the discussion following Eq. (3.102))

Comparison of $\eta^{(F)}_{mn}$, Eq. (4.27), with η'_{mn}, Eq. (3.75), reveals that model states appear in the former whereas eigenstates appear in the latter. Furthermore, while $\tilde{\Gamma}$ appears in the latter the analogous factor in the former is $[\hat{\Gamma}_n (1 - \sum_{n' > n} \mu_{nn'} M_{n'm} - C_{nn})]$.

With the $\eta^{(F)}$ of Eq. (4.26) one can, as demonstrated in [6], obtain an alternative representation of $T^{(F)}_{n,c}$, Eq. (4.21), namely,

$$T^{(F)}_{n,c} = \tau^{(F)}_{n,c} + \sum_{m < n} T^{(F)}_{m,c} \eta^{(F)}_{mn} \tag{4.28}$$

which should be compared with Eq. (3.6)).

We now consider the flow approach analog for the average width and the correlation width. The average width Γ_n of Section 3 is given in Eq. (3.50) as

$$\Gamma_n = \left\langle -2 \, \mathrm{Im} \, \langle \tilde{\Psi}_{ni} | \, d_n \mathcal{H}(P_{n-1}) d_n | \Psi_{ni} \rangle \right\rangle_i$$

where $(d_n \mathcal{H}(P_{n-1}) d_n)$ has two terms, Eq. (3.40),

$$d_n \mathcal{H}(P_{n-1}) d_n = d_n \mathcal{H}(Q_n) d_n + d_n V \mathcal{G}^{(+)}_{P_{n-1}} V \quad .$$

For the flow analog we simply replace the eigenstates by the model states $|\xi_{n\alpha}\rangle$ and average over α, so that

$$\Gamma_n^{(F)} = \left\langle -2 \left\langle \xi_{n\alpha} \right| Im(\mathcal{H}(Q_n) + V \mathcal{G}^{(+)}_{P_{n-1}} V) \left| \xi_{n\alpha} \right\rangle \right|^2 \Big\rangle_\alpha \qquad (4.29)$$

Note that in evaluating Eq. (4.29) all that is required is the imaginary part of $d_n \mathcal{H}(P_{n-1}) d_n$. To simplify matters, we assume in what follows that the V which appears in Eq. (4.29) is Hermitian, and we obtain

$$\Gamma_n^{(F)} \cong -2 \left\langle \left\langle \xi_{n\alpha} \right| Im \mathcal{H}(Q_{n+1}) + V (Im \mathcal{G}^{(+)}_{P}) V \left| \xi_{n\alpha} \right\rangle \right\rangle_\alpha \qquad (4.30)$$

As we proceed, we must exercise caution in the treatment of $Im \mathcal{G}^{(+)}_{P_{n-1}}$ since $\mathcal{G}^{(+)}_{P_{n-1}}$ is the Green's function for a non-Hermitian Hamiltonian, $P_{n-1} \mathcal{H}(Q_n) P_{n-1}$. It is proven in Appendix D that, for any non Hermitian Hamiltonian, $\mathcal{H}$, the Green's function given by

$$\mathcal{G}^{(+)} = (E^{(+)} - \mathcal{H})^{-1} \qquad (4.31)$$

has the following imaginary part

$$Im \mathcal{G}^{(+)} = -\pi \left(\sum_c |\Psi^{(+)c}\rangle\langle \Psi^{(+)c}| \right) + \mathcal{G}^{(+)} (Im \mathcal{H}) \mathcal{G}^{(+)\dagger} \qquad (4.32)$$

Here $(E - \mathcal{H}) \Psi^{(+)c} = 0$, and c labels the various continuum channels with energy E. For the case of a real potential Eq. (4.32) provides the familiar form, but for complex potentials the additional term from $Im \mathcal{H}$ provides a very important contribution to the width.

The imaginary parts of both $\mathcal{H}(Q_{n+1})$ and $\mathcal{H}(Q_n)$ which arise from the respective energy averages over I_{n+1} and I_n, are required in order to

evaluate the width $\Gamma_n^{(F)}$. By following the arguments given in Appendix D, we find

$$\mathrm{Im}\mathcal{H}(Q_{n+1}) = \sum_{\substack{m\alpha \\ (m>n)}} -\pi \frac{H|\xi_{m\alpha}\rangle\langle\xi_{m\alpha}|H}{D_m} \tag{4.33}$$

Likewise we obtain for $\mathrm{Im}\mathcal{H}(Q_n)$,

$$\mathrm{Im}\mathcal{H}(Q_n) = \sum_{\substack{m\alpha \\ (m\geq n)}} -\pi \frac{H|\xi_{m\alpha}\rangle\langle\xi_{m\alpha}|H}{D_m} \tag{4.34}$$

Notice that Eq. (4.33) differs from Eq. (4.34) by the inclusion of the m=n term in the sum.

When Eqs.(4.33) and (4.34) are inserted in Eq. (4.36) by making use of Eq. (4.32), we obtain the following form for the average width $\Gamma_n^{(F)}$,

$$\begin{aligned}\Gamma_n^{(F)} = &\sum_c 2\pi \left\langle \left| \langle \xi_{n\alpha}| V \left| \varphi_{P_{n-1}}^{(+)c} \right\rangle \right|^2 \right\rangle_\alpha \\ &+ \sum_{m>n} \Big\langle \Big[2\pi \left| \langle \xi_{n\alpha}| V \left| \xi_{m\beta} \right\rangle \right|^2 \frac{1}{D_m} \\ &+ 2\pi \left| \langle \xi_{n\alpha}| V \mathcal{G}_{P_{n-1}}^{(+)} V |\xi_{m\beta}\rangle \right|^2 \frac{1}{D_m} \Big] \Big\rangle_{\alpha,\beta} \\ &+ 2\pi \left\langle \left| \langle \xi_{n\alpha}| V \mathcal{G}_{P_{n-1}}^{(+)} V |\xi_{n\beta}\rangle \right|^2 \right\rangle_{\alpha,\beta}.\end{aligned} \tag{4.35}$$

Since we have assumed the matrix elements are randomly distributed, we can simplify the expression involving the sum over m, as

$$\sum_{m>n} \left\langle 2\pi \left| \langle \xi_{n\alpha}| V + V \mathcal{G}_{P_{n-1}}^{(+)} V |\xi_{m\beta}\rangle \right|^2 \right\rangle_{\alpha,\beta}$$

Using Eqs. (4.20), (4.23), (4.24) and (4.26) which define $M^{(F)}$, $\Gamma^{(F)}_{n,c}$, $\mu_{mn'}$, and $\eta^{(F)}_{mn}$, we find the following contributions to the average width.

$$\Gamma_n = \sum_c \Gamma^{\uparrow}_{nc} + \sum_{c,n'} \Gamma^{\uparrow}_{n'c} M_{n'n} \qquad (4.36)$$
$$+ \sum_m \Big(1 - \sum_{n'<n} \mu_{nn'} M_{n'n} - C_{nn}\Big) \hat{\Gamma}_n \eta_{nm}$$
$$+ \hat{\Gamma}_n \Big(\sum_{n'<n} \mu_{nn'} M_{n'n} + C_{nn} \Big)$$

where

$$C_{nn} = \frac{2\pi}{\hat{\Gamma}_n D_n} \Big\langle \Big| \langle \xi_{n\alpha} | V \mathcal{G}^{(+)}_{opt} V | \xi_{n\beta} \rangle \Big|^2 \Big\rangle_{\alpha,\beta} \qquad (4.37)$$

involves the external "self" mixing for the states of class n. In Eq. (4.36) the first two sums on the right hand side come from the first sum in Eq. (4.35); the third sum comes from the second sum, and the last term is from the last one in Eq. (4.36). The four contributions exhaust all of the processes in the flow approach which can occur upon leaving a state in class n: a) coupling directly to the continuum; b) coupling to the continuum via the class n' ($n' < n$); c) coupling to class m ($m > n$); d) coupling back to class n via the classes n' ($n' < n$) or via the continuum (C_{nn}). All but the last were enumerated in Ref. [6].

Finally we consider the flow analog to the correlation width, $\tilde{\Gamma}_h$, of Section 3. For this comparison we define $\tilde{\Gamma}_h$ by the following equation

$$\tilde{\Gamma}^{-1}_h = \frac{D_n}{2\pi} \Big\langle \sum_i \langle \tilde{\Psi}_{ni} | \; \frac{1}{E - H\!\!\!|} \; \frac{1}{E - H\!\!\!|^+} \; | \Psi_{ni} \rangle \Big\rangle_I \qquad (4.38)$$

which is equivalent to $\tilde{\Gamma}^{-1} \equiv \Big\langle \frac{1}{\Gamma_{ni}} \Big\rangle_i$ * used in Section 3, if $H\!\!\!|$ represents the effective Hamiltonian $d_n \mathcal{H}(P_{n-1}) d_n$ of Eq. (3.40) for which $|\Psi_{ni}\rangle$

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* See, however, Ref. [11].

are the eigenstates. For the $\tilde{\Gamma}^{(F)}$ we substitute model states $|\xi_{n\alpha}\rangle$ for the eigenstates in Eq. (4.38),

$$\tilde{\Gamma}_n^{(F)-1} \equiv \frac{D_n}{2\pi} \left\langle \sum_\alpha \langle \xi_{n\alpha}| \frac{1}{E - H} \frac{1}{E - H^\dagger} |\xi_{n\alpha}\rangle \right\rangle_I \tag{4.39}$$

Since the model states are not eigenstates of H, it is convenient now to break H into a part diagonal in $|\xi_{n\alpha}\rangle$ (called H_D),

$$H_D = \sum_\alpha |\xi_{n\alpha}\rangle \eta_\alpha^n \langle \xi_{n\alpha}| , \tag{4.40}$$

where

$$\eta_\alpha^n = \langle \xi_{n\alpha}| H |\xi_{n\alpha}\rangle \equiv E_{n\alpha} - \frac{i\Gamma_{n\alpha}}{2} \tag{4.41}$$

and a part non-diagonal in $|\xi_{n\alpha}\rangle$ (called H_{ND}),

$$H_{ND} = \sum_{\alpha \neq \beta} |\xi_{n\alpha}\rangle W_{\alpha\beta} \langle \xi_{n\beta}| \tag{4.42}$$

where

$$W_{\alpha\beta} = \langle \xi_{n\alpha}| H |\xi_{n\beta}\rangle \tag{4.43}$$

Proceeding with the evaluation of Eq. (4.39) we make several approximations which are discussed in Appendix D. The critical assumptions concern the random nature of the matrix elements $W_{\alpha\beta}$ and the narrow distribution of $\Gamma_{n\alpha}$'s. We show in Appendix D that

$$\tilde{\Gamma}^{(F)} = \Gamma_n^{(F)} - \Gamma_n^{(F)} \left(\sum_{n' \lesssim n} \mu_{ni} M_{in} + C_{nn} \right) . \tag{4.44}$$

which shows that $\tilde{\Gamma}^{(F)}$ is less than $\Gamma_n^{(F)}$ by precisely the self coupling contributions. Now combining Eq. (4.44) with Eq. (4.36) we have

$$\tilde{\Gamma}_n^{(F)} = \left(\sum_c \Gamma_{nc}^{(F)\uparrow} + \sum_{c,n'} \Gamma_{n'c}^{(F)\uparrow} M_{n'n}^{(F)} \right) + \tilde{\Gamma}_n^{(F)} \sum_\nu \eta_{n\nu}^{(F)} \tag{4.45}$$

which can be rewritten as

$$\frac{2\pi}{D_n}\,\tilde{\Gamma}^{(F)}\,(1-\sum_{\nu}\eta^{(F)}_{n\nu}) = \sum_{c}(\tau^{(F)}_{n,c}+\sum_{n'}\tau^{(F)}_{n',c}\;M^{(F)}_{n'n}) \qquad (4.46)$$

$$= \sum_{c} T^{(F)}_{n,c}$$

The sum $\sum_{c} T^{(F)}_{n,c}$ is the one that appears in Eq. (4.4).

To obtain the quantal expression for the FA fluctuation cross section we simply collect the relevant factors developed above. The flow approach partial cross section of Eq. (4.4)

$$\sigma^{(F)}_{n,cc'} = \frac{T^{(F)}_{nc}\;T^{(F)}_{nc'}}{\sum_{c''} T^{(F)}_{nc''}}\;(1-\eta^{(F)}_{n\nu})$$

can then be written, with Eq. (4.46), as

$$\sigma^{(F)}_{n,cc'} = \frac{T^{(F)}_{nc}\;T^{(F)}_{nc'}}{\frac{2\pi}{D_n}\,\tilde{\Gamma}^{(F)}} \qquad (4.47)$$

Next with the definitions of Eq. (4.18) we obtain

$$\sigma^{(F)}_{n,cc'} = \frac{\frac{2\pi}{D_n}\langle|\gamma_{n\alpha}|^2\rangle_{\alpha}\,\langle|\gamma_{n\beta}|^2\rangle_{\beta}\frac{2\pi}{D_n}}{\frac{2\pi}{D_n}\,\tilde{\Gamma}^{(F)}} \qquad (4.48)$$

Then, with Eq. (4.39) we find

$$\sigma^{(F)}_{n,cc'} = \langle|\gamma_{n\alpha}|^2\rangle_{\alpha}\left\langle\sum_{\alpha}\langle\xi_{n\alpha}|\left|\frac{1}{E-H}\right|^2|\xi_{n\alpha}\rangle\right\rangle_{I}\langle|\gamma_{n\beta}|^2\rangle_{\beta} \qquad (4.49)$$

Finally, with the assumption that the matrix elements in the model basis are randomly distributed we can show, that Eq. (4.49) for $\sigma^{(F)}_{n,cc'}$ is equivalent to the results of Section 3, namely,

$$\sigma^{(F)}_{n,cc'} = \left| \langle \varphi^{(-)c'}_{P_{n-1}} | \, V \, d_n \, \frac{1}{E-H} \, d_n \, V | \varphi^{(+)c}_{P_{n-1}} \rangle \right|^2 = \sigma^{f\ell}_{n,cc'} \, . \tag{4.50}$$

We have found above that we can associate "quantal" expressions in the flow approach with each of the factors identified with the nested average in Section 3, provided we replace the biorthogonal eigenstates $|\Psi_{ni}\rangle$ by model states $|\xi_{n\alpha}\rangle$, and assume that matrix elements involving these model states are of a random nature. The latter assumption is especially important in obtaining the identification in Eq. (4.50). The necessity for this assumption would indicate a possible fundamental difference between the formalism of Section 3 and that of the flow approach and by implication [6] the work of AWM [4]. This difference is discussed in Section 4.3.

4.2. Comparison with the Results of Other Formulations

As we mentioned in the Introduction, several different approaches [1, 3, 4] have been advanced for treating multistep compound processes. It is therefore of interest to understand the relationship among the final results of these theories.

In this subsection we wish to emphasize the importance of the S-matrix autocorrelation function (1.2, 1.3) in establishing such connections and to point out that the fluctuation cross section alone is not sufficient for this purpose.

In a recent paper, Lane [12] has attempted to relate the $\sigma^{f\ell}_{cc}$, calculated in different approaches for a particular (isospin mixing) 2-class example of multistep compound processes. Ref. (7a) subsequently showed that conclusions drawn from such a comparison are not unique if based on $\sigma^{f\ell}_{cc}$, alone, but it

also demonstrated that they do become unique if the ε-dependence of $C_{cc'}(\varepsilon)$, the auto-correlation function, is included in the comparison. In the present section we extend this latter comparison between the NA and AWM approaches. Unfortunately the FKK approach [3] cannot be included in this comparison because it has not been extended to calculate the auto-correlation function. However, a detailed comparison is not really necessary, since the FKK results are "included" in those of the NA model, and can be obtained from it directly, simply by imposing the additional restriction of the chaining assumption, as was shown in ref. 5. (See also further details given in Section 5.)

The result given by Harney et al. [4b] for $\sigma^{f\ell}$ and $C^S(\epsilon)$ can be written in the form

$$\sigma^{f\ell} = \langle \hat{\tau} | M^{-1} | \hat{\tau} \rangle , \tag{4.51}$$

and

$$C^S(\epsilon) = \langle \hat{\tau} | (M + 2\pi i \epsilon I)^{-1} | \hat{\tau} \rangle , \tag{4.52}$$

where $\langle \hat{\tau}| = (\hat{\tau}_1, \hat{\tau}_2, \ldots)$ is a row vector and $\hat{\tau}_i = \sqrt{D_i}\,\tau_i$ with τ_i the penetration factor and D_i the spacing for level class i. (The matrix M_{ij} is labelled by class indices.) The fluctuation cross section, $\sigma^{f\ell}$, can be cast into the structural form of Eq. (1.1) provided M can be brought into a diagonal form, m, by a congruence transformation (not unique) $A^T M A = m$ (or $M^{-1} = A m^{-1} A^T$). In that case $\sigma^{f\ell}$ can be written in the single-sum form of Eq. (1.2)

$$\sigma^{f\ell} = \sum_j X_j' X_j' \tag{4.53}$$

where $X_j' = \sum_k \sqrt{1/m_{jj}}\,(A^T)_{jk}\,\hat{\tau}_k$.

Clearly, to obtain the structural form of both Eqs. (1.1) and (1.2) for <u>both</u> $\sigma^{f\ell}$ and $C^S(\epsilon)$ requires, in addition, that A be an orthogonal transformation, O, so that $O^T(M + i2\pi\epsilon I)O = m' + i2\pi\epsilon I$. The elements of m' are the eigenvalues of M. This transformation is unique and leads to terms which we may call eigenclasses of M. If $|n\rangle$ are eigenvectors of M with eigenvalues $2\pi\tilde{\Gamma}$, then,

$$C^S(\epsilon) = \sum_n \frac{\langle \hat{\tau} | n \rangle \langle n | \hat{\tau} \rangle}{2\pi \tilde{\Gamma}_n + 2\pi i \epsilon} \tag{4.54}$$

$$= \sum_n \frac{X_n X_n}{1 + i\epsilon/\tilde{\Gamma}_n}$$

where $X_n = (2\pi \tilde{\Gamma}_n)^{-1/2} \sum_k \langle n|k \rangle \hat{\tau}_k$.

These X_n's then provide representations of both Eq. (1.1) and (1.2). We emphasize that the requirement for achieving the structural form of Eq. (1.2) for $C^S(\epsilon)$ leads to a unique set of X_n's. The pole structure of $C^S(\epsilon)$ allows for a simple physical interpretation for each term, namely, each is associated with the characteristic life time (i.e., $\hbar/\Gamma_n$) for that class. The requirement of achieving the structural form of Eq. (1.1) for $\sigma^{f\ell}$ does not lead to a unique set of X_n's with identifiable properties.

