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THE WORK OF J.A. SWIECA WITHIN TWO DECADES OF DEVELOPMENTS IN QUANTUM FIELD THEORY

by

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From the Principles of General Quantum Field Theory towards New Dynamical Intuition from Model Studies

The work of J. A. Swieca within two decades of developments in Quantum Field Theory

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INTRODUCTION

The scientific work of J.A. Swieca* constitutes a fascinating bridge between the thorough investigations of the general principles of Quantum Field Theory carried out in the late 50's and early 60's and the more recent attempts to understand the dynamical subtleties of the relation between particles and fields.

In order to recapture the motivations of a young theoretician who entered active research at the beginning of the 60's it is helpful to start with a panoramic view of Quantum Field Theory in those days.

Several years after the impressive success of perturbative renormalization theory in Quantum Electrodynamics, physicists started to question the adequacy of the Lagrangian approach for other interactions, in particular strong interactions. The first step taken was to liberate the principles underlying the Lagrangian approach from their perturbation wrapping. These attempts culminated later on in the framework of Wightman(1) and Haag(2) and that of Lehmann, Symanzik and Zimmermann (3). In the first one, emphasis was placed on vacuum expectation values of fields (or local observables) and their properties whereas in the LSZ theory the cornerstone was the asymptotic condition for "interpolating" fields, thus relating fields with particles. Some years later Haag(4) and Ruelle(5) as well as Hepp(6) demonstrated that if there are no zero mass particles in the spectrum, the LSZ asymptotic properties (without the asymptotic completeness) can actually be derived from the locality properties of fields. In addition to general structural theorems

* deceased on Dec. 22nd 1980

as TCP, Spin and Statistics and generalizations involving internal symmetries, this framework of General Quantum Field Theory furnished the foundation of Dispersion Relations. The distrust and to a certain degree misunderstanding of QFT among some physicists was so great that attempts were made, to disconnect the Dispersion Approach to elementary particle physics as the "S - Matrix Bootstrap" from the "contaminated" Field Theory . Even though those ideas are almost forgotten, they played a certain role in the 60's and sometimes even led to useful observations which were later on incorporated into the mainstream of QFT.

Only at the end of the 60's it becomes abundantly clear that the short distance singularities of QFT, far from threatening the mathematical existence of the theory, were actually necessary for its internal consistency; renormalization theory became synonymous with the study of short distance properties. The results of Wilson(7) and other physicists(8) played an important role in regaining confidence, not only in the general principles, but even in the Lagrangian approach which thus reached a new level of sophistication.

These remarks furnish the background for understanding the motivation behind part of Swieca work, the part which I will present under the heading of "Structural Theorems in QFT". These results are largely independent on Lagrangian models.

In order to appreciate the other part of his work which I will discuss under the heading "model studies as a laboratory for developing and testing new dynamical ideas" it may be helpful to point out that during the 70's the emphasis in

QFT changed from short-distances towards properties of physical states. The driving motor for this was the pressing need, especially posed by non abelian gauge theories, for understanding the relation of Lagrangian fields to the physical spectrum in a more profound way. After the renormalization properties (9) (including the observation of asymptotic freedom (10)) were clarified notably by 't Hooft, many physicists concentrated their attention to the vacuum and particles properties of these gauge models.

In this context it seems to me remarkable that by pressing the internal logic of a two dimensional gauge model (11) André Swieca together with John Lowenstein were able to capture some aspects of many of the modern concepts as Θ vacua, the U(1) problem, charge neutrality and confinement long before these words were coined. This line of research later on was refined in a series of papers dealing with the functional integral approach (12); the issue of "screening versus confinement" (13) and the U(1) problem in a solvable model with mass transmutation and its relation to fractional winding (14). I will have to say something on this work in the second part.

Being convinced that the study of models furnish a useful laboratory for new dynamical ideas, André enjoyed thoroughly the discovery of a certain class of nontrivial two-dimensional models whose S-matrix and Form-factors became computable. He realized that the appearance of exotic statistics (15) in some of these models is a manifestation of the order - disorder duality of Statistical Mechanics of which lattice model studies were first performed by Kadanoff and colaborators (16).

In the last year of his life, he was particularly interested in understanding the rather subtle renormalization aspects of kinks and disorder fields in the euclidean functional integration approach.

I. STRUCTURAL THEOREMS in QFT

At the beginning of the 60's Nambu and Goldstone discovered that spontaneously broken symmetries in theories of short range (local Lagrangian) interactions are always accompanied by the appearance of zero mass bosons.

Symmetries in QFT in those days were discussed in complete analogy to symmetries in classical field theories. The starting point was a Lagrangian

$$L(\phi_i, \partial_\mu \phi_i) \tag{1}$$

leading via the principle of minimal action to the Euler - Lagrange equation

$$\frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial \partial_\mu \phi_i} - \frac{\partial L}{\partial \phi_i} = 0 \tag{2}$$

The invariance of the Lagrangian (1) under a N-parametric invariance group G (for simplicity we restrict our attention to linear realization i.e. matrix groups):

$$\phi_i \longrightarrow V_{ij} (\lambda_1 \ldots \lambda_N)\ \phi_j \tag{3}$$

with generators: $\frac{d}{d\lambda_k} V/_{\lambda=0} = i\ I^k$ (4)

leads the Euler-Lagrange equations invariant under G and to N conserved currents given by Noether's theorem

$$I_\mu{}^k = -\ i \frac{\partial L}{\partial \partial_\mu \phi_\ell} I^k{}_{\ell m}\ \phi_m = -i \frac{\partial L}{\partial \partial_\mu \phi} I^k \phi \tag{5}$$

The Poisson bracket relation at equal times

$$\{I_0{}^k(\vec{x}),\ \phi_i(\vec{y})\}_p = i\ I_{ij}{}^k\ \phi_j(\vec{y})\delta(\vec{x}-\vec{y}) \qquad (6)$$

and hence for the conserved charge

$$Q^k = \int I_0{}^k\ d^3x \qquad (7)$$

the relation

$$\{Q^k,\ \phi_i(\vec{y})\} = i\ I^k{}_{ij}\ \phi_j(\vec{y}) \qquad (8)$$

are a consequence of the classical canonical formalism

$$\{\phi_i\ (\vec{x}), \pi_j\ (\vec{y})\}_p = \delta_{ij}\ \delta(\vec{x}-\vec{y}) \qquad (9)$$

with

$$\pi_j = \frac{\partial L}{\partial \partial_0 \phi_j}$$

Via exponentiation of the charges one obtains a representation of G in the phase space of the classical field theory.

It was standard praxis prior to Nambu's and Goldstone's observation to obtain the construction of unitary operators in Hilbert space $U(\lambda)$ implementing the substitution law

$$U(\lambda)\ \phi_i(x)\ U^+(\lambda) = V_{ij}{}^{-1}(\lambda)\ \phi_j(x) \qquad (10)$$

by replacing Poisson brackets simply by commutator brackets ,
thus writing:

$$U(\lambda) = e^{i\lambda_k Q^k} \tag{11}$$

with

$$Q^k = \int I_0{}^k(x)\ d^3x \tag{12}$$

and the equal-time commutator relation:

$$\left[I_0{}^k(\vec{x})\ \phi_i(\vec{y})\right] = -\ I_{ij}{}^k\ \phi_j\ (\vec{y})\ \delta(\vec{x}-\vec{y}) \tag{13}$$

This formal procedure of constructing unitary symmetry operator by simply copying the classical steps is correct in a quantum theory with a finite number of degrees of freedom i.e. Quantum Mecanics.As a result of the uniqueness theorem of John v. Neumann (every irreducible representation of the canonical commutation relation is unitarily equivalent to Schrödinger's), an algebraic symmetry i.e. an invariance of the Lagrangian and canonical relation under a symmetry group is always implementable by a unitary operator $U(\lambda)$. This is the basis of Wigners analysis of symmetries in Quantum Mechanics.

The situation is different in QFT. Fortunatly, in order to understand symmetries in QFT,we do not have to study the intricasies of canonical representation theory. It is sufficient to be aware of two aspects in which the QFT discussion deviates from classical field theory as well as from quantum mechanic:
1.) The Lagrangian, the equation of motions and the definition

of currents involve product of field operators at the same point and therefore are ill - defined quantities whose proper meaning should be obtained by limiting procedures starting from diffe - rent space-time points.

2.) The construction of the "classical" charge [12] from its density requires the vanishing of the field of large distances, a requirement which always can be fullfilled by appropriately re - stricting the Cauchy data of classical solutions.

The existence of particle-antiparticle fluctuations occuring all over space (translational invariance) in QFT pre - vents the general use of eq. (7) as a definition of a well de - fined charge operator. Even in the absence of spontaneous sym - metry breaking the convergence properties of this integral depend in a very subtle way on the properties of the states[20].

The short distance properties[7] had been understood in the frame work of the operator short-distance algebras by the end of the 60's . Apart from the anomaly phenomenon, which from a certain point of view has a classical interpretation[21] they do not enter the discussion of spontaneous symmetry breaking. It is rather through the fluctuation properties 2.) , which fall into the category of long distance behaviour, that a per - fectly conserved quantum Noether current may lead to nonexis - tent charges and spontaneously broken symmetries. Thus the Nambu Goldstone phenomenon is an evasion of the Wigner quantum mechanical symmetry mechanism due to subtle property of field theoretic fluctuations and as such very basic to elementary particle physics.

The standard argument[22] concerning spontaneous symme - try breaking and Nambu-Goldstone bosons is abstracted from the

O(N) Sigma model Lagrangian. Consider the renormalizable O(N) symmetric Lagrangian.

$$L = \partial_\mu \phi_i \partial^\mu \phi_i - V(\phi) \tag{14}$$

$$V(\phi) = \lambda (\phi_i \phi_i - \beta)^2 \tag{15}$$

this Lagrangian has a $\frac{O(N)}{O(N-1)}$ manifold of classical minima . Quasiclassicaly one constructs a QFT by relating one of the minima(23), say

$$\phi_{min} = \sqrt{\beta} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \tag{16}$$

to the field theoretic vacuum in zero order

$$\langle\phi_i\rangle_{\ell.o.} = \sqrt{\beta}\ \delta_{iN} \tag{17}$$

Performing a shift in the Lagrangian

$$\phi_i \longrightarrow \hat{\phi}_i + \langle\phi_i\rangle \tag{18}$$

one finds a Lagrangian in $\hat{\Phi}$ which admits a renormalized perturbation series(23). There exist Noether currents I_μ^k which are conserved in every order of renormalized perturbation theory(23). The model contains N-1 zero mass bosons and the symmetry is broken as result of (17). In order to relate these two properties , knowledge of its detailed the dynamical structure is not required. The standard argument is the following[22]. The vacuum expectation value of (13) yields

$$<|I_0{}^k(\vec{x}),\ \phi_i(\vec{y})|> = - I^k{}_{iN} <\phi_N> \delta(\vec{x}-\vec{y}) \qquad (19)$$

On the other hand, the Källen-Lehmann representation for the two point function:

$$<\left[I_\mu{}^k(x),\ \phi_i(y)\right]> = i\ \partial_\mu \int \Delta(x-y;\kappa^2)\rho^{\kappa i}(\kappa^2)d\kappa^2 \qquad (20)$$

with the conservation $I_\mu{}^k$:

$$\kappa^2\ \rho^{ki}(\kappa^2) = 0 \qquad \text{i.e. } \rho^{ki}(\kappa^2) = c^{ki}\delta(\kappa^2) \qquad (21)$$

yields,together with (14)

$$c^{ki} = - I^k{}_{iN} <\phi_N> \qquad (22)$$

i.e. the existence of N-1 Nambu-Goldstone bosons related to the N-1 unbroken directions.

The argument may be easily generalized to renormalizable Lagrangian with other symmetry groups and unbroken sub-groups. The essential mathematical input is the existence of non vanishing expectation values of elementary (canonical) fields. The shortcomings of this method as an argument for the general relation between spontaneously broken symmetries and Nambu-Goldstone bosons are the following:

1. For composite fields, commutation relation of the form (13)

are not a priori reasonable. The use of operator short distance expansion for the construction of local composite fields and products with currents, which many years after the Nambu-Goldstone observation were investigated by Wilson[7], with sufficiently many additional assumptions would perhaps allow for a more general argument along the above lines.

2. In the case of no symmetry breaking which should be suitably formalized mathematically, one really would like to have an argument in favour of hermitean charge operators as generators of the corresponding finite symmetry transformations.

3. The renormalized Lagrangian perturbation theory of a spontaneous broken symmetry situation does not allow a conclusion concerning the true existence of the broken symmetry phase.

The problem of existence is the most difficult one. With the help of techniques known in statistical mechanics, field theoretists were able to make some progress[24]. In a way this problem does not concern the Nambu-Golstone theorem, because the existence of the broken symmetry phase enters as an assumption.

Shortly after proofs running along the indicated lines were given, Swieca[25] and Ezawa[26] and Swieca gave a proof using more powerful techniques and thus removing the shortcomings 1) and 2). The conceptual-mathematical basis of this proof is contained in prior work of Haag Kastler and Swieca[27]. One difference is that the Wightman framework is used instead of the C* algebra methods and the new important ingrediente is the use of a more powerful spectral representation, that of Jost-Lehman-Dyson[28]. I will indicate the main steps of the derivation. One starts from local Wightman polynomials of the form:

$$A = \sum_{0}^{N} \int h_n(x_1 \ldots x_n)\ \phi(x_1)\ldots\phi(x_n)\ dx_1 \ldots dx_n \qquad (23)$$

where h_n are test functions from the Schwartz class of compact support $\mathcal{D}$. In this way one obtain well defined operators which are affiliated with a compact space-time region and which, if applied to the vacuum, generate a dense set of states[29]. We suppress in our notation all dependence of the fundamental field ϕ on indices. AT its most basic level a symmetry of Q.F.T. is a correspondence

$$A \longrightarrow A_\lambda \qquad (24)$$

induced by (3) which leaves invariant equation of motions, and the Lagrangian. However from the point of view of observable consequences (particle multiplets, symmetry relations of cross sections etc) one has to elevate this algebric symmetry, as Wigner did in Quantum Mechanics, to a unitary operator $U(\lambda)$ in the physical state space

$$U(\lambda)\ A\ U^{+}(\lambda) = A_\lambda \qquad (25)$$

$$U(\lambda)|0> = |0> \qquad (26)$$

Formaly $U(\lambda)$ is expected to have the form (11) Q being related to a conserved current with

$$< \frac{dA\lambda}{d\lambda}\Big|_{\lambda=0} > = <i\,[Q,A]> = 0$$

Since A has compact support say θ, any operator which is localized in the causal complement θ_c of θ commutes with A. Therefore the unbroken symmetry should be characterized in terms of:

$$<\left[I^0(f_d, f_R), A\right]>_{R>R_0} = 0 \qquad (27)$$

for all Wightman polynomials A (23).
The $I^0(f_d, f_{R_0}) = \int I^0(x_0,\vec{x}) f_d(x_0) f_{R_0}(\vec{x})$ is effectively the relevant part of the charge which does not commute with A.
$f_R(\vec{x})$ is a smooth test function

$$f_R(\vec{x}) = \begin{cases} 1 & |\vec{x}| < R \\ 0 & |\vec{x}| > R + \varepsilon \end{cases} \qquad (28)$$

thus preventing violent surface effects and

$$\begin{aligned} f_d(x_0) &= 0 \qquad |x_0| > d \\ \int f_d(x_0) &= 1 \end{aligned} \qquad (29)$$

is a smooth compact support interpolation of the δ-function in time which for noncanonical field operator as the current is necessary in order to obtain a finite operator. R_0 is simply the radius beyond which one enters the θ_c region (augmented by d where 2d is the thickness of the time smearing). The independence on R, once R is larger then R_0, is a trivial consequence of causal commutativity whereas the independence on f_d is simply obtained by using first the conservation law (with say d>d')

$$I^0(f_d, f_R) - I^0(f_d', f_R) = I^0(\hat{f}, \vec{\nabla} f_R) \tag{30}$$

$$\hat{f}(x_0) = \int_{-\infty}^{x_0} (f_d(x'_0) - f_d'(x'_0))dx'_0 \qquad \mathcal{D}_d$$

and the causality

$$|I^0(\hat{f}, \nabla f_R), A| = 0 \qquad R>R_0 \tag{31}$$

One says that a theory exhibits spontaneous symmetry breaking if for a conserved current there exists an A such that

$$\lim_{R\to\infty} <[I^0(f_d, f_R), A]> \neq 0 \tag{32}$$

Part of the Nambu-Goldstone theorem is the

Lemma 1 A spontaneously broken symmetry in the sense of (32) requires the existence of a Nambu-Goldstone boson.

The remainder is contained in

Lemma 2 If (27) holds, the formula

$$Q \quad A \; |0> \;=\; [I^0(f_d, f \;), A]_{R>R_0} \; |0>$$

defines a charge operator (on the dense set of local states) whose exponentiation yields a one parametric symmetry (sub) group. In order to prove Lemma 1, Swieca used the Jost-Lehman-Dyson representation for the commutator in (32) in the form derived by Araki Hepp and Ruelle[30].

$$<0|\left[j^0(x),A\right]|0> = \int_0^\infty d\mu^2 \int d^3y\, \Delta(\vec{x}-\vec{y},x_0,\mu^2)\rho_1(\mu^2,\vec{y}) \qquad (33)$$

$$+ \int_0^\infty d\mu^2 \int d^3y \frac{\partial}{\partial x^0} \Delta(\vec{x}-\vec{y},x_0,\mu^2)\rho_2(\mu^2,\vec{y})$$

$\rho_i(\mu^2,\vec{y})$ are measures in μ^2 having compact support in $\vec{y}$, this support being related to that of A. They can be split:

$$\rho_i(\mu^2,\vec{y}) = \overline{\rho}_i(\mu^2)\delta^3(\vec{y}) + \vec{\nabla}\,\vec{G}(\mu^2,\vec{y}) \qquad (34)$$

$$\overline{\rho}_i(\mu^2) = \int \rho_i\,(\mu^2,\vec{y})\,d^3y \qquad (35)$$

$\vec{G}_i(\mu^2,\vec{y})$ has same support in $\vec{y}$ as ρ_i.

The conservation law and causality yields

$$\frac{d}{dx_0} <0\,|\left[j^0(x_0,f_R),\,A\right]|0>_{R>R_0(x_0)} = 0 \qquad (36)$$

and therefore

$$\int_0^\infty d\mu^2\, \overline{\rho}_{\left\{\begin{smallmatrix}1\\2\end{smallmatrix}\right\}}(\mu^2) \qquad \begin{Bmatrix}\cos\mu x_0\\ \mu\sin\mu x_0\end{Bmatrix} = 0$$

which is only consistent with

$$\overline{\rho}_1 = 0$$

$$\overline{\rho}_2 = \lambda\,\delta(\mu^2) \quad , \lambda\neq 0 \text{ from } (32)$$

λ can also be written $(x_0=0)$

$$\lambda = \int_0^{M^2} d\mu^2 \int \rho_{\ell} (\mu^2, \vec{y}) \; g \; (\vec{y}) \; d^3 y \tag{38}$$

$$= <0|j^0(0,g) \; P(M^2) \; A| \; > - <0|A \; P(M^2) \; j^0(0,g)|0)$$

Here g is a 0-test function with $g(\vec{y}) = 1$ in the region of supp $\vec{G}$.

$P(M^2)$ is the projector on to the subspace with a mass $\leq M^2$.

The validity for every $M^2 > 0$ leads to the existence of a discret zero mass intermediate state.

Note that the *L*-covariant properties of the conserved current were not used up to this point. In case I_μ behaves like a vector (i.e. has no further suppressed *L*-indices) the intermediate state is necessarily a scalar boson.

In the case of spontaneous symmetry breaking there exists therefore a Wightmann polynomial (i.e. a product of fields) A, which couples the Nambu-Goldstone boson to the vacuum:

$$\lim_{\vec{p} \to 0} < p| \; A \; | \; 0 > = c \neq 0 \tag{39}$$

This is impossible in two-dimensional space time since (39) implies a infrared divergence in the two point function of A. This impossibility of two-dimensional spontaneous symmetry breaking was known to André and is implicit in his proof. Within the context of the standard method of proof it was derived by Coleman (31).