In order to make our discussion more specific we consider the recipe proposed by Lane [12] for relating the results of Harney et al. [4b] with those of Grimes et al. [4c]. This recipe amounts to the choice of a specific transformation A. The matrix M in Harney's result is given by

$$M = 2\pi \begin{pmatrix} \Gamma_1 & -\sqrt{\Gamma_1^{\downarrow}\Gamma_2^{\downarrow}} \\ -\sqrt{\Gamma_1^{\downarrow}\Gamma_2^{\downarrow}} & \Gamma_2 \end{pmatrix} \tag{4.55}$$

where Γ_i and $\Gamma_i^{\downarrow}$ are widths given in Ref. [4]. Lane's recipe is equivalent to diagonalizing M through $A^T M A$ with

$$A = \begin{pmatrix} (1-\mu)/\sqrt{D_1} & \mu/\sqrt{D_1} \\ 0 & 1/\sqrt{D_2} \end{pmatrix} \tag{4.56}$$

where $\mu = \Gamma_1^{\downarrow}/\Gamma_1$ and D_i are the class level spacings. This matrix A allows one to obtain a result in the form found by Grimes et al. [4c]

$$\sigma^{f\ell} = X_1' X_1' + X_2' X_2' \tag{4.57}$$

with

$$(X_1')_{cc} = \frac{(1-\mu)^{1/2}\tau_{1c}}{(\sum_{c''}\tau_{1c''})^{1/2}}; \qquad (X_2')_{cc} = \frac{\mu\,\tau_{1c} + \tau_{2c}}{(\sum_{c''}\mu\,\tau_{1c''} + \tau_{2c''})^{1/2}} \tag{4.58}$$

where τ_{1c} are the transmission functions. Notice that A in Eq.(4.56) does <u>not</u> provide an orthogonal transformation and thus the X_i's are not associated with eigenclasses.* In the special case, however, that the widths appearing in M (Eq.4.55)) are such that $\Gamma_1/\Gamma_2 \gg 1$ and $D_1/D_2 \gg 1$, it can be shown that the unique, orthogonal transformation of M does provide precisely the same forms for X as given in Eq.(4.58). In that case the correlation lengths become Γ_1 and Γ_2 $(1 - \frac{\Gamma_1^{\downarrow}\,\Gamma_2^{\downarrow}}{\Gamma_1\,\Gamma_2})$.

We turn to the question considered by Lane regarding relations between the transmission coefficients τ_{nc} which appear in various approaches to the multistep problem. Lane [11] in order to relate the notations of Harney et al.[4b] and Grimes et al., [4c] made the plausible assumption that their τ's were the same and hence concluded that their μ's were, as well. Whatever τ's may be introduced, each X_n will be a linear combination of them, $X_n = \sum_i \langle n|i\rangle\tau_i$. Since the choice of states $|i\rangle$ which define the τ's is tantamount to choosing a specific model for calculating them, many such models are possible. Although it may be illu-

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*The relationship between X' and X (the eigenclass expression) is explicitly given by $X = (m)^{-1/2}\,A^T 0 (m')^{1/2}\,X'$.

minating to relate one model to another, the above discussion suggests that the only quantities which can be extracted from experimental data are the positions and residues of the poles of $C^{S}_{cc'}(\epsilon)$, i.e., the coherence widths $\tilde{\Gamma}_n$'s and the eigenclass X_n's.

An important conclusion that we have reached is that consideration of the S-matrix auto-correlation function in addition to the averaged fluctuation cross section would assist in interpreting, and comparing among, the results of the different theoretical approaches to multistep compound processes. This is quite clear from the general forms of $\sigma^{f\ell}_{cc'}$ and $C^{(S)}_{cc'}(\epsilon)$ given in Eqs. (1.1) and (1.2), through which one immediately recognizes an inherent nonuniqueness in the form of $\sigma^{f\ell}_{cc'}$ if considered by itself. This is so since there are many ways [7.a], of writing $\sigma^{f\ell}_{cc'}$ as a sum of products of two X_n's, one attached to the incident channel, c, and the other to the outgoing channel, c'. Of all these X_n's sets one identifies the physical one as being the factor in the residues at the poles of the S-matrix auto-correlation function. This last observation constitutes the sufficient condition that one needs to impose on the X-matrix, which taken together with the necessary condition implied by the form of $\sigma^{f\ell}_{cc'} = \sum_n X_{n,cc} X_{n,c'c'}$ would determine the physical X-matrix uniquely.

4.3 General Remarks

In subsection 4.1, we presented a rather detailed comparison between the nested-average model (NA) and the probability flow approach (FA). We accomplished that by first defining, what was called, the quantal version of the FA quantities, Eqs. (4.10),(4.18),(4.19) and (4.20) which are the equivalent to the NA quantities, Eq.(4.49), (4.73),(3.74) and (3.77) and subsequently demonstrating that they do satisfy the FA relations, Eqs. (4.5), (4.6) and (4.7). Since the quantal versions of the FA quantities are obtained from the NA quantities merely by replacing the eigenstates $\{\psi_{ni}, \tilde{\psi}_{ni}\}$ in the latter by the model states $\{\chi_{n\alpha}\}$ we consider the results of subsection (4.1) as constituting a formal connection between the NA and the FA (and thus AWM) and providing a physical interpretation of the results of the NA in terms of probability flow. Any attempt to going further than that and trying to make term-by-term "numerical" comparison, is bound to lead to conceptual and mathematical difficulties which are discussed briefly in this subsection.

In the nested-average approach presented in Section 3, the statistical assumptions were made on the parameters of the S-matrix. On the other hand, in a formalism like AWM [4], the statistical "ansatz" was made much earlier in the theory, namely, on the matrix elements of the interaction between model states. To a certain extent, the FA is closer in spirit to AWM, in the sense that the various transition rates are defined for model states, not for the eigenstates. Therefore, any attempt at relating the nested-average approach with AWM or FA will involve,necessarily,a study of the relation of model vs. eigenstates of the problem. As we have

seen in detail in Section 3, finding the poles and residues of the S-matrix is equivalent to diagonalizing a complex symmetric matrix (see Eq. (3.45)), which can be accomplished by means of a complex orthogonal transformation. The implications of the above fact can be illustrated in the particular case in which we have only one class of levels.

If we call $O_{i\lambda}$ the complex orthogonal matrix referred to above, the relation between the $g_{i,c}$ of Eq. (3.16) and the corresponding quantities $\gamma_{\lambda,c}$ defined for model states λ is

$$g_{i,c} \sum_{\lambda=1}^{N} O_{i\lambda}\gamma_{\lambda,c} \quad ; \tag{4.59a}$$

$$\gamma_{\lambda,c} = \sqrt{2\pi}\langle \xi_\lambda | V | \varphi^{(+)c}\rangle \tag{4.59b}$$

As an example, consider the $\gamma_{\lambda,c}$, $\lambda=1,\ldots,N$ as a set of N random numbers, which are transformed by means of a fixed matrix $O=\langle\tilde{\Psi}|\xi\rangle$ into another set of N random numbers $g_{i,c}$ $i=1,\ldots,N$.

Let us now perform the following two calculations:

1) Assume that the ensemble average of $\gamma_{\lambda,c}\gamma^{*}_{\mu,c}$ is:

$$\langle \gamma_{\lambda c}\gamma^{*}_{\mu c}\rangle = \delta_{\lambda\mu}\langle|\gamma_{\lambda c}|^2\rangle = \delta_{\lambda\mu}\langle|\gamma_c|^2\rangle \tag{4.60}$$

where $\langle|\gamma_{\lambda c}|^2\rangle = \langle|\gamma_c|^2\rangle$ is assumed to be independent of the index λ. Let us now calculate the ensemble average of $|g_{ic}|^2$, using (4.19) and (4.20). We find

$$\langle|g_{i,c}|^2\rangle = N_i\langle|\gamma_c|^2\rangle \tag{4.61}$$

where

$$N = \left\langle \sum_\lambda |O_{i\lambda}|^2 \right\rangle_i = \left\langle \sum_\lambda |\langle\tilde{\Psi}_i|\xi_\lambda\rangle|^2 \right\rangle = \langle\langle \tilde{\Psi}_i|\tilde{\Psi}_i\rangle\rangle_i \geq 1 \tag{4.62}$$

The inequality (4.22) is a consequence of Eq. (3.10).

2) Assume now that the ensemble average of $g_{ic}g^{*}_{jc}$ is

$$\left\langle g_{ic}g^{*}_{ic}\right\rangle = \delta_{ij}\left\langle |g_{ic}|^2\right\rangle = \delta_{ic}\left\langle |g_c|^2\right\rangle \tag{4.63}$$

where $\left\langle |g_{ic}|^2\right\rangle = \left\langle |g_c|^2\right\rangle$ is assumed to be independent of the index i. We now calculate the ensemble average of $|\gamma_{\lambda c}|^2$, using the inverse of (4.19a), namely,

$$\gamma_{\lambda c} = \sum_i O_{\lambda i} g_{ic} = \sum_i \langle \xi_\lambda | \psi_i \rangle\, g_{ic} \tag{4.64}$$

together with the assumption (4.23). We find

$$\left\langle |\gamma_{\lambda c}|^2\right\rangle = N' \left\langle |g_c|^2\right\rangle \tag{4.65}$$

where

$$N' = \left\langle \langle \psi_i | \psi_i \rangle \right\rangle_i \geq 1 \tag{4.66}$$

We can write Eq. (4.65) in the form

$$\left\langle |g_c|^2\right\rangle = \frac{1}{N'}\left\langle |\gamma_{\lambda c}|^2\right\rangle \tag{4.67}$$

where $(1/N') \leq 1$.

If we now compare the two calculations, namely, Eqs. (4.61) and (4.67) and <u>insist</u> that they should give the same result, we are forced to conclude that

$$N' = N = 1 \ . \tag{4.68}$$

Although the result (4.68) is certainly attainable, we know [15] that, in general, (4.68) is not true. In particular, the calculations of Ref. [15] show that for strong absorption N, $N' > 1$.

In conclusion, statistical assumptions made on the $\gamma_{\lambda c}$'s calculated with model states may not be consistent with similar assumptions made

on the g_{ic}'s, calculated with eigenstates. Consequently a direct comparison of results based on using model states as in AWM (or equivalently the flow approach), with those based on using the true eigenstates of the system as in Section 3, requires very special care.

One such comparison is made in ref. [20], between the S-matrix auto-correlation function of the present NA approach and that of the AWM approach. It is there noted that the multi-class AWM auto-correlation function is given, in [4b], as a bi-linear expression in the transmission coefficients from the (model) classes to the channels, and a linear transformation to "eigenclasses" brings it into the class-diagonal form of Eq. (2.14),

$$C^{s}_{cc'}(\varepsilon) = \sum_{\alpha} \sigma^{f\ell}_{\alpha,cc'}(\mathrm{AWM}) \frac{\tilde{\Gamma}^{\mathrm{AWM}}_{\alpha}}{\tilde{\Gamma}^{\mathrm{AWM}}_{\alpha} + i\varepsilon} \quad . \tag{4.69}$$

If Eqs. (2.14) and (4.69) are sufficiently accurate that their analytic continuations to complex ε can be equated, then from the equality of the positions and residues of their poles in ε we conclude that

$$\tilde{\Gamma}^{\mathrm{NA}}_{n} = \tilde{\Gamma}^{\mathrm{AWM}}_{n} \quad , \tag{4.70a}$$

and

$$\sigma^{f\ell}_{n,cc'}(\mathrm{NA}) = \sigma^{f\ell}_{n,cc'}(\mathrm{AWM}) . \tag{4.70b}$$

It does not, however, appear possible to conclude that the transmission coefficient $\tau_{n,cc}$ themselves are identical in the two theories, which is satisfactory, for the very definitions of the classes will in general differ in the two approaches. This was the point of the comparison between Eqs.

(4.61) and (4.67), which even suggests a possible incompatibility between the statistical assumptions employed in the two approaches.

It would be extremely interesting to find out whether a careful analysis of experimental data can shed some light on the above questions, and teach us something about the distribution of the overlapping resonance widths.

5. APPLICATIONS

As was made clear throughout our discussion, the energy auto-correlation function is the fundamental theoretical and experimental vehicle that could, in principle, sort out the intermediate-structure content of multistep compound processes. This sorting out becomes quite clear in the nested-doorway approximation in which one distinguishes among classes of resonances through their corresponding average correlation widths.

Although the above conclusions were reached assuming the validity of the nested-doorway approximation, i.e., $\tilde{\Gamma}_1 \gg \tilde{\Gamma}_2 \ldots \gg \tilde{\Gamma}_N$, we have evidence to suggest that they are of a more general nature, (see below) Once this is recognized, one might then resort to simple models to actually evaluate

the X_n's that appear in $\sigma^{f\ell}_{cc'}$ and $C_{cc'}(\epsilon)$. This will be needed for a consistent analysis of experiments aimed at extracting the correlation widths, $\tilde{\Gamma}_n$. In the form of the exciton model considered by the MIT group [3] one can calculate the model transmission coefficient, τ_n's and the mixing parameters, μ_{ij}. From the μ_{ij}'s one may then proceed to calculate the η's and M's defined in Eqs. (4.6) and (4.7) finally, using the fundamental relation ($T = \sum \tau M$), the transmission coefficients, T_n can be evaluated. From the T_n's, $\sigma^{f\ell}_{n,cc'}$ is then easily obtained (see Eq. (2.29)). Clearly, to make such a calculation feasible, several approximations are in order. These approximations have been adapted by FKK and they amount basically to a weak coupling limit whereby the η's are replaced by the μ's throughout (which is valid if one ignores the upward coupling, $\mu_{nn'}$; $n' < n$ (see Subsection 4.1)). Furthermore, the two-body nature of the residual interaction dictates the way class n is coupled to more complicated classes (the chaining hypothesis of FKK would imply that n couples downward directly only to class n+1). Finally, one may also use the fact that for reactions with light projectiles (e.g., nucleons) the incident channel couples predominantly to the simplest of all doorway classes (e.g., 3 exciton-states). With the above approximations, the transmission coefficients in the entrance channel becomes

$$T_{n,cc} \approx \delta_{n1}\,\tau_{1,cc} + \tau_{1,cc} \prod_{i=1}^{n-1} \mu_{i,i+1} \qquad (5.1)$$

Note that only τ_1 appears in Eq. (5.1), implying the strict doorway assumption. As for the outgoing channel transmission coefficient, $T_{n,c'c''}$, the kind of channel considered would dictate the class of

doorways that couples more strongly to it vis.,

$$T_{n,c'c'} \approx \gamma_{n,c'c'} \tag{5.2}$$

Finally, FKK [3] construct the fluctuation cross section $\sigma^{f\ell}_{n,cc'}$ as

$$\sigma^{(J)\,f\ell}_{n,cc'} = \left(1 - \mu^{(J)}_{n,n+1}\right) \frac{T^{(J)}_{n,cc}\; T^{(J)}_{n,c'c'}}{\sum_{c''} T^{(J)}_{n,c''c''}} \tag{5.3}$$

where they use (5.1) for $T_{n,cc}$ and (5.2) for $T_{n,c'c'}$ and in the sum. Consequently time reversal invariance of $\sigma^{f\ell}_{n,cc'}$ is broken. Notice that in Eq. (5.3) we have explicitly indicated the total angular momentum, J, that should label all quantities. To make this paper as self-contained as possible, we list in Appendix E some of the pertinent expressions for the γ_n's and $\mu_{n,n+1}$'s that show explicitly their dependence on the strength of the residual interaction as well as on the number of excitons assumed present in class n and the details of the angular momentum couplings. The detailed derivations are found in Ref. [3].

In confronting theory with experiment, we suggest using Eq. (5.3) for the factor $X^J_{n,cc}\, X^J_{n,c'c'}$ to calculate the spectrum of outgoing particles and thus fix the strength of the residual interaction as well as the number of steps (terms) required. Once this is done then one may use these calculated $X_n X_n$ terms to construct the auto-correlation function with the correlation widths, $\tilde{\Gamma}_n$, left as parameters to be extracted by fitting the experimentally determined $C_{cc'}(\epsilon)$. This procedure should

supply a consistent check on the theory.

Alternatively one may use the $X_n X_n$ factors in the autocorrelation function as parameters, assuming a priori knowledge of the $\widetilde{\Gamma}_n$'s, and subsequently compare these adjusted parameters with the calculated ones used in reproducting the average spectrum, i.e., $\sigma^{f\ell}_{cc'}$. This last one is the procedure followed by the Milan group.

Although we have presented a rather detailed account of the results of FKK [3], which were based on weak absorption, we do believe, however, that some of the restrictions inherent in the FKK results, may be easily removed. One such restriction is the use, for $T_{n,c'c'}$ and $\sum_{c'} T_{n,c'c'}$, the expression (5.2), which resulted in the time reversal non-invariant form of $\sigma^{f\ell}_{n,cc'}$, Eq. (5.3). By using our result for $T_{n,cc}$, Eq. (3.87), both for the incident and the outgoing channel as well as for $\mathrm{Tr}\,\underset{\sim}{T}$, we recover time reversal invariance. The chaining approximation may also be reasoned easily this way. Improvements may easily be made by using instead of μ, the η as given in Eq. (4.6).

To summarize, in the absence of the autocorrelation measurement, the only available information, namely, $\sigma^{f\ell}_{cc'}$, involves the sum over all classes (Eq. (1.1)). It is therefore of paramount importance to extend the preequilibrium studies to involve experimental investigation of the cross-section autocorrelation function as it would supply useful information about the individual terms, $X_n X_n$, in the sum. Furthermore, through the approximate relation $\frac{2\pi\widetilde{\Gamma}_n}{D_n} \simeq (\mathrm{Tr}\, X_n)^2$ one may double check the results, by using the fitted $\widetilde{\Gamma}_n$'s.

6. DISCUSSION AND CONCLUSIONS

We have succeeded in constructing a formalism that describes the multistep compound contribution to preequilibrium processes, when the compound system exhibits intermediate structure with correlation widths which are widely different from one another. However, the recent analysis of the Milan group [2] suggests a wider range of applicability. In their attempt to analyze the $^{27}Al(^{3}He,p)$ reaction with a simple Lorentzian form for the autocorrelation function, Bonetti et al. [19] found it necessary to use an unacceptably large number of correlation widths, ranging in values between 55 keV to 230 keV. They subsequently reanalyzed [2b] their data with our generalized autocorrelation function, taking the $X_n X_n$'s as parameters. The assumed values of the $\tilde{\Gamma}_n$'s were 230 keV and 55 keV in <u>all</u> excitation function analyzed. The larger value of 230 keV is attributed to the simplest class of resonances (a 5-exciton doorways), whereas the smaller $\tilde{\Gamma}_n$ of 55 keV is attributed to what is called the γ-process[3] which, in our model refers to the most complicated, equiliberated stage or N-stage. These values are quite consistent with simple estimates based on the exciton model. Some of their fit curves are shown in Figs. 1 and 2.

The results of the Milan group suggests the kind of analysis we envisage using our theory. Namely, after fixing the values, and more importantly the number, of the $X_n X_n$ terms (i.e., the number of classes) in $\sigma^{f\ell}_{cc'}$ required to account for the spectrum of outgoing particles in the different exit channels [19], one would then postulate the existence of as many correlation widths as there are classes. Subsequent analysis with the assumed form of $C_{cc'}(\varepsilon)$ should then furnish a <u>unified</u> description of all

excitation functions considered.