Let us now make some brief comments on the second Lemma. This lemma has a moderately simple proof in the case of

the mass gap hypothesis.
For special quasi-local operator A the formula(34)

$$\lim_{R\to\infty} <0|A\ I^0(f_d,\ f_R)|0> = 0 \tag{40}$$

is a rather easy consequence of (27). It is only necessary to demonstrate that (27) has a generalization for quasilocal A's and convince oneself that by using the spectral gap there exists quasilocal A which applied once to the vacuum create a one particle state and whose hermitean adjoint annihilates the vacuum. The next step is the convergence:

$$\lim_{R\to\infty} <0|A\ I^0(f_d,\ f_R)\ B|0> = \lim_{R\to\infty} <0|A\ \left|I^0(f_d,\ f_R),\ B\right||0> \tag{41}$$

i.e. the existence of $\lim_{R\to\infty} I^0(f_d,f_R)$ between quasilocal states. In order to exponentiate this operator one has to enter the technically rather complicated discussion of essencial selfadjoint - ness on the dense domain of quasilocal states(32). For internal symmetries which do not change the localization properties of states, Swieca used the fact that $A|0>$ is an analytic vector for Q which leads to the convergence

$$U(\lambda)\ A|0> = \sum \frac{(i\ Q)^n}{n!}\ A|0> \tag{42}$$

The discussion without the spectral gap (i.e. QED) and the construction of exponentials for space-time symmetries is more involved and model-dependent. An especially interesting case will be discussed later in connection with global conformal symmetry. Then the resulting global representations turn out to be representations of the covering group with operator phases (i.e. reducible representation) for the center.

After the spontaneously broken symmetry situation was reasonably well understood in the relativistic case, Swieca[33] studied this problem for nonrelativistic many body problem. For this purpose it is illustrative to consider the Fourier-transform

$$L(\vec{p},p_0) = \int \langle 0| \left[j^0(\vec{x},x_0)A \right] |0\rangle \, e^{-i\vec{p}x + ip_0x_0} d^4x$$

By enterily formal manipulations (dropping boundary terms after using the conservation law) one obtains

$$\lim_{\vec{p}\to 0} \quad p_0 \, L(\vec{p},p_0) = 0 \tag{43}$$

and hence

$$L(0,p_0) = \lambda\delta(p_0) \tag{44}$$

This zero energy excitation is the $\vec{p} = 0$ part of an excitation branch only if L can be written as $g(\vec{p},p_0 - E(p))$ (or a sum of such function with different dispersions) where g is smooth in the first variable. The use of the spectral representation (33) shows that with $E(p) = \pm|\vec{p}|$ this is the case in relativistic causal models. Smoothness properties in p-space are related to fall-off properties in x-space. Swieca showed[34] that with

$$\lim_{\vec{x}\to\infty} \vec{x}^2 \langle 0| \left[U(\vec{x})P_1 \; U^+(\vec{x}),P_2 \right] |0\rangle = 0 \tag{45}$$

in particular for: $U(\vec{x})P_1\ U^+(\vec{x}) \to j^\mu(x^0,\vec{x})$ where P_i are quasi-local polynomials, the above formal considerations leading to a continuous g can be legitimized.

So the relevant question in connection with nonrelativistic theories is : what property of the interaction say for

$$H = \int \frac{\vec{\nabla}\psi\ \vec{\nabla}\psi}{2m}\, d^3x + \int \psi^+(x)\psi^+(y)V(\vec{x}-\vec{y})\psi(\vec{x})\psi(\vec{y})\ d^3x\ d^3y - \mu N \tag{46}$$

μ = chemical potential, N = particle number operator.
will lead to (45)

Swieca showed that the potential V has to decrease at infinity faster then Coulomb. It is well known[35] that for Coulombic ranged potentials the "would be"Nambu-Goldstone exitations may be transmuted into plasmonic excitations with a finite energy gap above the ground state.

In any many body system with a finite density,Galilei invariance is always spontaneously broken; this is a consequence of the velocity term in:

$$<\vec{j}> \longrightarrow <\vec{j}> + \vec{v}<\rho>$$

So there are always phonon like exitations. Using the techniques of sum rules Swieca showed the following theorem[33,34]

<u>Theorem :</u> For $\frac{1}{r^{1+\varepsilon}}\ V(r) \xrightarrow[\infty]{} 0$

one obtains for the spectral density

$$d\nu_p(\omega) = d\mu_p(\omega) + d\mu_{-p}(\omega)$$

with $$\langle\Omega|\rho(\vec{x},0)\ \rho\ (0,0)|\Omega\rangle = \int_0^\infty e^{i\vec{p}x} d\mu_p(\omega)\ d^3p$$

for $\vec{p}\to 0$ a concentration of weight at the origin:

$$\lim_{\vec{p}\to 0} \frac{\int_{a^2}^{\infty} d\nu_p(\omega)}{\int_0^{a^2} d\nu_p(\omega)} = 0\ ,\ a^2>0 \text{ arbitrary}$$

In order to alledge that these excitations have a quasiparticle nature one needs further dynamical information.

Thus in contradistinction to the relativistic case short range many body interactions <u>always</u> imply the existence of Nambu-Goldstone excitations,those of broken Galilei-invariance. To obtain additional information on the zero energy excitation spectrum from other spontaneously broken symmetries is a delicate and certainly very model-dependent matter.

What does happen to the charges in a relativistic theory with long range interactions? The only known relativistic models in this category are <u>gauge theories</u>. It had been known for some time that there are two types of abelian gauge theories with entirely different physical behavior . In conventional gauge theories as QED the identically conserved renormalized current j_ν

$$\partial^\mu F_{\mu\nu} = j_\nu \tag{47}$$

leads to a nontrivial charge formally given by (12). Using a physical description of the theory in which no unphysical states appear (example: The Coulomb gauge) one immediatly realizes that a physical charge raising operator cannot be local with respect to the electric field strength

$$\left| E(\vec{x}),\ \psi^{phys}(\vec{y}) \right| \neq 0 \qquad (x-y)^2 < 0 \tag{48}$$

In fact the Gauss law requires $\frac{1}{r^2}$ fall-off of this commutator. In the usual covariant gauge formalism (e.i Gupta-Bleuler) the locality is artificially obtained at the expense of ghost states i.e. the formally local operator $\psi(x)$ applied to the vacuum leads out of the physical Hilbert space. In QED the physical electron states carry a charge

$$<p\ |Q\ |p'> \ = <p\ |p'> \ \ G(0)$$

Here $G(p-p')^2)$ is the physical form-factor
For a scalar particle (for simplicity of illustration):

$$<p\ |j^{\mu}(0)\ |p'> = (p+p')_{\mu}\ G(t) \tag{49}$$

In 1964 Higgs[36] proposed a completely different abelian gauge model which is formally obtained from scalar QED by allowing the scalar field to develop a nonvanishing expectation value via the nontrivial minima of a potential (15). This model has a physical spectrum of finite mass particle (i.e. the photon turns into a relativistic plasmon) and the charge of all physical particles is zero as à result of the vanishing of the zero transfer formfactor. The formal mathematical aspects of this model, including its renormalization theory, are well known. Because of the formal analogies with the Nambu-Golstone models, the Higgs model has been often refered to as spontaneously broken gauge model.

In 1976 Swieca[37] proved a general structural theorem relating the mass spectrum with charge sectors in theories with identically conserved U(1) currents (47)

Consider the form-factor of $F^{\mu\nu}$:

$$<p|\ F^{\mu\nu}\ |p'> = \left[(p-p')^{\mu}\ (p+p')^{\nu} - (p+p')^{\mu}\ (p-p')^{\nu}\right]\ F(t) \tag{50}$$

From (47) we obtain

$$F(t) = i\ \frac{G(t)}{t} \tag{51}$$

If the states carry a non trivial charge, we have $G(0)\neq 0$, and the pole in the photon-vertex may be taken as an in dication of zero mass photon state. However the dispersion theo retical formalism linking poles in on-shell quantities with phy sical particles is only valid if the particle $|p>$ pocesses a lo cal interpolating field, which, as we have already stated, is not the case in QED. So one has to find a method avoiding any prejudice suggested by dispersion theory. For the purpose Swieca studied the commutator

$$C^{i}(\vec{x}) = <\vec{p}=0|\ \left[F^{oi}(\vec{x}),\ j^{i}(g)\right]\ |\vec{p}=0> \tag{52}$$

If one wants proper states, one should imagine the $|\vec{p}=0>$ as being normalized packets which are narrowly centered a round $\vec{p}=0$. In a asymptotically complete theory <u>without zero mass states</u> there is necessarily a mass gap between the one particle hyperboloid and the continuum in the"would be"charge sector.With such a gap one easily finds a fast decreasing smearing function g in x-space with the following p-space properties:

$$\tilde{g}(p) = 0 \qquad |p_0|>\delta, \quad \delta < \text{mass gap}$$

$$\tilde{g}(p) = \tilde{g}(-p)$$

$$\tilde{g}(0) = 1$$

$$j^i(g) = \int j^i(y)\, g(y)\, d^4y$$

Local commutatives now yields

$$|C^i(\vec{x})| < \frac{A_k}{|\vec{x}|^k}, \qquad \text{any } k \tag{54}$$

This is now confronted with the direct calculation (only one-particle intermediate states contribute in a theory with mass gap)

$$C(\vec{x}) = 4mi \int d^3p \, \frac{e^{i\vec{p}\vec{x}}}{\sqrt{\vec{p}^2+m}} \; \frac{G\;(t)}{t} \; \tilde{g}(\vec{p}, \;\; \sqrt{\vec{p}^2+ m^2}-m)p$$

$$\lim_{\vec{x} \to \infty} C^i(x) = i \;\; G^2(0) \;\; \frac{2\pi^{n/2}\Gamma(n/2)}{|\vec{x}|^{n+2}} \;(n\; x^{i2} - |\vec{x}|^2) \tag{55}$$

$$n+1 = \text{dim space time}$$

Hence for QFT in more than two dimensions the compatibility demands

$$G(0) = 0$$

i.e. charge neutrality. The charge of an identically conserved current is therefore screened, only if there are true photons do there exist nontrivial charge sectors.

Swieca noted that, similar to the Nambu-Goldstone situation, the two-dimensional situation is exceptional since any conserved current may be written in the Maxwellian form with

$$F_{\mu\nu} = \varepsilon_{\mu\nu}\, \phi \tag{56}$$

There are many models with conserved currents and mass gaps e.g.[38]

$$\bar{\psi}\gamma_\mu \psi = \frac{1}{\beta}\, \varepsilon_{\mu\nu}\, \partial^\nu \phi \tag{57}$$

ϕ = Quantum Sine-Gordon field.

It is interesting to understand the screening properties of the abelian Higgs model in more detail. Perturbatively , the two fundamental particles of the model are the massive vector meson and the Higgs meson. Their gauge invariant local interpolating fields are:

$$F_{\mu\nu} \quad , \quad \phi^+\phi \tag{58}$$

the composite field developing a nonvanishing expectation value. The formal language of broken gauge symmetry is physically somewhat misleading. In contrast to the Nambu-Goldstone situation when the symmetry is broken:

$$\int j_0(x)\, d^3x \quad \mathcal{H}_{phys} = \infty \tag{59a}$$

as a result of long range properties of states (even, if one handles the integral appropriately!), in the Higgs model we simply obtain

$$\int j_0(x)\,d^3x \quad H_{phys} = 0 \qquad (59b)$$

The resulting picture is in complete harmony with the"first law" of gauge theories:
"Gauge symmetries" of the second kind can not be broken because they do not constitute physical symmetries but rather a mathematical formalism by which the physical content is separated from the spurious properties of the mathematical description.

The formulation of the dynamical laws of gauge models solely in terms of physical (local) observables is presumably a very difficult task and anyhow has never been achieved in mathematically managable form.

A simple derivation of this almost philosophical point as a consequence of the mathematical consistency has been given by Elitzur and Lüscher(39) in the context of lattice gauge theories. This view point was known for a long time(40) to most physicists with a background in General QFT. I remember discussions with André which we had more than 10 years ago. It is interesting to recall that one of the inventors of the minimal model of eletro-weak interactions at a High-Energy Conference(41) called it a "moot point". Recently 't Hooft(42) made the screening aspects of the Salam-Weinberg model more explicite by exhibiting interpolating SU(2) neutral composite fields for all the physical particles appearing in every order of renormalized perturbation theory:

physical Higgs particles: $\phi^+\phi$, $\varepsilon_{ij}\phi_i\phi_j$, $\varepsilon_{ij}\phi_i^+\phi_j^+$

physical vector mesons: $\varepsilon_{ij}\,\phi_i(D_\mu\phi)_j$, $\varepsilon_{ij}\,\phi_i^+(D_\mu\phi)_j^+$

lin comb. of $(\phi^+ D_\mu\phi, B_\mu)$

photon: lin comb. of $(\phi^+ D_\mu\phi, B_\mu)$

ν phys: $\phi^+\psi_L$

e_{phys}: $\varepsilon_{ij}\,\Phi_i\,\Psi_{Lj}$, Ψ_R

Here ψ_l and ψ_R are the left handed doublet resp. the right handed singlet and B_μ is the gauge potential of the U(1) factor in the SU(2)xU(1) Salam-Weimberg model.

Does this picture hold if simple gauge groups are "spontaneously broken"? As an example consider the Giorgi-Glashow O(3) model; it is believed that the physical particles (without fermions) consist of the Higgs particle, a charged massive W-meson and the photon. The previous construction of local interpolating gauge invariant polynomials which perturbativly couple the vacuum with the corresponding one-particle states only works for the Higgs particle and the photon: $\phi^+\phi$ and $F_{\mu\nu}{}^a\,\phi^a$. The relevant question is: can one generate ordinary electromagnetism with charge sector for the W's from such a "spontaneous gauge symmetry breaking" or are the W's dynamically screened or confined? In more technical terminology: does the Bloch-Nordsiek exponentiation of infrared-singularities occur, thus leading to physical cross section with a finite resolution for the photon cloud ? Recent results[43] concerning the occurance of non-Bloch-Nordsiek terms (non-leading infraved singularities) as a result of the nonabelian nature of the gauge group may be relevant for the identification of electromagnetism from simple gauge groups. The question whether the language of "spontaneously broken gauge symmetries" is a consistent, albeit somewhat formal terminology hinges very much on the outcome of such investigations. Up to now, ignoring some incorrect discussion of QCD_2[44], this terminology has not led to misleading conclusions. The theorem of Swieca is applicable to the O(3) model and other gauge models with simple nonabelian gauge groups with monopoles; not to the problem of colour but rather to infrared properties of the monopole sectors. With

$$k^{\mu} = \partial_{\nu} \varepsilon^{\mu\nu\varkappa\lambda} F_{\varkappa\lambda}{}^{a} \phi^{a} \tag{60}$$

we have a gauge invariant identically conserved current whose non-trivial charge requires the presence of "photons".

It should be clear that Swieca's theorem does not exclude the occurance of sectors in massive theories whose conserved current is not identically conserved. For example in QED with a massive photon put "by hand", there exists, in addition to the identically conserved Maxwellian current another conserved (but not identically) U(1) current giving rise to sectors. We speculated before that nonabelian simple gauge group models with an incomplete Higgs mechanism may have certain aspects in common with QCD type models. Hence it is interesting to know whether color neutrality of physical states is a general feature of nonabelian gauge theories. This problem of "Kinematical color neutrality" was discussed occasionally among Swieca and collaborators. The consensus was that such a property pervades all non-abelian gauge theories, "broken" or "unbroken". The argument is as follows .

Consider a lattice gauge theory (for convenience) in the time like gauge and the Hamiltonian formulation[45].

$$U(\text{time link}) = 1 \text{ , i.e. formally } A_0^{\,a} = 0 \qquad (61)$$

In such a formulation one can introduce field strength $\vec{E}^a$ which still transform under spatial gauge transformations:

$$U_{op}(\Lambda)\ \vec{E}^a_{(\vec{x})}\ U^+_{op}(\Lambda) = \left\{\Lambda^{-1}_{\vec{x}}\right\}_{ab}\ \vec{E}^b \qquad (62)$$

On the other hand any physical (= gauge invariant) state on a lattice may be obtained by performing "gauge averaging" starting from arbitrary states.

$$|\psi phys > = \int U_{op}(\Lambda)\ |\psi>\ \prod_x d\Lambda_x \qquad (63)$$

$d\Lambda_x$ = normalized Haar measure

The volume of the gauge group on a finite lattice is finite and hence this averaging is well defined. Using the invariance of $|\psi phys>$ under $U_{op}(\Lambda)$ and taking the expectation value of (62) between physical states one obtains a consistency condition.

$$<\psi phys\ |\vec{E}\ |\psi\ phys> = 0 \qquad (64)$$

The infinitesimal generator of $U(\Lambda)$ is the lattice version of the Gauss operator "div$\vec{E}^a - \rho^a$ (ρ^a contains the density coming from covariant derivations) and the gauge invariance of $|\psi phys>$

is simply the validity of the Gauss law between physical states. In the temporal gauge we obtain therefore:

$$<\psi phys.|Q^{a}|\psi phys.> = \oint d\vec{S} <\psi phys.|\vec{E}^{a}(x)|\psi phys.> = 0 \tag{65}$$

It is technically difficult to generalize this construction of physical states to the continuum. Such an idea cannot work if one includes the vector-potentials in the list of operators; because of its additive piece in the gauge transformation it can never have finite matrix elements in $H_{phys.}$ This is the reason why even in abelian theories one has to resort to a more sophisticated construction viz. Gupta-Bleuler , if one wants to incorporate vector-potentials. For objects which only rotate under gauge transformations this most visible obstacle is not there. I do not know whether the Faddeev-Popov framework or other constructions allow to introduce finite matrix elements $\vec{E}$'s in a suitably defined physical Hilbert space.

Such a "kinematical colour neutrality" does not resolve the problem of "screening versus confinement". In the Salam - Weinberg model this neutrality is achieved by screening. In the QCD the mechanisms is believed to be confinement. This problem of screening and confinement was the prime motive for carrying out rather detailed investigations in two-dimensional gauge models . We will return to this in the next section.

There are many more structural properties of QFT which Swieca investigated.

Together with R.Haag(46) he tackled the very difficult problem of asymptotic completeness. On intuitive reasoning it was expected that a certain property corresponding to the fact that a finite volume of (classical) phase space contains a finite number of quantum states, appropriately formulated in QFT and there called the "compactness property", should play an important role for asymptotic completeness. Indeed certain (non-Lagrangian) models of Wightman fields which fulfilled all "axioms" except the compactness property were shown to be asymptotically incomplete. Many years later rather trivial applications of this compactness were made in two body nonrelativistic potential scattering(47). A "geometric scattering method"(48) based on the Haag-Swieca compactness property becomes popular. However even in higher body potential scattering the problem of asymptotical completeness based on the geometry methods is physically subtle and mathematically complicated.

Another interesting structural problem arose in connection with the so called "short distance algebra" of Wilson and Kadanoff(49). Within the context of renormalized perturbation theory the definition of renormalized composite operators and their short distance properties was investigated most thoroughly notably by Zimmermann(8) and Lowenstein(50). The result is a Wilson-Kadanoff short distance algebra in the following sense:

$$C_i(x)\ C_k(y) = \sum_{1}^{N} f_{ik}{}^{\ell}(x-y)\ C_\ell(y) + R_k(x,y) \qquad (66)$$

Here C_i is a complete set of dynamically independent (i.e. The ideal defined by the equations of motion is divided out) composite fields constructed from products of basis fields (which are included in the denumerable list of C_i) and derivatives. The series of the right hand side is asymptotically converging: The remainder term R_N vanishes faster than any preassigned power.

$$R_N(x,y) = O(|x_\mu - y_\mu|^{n(N)}) \qquad (67)$$

if one increases N correspondingly. For simplicity of notation we have absorbed all internal and Lorentz indices into i,k,l.