The reaction studied by the Milan group involves a light target (^{27}Al). It would be interesting to extend the analysis for other energy resolutions, to really see whether the compound nucleus correlation width $\tilde{\Gamma}_N$ disappears and only the remaining $\tilde{\Gamma}_n$'s show up. This could also be studied by changing the mass of the target. We believe that a more direct demonstration of the presence of a heirarchy of correlation widths would be more easily attained in reactions involving heavier target nuclei. As one knows, the equilibrium correlation width, $\tilde{\Gamma}_N$, is related to the mass number, A, and excitation energy, E, according to the Fermi gas model result [29]

$$\tilde{\Gamma}_N = 14 \exp\left[-4.69 \ (A/E)^{1/2}\right] \text{ MeV}. \tag{6.1}$$

Thus with not too high an excitation energy, and with mildly large mass number $\tilde{\Gamma}_N$ should become quite small. In such cases, the autocorrelation function, constructed with energy steps larger than $\tilde{\Gamma}_N$, should exhibit only intermediate structure-related correlation widths. This would indicate the presence (or absence!), in a given nuclear reaction, of the heirarchy of coherence widths referred to above.

The planned experiments with intermediate-mass target nuclei at Milan and São Paulo should supply quite an interesting test of our theory. It would also be worthwhile analyzing cases involving heavy projectiles. A study of reactions involving heavy ions of mass numbers around 12 is underway at Oak Ridge and ININ-Mexico.

On the theoretical side, we have considered in some detail the relations between the three major approaches to multistep compound reactions: the nested-average (NA) model, the MIT approach (FKK) and the Heidelberg approach (AWM).

All three are based on the assumption of doorway classes to define the reaction stages, and all three agree in their conclusions about the structure which these classes impose on the form of the expressions for the fluctuation cross sections proceeding through these classes. All three, that is, describe the entrance of flux from one channel, its percolation through the system of doorways, and its exit back into the channels. All three approaches begin with fully quantum-mechanical formulations,but, as ref. [6] has emphasized, their end product, obtained as a result of a variety of statistical approximations related to energy-averaging, is a classical flow pattern which is Markovian in its class-class connections, and unitary, i.e., probability-conserving. It is by no means trivial that all three approaches should have led to this same structure, for their starting-points are very different and their approximations are not obviously equivalent. Indeed, the results are changed substantially if the approximations are altered (e.g., by inclusion of higher-order terms in 1/Tr(P) in the AWM approach). Our conjecture is that this unanimity has resulted from the physical similarities in the spirit of the approximations which were employed by the various groups, which one might characterize as being "maximally statistical" within the constraints of unitarity. Maximal in the sense that, if all three groups were to attempt to include corrections by retreating from this "Hauser-Feshbach"-like limit, it seems almost certain that their different approaches would lead them to far less unanimous conclusions.

Finally, as we mentioned in the Introduction, we have not considered, in the present paper, the multistep direct contributions to the average cross section. It would be interesting to explore the application of the techniques developed in this paper to the analysis of multistep direct reactions.

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APPENDIX A

THE EXTENSION OF THE KKM ANALYSIS TO THE CALCULATION OF $\langle(\sigma_n^{f\ell})_{cc'}\rangle_{I_1}$

The purpose is to evaluate $\left\langle \sigma^{f\ell}_{n,cc'} \right\rangle_{I_1}$ averaging directly over the largest interval. Let us take the case of only 2 classes of levels: doorways and fine-structure status: For simplicity in notation, the index d runs over the levels of the first class and q over those of the second class. The KKM procedure can be carried out in the way indicated below. We first calculate

$$\left\langle \left|S^{f\ell}_{2,cc'}\right|^2 \right\rangle_{I_1} = \left\langle \sum_{qq'} \frac{g_{qc}(E)\, g_{qc'}(E)\, g^*_{q'c}(E)\, g^*_{q'c'}(E)}{(E-\mathcal{E}_q)(E-\mathcal{E}^*_{q'})} \right\rangle_{I_1} \approx \tag{A.1}$$

$$\approx \left\langle \sum_q \frac{|g_{qc}(E)|^2 |g_{qc'}(E)|^2}{(E-E_q)^2 + (\Gamma_q/2)^2} \right\rangle_{I_1} = \sum_q \frac{2\pi}{\Gamma_q} \frac{1}{I_1} \int_{E-\frac{I_1}{2}}^{E+\frac{I_1}{2}} dE' \, \frac{\Gamma_q/2\pi}{(E-E_q)^2+(\Gamma_q/2)^2} |g_{qc}(E)|^2 |g_{qc'}(E)|^2$$

where the randomness in signs of the g_{qc}'s was used to drop the terms with $q\neq q'$. The energy dependence in the $g_{ac}(E)$'s occurs on the energy scale of the first class. The average over I_1 was performed with a box weighting function. The Lorentzian that occurs in the integrand has a width Γ_q which satisfies $\Gamma_q \ll \Gamma_d \ll I_d$; Γ_q is therefore much smaller than the energy scale over which the g(E) varies, so that the integral will essentially select the energy $E'=E_q$, except for a few E_q in the edges of the interval. We then have

$$\left\langle \left|S^{f\ell}_{2,cc'}\right|^2 \right\rangle_{I_1} \approx \sum_q \frac{2\pi}{\Gamma_q I_1} |g_{qc}(E_q)|^2 |g_{qc'}(E_q)|^2 =$$

$$= \frac{2\pi}{D_2} \left\langle \frac{1}{\Gamma_q} |g_{qc}(E_q)|^2 |g_{qc'}(E_q)|^2 \right\rangle_{q\in I_1} \tag{A.2}$$

$$\approx \frac{2\pi}{D_2 \tilde{\Gamma}_2} \left\langle |g_{ac}(E_q)|^2 |g_{qc'}(E_q)|^2 \right\rangle_{q\in I_1}$$

We now expand the states q in a basis set of states $|Q\rangle$ that diagonalize H_{QQ}

$$|q\rangle = \sum_Q C_Q^q \, |Q\rangle \tag{A.3a}$$

$$g_{qc}(E) = \langle \varphi_{P_1}^{(-)c}(E) | V | q \rangle = \sum_Q C_Q^q \langle \varphi_{P_1}^{(-)c}(E) | V | Q \rangle$$
$$\equiv \sum_Q C_Q^q \, g_{Qc}(E) \tag{A.3b}$$

We now insert (A3b) in (A2):

$$G_{cc'} \equiv \langle |g_{qc}(E_q)|^2 |g_{qc'}(E_q)|^2 \rangle_{q \in I_d} = \tag{A.4}$$

$$= \sum_{Q_1 Q_2 Q_3 Q_4} \left\langle C_{Q_1}^q \, C_{Q_2}^{q*} \, C_{Q_3}^q \, C_{Q_4}^{q*} \, g_{Q_1 c}(E_q) \, g_{Q_2 c}^*(E_q) \, g_{Q_3 c'}(E_q) \, g_{Q_4 c'}^*(E_q) \right\rangle_{q \in I_d}$$

Because of the complex many-body nature of the Q-states, we argue that the C's have random phases, which select the combinations $Q_1 = Q_2$, $Q_3 = Q_4$ or $Q_1 = Q_4$, $Q_2 = Q_3$. Then

$$G_{cc'} = \left\langle \sum_Q |C_Q^q|^2 |g_{Qc}(E_q)|^2 \sum_{Q'} |C_{Q'}^q|^2 \, |g_{Q'c'}(E_q)|^2 \right\rangle_{q \in I_1} +$$
$$+ \left\langle \sum_Q |C_Q^q|^2 \, g_{Qc}(E_q) \, g_{Qc'}^*(E_q) \sum_{Q'} |C_{Q'}^q|^2 \, g_{Q'c'}(E_q) g_{Q'c}^*(E_q) \right\rangle_{q \in I_1} \tag{A.5}$$

If there were no doorways present, the $g_{Qc}(E_q)$ would not depend on E_q; the sums occurring in Eq. (A5) would then be uncorrelated and they would factorize, yielding the $X_{cc}X_{c'c'} + X_{cc'} X_{c'c}$ structure, just as

in KKM. If doorways are present, they may produce correlations through the E_q-dependence which can prevent this factorization. To investigate this, we need the explicit E_q-dependence; it can be obtained from the analysis of Section 3, which gives

$$g_{Qc}(E_q) = \langle \phi^{(-)}_{p,c} | V_{p+d,q} | Q \rangle$$

$$+ \langle \phi^{(-)}_{p,c} | V_{pd} \frac{1}{E_q - d\mathcal{H}(q)d - V_{dp} \frac{1}{E_q - p\mathcal{H}(q+d)p} V_{pd}} \left[1 + V_{dp} \frac{1}{E_q - p\mathcal{H}(q+d)p} \right] V_{p+d,q} | Q \rangle$$

or, introducing a complete set of eigenstates in d space:

$$g_{Qc}(E_q) = g^{opt.}_{Qc} + \sum_d \frac{f_{d,Qc}}{E_q - E_d + i\Gamma_d/2} ,$$

In the extreme case of an isolated doorway and taking $g^{opt}_{Qc} = 0$, Eq. (A.7) implies $|g_{Qc}(E_q)|^2 = |f_{d,Qc}/f_{d,Q'c'}|^2 \, |g_{Q'c'}(E_q)|^2$, i.e., that all g's have the same E_q-dependence. In this instance they are completely correlated, and the sums in Eq. (A.5) do not factorize. However, the case under consideration here is the one in which many doorways are present and are themselves overlapping. This will randomize the phase of the $f_{d,Qc}$ as a function of Q (enough, in fact, that $\langle g_{Qc} \rangle_Q = 0$) so that the $g_{d,Qc}(E_q)$ lose their correlations, and the lack of correlations between the C^q_Q's then leads, exactly as in KKM, to the desired factorization. That is,

$$G_{cc'} \approx \left\langle \sum_Q |C^q_Q|^2 |g_{Qc}(E_q)|^2 \right\rangle_{q \varepsilon I_1} \left\langle \sum_{Q'} |C^q_{Q'}| \, |g_{Q'c}(E_q)|^2 \right\rangle_{q \varepsilon I_1} +$$

$$\left\langle \sum_Q |C_Q^q|^2 \, g_{Qc}(E_q) g^*_{Qc'}(E_q) \right\rangle_{q\in I_1} \left\langle \sum_{Q'} |C_{Q'}^q|^2 \, g_{Q'c'}(E_q) \, g^*_{Q'c}(E_q) \right\rangle_{q\in I_1}$$

$$\approx \left\langle |g_{qc}(E_q)|^2 \right\rangle_{q\in I} \left\langle |g_{qc'}(E_q)|^2 \right\rangle_{q\in I_1} + \left\langle g_{qc}(E_q) \, g^*_{qc'}(E_q) \right\rangle_{q\in I_1}$$

$$\cdot \left\langle g_{qc'}(E_q) \, g^*_{qc}(E_q) \right\rangle_{q\in I_1}$$

and defining

$$\left\langle X_{2,cc'} \right\rangle_{I_1} = \sqrt{\frac{2\pi}{\Gamma_2 D_2}} \left\langle g_{qc}(E_q) \, g^*_{qc'}(E_q) \right\rangle_{q\in I_1}$$

we finally have

$$\left\langle |S^{f\ell}_{2,cc'}|^2 \right\rangle_{I_1} = \left\langle X_{2,cc} \right\rangle_{I_1} \left\langle X_{2,c'c'} \right\rangle_{I_1} + \left\langle X_{2,cc'} \right\rangle_{I_1} \left\langle X_{2,c'c} \right\rangle_{I_1} \tag{A.10a}$$

The $\langle X_{2,cc'} \rangle_{I_1}$ are the quantities calculated in the second part of Subsection 3.3.

Notice that from Eq. (A5) we obtain directly

$$\left\langle |S^{f\ell}_{2,cc'}|^2 \right\rangle_{I_1} = \left\langle X_{2,cc}\, X_{2,c'c'} \right\rangle_{I_1} + \left\langle X_{2,cc'}\, X_{2,c'c} \right\rangle_{I_1} \qquad \text{(A.10b)}$$

where

$$X_{2,cc'} = \sqrt{\frac{2\pi}{\tilde{\Gamma}_2 D_2}} \left\langle g_{qc}(\epsilon_q)\, g^*_{qc'}(\epsilon_q) \right\rangle_{q\epsilon I_2} .$$

Therefore our result (A.10a) implies the statement that the average of the X's that appears in (A.10b) factorizes as in (A.10a). The difference between (A.10a) and (A.10b) is clearly of higher order in $\frac{D_1}{\Gamma_1}$ (which we assumed to be small) and in $1/\mathcal{N}$ (which is also small in the KKM analysis). But this has also the following implication. We can write the average $\left\langle \underset{\sim}{P}_2 \right\rangle_{I_1}$ as

$$\left\langle \underset{\sim}{P}_2 \right\rangle_{I_1} = \left\langle \underset{\sim}{X}_2 \,\mathrm{Tr}\, \underset{\sim}{X}_2 \right\rangle_{I_1} = \left\langle \underset{\sim}{X}_2 \right\rangle \mathrm{Tr} \left\langle \underset{\sim}{X}_1 \right\rangle \qquad \text{(A.12)}$$

where we have assumed that the number of channels, $\mathcal{N}$, is large. Therefore when the cross section is written as an average involving penetration matrices, as in Eq. (2.26), the average can be taken on each factor. This last fact can also be shown by evaluating the average explicitly, as we shall see below.

The definition of P_2 is

$$\underset{\sim}{P}_2 = \underset{\sim}{1} - \underset{\sim}{\bar{S}}_2^{\dagger}\,\underset{\sim}{\bar{S}}_2 \,, \tag{A.13}$$

where $\underset{\sim}{\bar{S}}$ still varies on the energy scale of the doorways and can be written as Eq. (2.2)

$$\underset{\sim}{\bar{S}}_2 = \underset{\sim}{\bar{S}}_1 + \underset{\sim}{S}^{f\ell} \tag{A.14}$$

Substituting in (A.11)

$$\underset{\sim}{P}_2 = \underset{\sim}{1} - (\underset{\sim}{\bar{S}}_1 + \underset{\sim}{S}_1^{f\ell})\,(\underset{\sim}{\bar{S}}_1 + \underset{\sim}{S}_1^{f\ell}) \,. \tag{A.15}$$

The quantity to be studied is then ($\mathcal{N}$ being the number of channels)

$$\left\langle \frac{P_{2,cc}P_{2,c'c'}}{\mathrm{Tr}P_2} \right\rangle_{I_2} =$$

$$= \left\langle \frac{\left[1-\sum_a (\bar{S}_{1,ac}+S^{f\ell}_{1,ac})^*(\bar{S}_{1,ac}+S^{f\ell}_{1,ac})\right]\left[1-\sum_b (\bar{S}_{1,bc'}+S^{f\ell}_{1,bc'})^*(\bar{S}_{1,bc'}+S^{f\ell}_{1,bc'})\right]}{N\left[1-\frac{1}{N}\sum_{\alpha\beta}(\bar{S}_{1\alpha\beta}+S^{f\ell}_{1,\alpha\beta})^*(\bar{S}_{1,\alpha\beta}+S^{f\ell}_{1,\alpha\beta})\right]} \right\rangle$$

$$= \frac{1}{\mathcal{N}}\Bigg\{1 - \left\langle \sum_a (\bar{S}_{1,ac}+S^{f\ell}_{1,ac})^*(\bar{S}_{1,ac}+S^{f\ell}_{1,ac}) \right\rangle - \left\langle \sum_b (\bar{S}_{1,bc'}+S^{f\ell}_{1,bc'})^*(\bar{S}_{1,bc'}+S^{f\ell}_{1,bc'}) \right\rangle$$

$$+ \left\langle \sum_{ab} (\bar{S}_{1,ac}+S^{f\ell}_{1,ac})^*(\bar{S}_{1,ac}+S^{f\ell}_{1,ac})(\bar{S}_{1,bc'}+S^{f\ell}_{1,bc'})(\bar{S}_{1,bc'}+S^{f\ell}_{1,bc'}) \right\rangle + \tag{A.16}$$

$$+ \frac{1}{\mathcal{N}^2}\left\langle \sum_{\alpha\beta} (\bar{S}_{1,\alpha\beta}+S^{f\ell}_{1,\alpha\beta})^*(\bar{S}_{1,\alpha\beta}+S^{f\ell}_{1,\alpha\beta}) \right\rangle -$$

$$- \frac{1}{\mathcal{N}^2}\left\langle \sum_{a\alpha\beta} (\bar{S}_{1,ac}+S^{f\ell}_{1,ac})^*(\bar{S}_{1,ac}+S^{f\ell}_{1,ac})(\bar{S}_{1,\alpha\beta}+S^{f\ell}_{1,\alpha\beta})^*(\bar{S}_{1,\alpha\beta}+S^{f\ell}_{1,\alpha\beta}) \right\rangle -$$

$$- \frac{1}{N} \left\langle \sum_{b\alpha\beta} (\bar{S}_{1,bc'} + S^{f\ell}_{1,bc'})^* (\bar{S}_{1,bc'} + S^{f\ell}_{1,bc'}) (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta})^* (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta}) \right\rangle$$

$$+ \frac{1}{N^2} \left\langle \sum_{\alpha\beta\gamma\delta} (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta})^* (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta}) (\bar{S}_{1,\gamma\delta} + S^{f\ell}_{1,\gamma\delta})^* (\bar{S}_{1,\gamma\delta} + S^{f\ell}_{1,\gamma\delta}) \right\rangle \qquad (A.16)$$

$$+ \ldots \Big\}$$

The terms 1,2,3 and 5 inside the bracket are already in the form that would appear in an expansion of $\langle P_{2,cc} \rangle_{I_1} \langle P_{2,c'c'} \rangle_{I_1} / \mathrm{Tr} \langle P_{\sim 2} \rangle_{I_1}$. We shall thus concentrate on the analysis of the terms that involve fourth powers of S, which we shall call 4, 6, 7 and 8, in the order in which they appear above. Term 4 gives

$$\begin{aligned}
(4) = {} & \sum_a |\bar{S}_{1,ac}|^2 \sum_b |\bar{S}_{1,bc'}|^2 + \sum_a |\bar{S}_{1,ac}|^2 \sum_b \langle S^{f\ell *}_{1,bc'} S^{f\ell}_{1,bc'} \rangle + \\
& + \sum_{ab} \bar{S}^*_{1,ac} \bar{S}^*_{1,bc'} \langle S^{f\ell}_{1,ac} S^{f\ell}_{1,bc'} \rangle + \sum_{ab} \bar{S}^*_{1,ac} \bar{S}_{1,bc'} \langle S^{f\ell}_{1,ac} S^{f\ell *}_{1,bc'} \rangle \\
& + \sum_{ab} \bar{S}^*_{1,ac} \langle S^{f\ell}_{1,ac} S^{f\ell *}_{1,bc'} S^{f\ell}_{1,bc'} \rangle + \\
& + \sum_{ab} \bar{S}_{1,ac} \bar{S}^*_{1,bc'} \langle S^{f\ell *}_{1,ac} S^{f\ell}_{1,bc'} \rangle + \sum_{ab} \bar{S}_{1,ac} \bar{S}_{1,bc'} \langle S^{f\ell}_{1,ac} S^{f\ell *}_{1,bc'} \rangle \\
& + \sum_{ab} \bar{S}_{ac} \langle S^{f\ell *}_{1,ac} S^{f\ell}_{1,bc'} S^{f\ell}_{1,bc'} \rangle + \sum_{ab} \bar{S}^*_{1,bc'} \langle S^{f\ell *}_{1,ac} S^{f\ell}_{1,ac} S^{f\ell}_{1,bc'} \rangle \\
& + \sum_a \langle |S^{f\ell}_{1,ac}|^2 \rangle \sum_b |\bar{S}_{1,bc'}|^2 + \sum_{ab} \bar{S}_{1,bc'} \langle S^{f\ell *}_{1,ac} S^{f\ell}_{1,ac} S^{f\ell *}_{1,bc'} \rangle + \\
& + \sum_{ab} \langle S^{f\ell *}_{1,ac} S^{f\ell}_{1,ac} S^{f\ell}_{1,bc'} S^{f\ell *}_{1,bc'} \rangle . \qquad (A.17)
\end{aligned}$$

We neglect the terms involving $\langle S^{f\ell}_1 S^{f\ell}_1 \rangle$ and $\langle S^{f\ell}_1 S^{f\ell}_1 S^{f\ell}_1 {}^* \rangle$, so that

$$(4) \approx \sum_a |\bar{S}_{1,ac}|^2 \sum_b |\bar{S}_{1,bc'}|^2 + \sum_a |\bar{S}_{1,ac}|^2 \sum_b \left\langle S^{f\ell}_{1,bc'} \; {}^{*} \; S^{f\ell}_{1,bc'} \right\rangle +$$

$$+ \sum_a \left\langle S^{f\ell\,*}_{1,ac} \quad S^{f\ell}_{1,ac} \right\rangle \sum_b |\bar{S}_{1,bc'}|^2 + \qquad (A.18)$$

$$+ \sum_{ab} \bar{S}^{*}_{1,ac} \; \bar{S}_{1,bc'} \left\langle S^{f\ell}_{1,ac} \; S^{f\ell\,*}_{1,bc'} \right\rangle + \sum_{ab} \bar{S}_{1,ac} \; \bar{S}^{*}_{1,bc'} \left\langle S^{f\ell\,*}_{1,ac} \quad S^{f\ell}_{1,bc'} \right\rangle +$$

$$+ \sum_{ab} \left\langle S^{f\ell\,*}_{1,ac} \quad S^{f\ell}_{1,ac} \quad S^{f\ell\,*}_{1,bc'} \quad S^{f\ell}_{1,bc'} \right\rangle .$$

The order of magnitude of the above terms can be estimated as follows:

$$(4)_1 = (\bar{S}_1 \quad \bar{S}_1^{\dagger})_{cc} \, (\bar{S}_{1,} \quad \bar{S}_{1,}^{\dagger})_{c'c'} \sim 1 \qquad (A.19)$$

$$(4)_2 = (\bar{S}_1 \quad \bar{S}_1^{\dagger})_{cc} \sum_b (X_{1,bb} X_{1,c'c'} + X_{1,bc'} X_{1,c'b}) =$$

$$= (\bar{S}_1 \quad \bar{S}_1^{\dagger})_{cc} \left[X_{1,c'c'} \mathrm{Tr}\, X_1 + ((X_1)^2)_{c'c'} \right] \sim 1 + \frac{1}{\mathcal{N}} \sim 1 , \qquad (A.20)$$

considering that $X \sim P/\sqrt{\mathrm{Tr}\, P}$ is basically of order $1/\sqrt{\mathcal{N}}$. A similar analysis applies to $(4)_3$.

$$(4)_4 = \sum_{ab} \bar{S}_{1,ac} \, \bar{S}_{1,bc'} (X_{1,ab} \, X_{1,cc'} + X_{1,ac'} X_{1,cb}) =$$

$$= (\bar{S}_{1,}^{\dagger} \quad X_1 \, \bar{S}_1 \,)_{cc'} \, X_{1,cc'} + (\bar{S}_1^{\dagger} \quad X_1)_{cc'} \, (X_1 \, \bar{S}_1 \,)_{cc'} \sim \frac{1}{\mathcal{N}} \qquad (A.21)$$

A similar analysis applies to $(4)_5$. For $(4)_6$ we shall apply the rule found in Appendix B

$$(4)_6 \approx \sum_{ab} \left[\left\langle S^{f\ell}_{1,ac} \; {}^{*} \; S^{f\ell}_{1,ac} \right\rangle \left\langle S^{f\ell}_{1,bc'} \; {}^{*} \; S^{f\ell}_{1,bc'} \right\rangle + \right. \qquad (A.22)$$

$$\left. \left\langle S^{f\ell}_{1,ac} \; {}^{*} \; S^{f\ell}_{1,bc'} \right\rangle \left\langle S^{f\ell}_{1,ac} \quad S^{f\ell\,*}_{1,bc'} \right\rangle \right] =$$

$$= \sum_{ab} \left[(X_{1,aa} X_{1,cc} + X_{1,ac} X_{1,ca})(X_{1,bb} X_{1,c'c'} + X_{1,bc'} X_{1,c'b}) + \right.$$

$$\left. + X_{1,ab} X_{1,cc'} + X_{1,ac} X_{1,cb})(X_{1,ab} X_{1,cc'} + X_{1,ac'} X_{1,cb}) \right] =$$

$$= \left[X_{1,cc} \mathrm{Tr}\, X_1 + ((X_1)^2)_{cc} \right] \left[X_{1,c'c'} \mathrm{Tr}\, X_1 + ((X_1)^2)_{c'c'} \right] +$$

$$+ (X_{1,cc'})^2 \mathrm{Tr}(X_1 X_1^\dagger) + X_{1,cc'} (X_1 X_1^\dagger X_1)_{cc'} +$$

$$+ X_{1,cc'} (X_1 \overbrace{X_1^\dagger})_{cc'} + (X_1 X_1^\dagger)_{cc} (X_1^\dagger X_1)_{c'c'}$$

$$= \left[X_{1,cc} \mathrm{Tr}\, X_1 + ((X_1)^2)_{cc} \right] \left[X_{1,c'c'} \mathrm{Tr}\, X_1 + ((X_1)^2)_{c'c'} \right] + O\left(\frac{1}{\mathcal{N}}\right)$$

$$\sum_{ab} \left\langle S_{1,ac}^{f\ell,*} S_{1,ac}^{f\ell} \right\rangle \left\langle S_{1,bc'}^{f\ell *} S_{1,bc'}^{f\ell} \right\rangle \approx 1$$

Collecting terms we then find

$$(4) = \left\langle \sum_a (\bar{S}_{1,ac} + S_{1,ac}^{f\ell})^* (\bar{S}_{1,ac} + S_{1,ac}^{f\ell}) \right\rangle \times$$

$$\times \left\langle \sum_b (\bar{S}_{1,bc'} + S_{1,bc'}^{f\ell})^* (\bar{S}_{1,bc'} + S_{1,bc'}^{f\ell}) \right\rangle + O\left(\frac{1}{\mathcal{N}}\right), \qquad (A.23)$$

which is again in the form that would appear in an expansion of

$$\langle P_{2,cc} \rangle_{I_1} \langle P_{2,c'c'} \rangle_{I_1} / \mathrm{Tr} \langle P_2 \rangle_{I_1}$$

Terms 6, 7 and 8 of Eq. (A.16) are exactly of the same form as term 4 we have just calculated (except for relabelling of the indices), with extra summations and factors $1/\mathcal{N}$. We have

$$(6) = -\left\langle \sum_a (\bar{S}_{1,ac} + S_{1,ac}^{f\ell})^* (\bar{S}_{1,ac} + S_{1,ac}^{f\ell}) \right\rangle \times$$

$$\times \left\langle \frac{1}{\mathcal{N}} \sum_{\alpha\beta} (\bar{S}_{1,\alpha\beta} + S_{1,\alpha\beta}^{f\ell})^* (\bar{S}_{1,\alpha\beta} + S_{1,\alpha\beta}^{f\ell}) \right\rangle + O\left(\frac{1}{\mathcal{N}}\right) \qquad (A.24)$$

$$(7) \quad = -\left\langle \sum_b (\bar{S}_{1,bc'} + S^{f\ell}_{1,bc'})^* \, (\bar{S}_{1,bc'} + S^{f\ell}_{1,bc'}) \right\rangle \times$$
$$\times \left\langle \sum_{\alpha\beta} (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta})^* \, (\bar{S}_{1,\alpha\beta} + S^{f\ell,d}_{1,\alpha\beta}) \right\rangle + O\left(\frac{1}{N}\right) \tag{A.25}$$

$$(8) \quad = \left\langle \sum_{\alpha\beta} (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta})^* \, (\bar{S}_{1,\alpha\beta} + S^{f\ell}_{1,\alpha\beta}) \right\rangle \times$$
$$\times \left\langle \frac{1}{N} \sum_{\gamma\delta} (\bar{S}_{1,\gamma\delta} + S^{f\ell}_{1,\gamma\delta})^* \, (\bar{S}_{1,\gamma\delta} + S^{f\ell}_{1,\gamma\delta}) \right\rangle + O\left(\frac{1}{N}\right) \tag{A.26}$$

The treatment of the higher order terms in (A.16) would proceed along similar lines.

We have thus proved that, for a large number of open channels , we have, approximately

$$\left\langle \frac{P_{2,cc} P_{2,c'c'}}{\mathrm{Tr}\, P_2} \right\rangle_{I_1} = \frac{\langle P_{2,cc} \rangle_{I_1} \langle P_{2,c'c'} \rangle_{I_1}}{\langle \mathrm{Tr}\, P \rangle_{I_1}} \tag{A.27}$$

Finally, we prove that, in the case of no direct reactions between channels c and c',

$$|\langle X_{cc'} \, X_{c'c} \rangle_{I_1}| \ll \langle X_{cc} \, X_{c'c'} \rangle_{I_1} \, , \quad c \neq c' \tag{A.28}$$

so that the second, or "off-diagonal" term of Eq. (2.15a) or (2.26b) can, in this case, be dropped for $c \neq c'$.

To simplify the notation we consider only the case of two classes of states, identified by their running indices q and d, and start with the optical background representation of g_{qc} given by Eqs. (3.65) and (3.66), which we write as

$$g_{qc} = g^{opt}_{qc} + \sum_d \frac{g_{dc} \, V_{dq}}{E_\nu - \xi_d} \, , \tag{A.29}$$

$$\text{with} \quad g_{qc}^{opt} = \sqrt{2\pi}\, \langle \psi_{opt}^{(-)c} \,|V_{pq}|\, g \rangle . \tag{A.30}$$

As usual we assume random phases for $g_{cc'}$, $\langle g_c \rangle_{I_d} = 0$, so that

$$\begin{aligned}(X_q)_{cc'} &= \sqrt{\frac{2\pi}{D_q \tilde{\Gamma}_q}}\, \langle g_{qc}\, g^{*}_{qc'} \rangle_{I_q} \\ &\approx (X_q^{opt})_{cc'} + \sqrt{\frac{2\pi}{D_q \tilde{\Gamma}}} \sum_d \left\langle \frac{g_{dc}\, g^{*}_{dc'}\, |V_{dq}|^2}{|E_q - \xi_d|^2} \right\rangle_{I_q}\end{aligned} \tag{A.31}$$

Since $(X_q^{opt})_{cc'}$ is defined exactly as in KKM, it vanishes for $c \neq c'$ if there are no direct reactions between these channels. The remaining term is a sum of positive-definite quantities if $c = c'$, but of fluctuating quantities if $c \neq c'$. In this latter case,

$$\begin{aligned}\left\langle (X_q)_{cc'}\, (X_q)_{c'c} \right\rangle_{I_1} &= \frac{2\pi}{D_q \tilde{\Gamma}_q} \sum_d \sum_{d'} \frac{g_{dc}\, g^{*}_{dc'}\, g^{*}_{d'c}\, g_{d'c'}\, |V_{dq}|^2}{|E_q - \xi_d|^2\, |E_q - \xi_{d'}|^2} \\ &\approx \frac{2\pi}{D_q \tilde{\Gamma}_q} \sum_d \frac{|g_{dc}|^2\, |g_{dc'}|^2 |V_{dq}|^4}{|E_q - \xi_d|^4}\end{aligned} \tag{A.32}$$

This sum contains only $\sim (\Gamma_d/D_d)$ terms, whereas in contrast $X_{q,cc}\, X_{q,c'c'}$ will contain $\sim (\Gamma_d/D_d)^2$ terms and hence be $\mathcal{O}(\Gamma_d/D_d)$ larger. For this reason, $X_{q,cc'}\, X_{q,c'c}$ is negligible relative to $X_{q,cc}\, X_{q,c'c'}$.

APPENDIX B

AVERAGES OF PRODUCTS OF $S^{f\ell}$ MATRIX ELEMENTS

In Section 2 we needed averages of products of S-matrix elements, in order to calculate average cross sections and the cross section auto-correlation function. In this appendix we shall indicate how this can be accomplished by making use of reasonable statistical assumptions on the KKM terms that builds up S .

Consider the energy average over an interval of a function f(E)

$$\left\langle f(E) \right\rangle_I = \int W_I(E-E')\ f(E')dE' \qquad (B.1)$$

where W_I is a weighting function of width I.

We shall first consider the average over n points ($n \gg 1$), $E_1, E_2, \ldots$, well separated compared with the correlation length $\tilde{\Gamma}$. We shall denote it by

$$\overline{f(E)} \equiv \frac{1}{n}\sum_{k=1}^{n} f(E_k) \quad . \qquad (B.2)$$

Let us assume that f(E) is random enough and has uniform statistical properties so that the average (B.2) does not depend very much on the choice of the n points. If this happens it is clear that (B.2) will almost coincide with energy average (B.1).*

In what follows we shall take, as the function f to be averaged, products of matrix elements $S^{f\ell}_{ab}$. In the KKM formalism, $S^{f\ell}_{ab}$ can be written

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*Notice that $f(E_1)$, $f(E_2)$ etc. define an ensemble of f's and what we are stating is precisely a condition for the validity of the ergodic theorem.

as a sum over poles, which we shall generally denote by q

$$S^{f\ell}_{ab}(E_k) = \sum_q S^{f\ell}_{ab,q}(E_k) \quad . \tag{B.3}$$

For a given E_k, Fig. 5 shows the absolute value of the terms that build up the sum (B.3). The average of (B.3), as defined in (B.2), is

$$\overline{S^{f\ell}_{ab}(E)} = \frac{1}{n} \sum_{k=1}^{n} \sum_q S^{f\ell}_{ab,q}(E_k) \tag{B.4}$$

and we know that this vanishes by the KKM construction. It is convenient to group the terms that appear above, so that we first sum over all the states q labelled with a "0" in Fig. 5 (the state closest to E_k), then over all those labelled with a "1", etc. In a sense, this introduces an ensemble of states "0", one of states "1", etc. Consider then a fixed q, in the sense of this new labelling; we shall make the reasonable assumption that

$$\overline{S^{f\ell}_{ab,q}} \equiv \frac{1}{n} \sum_k S^{f\ell}_{ab,q}(E_k) = 0 \quad . \tag{B.5}$$

This has as a consequence that (B.4) vanishes.

The next degree of complication is the average of quadratic functions of S, i.e.,

$$\overline{S^{f\ell\,*}_{ab}(E)\, S^{f\ell}_{cd}(E')} \equiv \frac{1}{n} \sum_{k=1}^{n} S^{f\ell\,*}_{ab}(E_k)\, S^{f\ell}_{cd}(E'_k), (E_k - E'_k = \epsilon = \text{fixed})$$

$$= \frac{1}{n} \sum_{k=1}^{n} \sum_{qq'} S^{f\ell\,*}_{ab,q}(E_k)\, S^{f\ell}_{cd,q'}(E'_k) \quad . \tag{B.6}$$

The various contributions are shown in Fig. 6. In analogy with what we did in connection with Eq. (B.4), we can group the terms that appear in (B.6), so that we first sum over all the pairs labelled as "0,0" in the figure,..., over all the pairs "0,12", etc. Consider then a fixed pair

$q \neq q'$, in the sense of this new labelling. We shall make the following assumption of statistical independence

$$\overline{S^{f\ell\,*}_{ab,q}(E)\; S^{f\ell}_{cd,q'}(E')} \equiv \frac{1}{n}\sum_{k=1}^{n} S^{f\ell\,*}_{ab,q}(E_k)\; S^{f\ell}_{cd,q'}(E'_k)$$
$$= \overline{S^{f\ell\,*}_{ab,q}}\;\; \overline{S^{f\ell}_{cd,q'}} = 0\,, \qquad q \neq q' \tag{B.7}$$

where we have applied (B.5) in the last step. We then have the following implication for the average in Eq. (B.6):

$$\overline{S^{f\ell\,*}_{ab}(E)\; S^{f\ell}_{cd}(E')} = \sum_{qq'} \overline{S^{f\ell\,*}_{ab,q}(E)\; S^{f\ell}_{cd,q'}(E')} = \sum_{q} \overline{S^{f\ell\,*}_{ab,q}(E)\; S^{f\ell}_{cd,q}(E)} \tag{B.8}$$

Let us next consider the fourth order in $S^{f\ell}$:

$$\overline{S^{f\ell\,*}_{ab}(E)\; S^{f\ell\,*}_{cd}(E)\; S^{f\ell}_{ef}(E')\; S^{f\ell}_{gh}(E')}$$
$$= \sum_{q_1 q_2 q_3 q_4} \overline{S^{f\ell\,*}_{ab,q_1}(E)\; S^{f\ell\,*}_{cd,q_2}(E)\; S^{f\ell}_{ef,q_3}(E')\; S^{f\ell}_{gh,q_4}(E')} =$$
$$= \sum_{q_1=q_2\neq q_3\neq q_4} + \left[\sum_{\substack{q_1=q_2 \\ \neq \\ q_3=q_4}} + \sum_{\substack{q_1=q_3 \\ \neq \\ q_2=q_4}} + \sum_{\substack{q_1=q_4 \\ \neq \\ q_2=q_3}} \right] \tag{B.9}$$
$$\left[\sum_{\substack{q_1=q_2=q_3 \\ \neq \\ q_4}} + \ldots + \sum_{\substack{q_2=q_3=q_4 \\ \neq \\ q_4}} \right] + \sum_{q_1=q_2=q_3=q_4}$$

We analyze each one of the above terms, with the assumption that for different indices we have statistical independence, as we had in (B.7) for products of two S's :

$$(B.9)_1 = \sum_{q_1 \neq q_2 \neq q_3 \neq q_4} \overline{S^{f\ell}_{ab,q_1}(E)^*}\ \overline{S^{f\ell}_{cd,q_2}(E)^*}\ \overline{S^{f\ell}_{ef,q_3}(E')}\ \overline{S^{f\ell}_{gh,q_4}(E')} = 0 \tag{B.10}$$

$$(B.9)_2 = \sum_{q_1 \neq q_3} \overline{S^{f\ell}_{ab,q}(E)^* S^{f\ell}_{cd,q_1}(E)^*}\ \overline{S^{f\ell}_{ef,q_3}(E')\, S^{f\ell}_{gh,q_3}(E')} \approx 0, \tag{B.11}$$

since in the spirit of KKM, averages that involve $S^{f\ell}\, S^{f\ell}$ (instead of $S^{f\ell *}\, S^{f\ell}$) are neglected.

$$(B.9)_3 = \sum_{q_1 \neq q_2} \overline{S^{f\ell *}_{ab,q_1}(E)\, S^{f\ell}_{ef,q_1}(E')}\ \overline{S^{f\ell *}_{cd,q_2}(E)\, S^{f\ell}_{gh,q_2}(E')} \tag{B.12}$$

$$(B.9)_4 = \sum_{q_1 \neq q_2} \overline{S^{f\ell *}_{ab,q_1}(E)\, S^{f\ell}_{gh,q_1}(E')}\ \overline{S^{f\ell *}_{cd,q_2}(E)\, S^{f\ell}_{ef,q_2}(E')}\ . \tag{B.13}$$

The terms $(B.9)_5, \ldots (B.9)_{10}$ vanish, since they are of the type

$$\sum_{q_1 \neq q_4} \overline{S^{f\ell *}_{ab,q_1}(E)\ S^{f\ell *}_{cd,q_1}(E)\ S^{f\ell}_{ef,q_1}(E')}\ \overline{S^{f\ell}_{gh,q_4}(E')} = 0 \tag{B.14}$$

$(B9)_{11}$ involves N terms, N being the number of levels. On the other hand, if the terms in $(B9)_3$ and $(B9)_4$ do not vanish, there are N(N-1) of them. Since $N \gg 1$, we only keep the latter. Within the same approximation, we may add to $(B9)_3$ and $(B9)_4$ the contribution $q_1 = q_2$. Comparing with (B8) we finally have

$$\left\langle S_{ab}^{f\ell\,*}(E)\;\; S_{cd}^{f\ell *}(E)\; S_{ef}^{f\ell}(E')\; S_{gh}^{f\ell}(E')\right\rangle \approx$$

$$\approx \left\langle S_{ab}^{f\ell\,*}(E)\; S_{ef}^{f\ell}(E')\right\rangle\left\langle S_{cd}^{f\ell\,*}(E)\; S_{gh}^{f\ell}(E')\right\rangle \;+$$

$$+\left\langle S_{ab}^{f\ell\,*}(E)\; S_{gh}^{f\ell}(E')\right\rangle\left\langle S_{cd}^{f\ell\,*}(E)\; S_{ef}^{f\ell}(E')\right\rangle \quad , \qquad \text{(B.15)}$$

which is the result found by AWM [4]. The extension to higher powers of S proceeds exactly along the same lines.