In a scale invariant QFT there exists a tight relation between the operator scale dimension of the C_i's and the singularities and the directional dependence of the coefficient functions. In fact by making assumptions on the transformation properties and the number of "relevant" operator with dim ≤ 2 for a short distance algebra in two dimensions Kadanoff( ) proposed a derivation of the critical indices of the Ising model (abandoning the lattice by passing to the scale invariant limit of the model). These ideas of using properties of scale invariant limits in order to obtain dynamical informations are the basis of the closely related "conformal bootstrap" program of Migdal(51) and Polyakov(52). Swieca started to get interested in conformal invariant QFT around 1972. By that time the causality aspects of global conformal transformations were already understood(53). However apart from some trivial cases, the form of the finite conformal substitution law, which in principle follows from the infinitesimal relation, was not known. In two papers(54) of Swieca in collaboration with others authors it was demonstrated that in local QFT one obtains representations of the (infinite sheeted covering group $\widetilde{SO}(D,2)$ of the conformal group SO(D,2) (D=dim. space-time). For irreducible representations the local field A naturally decomposes into non-local components:

$$A(x) = \hat{\int_0} d\xi \; A^{\xi}(x) \qquad (68)$$

(the ξ-integral being a sum in all explicitly studied cases), such that the conformal transformation law for each irreducible component is (we restrict our attention to proper conformal transformation corresponding to the parameter b_μ).

$$U(b)\ A^{\xi}(x)\ U^{+}(b) = \frac{1}{\sigma_{+}(b,x)^{\dim A-\xi}\ \sigma_{-}(bx)^{\xi}}\ A^{\xi}(x_T) \tag{69}$$

with $x_T = \frac{x-bx^2}{\sigma(b,x)}$, $\sigma(b,x) = 1-2bx + b^2x^2$

and $(\sigma\pm)^{\lambda}$ being the analytic continuation of the corresponding euclidean expressions with the $\pm i\varepsilon$ Wightman prescriptions

The ξ spectrum which also appears in the center of the conformal group law:

$$Z\ A^{\xi}(x)\ Z^{+} = \exp-i\pi(\dim A - 2\xi)\ A^{\xi}(x) \tag{70}$$

is intimately related to the dimensional spectrum of the theory . For free fields the ξ-decomposition of A is the same as the decomposition into creation and annihilation parts. The fact that the integration of infinitesimal transformation properties of QFT may lead to ray representations of the covering group is interesting in itself. Without this mechanism it would appear as a miracle that for example quantum solitons in the O(N) Gross-Neven model transform as iso-spinors(55). In a subsequent publication Swieca(56) and collaborators investigated the validity of global conformal operator expansions. On the vacuum they have the form:

$$A^{\xi}(x_1)\ B^{0}(x_2)|0\rangle = \sum_{|N|}\int K_{[N]}^{\xi,0}(x_1-x_3,x_1-x_3)\,C^{0}_{[N]}(x_3)dx_3|0\rangle \tag{71}$$

Here the kernels K are some kind of globally conformal invariant vertex function. On the vacuum only the $\xi = 0$ compo - nent contributes, on other states (for example those generated by the application of local fields) also other ξ-components participate. These expansions for free zero mass fields and the Thirring model turn out to be convergent whereas the local Wilson-Kadanoff expansions are (even for free fields!) only asymptoti - cally convergent.

Hence we believe that such global expansions exist without convergence problem. For the global expansion on the vacuum it is fairly easy to give explicit formulas for K(56), this turning into a more difficult task away from the vacuum(56). These global conformal operator expansion have their euclidean counterpart in the euclidean conformal bootstrap program in the form developed by Mack(57).

The massless Thirring model furnishes a nontrivial solution of this program in two dimensions. For every real spin (the two dimensional L-group is abelian) and sufficiently posi - tive dimension (depending on the spin) the Thirring model in the general form as discussed by Klaiber(58) solves the bootstrap equations. We thought (at the time when we worked on these problems) that this is the only solution. However, recently it became clear to us that there are many more conformally covariant solutions. I will return to this point towards the end of the second section in connection with the euclidean functional integral construction for Kinks.

In the days of the conformal bootstrap program we were interested to understand whether such converging global operator expansions of the form (71) may hold more generally in any Wightman theory. They certainly are valid for massive free field and their composites . Establishing such expansions would be surely of theoretical as well as practical use. Theoretically it is of great interest to resolve a general QFT in terms of 3-point function of the composite fields. Practically they may serve to explore those regions of momentum space which remained unaccessible by using Callan-Symanzik[60] techniques together with Wilson-Kadanoff short distance expansion (i.e. infrared factorization regions in QCD).

We did not investigate these problems on a profound level, because after 1974 there emerged other very interesting problems in QFT related to the vacuum and particle structure.

There are three papers of Swieca and collaborators falling into this category of structural investigations which are concerned with stability and causality problems. In two of those publications these problems are investigated in field theories with time dependent and stationary external potentials. This work is an extension of that of Schiff, Snyder and Weinberg[61] and of Velo and Zwanziger[62]. Some of the mathematical methods were later used by Fulling[63] in his treatment of the Hawking effect.

The third paper on causality is motivated by preceeding work of Lee and Wick[6]. These authors introduced complex poles in an S-matrix formulation. Swieca and Marques[65] studied these problems in a more field theoretical setting using the Yang-Feldman equation. Although in their approach there was no problem with unitarity and Lorentz invariance, they showed that the basic microscopic causalities of the propagation are enhanced through the contribution of virtual states and generally lead to an unacceptable deviation from macro-causality.

MODEL STUDIES AS A LABORATORY FOR NEW IDEAS ON DYNAMICAL PROPERTIES OF QFT.

At the beginning of the 70's a renewed interest in the age-old difficult problem of QFT: the connection of particles and fields began to develop. Here the QFT of the 50's and 60's had little to offer; pertubative Lagrangian QFT only accounted for those particles which had a sufficiently simple relation to the Lagrangean fields. On the other hand the approach of QFT based on general physics postulates (sometimes referred to as Axiomatic QFT) was too inespecific. In the LSZ- and Wightman schemes particles played essentially (apart from perhaps Nambu-Goldstone bosons) a phenomenological role; together with the causality properties and commutation properties of charges or currents one was able to obtain Dispersion Relations, Sum Rules and All That. "Constructive QFT", closely related to Axiomatic QFT, was unable to produce new intuition for "peculiar"(from the conventional viewpoint) dynamical proper-ies of the physical state space, e.g. θ-vacua, kinks, solitons order-disorder duality etc. This is not to say that those new structures could not, with modest ease, be incorporated in QFT. One of the first models investigated with this specific purpose in mind was QED_2. This model was already(66) introduced by Schwinger in 1962 as an illustration of his speculation that U(1) Gauge theories can exist in another phase than the QED phase and that the massless of the photon is not an automatic consequence of this principle of gauge invariance of the 2nd kind.

In modern functional language Schwinger's observation can be paraphrased in the following way. Consider[67] the functional determinant of the two dimentional euclidean Dirac operator:

$$\frac{\text{" det } i \not{D}\text{"}}{\det i \not{\partial}} = e^{-\Gamma}, \quad \not{D} = i\gamma_\mu(\partial^\mu - ieA^\mu) \tag{1}$$

How can one define this formal object? In order to obtain a definition for sufficiently general A_μ's one has to go beyond the Fredholm method.

There are two known ways:

1) Use the conformal invariance of the massless Dirac equation in order to pass the compactfied euclidean space: $R^2 \to R^2_c = S^2$ Verify that all "classical" quantities (e.i. Green funtion G_c) of the compactified eigenvalue equation (R=radius of S^2)

$$i\not{D}\Psi_k = \lambda_k \frac{1}{R^2+x^2}\Psi_k, \quad (\psi,\phi) = \int \psi^+(x)\phi(x)\frac{d^2x}{R^2+x^2} \tag{2}$$

$$G_c = \sum_k{}' \frac{\psi_k(x)\psi_k^+(y)}{\lambda_k} \quad (' = \text{omission of } \lambda_k=0) \tag{3}$$

are the same (apart from conformal factors) as those of the R^2 theory. Note: this holds only in the absence of zero modes $\lambda_k=0$ for which the precise condition is:

$$\nu = \frac{e}{2\pi}\int F_{12}d^2x = e\tilde{F}_{12}(0) = 0$$

Non-classical quantities as the logarithm of the determinant Γ are defined as:

$$\Gamma = \xi'(0,A_\mu) - \xi'(0,0) + (\xi(0,A_\mu) - \xi(0,0))\ln\,\mu \tag{5}$$

with $$\xi(s,A_\mu) = \sum_k{}' \frac{1}{\lambda_k^{\,s}} \tag{6}$$

The ξ functions has enough meromorphic properties in order to allow the analytic continuation necessary for de - fining Γ. This definition is reasonable because

(a) it reduces to the usual one for finite determinants in which case an analytic continuation is unnecessary

(b) In the absence of zero modes(5) obeys the Schwinger variational calculus e.g.

$$j_\mu^{\,ind}(x) = \frac{\delta\Gamma}{\delta A_\mu(x)}$$

when the left hand side is the (independently defined) induced current.

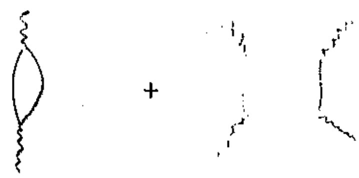
The same formula (5) is obtained if one uses Pauli-Villas regularization(67).

The result of an explicit calculation

$$\Gamma = \frac{e^2}{2\pi}\int A_\mu(z)\,A^\mu(z) + \text{contr. from zero modes}$$

could have been anticipated (apart from the nonperturbation zero mode contribution) on the basis that in the Feymman-representative of Γ

Γ :



+.....

only the first term survives as a result of the vanishing of the symmetric part of the traces containing more than two two-dimensional γ-matrices.

For configuration with $\nu=n$ (nontrivial winding) the first integral in (8) is defined by a "finite part" prescription using a split :

$$A_\mu = \hat{A}_\mu + \frac{n}{e} \varepsilon_{\mu\nu} \partial^\nu \ln x^2 \qquad (9)$$

Now we briefly mention the second method which is in spirit closer to the thermodynamic limit method of Statistical Mechanics.

2) Study the Dirac equation

$$i \not{D}\psi_k = \lambda_k \psi_k \qquad (10)$$

as a boundary value problem and compute the determinant as in(5).

This approach is <u>very subtle</u> since the type of boundary condition consistent with γ_5 and C-invariance is necessarily nonlocal(68) . The form of the determinant is similar to (8) but there is yet another contribution from the boundary.