APPENDIX C

OPTICAL BACKGROUND REPRESENTATION OF THE MATRIX PROPAGATOR $\mathcal{G}^{(+)}_{P_{n-1}}$

In this appendix we give a detailed derivation of Eqs. (3.92) to (3.95). Consider the following operator of Eq. (3.38)

$$\mathcal{G}^{(+)}_{P_{n-1}} = \left[E^{(+)} - P_{n-1}\mathcal{H}(Q_n)P_{n-1}\right]^{-1} \tag{C.1}$$

where $P_{n-1}\mathcal{H}(Q_n)P_{n-1}$ is an effective operator for the space P_{n-1}. Now we decompose $P_{n-1} = D_{n-1} + p$ with $D_{n-1} = d_1 + d_2 + \ldots d_{n-1}$. Then

$$\begin{aligned} P_{n-1}\mathcal{H}(Q_n)P_{n-1} &= \left[p\mathcal{H}(Q_n)p + D_{n-1}\mathcal{H}(Q_n)D_{n-1}\right] + \\ &\quad \left[p\mathcal{H}(Q_n)D_{n-1} + D_{n-1}\mathcal{H}(Q_n)p\right] \\ &\equiv \mathcal{H}_o(Q_n) + \mathcal{H}'(Q_n) \end{aligned} \tag{C.2}$$

where $\mathcal{H}_o(Q_n)$ refers to the first term in brackets in Eq. (C.2) and $\mathcal{H}'(Q_n)$ refers to the second term. Making use of the following relation

$$\mathcal{G}^{(+)}_{P_{n-1}} = \frac{1}{E^{(+)} - \mathcal{H}_o(Q_n)} + \frac{1}{E^{(+)} - \mathcal{H}_o(Q_n)}\mathcal{H}'(Q_n)\mathcal{G}^{(+)}_{P_{n-1}} \tag{C.3}$$

we could then calculate the elements of the matrix $\mathcal{G}^{(+)}_{P_{n-1}}$, i.e., $p\mathcal{G}^{(+)}_{P_{n-1}}p$, $p\mathcal{G}^{(+)}_{P_{n-1}}D_{n-1}$, etc.

$$p\mathcal{G}^{(+)}_{P_{n-1}}p = \frac{1}{E^{(+)} - p\mathcal{H}_o(Q_n)p} + \frac{1}{E^{(+)} - p\mathcal{H}_o(Q_n)p}\, p\mathcal{H}'(Q_n)\, D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}}p \tag{C.4}$$

$$P\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} = \frac{1}{E^{(+)} - P\mathcal{H}(Q_n)P} P\mathcal{H}(Q_n) D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} \qquad (C.5)$$

$$D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}} P = D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} \mathcal{H}(Q_n) P \frac{1}{E^{(+)} - P\mathcal{H}(Q_n)P} \qquad (C.6)$$

$$D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} = \frac{1}{E^{(+)} - D_{n-1}\mathcal{H}_0(Q_n)D_{n-1}} \qquad (C.7)$$

$$+ \frac{1}{E^{(+)} - D_{n-1}\mathcal{H}_0(Q_n)D_{n-1}} D_{n-1}\mathcal{H}(Q_n) P \mathcal{G}^{(+)}_{P_{n-1}} D_{n-1}$$

The above set of equations constitutes a solution since we can, for example, insert (C5) into (C7) and obtain an uncoupled equation for $D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1}$. The same can be done for the other elements of $\mathcal{G}^{(+)}_{P_{n-1}}$. For our purposes, however, it would be more interesting to write $P\mathcal{G}^{(+)}_{P_{n-1}} P$ as a sum of an average optical Green's function $\mathcal{G}^{(+)}_{opt}$ and a fluctuation Green's function (due to the presence of the $d_1, \ldots d_{n-1}$ classes of doorways). Our final aim is to express Eqs. (C4) - (C7) in terms of $\mathcal{G}^{(+)}_{opt}$ and another Green's function associated with the propagation in the D_{n-1} - subspace (where all classes in D_{n-1} now would acquire an escape width). The above manipulations can be accomplished again with the KKM reduction scheme. To see this we first write Eqs. (C.4) and (C.7) in the following forms

$$(E - P\mathcal{H}(Q_n)P)\, P\mathcal{G}^{(+)}_{P_{n-1}} P = 1 + P\mathcal{H}(Q_n)\, D_{n-1} P\mathcal{G}^{(+)}_{P_{n-1}} P \qquad (C.8)$$

$$(E - D_{n-1}\mathcal{H}_o(Q_n)D_{n-1})\, D_{n-1}\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} = 1 + D_{n-1}\mathcal{H}'(Q_n)\, p\mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} \qquad (C.9)$$

Using Eq. (C6) in (C8) we obtain an effective equation for $p\mathcal{G}^{(+)}_{P_{n-1}} p$

$$(E - p\mathcal{H}_o(Q_n)p)\, p\mathcal{G}^{(+)}_{P_{n-1}} p = 1 + p\mathcal{H}'(Q_n) D_{n-1} \frac{1}{E - D_{n-1}\mathcal{H}_o(Q_n)D_{n-1}} Q_{n-1}\mathcal{H}'(Q_n)p$$
$$p\mathcal{G}^{(+)}_{P_{n-1}} p \qquad (C.10)$$

We now define the optical Green's function $\mathcal{G}^{(+)}_{opt}$ which satisfies the following equation

$$\left[E - p\mathcal{H}_o(Q_n)p - p\mathcal{H}'(Q_n)D_{n-1}\left\langle \frac{1}{E - D_{n-1}\mathcal{H}_o(Q_n)D_{n-1}}\right\rangle_{I_1} D_{n-1}\mathcal{H}'(Q_n)p\right]\mathcal{G}^{(+)}_{opt} = \mathbb{1} \qquad (C.11)$$

where the average above indicates that all doorway classes in D_{n-1} have been represented by an effective, smooth, interaction. Clearly $\mathcal{G}^{(+)}_{opt} = \langle p\mathcal{G}^{(+)}_{P_{n-1}} p\rangle_{I_1}$. Simple KKM manipulations done on Eq. (C10) would then permit us to write for $p\mathcal{G}^{(+)}_{P_{n-1}} p$ the following

$$p\mathcal{G}^{(+)}_{P_{n-1}} p = \mathcal{G}^{(+)}_{opt} + \mathcal{G}^{(+)}_{opt} V_{pD_{n-1}} \mathcal{G}_{D_{n-1}} V_{D_{n-1}} \mathcal{G}^{(+)}_{opt} \qquad (C.12)$$

where the form factors $V_{pD_{n-1}}$ are defined by

$$V_{pD_{n-1}} \equiv p\mathcal{H}'(Q_n) D_{n-1} \left[\frac{i I_1/2}{E - D_{n-1}\mathcal{H}_o(Q_n) D_{n-1} + iI_1/2}\right]^{1/2} \qquad (C.13)$$

and the propagator $\mathcal{G}_{D_{n-1}}$ is

$$\mathcal{L}_{D_{n-1}} = \left[E - \mathcal{D}_{n-1} \mathcal{H}(Q_n) \mathcal{D}_{n-1} - V_{D_{n-1}p} \mathcal{G}^{(+)}_{opt} V_{pD_{n-1}} \right]^{-1} \tag{C.14}$$

With the above optical background representation for $p\mathcal{L}^{(+)}_{P_{n-1}} p$ we can reexpress Eqs. (C5), (C6) and (C7) as follows (up to the square root factor appearing in (C13))

$$p\mathcal{L}^{(+)}_{P_{n-1}} D_{n-1} = \mathcal{G}^{(+)}_{opt} V_{pD_{n-1}} \mathcal{L}_{D_{n-1}} \tag{C.15}$$

$$D_{n-1}\mathcal{L}^{(+)}_{P_{n-1}} p = \mathcal{L}_{D_{n-1}} V_{D_{n-1}p} \mathcal{G}^{(+)}_{opt} \tag{C.16}$$

and

$$D_{n-1}\mathcal{L}^{(+)}_{P_{n-1}} D_{n-1} = \mathcal{L}_{D_{n-1}} \tag{C.17}$$

which are (together with (C12)) just Eqs. (3.92) to (3.95).

APPENDIX D

FA CALCULATION OF IM $\mathcal{G}^{(+)}_{M}(F)$ AND $\tilde{\Gamma}^{(F)}$

In this appendix we present a detailed derivation of some of the flow approach quantities used in Subsection 4.1.

D.1 Im $\mathcal{G}^{(+)}$ for Non-Hermitian Hamiltonians

We shall derive here a useful representation for the imaginary part of the Green's function associated with an optical Hamiltonian, i.e., one having a complex potential. Let us represent the optical Hamiltonian by

$$\mathcal{H}_{opt} = R + iW \tag{D.1}$$

where R and W are each Hermitian. Let us also define the following two Green's functions:

$$\mathcal{G}^{(+)} = \left(E^{(+)} - \mathcal{H}_{opt}\right)^{-1} \tag{D.2}$$

$$G^{(+)} = \left(E^{(+)} - R\right)^{-1} \tag{D.3}$$

Then from Eqs. (D.1) to (D.3)

$$\mathcal{G}^{(+)} = G^{(+)} + \mathcal{G}^{(iW)}\, G^{(+)} \,. \tag{D.4}$$

The continuum eigenstates associated with $\mathcal{H}_{opt}$ and R, called here $\{\phi_c^{(+)}, \tilde{\phi}_c^{(+)}\}$ and $\chi_c^{(+)}$ are defined by

$$(E^{(+)} - \mathcal{H}_{opt})\,|\phi_c^{(+)}\rangle = 0 \quad ; \quad (E^{(+)} - \mathcal{H}_{opt}^{+})\,|\tilde{\phi}_c^{(+)}\rangle = 0 \tag{D.5}$$

$$(E^{(+)} - R)\,|\chi_c^{(+)}\rangle = 0 \quad , \tag{D.6}$$

with the eigenstates for the two Hamiltonians related as follows

$$|\Phi_c^{(+)}\rangle = (1 + \mathcal{G}(iW))\,|\chi_c^{(+)}\rangle \tag{D.7a}$$

$$\langle\Phi_c^{(+)}| = \langle\chi_c^{(+)}|\,(1 + (iW)^\dagger \mathcal{G}^\dagger) \tag{D.7b}$$

With Eq. (D.4) we have

$$\mathrm{Im}\,\mathcal{G}^{(+)} = \mathrm{Im}\,G^{(+)} + \mathrm{Im}\,\mathcal{G}^{(+)}(iW)G^{(+)} . \tag{D.8}$$

We next define the quantity Δ as follows

$$\Delta \equiv \mathrm{Im}\,\mathcal{G}^{(+)} + \pi\sum_c |\Phi_c^{(+)}\rangle\langle\Phi_c^{(+)}| \tag{D.9}$$

Using Eq. (D.7) we find

$$\mathrm{Im}\,\mathcal{G}^{(+)} = -\pi\sum_c (1 + \mathcal{G}(iW))\,|\chi_c^{(+)}\rangle\langle\chi_c^{(+)}|\,(1 + (iW)^+\mathcal{G}^+) + \Delta$$

$$= \mathrm{Im}(1 + \mathcal{G}(iW))G^{(+)}(1 + (iW)^+\mathcal{G}^+) + \Delta \tag{D.10}$$

Finally, combining Eq. (D.8) and Eq. (D.10) we obtain, with the aid of Eq. (D.4),

$$\Delta = -\,\mathrm{Im}\,\mathcal{G}(iW)^\dagger\mathcal{G}^\dagger \tag{D.11}$$

$$= \mathcal{G}W\mathcal{G}^\dagger$$

Thus we have the desired representation used in Section 4.1

$$\mathrm{Im}\,\mathcal{G}^{(+)} = -\pi\sum_c |\Phi_c^{(+)}\rangle\langle\Phi_c^{(+)}| + \mathcal{G}^{(+)}\,\mathrm{Im}\,\mathcal{H}\,\mathcal{G}^{(+)\dagger} \tag{D.12}$$

Note that when $\mathrm{Im}\,\mathcal{H} \to 0$, the result is the usual one. Note in addition that it is $\sum_c |\Phi_c^{(+)}\rangle\langle\Phi_c^{(+)}|$ which appears in Eq. (D.12) and not $\sum_c |\Phi_c^{(+)}\rangle\langle\tilde{\Phi}_c^{(+)}|$, which in general is complex.

The optical potentials of this paper arise from the energy averaging procedures discussed above. Let us consider the result of this averaging when there is only one class of compound levels. For this case, using the conventional projection operators p and q, we obtain for the optical Hamiltonian

$$\mathcal{H}_{opt} = H_{pp} + H_{pq} \left\langle \frac{1}{E - H_{qq}} \right\rangle_I H_{qp} \tag{D.13}$$

so that $\mathrm{Im}\,\mathcal{H}_{opt}$ is given by

$$\mathrm{Im}\,\mathcal{H}_{opt} = \mathrm{Im}\,H_{pq} \left\langle \frac{1}{E - H_{qq}} \right\rangle_I H_{qp}$$

$$= - H_{pq} \frac{I/2}{(E - H_{qq})^2 + I^2/4} H_{qp} \tag{D.14}$$

Upon inserting the eigenstates of H_{qq}, $|\Psi_q\rangle$, and performing the sum over them, we obtain,

$$\mathrm{Im}\mathcal{H}_{opt} = - \frac{\pi}{D_q} H_{pq} \overline{|\Psi_{qi}\rangle\langle\tilde{\Psi}_{qi}|} H_{qp} \tag{D.15}$$

where D_q is the average level spacing for the states $|\Psi_{qi}\rangle$ and the bar indicates an average over these states. When Eq. (D.15) is combined with Eq. (D.12) we obtain for the specific Hamiltonian of Eq. (D.13)

$$\mathrm{Im}\mathcal{H}_{opt} = - \pi \sum_c |\phi^{(+)c}\rangle\langle\phi^{(+)c}|$$

$$- \pi \mathcal{L} H_{pq} \frac{\overline{|\Psi_{qi}\rangle\langle\tilde{\Psi}_{qi}|}}{D_q} H_{qp} \mathcal{L}^{\dagger} \tag{D.16}$$

D.2. Recursion Representation of $M^{(F)}_{nn'}$

In this appendix we demonstrate that M^{F}_{mn} given in Eq. (4.20), i.e., $\frac{D_m}{D_n}\left\langle \left| \langle \xi_{m\alpha} | \mathcal{G}_{D_{n-1}} V | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta}$, satisfies the iterative expression of Eq. (4.6) with the proper identification of μ_{mn}. First consider the Green's function $\mathcal{G}_{D_{n-1}}$ which appears in the definition of $M^{(F)}_{mn}$, Eq. (4.20). Let us partition the space D_{n-1} into d_m (any one of the classes contained in D_{n-1}) and the complementary space Δ_m, such that $d_m + \Delta_m = D_{n-1}$. It follows from the procedures of Appendix C that

$$d_m \left(\mathcal{G}_{D_{n-1}}\right) d_m = G_{d_m} + G_{d_m} d_m \mathcal{H} \Delta_m \mathcal{G}_{D_{n-1}} d_m \tag{D.17}$$

$$\Delta_m \left(\mathcal{G}_{D_{n-1}}\right) d_m = G_{\Delta_m} \Delta_m \mathcal{H} d_m \mathcal{G}_{D_{n-1}} d_m \tag{D.18}$$

where

$$G_{d_m} = (E - d_m \mathcal{H} d_m)^{-1} \tag{D.19}$$

$$G_{\Delta_m} = (E - \Delta_m \mathcal{H} \Delta_m)^{-1} \tag{D.20}$$

and where $\mathcal{H}$ is the effective Hamiltonian corresponding to the Green's function $\mathcal{G}_{D_{n-1}}$, in which the states of class n' ($n' \geq n$) appear only on the average. Next we define the following Green's function for class m

$$\mathcal{G}_{d_m} = \left[E - d_m \mathcal{H} d_m - d_m \mathcal{H} \Delta_m \langle G_{\Delta_m} \rangle_I \Delta_m \mathcal{H} d_m\right]^{-1} \tag{D.21}$$

We assume that the off diagonal elements of $d_m \mathcal{H} \Delta_m \langle G_{\Delta_m} \rangle_I \Delta_m \mathcal{H} d_m$ with respect to the model state basis, are zero as $\langle G_{\Delta_m} \rangle_I$ acts solely as a sink for these states. The widths, $\Gamma_{m\alpha}$ arising from the Hamiltonian in Eq. (D.21) can be expressed as

$$\Gamma_{m\alpha} = \Gamma^{\uparrow}_{m\alpha} + \Gamma_{m\alpha}(Q_n) + \Gamma_{m\alpha}(\Delta_m) \tag{D.22}$$

where $\Gamma_{m\alpha}\uparrow + \Gamma_{m\alpha}(Q_n)$ arise from the term $d_m \mathcal{H} d_m$ in the Hamiltonian Eq. (D.21) and $\Gamma_{m\alpha}(\Delta_m)$ is defined by

$$\Gamma_{m\alpha}(\Delta_m) = -2\,\mathrm{Im}\langle \xi_{m\alpha} | \mathcal{H} \left\langle \mathcal{G}_{\Delta_m} \right\rangle_I \mathcal{H} | \xi_{n\alpha} \rangle \tag{D.23}$$

With the Green's functions defined above in Eq. (D.17) - (D.21), we obtain

$$d_m \mathcal{G}_{D_{n-1}} d_m = \mathcal{G}_{d_m} + \mathcal{G}_{d_m} v \mathcal{G}_{\Delta_m} v \mathcal{G}_{d_m} \tag{D.24}$$

$$d_m \mathcal{G}_{D_{n-1}} d_m = \mathcal{G}_{d_m} v \mathcal{G}_{\Delta_m} \tag{D.25}$$

$$d_m \mathcal{G}_{D_{n-1}} d_m = \mathcal{G}_{\Delta_m} = (E - \Delta_m H \Delta_m - \Delta_m v \mathcal{G}_{d_m} v \Delta_m)^{-1} \tag{D.26}$$

In obtaining Eqs. (D.24) to (D.26) we used steps identical to those employed in Appendix C to obtain Eqs. (3.92) to (3.95). With Eqs. (D.24) - (D.26) we obtain the following representation of $\langle \xi_{m\alpha} | \mathcal{G}_{\Delta_{n-1}} v | \xi_{n\beta} \rangle$ by inserting a complete set of states,

$$\begin{aligned} &\langle \xi_{m\alpha} | \mathcal{G}_{D_{n-1}} v | \xi_{n\beta} \rangle \\ &= \sum_\gamma \langle \xi_{m\alpha} | \mathcal{G}_{d_m} | \xi_{m\gamma} \rangle \langle \xi_{m\gamma} | v | \xi_{n\beta} \rangle \\ &+ \sum_{s\neq m} \sum_\gamma \langle \xi_{m\alpha} | \mathcal{G}_{d_m} | \xi_{m\gamma} \rangle \langle \xi_{m\gamma} | v | \xi_{s\gamma'} \rangle \langle \xi_{s\gamma'} | \mathcal{G}_{D_{n-1}} v | \xi_{n\beta} \rangle \end{aligned} \tag{D.27}$$

We next square the expression in Eq. (D.27) and take averages to obtain

$$\begin{aligned} &\left\langle \left| \langle \xi_{m\alpha} | \mathcal{G}_{D_{n-1}} v | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} \\ &\approx \left\langle |\langle \xi_{m\alpha} | \mathcal{G}_{d_m} | \xi_{m\gamma} \rangle|^2 \right\rangle_{\alpha,\gamma} \left\langle |\langle \xi_{m\gamma} | v | \xi_{n\beta} \rangle|^2 \right\rangle_{\gamma,\beta} \\ &+ \sum_{\substack{s<n \\ (\neq m)}} \left\langle |\langle \xi_{m\alpha} | \mathcal{G}_{d_m} | \xi_{m\gamma} \rangle|^2 \right\rangle_{\alpha,\gamma} \left\langle |\langle \xi_{m\gamma} | v | \xi_{s\gamma'} \rangle|^2 \right\rangle_{\gamma,\gamma'} \\ &\times \left\langle |\langle \xi_{s\gamma'} | \mathcal{G}_{D_{n-1}} v | \xi_{n\beta} \rangle|^2 \right\rangle_{\gamma',\beta} \end{aligned} \tag{D.28}$$

In arriving at Eq. (D.28) we have kept only those terms which are manifestly positive definite. Using Eq. (D.21) for $\mathcal{L}_{d_m}$ we find

$$\left\langle \left| \langle \xi_{m\alpha} | \mathcal{L}_{d_m} | \xi_{m\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} \approx \frac{2\pi}{D_m \hat{\Gamma}_m} \tag{D.29}$$

where $\hat{\Gamma}_m^{-1} = \left\langle \frac{1}{\Gamma_{m\alpha}} \right\rangle_\alpha$ and $\Gamma_{m\alpha}$ is given in Eq. (D.22). With Eq. (D.28) and Eq. (D.29) we obtain

$$\begin{aligned} &\frac{D_m}{D_n} \left\langle \left| \langle \xi_{m\alpha} | \mathcal{L}_{D_{n-1}} v | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} \qquad (D.30) \\ &= \frac{2\pi}{D_n \hat{\Gamma}_m} \left\langle \left| \langle \xi_{m\alpha} | v | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} \\ &+ \sum_{s<n (s\neq m)} \frac{2\pi}{D_s \hat{\Gamma}_m} \left\langle \left| \langle \xi_{m\alpha} | v | \xi_{s\gamma} \rangle \right|^2 \right\rangle \frac{D_s}{D_n} \\ &\qquad \left\langle \left| \langle \xi_{s\gamma} | \mathcal{L}_{D_{n-1}} v | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\gamma,\beta} \end{aligned}$$

By identifying μ_{mn} as

$$\mu_{mn} = \frac{2\pi}{D_n \hat{\Gamma}_m} \left\langle \left| \langle \xi_{m\alpha} | v | \xi_{n\beta} \rangle \right|^2 \right\rangle_{\alpha,\beta} \tag{D.31}$$

We finally obtain,

$$M_{mn}^{(F)} = \mu_{mn} + \sum_{s<n} \mu_{ms} M_{sn}^{(F)} \tag{D.32}$$

Note that μ_{mn} involves averaged absolute squares of the matrix elements of the basic interaction v, Eq. (3.100) (which contains external mixing) between model states. Notice also that the Green's function $\mathcal{L}_{D_{n-1}}$ which appears in Eq. (4.20) accounts for the propagation among the classes $d_1 \ldots d_{n-1}$ in the evolution from class m to class n.

D.3 Flow Analog for Correlation Width

Here we construct the $\tilde{\tilde{\Gamma}}^{(F)}$ given in Eq. (4.39)

$$\tilde{\tilde{\Gamma}}^{(F)-1}_{n} = D_n/2\pi \left\langle\!\!\left\langle \xi_{n\alpha} \right| \frac{1}{E-H} \; \frac{1}{E-H^{\dagger}} \left| \xi_{n\alpha} \right\rangle\!\!\right\rangle_{\alpha}$$

We first break H into a part diagonal in the states $|\xi_{n\alpha}\rangle$ called H_D and a part nondiagonal called H_{ND}, defined by Eqs.(4.40) to (4.43)

$$H_D = \sum_{\alpha} |\xi_{n\alpha}\rangle \, \eta^{n}_{\alpha} \, \langle \xi_{n\alpha}|$$

where

$$\eta^{n}_{\alpha} = \langle \xi_{n\alpha} | H | \xi_{n\alpha} \rangle = E_{n\alpha} - \frac{i\Gamma^{(F)}_{n\alpha}}{2}$$

$$H_{ND} = \sum_{\alpha \neq \beta} |\xi_{n\alpha}\rangle \, W_{\alpha\beta} \, \langle \xi_{n\beta}|$$

where

$$W_{\alpha\beta} = \langle \xi_{n\alpha} | H | \xi_{n\beta} \rangle$$

We next expand $\frac{1}{E-H}$ in H_D and H_{ND}

$$\frac{1}{E-H} = \sum_{n=0} \left(\frac{1}{E-H_D} \; H_{ND}\right)^{n} \; \frac{1}{E-H_D} \tag{D.33}$$

so that $\tilde{\tilde{\Gamma}}^{(F)}_{n}$ is given by

$$\tilde{\tilde{\Gamma}}^{(F)-1}_{n} = \frac{D_n}{2\pi} \left\langle \sum_{\alpha} \langle \xi_{n\alpha} | \left(\frac{1}{E-H_D} + \frac{1}{E-H_D} \; H_{ND} \; \frac{1}{E-H_D} + \ldots\right) \right.$$
$$\left. \left(\frac{1}{E-H_D^{\dagger}} + \frac{1}{E-H_D^{\dagger}} \; H^{\dagger}_{ND} \; \frac{1}{E-H^{\dagger}_D} \; \ldots\right) | \xi_{n\alpha} \rangle \right\rangle_{I} \tag{D.34}$$

This expression can be rewritten in terms of η^{n}_{α} and $W_{\alpha\beta}$

$$\tilde{\tilde{\Gamma}}^{(F)-1}_{n} = \frac{D_n}{2\pi} \left\langle \sum_{\alpha,\beta} \left(\frac{\delta_{\alpha\beta}}{E-\eta^{n}_{\alpha}} + \frac{1}{E-\eta^{n}_{\alpha}} \; W_{\alpha\beta} \; \frac{1}{E-\eta^{n}_{\beta}} \right. \right. +$$

$$+\sum_{\gamma}\frac{1}{E-\eta_{\alpha}^{n}}\,W_{\alpha\gamma}\,\frac{1}{E-\eta_{\gamma}^{n}}\,W_{\gamma\beta}\,\frac{1}{E-\eta_{\beta}^{n}}+\cdots\Big)$$

$$\Big(\frac{1}{E-\eta_{\alpha}^{n*}}+\frac{1}{E-\eta_{\beta}^{n*}}\,W^{*}_{\beta\alpha}\,\frac{1}{E-\eta_{\alpha}^{n*}}+\sum_{\gamma'}\frac{1}{E-\eta_{\beta}^{n*}}\,W^{*}_{\beta\gamma'}\,\frac{1}{E-\eta_{\gamma'}^{n*}}\,W^{*}_{\gamma'\alpha}\,\frac{1}{E-\eta_{\alpha}^{n*}}+\cdots\Big)\Big\rangle_{I} \tag{D.35}$$

In taking the sums over $\alpha,\beta,\gamma,\ \ldots$ and in taking the energy average over I, we assume that only manifestly positive definite terms survive. Hence

$$\tilde{\Gamma}_{n}^{(F)-1}=\frac{D_n}{2\pi}\Big\langle\sum_{\alpha}\frac{1}{\left|E-\eta_{\alpha}^{n}\right|^{2}}+\sum_{\alpha}\frac{1}{\left|E-\eta_{\alpha}^{n}\right|^{2}}\sum_{\beta}\left|W_{\alpha\beta}\right|^{2}\frac{1}{\left|E-\eta_{\beta}^{n}\right|^{2}}$$

$$+\sum_{\alpha}\frac{1}{\left|E-\eta_{\alpha}^{n}\right|^{2}}\sum_{\beta}\left|W_{\alpha\beta}\right|^{2}\frac{1}{\left|E-\eta_{\beta}^{n}\right|^{2}}\sum_{\gamma}\left|W_{\beta\gamma}\right|^{2}\frac{1}{\left|E-\eta_{\gamma}^{n}\right|^{2}}+\cdots\Big\rangle_{I} \tag{D.36}$$

If we assume that the $\Gamma_{n\gamma}$'s, Eq. (4.41) are narrowly distributed, and approximately equal to $\hat{\Gamma}_{n}$

$$\sum_{\gamma}\frac{\left|W_{\beta\gamma}\right|^{2}}{(E-E_{n\gamma})^{2}+(\frac{\Gamma_{n\gamma}}{2})^{2}}\simeq\frac{2\pi}{D_n}\frac{1}{\hat{\Gamma}}\left\langle\left|W_{\beta\gamma}\right|^{2}\right\rangle_{\gamma} \tag{D.37}$$

Finally if we assume

$$\left\langle\left|W_{\beta\gamma}\right|^{2}\right\rangle_{\beta,\gamma}=\overline{W^{2}}\equiv\left\langle\left|W_{\beta\gamma}\right|^{2}\right\rangle_{\beta,\gamma} \tag{D.38}$$

We can sum the series in Eq. (D.36) to obtain

$$\tilde{\Gamma}_{n}^{(F)-1}=\frac{1}{\hat{\Gamma}_{n}\left(1-\frac{2\pi}{D_n}\frac{\overline{W^{2}}}{\hat{\Gamma}_{n}}\right)} \tag{D.39}$$

$$\widetilde{\Gamma}_n^{(F)} = \hat{\Gamma}_n - \frac{2\,\overline{W^2}}{D_n} \tag{D.40}$$

When the model states are the eigenstates of $d_n H d_n$ (where H is the full Hermitian-Hamiltonian) the nondiagonal elements $W_{\alpha\beta}$ are approximately given by

$$W_{\alpha\beta} = \langle \xi_{n\alpha} | V \mathcal{G}^{(+)}_{P_{n-1}} V | \xi_{n\beta} \rangle \tag{D.41}$$

(See Eq. (3.40)) From Eqs. (D.41) and (4.36) it then follows that

$$2\pi \frac{\overline{W^2}}{D_n} = \hat{\Gamma}_n \Big(\sum_{n' < n} \mu_{nn'} M_{n'n} + C_{nn} \Big) \tag{D.42}$$

Using Eq. (D.40) and Eq. (D.42)

$$\widetilde{\Gamma}_n^{(F)} = \hat{\Gamma}_n \Big(1 - \sum_{n' < n} \mu_{nn'} M_{n'n} - C_{nn} \Big) \tag{D.43}$$

and with $\hat{\Gamma}_n \simeq \Gamma_n^{(F)}$ we finally have

$$\widetilde{\Gamma}_n = \Gamma_n^{(F)} - \Gamma_n^{(F)} \Big(\sum_{n' < n} \mu_{nn'} M_{n'n} + C_{nn} \Big) \tag{D.44}$$

APPENDIX E

SUMMARY OF THE FKK RESULTS

The expressions for $\tau^{J}_{n;cc}$ and $\mu^{J}_{n,n+1}$ obtained by FKK [3] were based on a simple model for the excitation of the compound systems. To describe the system of p-particles-h-holes one uses the "equidistant" model for the density $\rho(E,J)$ of single particle levels at excitation energy E and angular momentum J vis

$$\rho(E,J) = g \frac{2J+1}{\sqrt{\pi}\,\sigma^3} e^{-\frac{(J+\frac{1}{2})^2}{\sigma^2}} \tag{E.1}$$

where g^{-1} is the constant average distance between levels and the J-dependence is suggested from the Fermi-gas model of the density of states. The parameters g and σ have the following approximate values

$$g \simeq \frac{3A}{4\pi^2} \text{ Mev}^{-1}$$

$$\sigma = [2\,C\,t\,(\frac{2}{gE})^{\frac{1}{2}}]^{\frac{1}{2}} \tag{E.2}$$

with

$$C \simeq \frac{A^{5/3}}{90} \text{ Mev}^{-1}$$

$$E \simeq \frac{A}{8} t^2 - t$$

Notice that at large A values, σ attains the E-independent value of 0.57 $A^{1/3}$. With the above density of single-particle levels one can construct the density of p-particle-h-hole states vis

$$\rho_{ph}(E,J) = \frac{g(gE)^{N-1}}{p!\,h!\,(N-1)!} R_N(J) \tag{E.3}$$

$$R_N(J) = \frac{2J+1}{\sqrt{\pi}\,N^{3/2}\sigma^3} e^{-(J+\frac{1}{2})^2/N\sigma^2}$$

where $N = p+h$ = total number of excitons. By using a simple δ-form for the residual interaction, the expressions for $\gamma^J_{n,cc}$ and $\mu^J_{n\ n+1}$ are found to be

$$\gamma^J_{1,cc} = \gamma^J_{1,(\ell,0)} \simeq \frac{2\pi}{\langle D^J_1 \rangle} \Gamma^J_{1(\ell,0)}$$

$$= \frac{(2\pi)^2 \theta_J^2}{\langle D^J_1 \rangle R_3(J)} \sum_{j_3 Q} (2Q+1)\ F(Q)(2j_3+1)R(j_3) \begin{pmatrix} J & j_3 & Q \\ 0 & 0 & 0 \end{pmatrix}^2 \qquad \text{(E.4)}$$

where θ_J is related to the strength, V_o, of the assumed δ-residual interaction via

$$\theta_J = \frac{V_o}{4\pi} \left(\frac{4}{3}\pi r_o^3\right) \int_o^\infty U_{\ell=J}\ U_{j_1} U_{j_2} U_{j_3}\ \frac{dr}{r^2} \qquad \text{(E.5)}$$

The U_j's are single particle radial wave functions (there are three of these specifying the 2p-1h doorway state) and $U_{\ell=J}$ is the radial wave function of the incoming projectile nucleon. The function $R_3(J)$ is defined in Eq. (E.3) and $F(Q)$ is given by

$$F(Q) = \sum_{j_1 j_2} (2j_1+1)(2j_2+1)R(j_1)R(j_2) \begin{pmatrix} j_1 & j_2 & Q \\ 0 & 0 & 0 \end{pmatrix}^2 \qquad \text{(E.6)}$$

The expression for the outgoing channel $\gamma_{n,c'}$ is similar to γ_1 except that, now, the spin of the final residual nuclear may differ from zero and the number of excitons in the final channel is generally large and different from the number of excitons present in the nth class of doorways by $\pm 2, 0$. In the case of inclusive spectra considered, the $\gamma_{n,c'c'}$

must be considered together with the density of levels in the residual nuclear (average over final channels) and therefore one should really calculate, instead of $\tau_{n,c'c'}$, the following

$$\left\langle \tau^{J}_{n,c'c'} \rho^{(U)}_{c'} \right\rangle_{c'} = \sum_{\nu=-2,0,2} \left\langle \tau^{J}_{n}(\ell,s;\nu)\, \rho^{\nu}_{s}(U) \right\rangle = \sum_{\nu} \overline{\tau_{n}(\ell,s,\nu;U)}\, \overline{\rho^{(\nu)}_{n}(U)} \qquad (E.7)$$

where ν labels the possible ways of connecting the N_n-excitons doorway, later to the final exit channel. As can be clearly seen in the above expression, three different processes contribute incoherently to the average decay of the nth class of doorways, namely a process in which the number of excitons contained in the compound system, N_n, is not changed as a consequence of the decay $n \rightarrow c'$, another process in which this number is changed by +2 and a third one where the change is -2.