This construction can now be used in order to compute euclidean correlation functions. The mass term in (8) will lead to a "plasmon", a model illustration of the "Schwinger mechanism". Assuming for the moment that there are no zero mass contribution for Γ and G one obtains the Mathews-Salam(69) rules for the integration over fermions with the help of the Grassmann rules.

$$\langle\psi(x_1)\ldots\psi(x_n)\ \psi^+(y_1)\ldots\ \psi^+(y_n)\ A_{\mu_1}(z_1)\ldots A_{\mu_m}(z_m)\rangle$$

$$= \frac{1}{Z}\int\left[dA_\mu\right]\ e^{-S_0(A)-m^2\int A_\mu^2}\ \prod_1^m A_{\mu_i}(z_i)\ \sum G(x_1,y_1,A_\mu)\ldots\ldots \tag{11}$$

The term in the sum represents the various Wick contractions be - tween ψ's and ψ^+'s using(67):

$$\underbrace{\psi(x)\ \psi^+}(y) = G(x,y) = G_0(x-y)\ \exp\ \mathrm{lin}\ (A_\mu)$$

This contribution may also be written in the compact form:

$$\langle\psi_0(x_1)\ldots\psi_0{}^+(y_1)\ldots\rangle\ \exp.\ \mathrm{lin.}\ (A_\mu) \tag{12}$$

where the first expectation value is that of free massless eucli- dean free spinor fields and the second factor contains the linear exponential external A_μ- dependence whichs will be called the in - duced part Γ_{ind}.

Hence the remaining integration is of the form

$$\frac{1}{Z}\int\left[dA\mu\right]\ \exp\ -\ S_0(A_\mu)\ -\ m^2\int A_\mu{}^2\ -\ \Gamma_{ind}(x_1\ldots y_1\ldots) \tag{13}$$

The easiest way to perform such Gaussian integrals is to write

$$A_\mu = A_\mu{}^{cl} + A_\mu{}^{fl} \tag{14}$$

where $A_\mu{}^{cl}$ is the (classical) minimum of the total induced action. Linear terms in $A_\mu{}^{cl}$ do not contribute as a result of the classi- cal equation for $A_\mu{}^{cl}$ and the quadratic terms are source-independent. Their contribution to the functional integral is therefore absorbed in the factor Z. In (13) one has the option of selecting different gauges by adding the appropriate gauge breaking terms. In the form (13) one obtains the expectation values in Schwinger's (transversal) gauge as (after continuation to Minkowski space):

$$< \psi(x_1)\ldots\psi(x_n)\overline{\psi}(y_1)\ldots\psi(y_n) = e^{iF(x,y)}W_o(x_1\ldots x_n,y_1\ldots y_n)$$

$$F(x,y)=\pi\Big\{\sum_{j<k}\Big[\gamma^5_{x_i}\gamma^5_{x_i}(i\Delta^{(+)}(x_j-x_k)-iD^{(+)}(x_j-x_k) + \gamma^5_{y_i}\gamma^5_{y_k}(i\Delta^{(+)}(y_j-y_k)-iD^{(+)}(y_j-y_k))\Big] + \sum_{j,k}\gamma^5_{x_i}\gamma^5_{y_k}(i\Delta^{(+)}(x_j-y_k)-iD^{(+)}(x_j-y_k)\Big\} \tag{15}$$

where $$i\Delta^{(+)}(x) = \frac{1}{2\pi}\int d^2p\, e^{-ipx}\,\delta(p^2-m^2)\theta(p_0) \tag{16a}$$

and $$iD^{(+)}(x) = \frac{1}{2\pi}\int d^2p\,\delta(p^2)\theta(p_0)\left[e^{-ipx}-\theta(\kappa-p_0)\right] \tag{16b}$$

is the infrared regularized zero mass two point function.

The Lowenstein(11)-Swieca analysis starts with these correlation function of Schwinger. The reason for being more careful than Schwinger in their derivation, in particular emphasazing the subtleties of zero modes, will only become clear later on.

Lowenstein and Swieca were able to unravel the structure of the underlying physical state space by a very ingenuous trick. This problem cannot be settled by referring to Swieca structural theorem of 1976 because of the two-dimensionality of the problem. Naive intuition would lead one to expect some kind of "screened" fermions. However the results of the LS investi -

gations give a more radical picture: the physical content is described just in terms of a massive bose field Σ, the same Σ which appears in the transversal massive A_μ^{tr}. In a suitable operator gauge, the so called "$\sqrt{\pi}$ - gauge" which apart from a Klein transformation is a unitary gauge similar to the one in the Higgs model, this physical content becomes very transparent.

Explicity one obtains[11]:

$$A_\mu^{\sqrt{\pi}} = -\frac{\sqrt{\pi}}{e}\,\varepsilon_{\mu\nu}\partial^\nu\Sigma \quad , \quad (\Box - m^2)\Sigma = 0 \qquad m = \frac{e}{\sqrt{\pi}} \tag{17a}$$

$$\psi^{\sqrt{\pi}} = \sqrt{\frac{x}{2\pi}}\, e^{-\frac{1}{4}\pi i\gamma^5}\, e^{i\sqrt{\pi}\gamma^5\Sigma(x)}\,\sigma \quad , \quad \sigma = \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} \tag{17b}$$

χ=infrared parameter in (16b)

Here σ is a two component constant unitary operator which commutes with Σ. The presence of such a constant "spurious" operator indicates a violation of the cluster decomposition property[70] as a consequence of a vacuum degeneracy. In a description based on a unique vacuum, the operator σ will just be numerical phases:

$$\sigma_{1,2}\ |vac;\theta_1,\theta_2> = e^{i\theta_{1,2}}\ |vac;\theta_1\theta_2> \tag{18}$$

The operator gauge transformation (which involves in addition a Klein-factor, being responsible for the change of statistics between the two irreducible Lorentz representations ψ_1 and ψ_2) has converted the original spinorial "quark" field into a bosonic field. This mechanism is related to the subsequently discovered "bosonization" of Mandelstam[71], a point which will be explained later on. Lowenstein and Swieca emphasized the fact that gauge invariant quantities e.g

$$\psi(x) \quad \exp ie \int_x^y A^\mu \, d\xi_\mu \; \psi^+(y) \tag{19}$$

suitably renormalized, are the same quantities in the $\sqrt{\pi}$ gauge as in covariant gauges. In the $\sqrt{\pi}$ gauge there exists no conserved axial current. The gauge invariant axial current satisfies the two-dimensional anomaly[72] equation

$$\text{with} \quad j_\mu{}^5 = \varepsilon_{\mu\nu} j^\nu = \partial_\mu \Sigma \tag{20a}$$

$$\partial^\mu j_\mu{}^5 = \frac{e^2}{4\pi} \, \varepsilon_{\mu\nu} \, F^{\mu\nu} \tag{20b}$$

This is the manifestation of the Schwinger-Higgs mechanism i.e. a breakdown of the formal chiral U(1) invariance:

$$\langle\bar\psi\,\psi\rangle = C\cos\theta \ , \ \langle\bar\psi\gamma_5\psi\rangle = C.\sin\theta \tag{21}$$

and the conversion of the photon into a Schwinger-Higgs plasmon. The "vacuum angle" $\theta=\theta_1-\theta_2$ appearing in the vacuum expectation values (21) of the gauge invariant composite operator superficially causes a violation of CP invariance for $\theta\neq0$. However by using chirally rotated fields:

$$e^{-i\gamma^5\theta}\;\psi \tag{22}$$

for the description of the model, one realizes that a physical CP invariance continues to exist at least in the massless version (vanishing Lagrangian quark masses) of QED_2.

If one would use another gauge different from the $\sqrt{\pi}$ gauge, there would be unphysical "ghost" states of zero mass and with negative metric. These states formally support a non-gauge invariant but conserved axial current, e.g. in the Lorentz gauge:

$$j_\mu^{5,n.g.} = j_\mu^5 - \frac{m}{\sqrt{\pi}} A_\mu \neq 0 \qquad (23)$$

However the "ghoststone" states[73] which this current generates if applied to the vacuum, are void of any physical meaning.

A good physical way to understand the basic difference between the Nambu-Goldstone and the Schwinger-Higgs chiral link is to think of two ferromagnets, one with a local interaction and the other with an interaction of such a long range that mean field theory becomes exact[70]. A particular vacuum can be labeled by the direction of symmetry breaking. In the local case it is well known that by switching on a magnetic field in a finite region in a direction different from the vacuum direction one will turn the vacuum direction inside this region; the energy of the partially changed vacuum is only different from zero around the boundary of that region. For long range interaction the energy increases with a larger power of the volum This difference in the energy balance is responsible for the fact that a short range ferromagnetic responds to an external agent by an alignment whereas the long range ferromagnetic is inert to such agents. This picture was known to Swieca for a long time; he pointed out to me the relevance of Haag's work[70] Later Kogut and Susskind[74] introduced the very appropriate nomenclature of "vacuum seizing" for the chiral properties of a Schwinger-Higgs vacuum.

In order to obtain a better understanding of those dynamical features which are not an artifact of the soluble massless QED_2 Swieca began to familiarize himself around 1977 with the euclidean functional techniques since they constitute the only known systematic way to relate Lagrangian with correlaction function. By that time it was already understood that QCD has a Θ angle which enters through topological properties of the euclidean functional integral.

Here the basic observation was that there exists an ambiguity in the quantization of classical Lagrangians by Feymann-Wiener path integrals. For example in QCD the pseudo-scalar density $F\tilde{F}$ has a representation (we use the SU(N) matrix formalism)

$$F\tilde{F} := \mathrm{tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu} = \partial_\mu I^\mu \tag{24}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\varkappa\lambda} F^{\varkappa\lambda} \quad , I^\mu = 2\mathrm{tr}\,\varepsilon^{\mu\nu\varkappa\lambda}\left[A_\nu \partial_\varkappa A_\lambda + \frac{2}{3} A_\nu A_\varkappa A_\lambda\right]$$

Hence, this density, added to the classical Lagrangian

$$L_\theta = L_{cl} + \theta F\tilde{F} \tag{25}$$

will not change the classical Euler-Lagrange equation although it may lead to θ - dependent correlationfunction via euclidean functional integrals if configuration with $q = \frac{-1}{16\pi^2}\int F\tilde{F} \neq 0$ turn out to be relevant. In that case one would like to interprete the contribution of one "winding number" q

$$\int \left[dA_\mu\right]_q \; e^{-\int L_\theta d^4x} \tag{26}$$

as a tunelling amplitude between two vacua

$$< n'|n >_{n'-n=q} \; e^{i\theta q} \qquad (27)$$

The θ vacuum is then defined as

$$|\theta> = \sum_n |n>e^{i\theta n} \qquad (28)$$

which leads to an interpretation of the functional integral as

$$<\theta'|\theta> = \delta(\theta'-\theta) \int \left[dA_\mu\right] e^{-\int L_\theta d^4 x}$$

Historically the θ vacuum structure of nonabelian gauge theories was first exposed in the temporal gauge where there exists enough gauge freedom to interprete the A_μ 's with a fixed winding number q (instantons) as interpolating configuration between topologically inequivalent classical n-vacua. These arguments do not hold in other gauges (viz. the Coulomb gauge with strong boundary condition) and in theories without gauge fields i.e. the two dimensional nonlinear O(3) Sigma model.

Generalizing from the quantization ambiguity of a quantum mechanical particle on a circle (75) Swieca and Rothe found an intrinsic way of introducing θ vacua (76).

In Quantum Mechanics with a simply connected configuration space, the validity of J. von Neumann's uniqueness theorem assures that there is no quantization ambiguity. If one were to change the standard representation by writing:

$$p_i = -i \frac{\partial}{\partial q_i} + A_i(q) \tag{30}$$

with $\partial_j A_i - \partial_i A_j = 0$

then the gauge transformation ϕ in $A_i = \partial_i \phi$ leads precisely to the unitary equivalence with the standard realization. In a multiple connected space the formal application of such a transformation leads to a multiple-valued wave function.

$$\psi \longrightarrow \psi'(q) = e^{-i\phi(q)}\psi(q) \tag{31}$$

This happens for example for a particle on a circle where $A = \theta/2\pi$ leads to $\phi(q) = \frac{\theta}{2\pi} q$

In passing we mention that by slightly generalizing the formula (30), in order to include "nontrivial bundles" with transition functions, one demonstrate that the most general canonical system on non-simply connected spaces is equivalent to this "gauge" form.

In QFT this construction has an analogue. An important class of quantization ambiguities is obtained via the existence of a topological density. In its most general form a topological density $Q(x)$ is a (pseudo) scalar composite field whose integral on euclidean space does not change under an infinitesimal variation in the manifold of field configurations. A mathematically more convenient definition is the language of differential (d+1) form (d+1 dimension of space-time).

There are many physically relevant topological densities, in addition to the density (24) appearing as the axial anomaly of a Dirac equation there are the Pontragin, Euler and Hirzebruch densities [77] of euclidean General Relativity and the topological densities of the various Sigma-models [78].

Let $\{\phi\,(\vec{x})\,\}$ be the physical configuration space at one time e.g. in gauge theories the class of all gauge field configuration which are equivalent under (nonsingular) gauge transformations at one time. Then an "angular variable" may be defined via:

$$q\,|\phi| = 2\pi \int_{\phi=0}^{\phi(\vec{x})} dx\; Q(x) \tag{32}$$

where the integration is carried out along a path connecting the two field configuration with the time variable parametrizing the path. This variable does not depend on the details of the path: under a infinitesimal variation it does not change. For path which can be continously transformed into each other , q has the same value. In particular for a closed path (assuming fall-off properties at spatial infinity) which is transversed in a finite time, the integral is a multiple of 2π (Q has been appropriately normalized) The case of an infinite time interval will be considered as a limiting case. Without such boundary condition one may encounter situations in which $q/2\pi$ has a fractional "quantization".

In analogy with the quantum mechanical case, one finds the quantization ambiguity:

$$\pi(\vec{x}) = -i \frac{\delta}{\delta\phi(\vec{x})} + \frac{\theta}{2\pi} \frac{\delta q[\phi]}{\delta\phi(\vec{x})} \tag{33}$$

resulting from a Lagrangian

$$\mathbb{L}\left[\phi,\dot{\phi}\right] = L_0\left[\phi \;\dot{\phi}\right] + \frac{\theta}{2\pi}\frac{d}{dt}\; q[\phi] \tag{34}$$

with the corresponding action:

$$S = S_0 + \theta \int dx \; Q(x) \tag{35}$$

There is a certain similarity with the quantum mechanical Aharonov-Bohm effect; there the θ has the interpretation of a magnetic flux, here the θ measures a "magnetic hyper-flux" through a (topological) hole in configuration space.

Note that $q\;[\phi]$ depends on certain topological aspects of the history of the path, not just on $\phi(\vec{x})$. It is convenient to use a parametrization of configuration space say $\phi \longrightarrow A$ whose different q - values can be associated with different values of the A - field configurations. This process of "unwinding" the configuration space will then permit an interpretation of non-trivial topological path configurations (semiclassical instantons) as links between inequivalent classical A - vacua.

This picture, which is usually enforced by imposing a temporal gauge condition, will then energy in a completely intrinsic, gauge invariant fashion. An illustration of this was given by H. Rothe and Swieca for the standard formulation of the O(3) Sigma model (76).

From this interpretation of θ as a quantization ambiguity it should be expected that θ has quite different renormalization properties than ordinary Lagrangian parameters i.e. coupling constants. We will return to this point.

The problem of integer versus fractional winding numbers is dynamical. For pure nonabelian gauge configurations without the presence of matters Marino and Swieca gave convincing arguments (79) albeit not a proof, that the spectrum of winding numbers allowed by the finiteness of the action is integer. With matter i.e. for the induced action, their arguments break down, and, as we will see later on, one encounters examples of non-integer winding numbers.

It is quite instructive to understand QED_2 and its generalizations within the euclidean functional integration. We have set up the necessary formalism already at the beginning of this section; the only missing piece in the induced action is the topological contribution:

$$\frac{e\theta}{4\pi} \int \varepsilon_{\mu\nu} F_{\mu\nu} \, dx^2$$

This time, of course, we will not throw away the zero mode contribution in the determinant and in the Green Functions. The induced action has now the form (for a quantity carrying a well defined chirality):

$$S_{ind}[A_\mu] = S_0 + m^2 \int A_\mu^2 + \text{zero mode contr.} \qquad (36)$$

$$+ i \frac{e\theta}{4\pi} \int F_{\mu\nu} \varepsilon_{\mu\nu} + \Gamma'_{ind}(x_1 \ldots, y_1 \ldots)$$

where Γ'_{ind} is the contribution of the external field dependence of the zero modes and the modified Green's functions. This formula results from a straightforward applications of the Grassmann fermion integration rules by incorporating the effect of zero modes which brings about a derivation from the Mathews-Salam formula (11). A close examination [80], which will not be carried out here, reveals that the zero mode contribution in Γ_{ind} and of the determinant compensate each other, thus leaving a Gaussian integration over A_μ which, for the gauge invariant quantities, gives precisely the same result as the "reduced vacuum formalism" of Lowenstein and Swieca, e.g.

$$<\overline{\psi}(x)\ \psi(x)\ldots.>_\theta \quad = <\theta|\overline{\psi}(x)\ \psi(x)\ldots.|\theta>_{L.S.} \qquad (37)$$

This model therefore shows a Higgs-Schwinger mechanism of chiral symmetry breaking: the photon acquires a mass, as observed by Schwinger, and the vacuum expectation values of chiral symmetry carrying operators as (21), are different from zero as a result of the Atiyah-Singer-'t Hooft ' zero modes. However the model is too unrealistic in order to shine any light on the real " chiral U(1) problem" as encountered in QCD. A two-dimensional model, which in certain aspects is somewhat close to QCD (asymptotic freedom mass-transmutation, nontrivial topological gauge classes) has been proposed and studied by K. Rothe and Swieca [13]. Its Lagrangian is obtained by combining that of the Gross-Neveu 4 - fermion coupling with QED_2.

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ + \frac{e}{\sqrt{N}} A\!\!\!/)\psi + \frac{g^2}{N}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma^5\psi)^2\right]$$
$$-\frac{ie}{\sqrt{N}}\ \frac{\theta}{4\pi}\ \varepsilon_{\mu\nu}\ F_{\mu\nu} \qquad (38)$$

The fermions are taken to belong to the fundamental U(N) representation and the factors $\frac{1}{\sqrt{N}}$, $\frac{1}{N}$ have been introduced for later convenience. In contrast to QCD_4 the model is not SU(N) chirally invariant but only exhibits U(1) chiral symmetry. In fact the only 4 - fermion interaction in d=2 which is SU(N)-chirally invariant is the Thirring interaction which, unfortunately does not lead to a mass transmutation.

A careful investigation of the pure Gross-Neveu model carried out by Koberle, Kurak and Swieca has shown [81]:

1.) The Gross-Neveu fields split into a noninteracting "infraparticle" factor carrying the U(1) x U(1) charge and a massive SU(N) field $\hat{\psi}$ with exotic statistics:

$$\psi_f(x) = \exp i \frac{\pi}{N}(\gamma^5\phi(x) + \int_{x_1}^{\infty} d\xi^1\partial_0\phi)\ \hat{\psi}_f \tag{39a}$$

$$\overline{\psi}\gamma_\mu\psi = - \frac{N}{\pi}\varepsilon_{\mu\nu}\partial^\nu\phi$$

$$\hat{\psi}_f = \sqrt{\frac{\mu}{2\pi}}\ e^{1/4 i\pi\gamma^5} \exp i\sqrt{\frac{\pi}{2}} \sum_{i_D} \lambda_{ff}^{i_D}\left[\gamma^5\phi^{i_D}(x) + \int \xi\partial_0\phi^{i_D}(x_0\xi)\right]$$

$$\frac{1}{2}\ \psi\lambda^{i_D}\gamma_\mu\psi = \frac{1}{\sqrt{\pi}}\ \varepsilon_{\mu\nu}\partial^\nu\phi^{i_D} \tag{39b}$$

λ^{i_D} = diagonal λ - matrices

2.) The $\hat{\psi}_f$ - field interpolates a known [82] factorizing S-matrix. The resulting unusual particle statistics (which may be rewritten in terms of ordinary statistics) is helpful in order to understand the bound-state structure: the antiparticle is a bound state of N-1 particles thus examplifying the SU(N) (instead of U(N)) invariance.