We list below the detailed expressions for $\tau_n(\ell,s,\nu;U)$ and $\overline{\rho^{(\nu)}_{n}}(U)$ obtained by FKK

$$\overline{\tau_{n}}(\ell,s,0;U) = \frac{2\pi}{\langle D_{nJ} \rangle}\, 2\pi\, \frac{(2\ell+1)(2s+1)}{R_N(J)}\, \vartheta^2_{\ell}(J) \sum_{Qj_3j_4} (2Q+1)F(Q)(2j_3+1)R(j_3)R_{N-2}(j_4) \begin{pmatrix} \ell & j_3 & Q \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} \ell & j_3 & Q \\ j_4 & \ell & s \end{Bmatrix}^2 \qquad (E.8)$$

$$\overline{\tau_n}(\ell,s,+2;U) =$$

$$\frac{2\pi}{\langle D_{nJ}\rangle}\; 2\pi\; \frac{(2\ell+1)(2s+1)}{R_N(J)}\; \Theta_\ell^2(U) \sum_{Qj_3j_4} (2Q+1)R(Q)(2j_3+1)F(j_3)$$

$$R_{N-1}(j_4) \begin{pmatrix} \ell & Q & j_3 \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} \ell & j_3 & Q \\ j_4 & J & s \end{Bmatrix}^2 \qquad \text{(E.9)}$$

$$\overline{\tau_n}(\ell,s,-2;U) = \frac{2\pi}{\langle D_{nJ}\rangle}\; 2\pi\; \frac{R_{N-3}(s)}{R_N(J)}\; \Theta_\ell^2(U)$$

$$\sum_{Qj_3} (2Q+1)F(Q)(2j_3+1)R(j_3) \begin{pmatrix} \ell & j_3 & Q \\ 0 & 0 & 0 \end{pmatrix}^2 \qquad \text{(E.10)}$$

and

$$\overline{\rho_n^{(0)}}(U) = gp\left(\frac{p-1}{2} + h\right)(N-1)\,\xi^{N-2}\left(1 - \frac{N-2}{N-1}\xi\right)$$

$$\overline{\rho_n^{(+2)}}(U) = \frac{gh\,p(p-1)}{4E}\; \frac{(N-1)!}{(N-4)!}\; \xi^{N-4}(1-\xi)^2$$

$$\overline{\rho_n^{(-2)}}(U) = g(gE)\,\xi^N \qquad \text{(E.11)}$$

with $\xi = U/E$ and $N = p+h$. Notice that the total transmission out of class n to channel c' through mode ν is given by $\sum_s \tau^J(\ell,s,\nu;U)$. Another important quantity in our theory is the mixing parameter $\mu^J_{n\ n+1}$ which is given by

$$\mu^J_{n,n+1} = \frac{\langle \Gamma_{nJ}{}^{\downarrow} \rangle}{\langle \Gamma_{nJ} \rangle} \qquad \text{(E.12)}$$

$$\Gamma_{nJ}{}^{\downarrow} = g\,\frac{(gE)^2}{2(n+1)}\; (2\pi) \sum_{Qj_3j_4} \frac{\textcircled{H}(j_3,Q)}{R_N(J)}\; R(Q)(2j_3+1)$$

$$\times F(j_3)R_{N-1}(j_4)\;\Delta(Q\ J\ j_4) \qquad \text{(E.13)}$$

and

$$\langle \Gamma_{nJ} \rangle = \langle \Gamma_{nJ}^{\downarrow} \rangle + \sum_{\nu=2,0,-2} \sum_{ls} \int_0^{E'} \frac{\langle D_{nJ} \rangle}{2\pi} \overline{\tau_n^J(l,s,\nu;U)}\, \bar{\rho}_n^{(\nu)}(U)\, dU \tag{E.14}$$

The symbol $\Delta(QJ\ j_4)$ is defined so that

$$\Delta(a\ b\ c) = \begin{cases} 1 & \text{for } |a-b| \leq c \leq a+b \\ 0 & \text{otherwise} \end{cases}$$

and $\textcircled{H}(j_3,Q)$ is given in terms of the radial integral for bound-bound transitions, $\theta^2_{l,B}$ (see Eq. (E.5))

$$\textcircled{H}(j_3,Q) = \sum_l (2l+1)\, \theta^2_{l,B} \begin{pmatrix} l & j_3 & Q \\ 0 & 0 & 0 \end{pmatrix}^2 \tag{E.15}$$

A useful approximationfor the radial integrals that appear in θ_J and θ_l would be to assume constant wave functions in the nuclear interior and zero outside. This is the approximation adapted by the MIT and the Milan group in their calculation of the averaged cross section. With the above formulae one should be able to calculate $X_{n,cc}\, X_{n,c'c'}$ as given in Eq. (5.3)

TABLE CAPTION

Table 1 Comparison of the fluctuation cross section and auto-correlation functions among the different approaches. The coefficients C_{nm} are the elements of the orthogonal transformation that diagonalizes the matrix M.

FIGURE CAPTIONS

Fig. 1 Correlation function for the reaction $^{27}A\ (^{3}He,p)^{29}Si$, leading to the (a) second and (b) fifth excited state of ^{29}Si. The continuous curves are Ericson's formula $C(\epsilon) = (\epsilon^2 + \tilde{\Gamma}^2)^{-1}$, with a correlation width of 55 keV and 230 keV, respectively (From Ref. [2b]).

Fig. 2 Correlation function for the reaction $^{27}A\ (^{3}He,p)^{29}Si$, leading to the fourth excited state of ^{29}Si. The continuous curve represents a plot of our Eq. (2.17) with $\tilde{\Gamma}_1 = 230$ keV, $\tilde{\Gamma}_2 = 55$ keV and $\sigma_1/\sigma_2 = 2.3$ (From Ref. [2b]).

Fig. 3a Partition of Hilbert space.

Left: usual Hauser-Feshbach partition.
Center: Nested doorway partition.
Right: Generalized Hauser-Feshbach partition used in the evaluation of $\sigma^{f\ell}_{n,cc'}$.

Fig. 3b Schematic representation of the various classes of levels used in the nested-average approach. In this diagram, class (n-1) is "above" class n, and (n+1) "below" it; we adhere to this terminology throughout.

Fig. 4 Schematic representation of $C(\epsilon,I)$ for two classes of levels with $\tilde{\Gamma}_2 \ll \tilde{\Gamma}_1$, where I is the energy averaging interval employed. The curve in the C-I plane is what is known as the "chair of Pappalardo".

Fig. 5 Schematic representation of the absolute values of the terms that build up (B3), for k=1 and k=2.

Fig. 6 Schematic representation of the absolute values of the factors $S_{ab}^{f\ell}(E_k)$, $S_{cd}^{f\ell}(E_k')$ of Eq. (B6), for k=1 and k=2.

TABLE 1

Grimes et al. [4c]

$$\sigma^{f\ell}_{cc'} = (1 - \mu_G)\frac{\tau^G_{1c}\tau^G_{1c'}}{\tau^G_1} + \frac{(\tau^G_{2c} + \mu_G\,\tau^G_{1c})(\tau^G_{2c'} + \mu_G\,\tau^G_{1c'})}{\tau^G_2 + \mu_G\,\tau^G_1}$$

Harney et al. [4b]

$$\sigma^{f\ell}_{cc'} = \langle \tau_c | M^{-1} | \tau_{c'} \rangle$$

(congruence transform)

$$= (1-\mu)\frac{\tau_{1c}\,\tau_{1c'}}{\tau_1} + \frac{(\tau_{2c} + \mu\,\tau_{1c})(\tau_{2c'} + \mu\,\tau_{1c'})}{\tau_2 + \mu\tau_1} \equiv \sum_{n=1}^{2} X_{nc}\,X_{nc'}$$

(orthogonal transform)

$$= \frac{(\sum_{n=1}^{2} C_{1n}\,\tau_{nc})(\sum_{m=1}^{2} C_{1m}\tau_{mc'})}{(\sum_{n=1}^{2} C_{1i}\,\tau_i)} + \frac{(\sum_{n=1}^{2} C_{2n}\,\tau_{nc})(\sum_{m=1}^{2} C_{2m}\,\tau_{mc'})}{(\sum_{n=1}^{2} C_{2i}\,\tau_i)} \equiv \sum_{n=1}^{2} \tilde{X}_{nc}\tilde{X}_{nc'}$$

$$C^S_{cc'}(\varepsilon) = \langle \tilde{\tau} | (M + 2\pi i \varepsilon I)^{-1} | \tilde{\tau}_{c'} \rangle = \sum_{n=1}^{2} \frac{\tilde{X}_{nc}\,\tilde{X}_{nc'}}{1 + i\varepsilon/\tilde{\Gamma}_n}$$

Hussein and McVoy [5]

$$\sigma^{f\ell}_{cc'} = \sum_n X_{nc}\,X_{nc'}$$

Friedman et al. [7]

$$C^S_{cc'}(\varepsilon) = \sum_n \frac{X_{nc}\,X_{nc'}}{1 + i\epsilon/\tilde{\Gamma}_n}$$

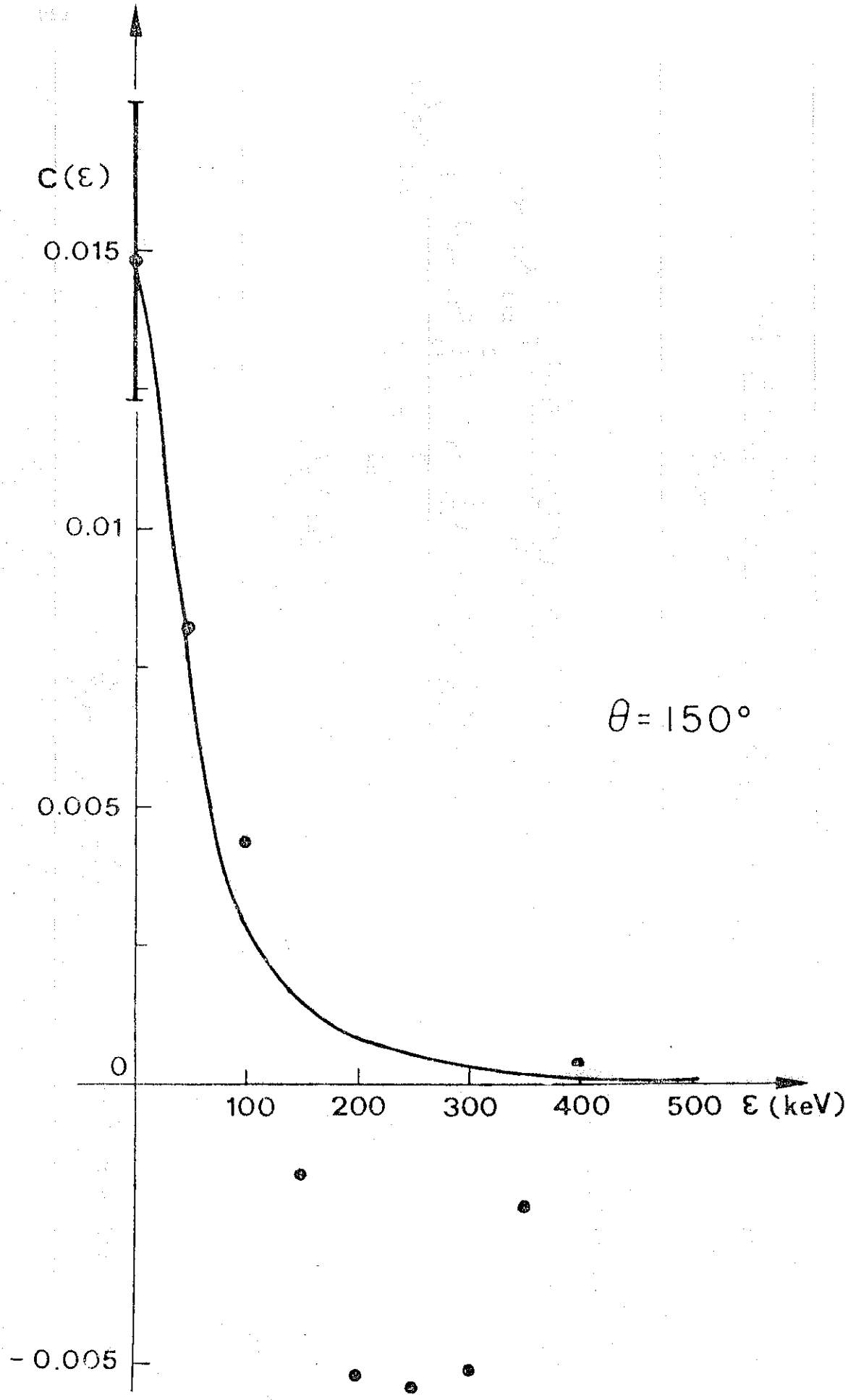

Fig. 1a

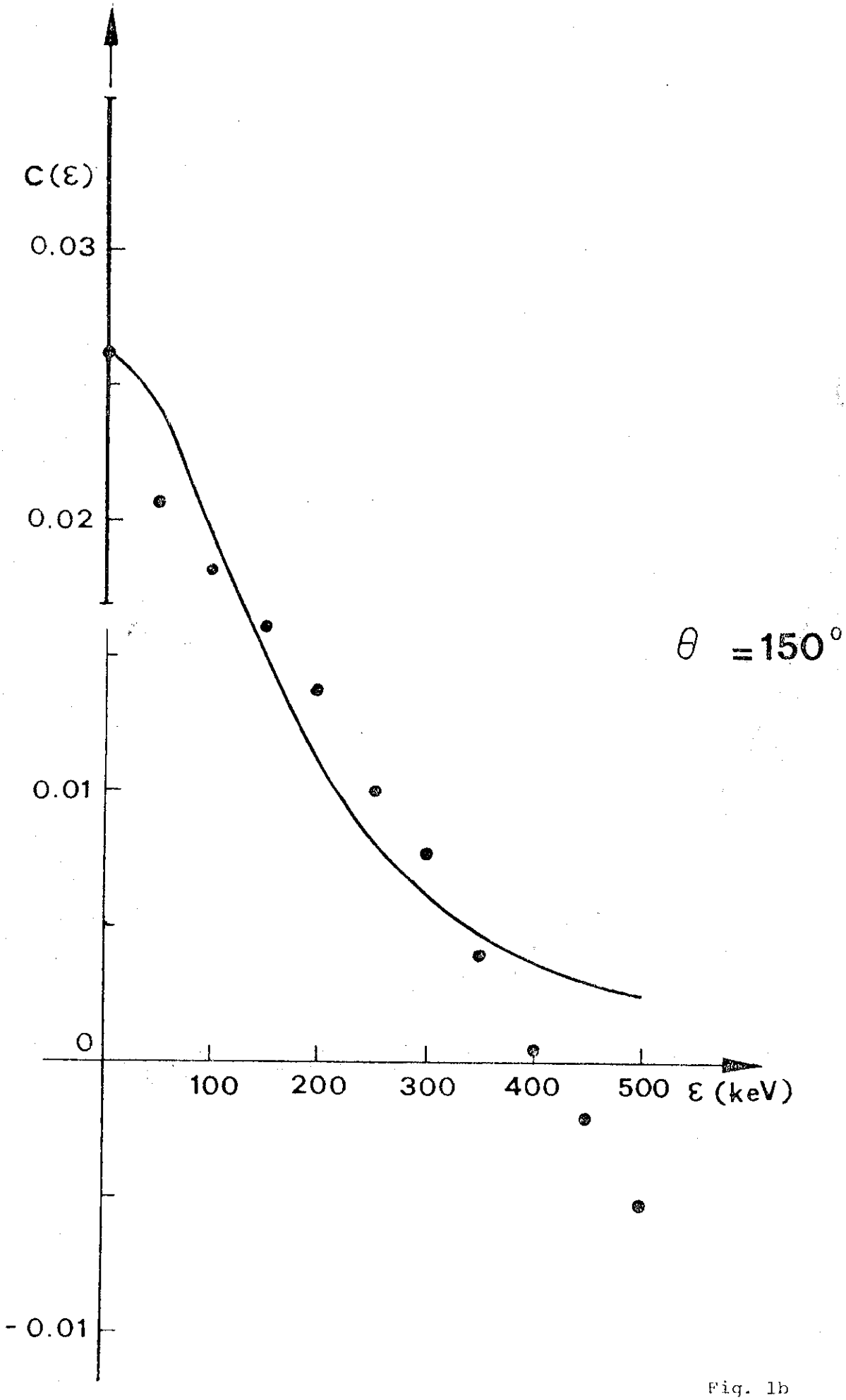

Fig. 1b

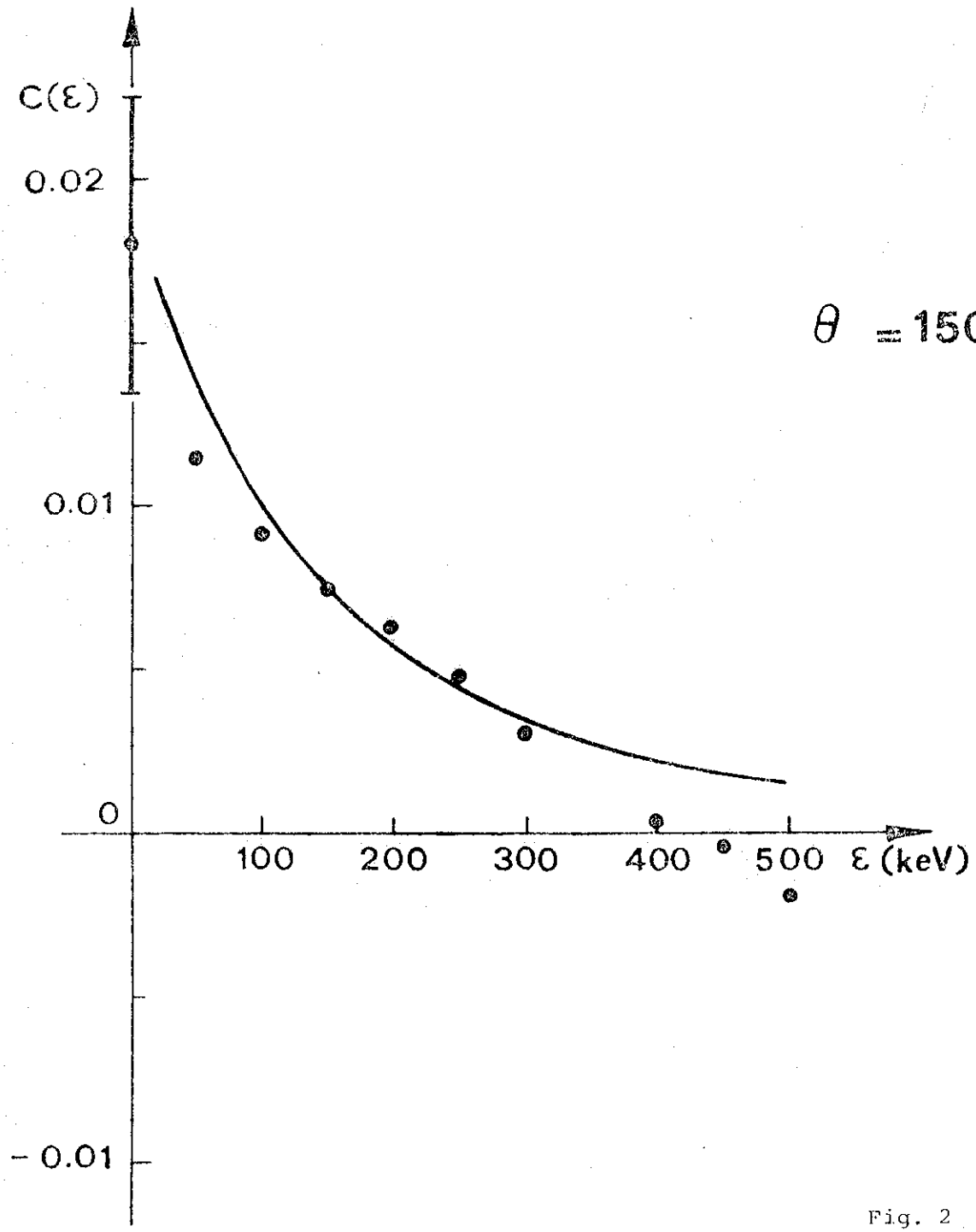

Fig. 2

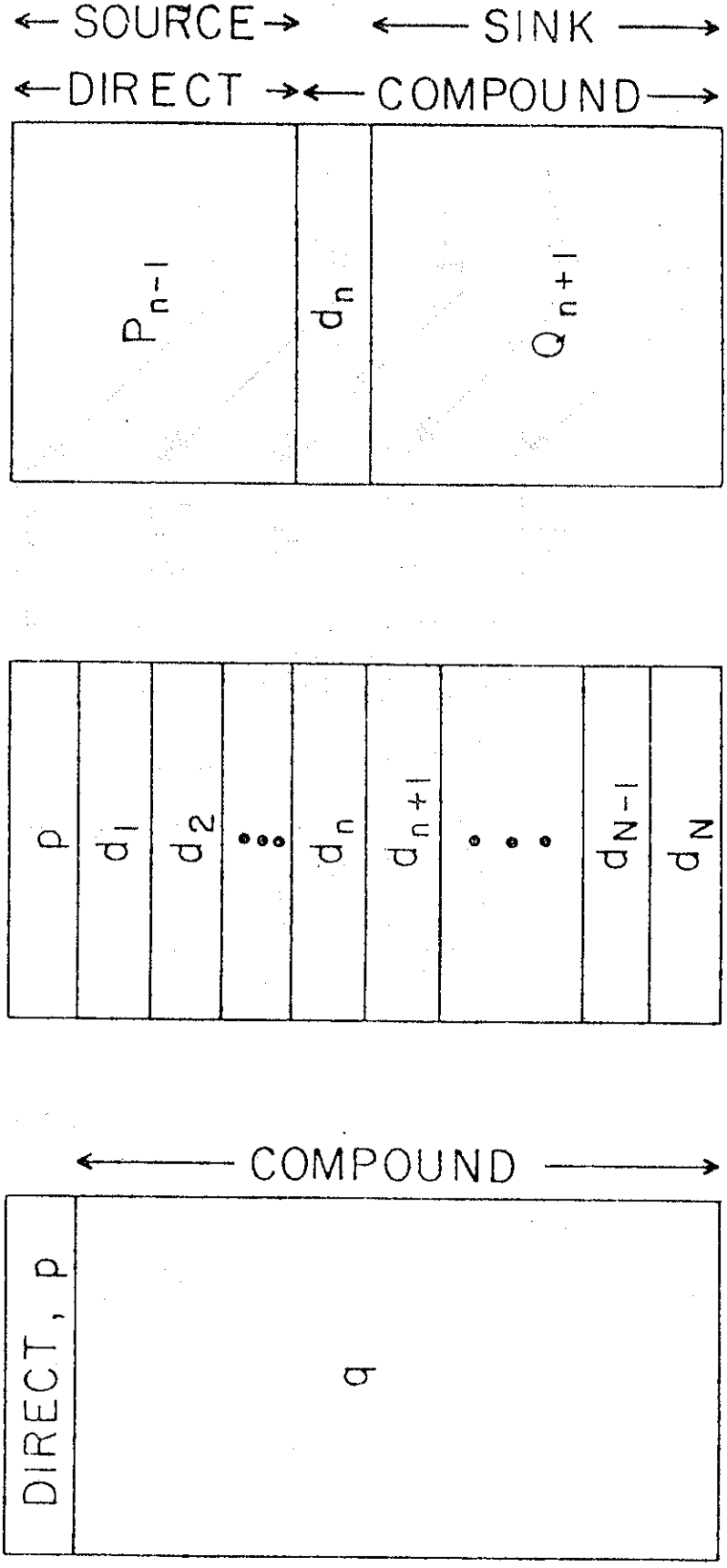
← SOURCE →
← SINK →
← DIRECT →
← COMPOUND →
P_{n-1}
d_n
Q_{n+1}
p
d_1
d_2
• • •
d_n
d_{n+1}
• • •
d_{N-1}
d_N
← COMPOUND →
DIRECT, p
q

Fig. 3d

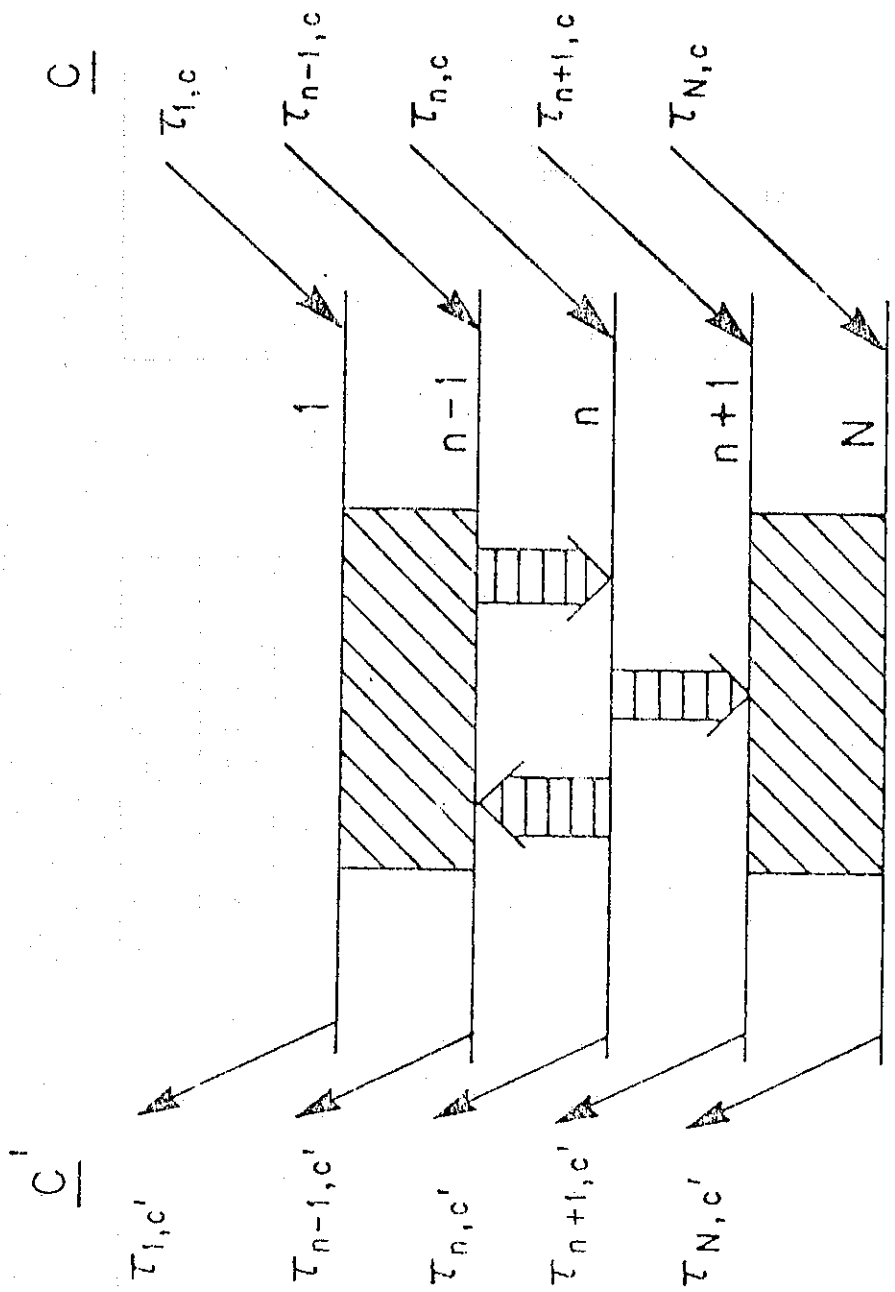
C
C'
$\tau_{1,c}$
$\tau_{n-1,c}$
$\tau_{n,c}$
$\tau_{n+1,c}$
$\tau_{N,c}$
1
n-1
n
n+1
N
$\tau_{1,c'}$
$\tau_{n-1,c'}$
$\tau_{n,c'}$
$\tau_{n+1,c'}$
$\tau_{N,c'}$

Fig. 3b

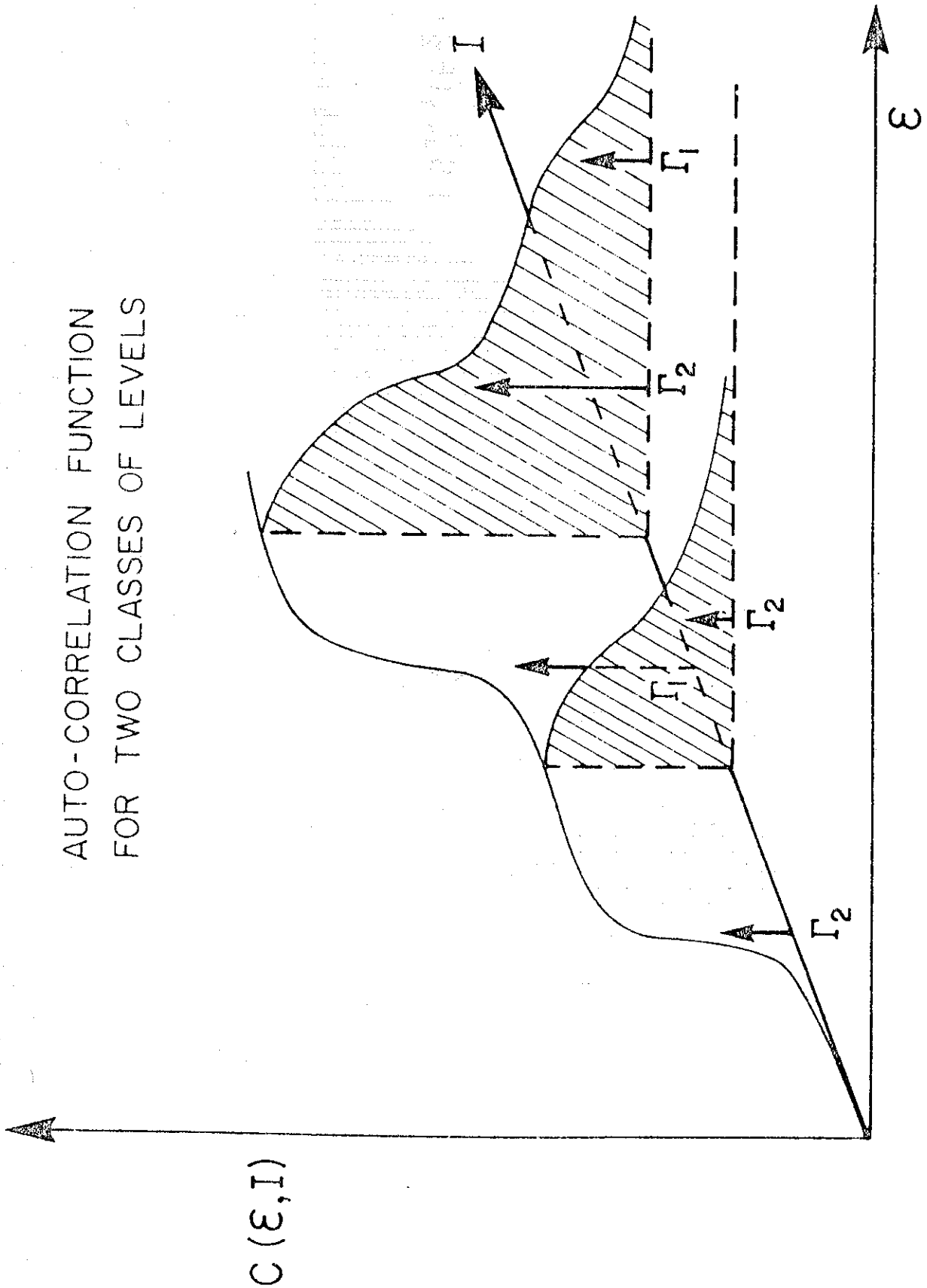
AUTO-CORRELATION FUNCTION
FOR TWO CLASSES OF LEVELS
C(ε,I)
I
ε
Γ1
Γ2
Γ2
Γ1
Γ2

Fig. 4

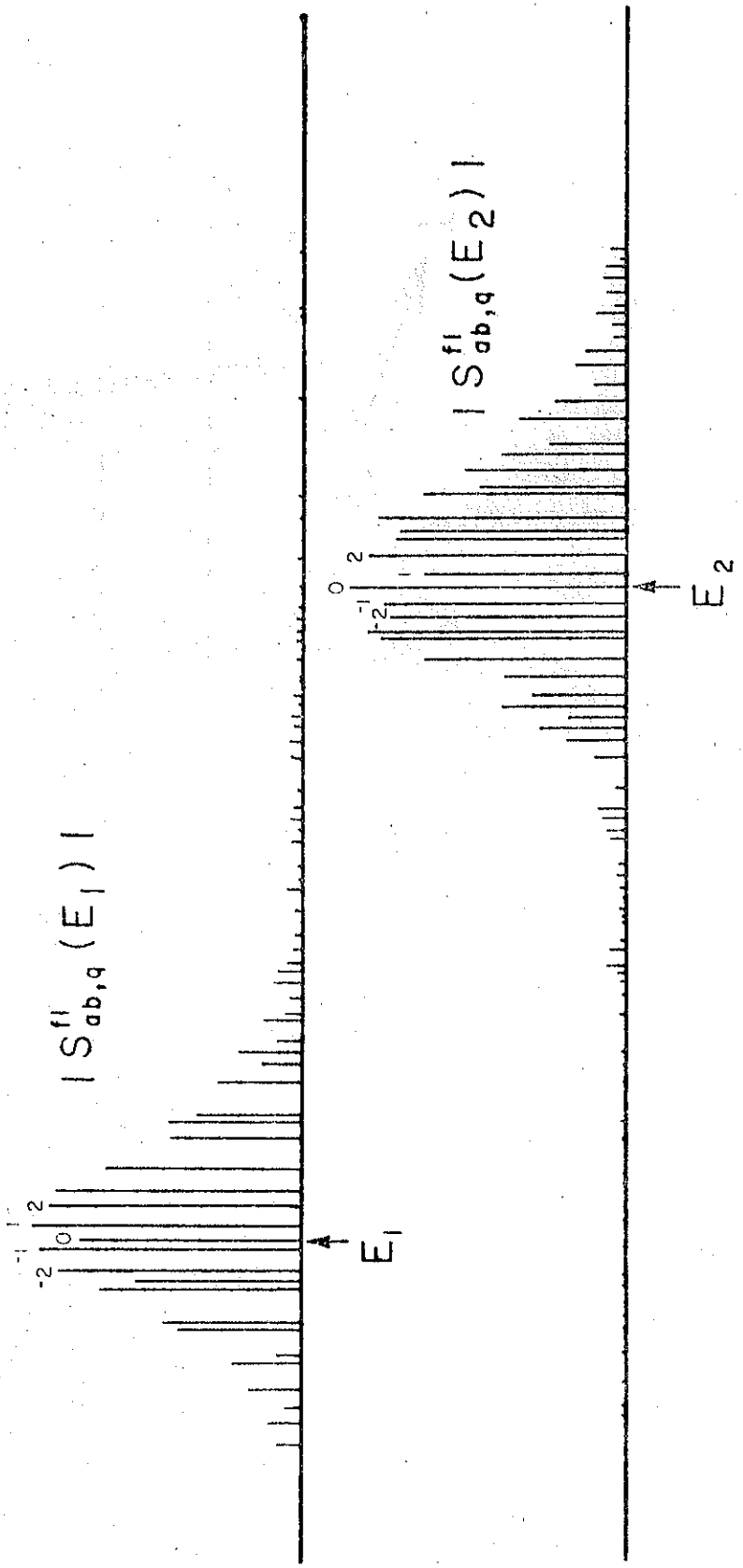

Fig. 5

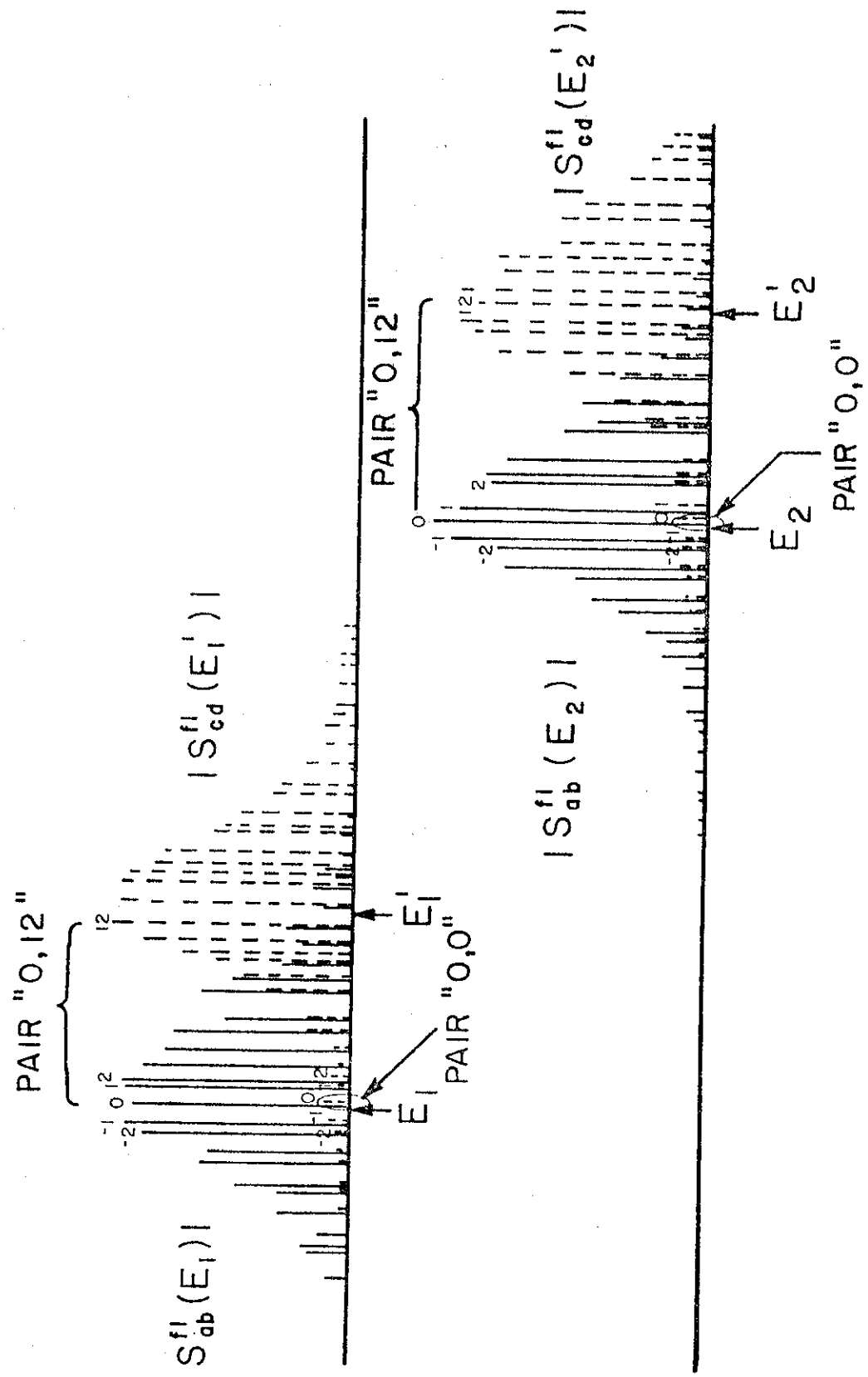
$|S^{fI}_{ab}(E_1)|$
$|S^{fI}_{cd}(E_1')|$
PAIR "0,12"
PAIR "0,0"
E_1
E_1'
$|S^{fI}_{ab}(E_2)|$
$|S^{fI}_{cd}(E_2')|$
PAIR "0,12"
PAIR "0,0"
E_2
E_2'

Fig. 6