The spontaneous mass generation in the Gross-Neveu model is, as expected, accompanied by the appearance of zero mass excitations. This is reminiscent of the Nambu -Goldstone mechanism. But as we have seen in our general discussion of section I, it does not lead to a spontaneous symmetry breaking. Zero mode excitations in two dimensions only arise from exponentials of zero mass fields and these infraparticle factors, far from destroying the U(1) x U(1) invariance, actually carry the corresponding selection rules

What happens now if the A_μ gets coupled to the U(1) part of the model? The "bosonization" formula (39) suggests that the Lagrangian has the form:

$$L = L_{U(1)} + L_{SU(N)} \qquad (40)$$

$$L_{U(1)} = \frac{1}{4} F^2_{\mu\nu} + \frac{1}{2} \partial_\mu\phi \partial_\mu\phi + \frac{ie}{\sqrt{\pi}} A_\mu \varepsilon_{\mu\nu}\partial_\nu\phi - \frac{ie}{\sqrt{N}} \frac{\theta}{4\pi} \varepsilon_{\mu\nu} F_{\mu\nu} \qquad (41a)$$

$$L_{SU(N)} = \frac{1}{2} \sum \partial_\mu\phi^{i_D} \partial_\mu\phi^{i_D} + \frac{g^2}{N} \left\{ \left(\sum_f \cos \sqrt{2\pi} \sum_{i_D} \lambda^{i_D}_{ff} \phi^{i_D} \right)^2 + \left(\sum_f \sin \sqrt{2\pi} \sum_{i_D} \lambda^{i_D}_{ff} \phi^{i_D} \right)^2 \right\} \qquad (41b)$$

As expected, the Higgs-Schwinger mechanism leads to a plasmon and the chirality will be broken. Consider as an example the one-point function of the chiral $\pm$ operator:

$$I_\pm = \frac{1}{N} \sum_f \overline{\Psi}_f \frac{1 \pm \gamma^2}{2} \psi_f \qquad (42)$$

The functional representation, after integration over the ϕ-field yields:

$$<\theta|I_{\pm}(x)|\theta> = <I_{\pm}(x)>^{GN} . \int [dA_\mu] e^{-Sind} \qquad (43)$$

$$Sind = \frac{1}{4}\int F_{\mu\nu}{}^2 d^2z + \frac{1}{2}m^2 \int A_\mu{}^2 d^2z \pm \frac{e}{\sqrt{N}} \int D(x-z)\varepsilon_{\mu\nu}F_{\mu\nu}(z)d^2z$$

$$+ i\frac{\theta}{4\pi}\frac{e}{\sqrt{N}}\int d^2z\, \varepsilon_{\mu\nu}F_{\mu\nu} \qquad (44)$$

The functional integral for the 1st Gross-Neveu factor involves nonpolynomial interaction terms in ϕ^{i_D} ; they can be expanded in $\frac{1}{N}$. For our purpose we only consider the 2nd U(1) factor. The "saturating" field configuration in terms of which the remainning integration can be explicity performed

$$A_\mu^{\left[\pm\frac{1}{N}\right]}(z) = \pm\frac{2\pi}{N}\frac{\sqrt{N}}{e}\varepsilon_{\mu\nu}\frac{\partial}{\partial z_\nu}(D(z) - \Delta(z,e^2/\pi)) \qquad (45)$$

carries <u>fractional</u> winding number $\frac{1}{N}$

$$\nu = \frac{1}{4\pi}\frac{e}{\sqrt{N}}\int \varepsilon_{\mu\nu}F_{\mu\nu} \doteq \pm\frac{1}{N} \qquad (46)$$

Using the well-known relation between the chirality transfer of operators (in our case $I_{\pm}$) and the winding number ν :

$$\Delta Q_5 = 2\nu N$$

we see that this fractional winding number is in perfect agreement with $\Delta Q_5 = \pm 2$ of the operators (42).

This result was obtained from "bosonization" whose deeper relation to the order - disorder concepts of statistical mechanics will be explained later on.

Our attempt [83] to understand this fractional winding with the use of the Dirac equation via the Atiyah - Singer - 't Hooft mechanism has failed. In that formalism the lowest non-vanishing composite expectation value is the flavor - determinant:

$$< \det_{f,f'} \bar{\psi}_f \frac{1+\gamma^5}{2} \psi_{f'} > \neq 0 \qquad (48)$$

thus indicating chiral breaking . This approach based on the compactification $R^2 \to R^2_c$, although not illegitimate, yields vacuum expectation values of $I_{\pm}$'s which violate the cluster properties. The expectation values in the irreducible θ - representation can then be determined using the operator formalism of Lowenstein-Swieca.

As a consequence of our failure to understand fractional winding numbers as the result of the Atiyah-Singer-'t Hooft mechanism on some Riemann surface as a covering space for (non canonical) fermions,[83] we became convinced that "bosonization" is essential for fractional winding.

There is another interesting message in (43): the $\frac{1}{N}$ expansion is only reasonable in those pieces of the correlation function which do not carry topology e.g. one obtains a misleading picture if one decomposes the second factor in (43) as a $\frac{1}{N}$ series expansion.

Now I would like to comment on the U(1) problem in QCD_4. Although in principle the Goldstone mechanism for the chiral U(1) part fails as a result of the axial anomaly and hence there exists no reason to expect[84] a ninth Nambu - Goldstone boson (say in case of $SU(3)_f$), it is another matter to really pinpoint the detailed dynamical mechanism yielding a "η - plasmon"[85] As the previous model the plasmon cannot be understood without the rather subtle infrared properties one expects in QCD_4 the dynamics, leading to a massive η to be inexorably linked with the Nambu - Goldstone infrared dynamics yielding massless chiral mesons. The importance of "bosonization" in the model case together with the observation that "bosonization" is part of a more fundamental order - disorder duality scheme nourishes the hope that a dual variable formalism incorporating spinor fields in addition to gauge fields should be the necessary ingredients for a QCD_4 Higgs - Schwinger mechanism including nonvanishing expectation values of $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$. Since we have nothing concrete to offer, on this point we content ourselves with some consistency considerations a la Crewther [85] which, as the reader will realize, deviate in content somewhat from those of Crewther.

The standard direction of SU(N) chiral symmetry breaking is:

$$\langle \psi^{+}_{L,j}\, \psi_{Ri} \rangle_\theta = C e^{-i\theta/N}\, \delta_{ij} \tag{49}$$

Consider first the ν dependence. The more general statement referring to the addition of an arbitrary SU(N) x SU(N) breaking direction in the Lagrangian, with a coupling constant that goes to zero, would yield an arbitrary SU(N) matrix V instead of δ_{ij} , which by going into an adjusted SU(N) x SU(N) frame, can always be chosen as (49). The direction in U(1) x U(1) however cannot be adjusted by an external agent it is rigid i.e. the θ vacuum is " seized ". The dependence of (49) on θ is the same as in the corresponding Rothe - Swieca model for the same reasons namely the validity of equ. (47). However in contrast to that model in which the fractional winding led to the enhanced θ - period $2\pi N$, the effective θ - period in QCD_4 suffers a reduction as a result of the chiral Z_n factor in the Nambu-Goldstone SU(N) chiral invariance:

in QCD_4 : Θ/N is counted mod $\frac{2\pi}{N}$ (50)

Now imagine that a Lagrangian quark mass term is added whose physical origin lies outside QCD. Because of the "vacuum seizure" such an outside term does not influence the U(1) chiral direction of the transmutated QCD mass. In other words, the relative angle between the external mass direction and the transmutated direction becomes a physically relevant quantity. Thus in a model in which all quarks have a Lagrangian mass the situation is very different from the massless case. It is impossible to find a chirally rotated interpolating field in terms of which the expectation value takes the standard form with $\theta = 0$. This raises the spectre of strong CP violations for Lagrangian mass operators which are "out of tune" with the chiral direction of the transmutated masses in the seized vacuum. At this point we should not forget the special nature of the Θ- parameter as a quantization ambiguity which led us to suspect that the renormalization aspects are different from those of ordinary Lagrangian parameters. Swieca pointed out to me the relevance of the work of Shifman, Vainshtein and Zakharov in this context. These authors showed [86]

a) The renormalized value of θ can always be adjusted to zero by choosing Pauli-Villars quark regulators whose mass - direction in chiral space is suitably adjusted.

b) With the help of a real (non-ghost) quark of a very large mass which is the only quark coupled to an Higgs "Axion", one can achieve $\theta = 0$ at the expense of arbitrary small physical effects in the observable energy region.

On several occasion André discussed with me Witten's ideas on the U(1) problem in the $\frac{1}{N}$ expansion. At that time we thought that such an approach could lead to a solution if

a) Only the non-topological part of the functional integrals is expanded in $\frac{1}{N}$.

b) The bound state picture can be understood in the $\frac{1}{N}$ systematics.

The second problem is perhaps the most difficult one. Even in simple models, in which the $\frac{1}{N}$ systematics is well understood in terms of finite number of Feynman diagrams (e.g. the CP^N Sigma model) it is not quite known, what threshold property in lowest nontrivial order of $\frac{1}{N}$ should be taken as an indication for the emergence of a bound state (87).

Leaving the U(1) problem aside, I now would like to comment on model studies of Screening versus Confinement (12). In order to have a clear cut distinction on the level of physical states, we take the following definition of confinement. Consider a Lagrangian gauge theory with a "gauged" quark spinor field ψ which has in addition a $SU(N)_F$ flavour index in the fundamental representation.

Def. Quarks are confined if the physical state - space does not contain states in the fundamental $SU(N)_F$ representation.

The Salam-Weinberg model does not confine in the sense of this definition. In that model the gauge color is called the "physical flavour", and therefore the added flavor index is a mathematical flavor index. Clearly the interpolating field:

$$\varepsilon_{ij}\,\phi_i\,\psi^a_{Lj}$$

(a = mathematical flavour) generates a physical state carrying fundamental mathematical flavour. Such a model we call "Screened".

According to Wilson (88), on important ingredient for the above mechanism of physical confinement is the so called static quark confinement. In a pure gauge theory without matter one studies the expectation value of the path ordered loop operator:

$$< P e^{i \oint A_\mu^a \lambda^a dx^\mu} > = C e^{-TV(L)} \tag{52}$$

If one chooses a rectangular loop of height T with $T \to \infty$ and width L, the V(L) defined by the right hand side can be shown to have the interpretation of an interaction energy between two external $q\bar{q}$ sources in the distance L. The desired "static confinement" behavior is for $L \to \infty$

$$V(L) = aL + O(L)\,,\ a = \text{string tension} \neq 0 \tag{53}$$

Recent studies in nonabelian lattice gauge theories including numerical Monte Carlo calculations have led physicists to believe that the string tension is non vanishing in the total range of the lattice coupling constant.(89) An analytic proof has as up to now not been given, although recent works of 't Hooft (90) and Mack (91) leave the impression, that one has come very close to a proof of (53).

Investigations of the physical confinement problem in the presence of matter are much more difficult; in particular the effective potential becomes a less useful theoretical concept since V(L), as a result of $q\bar{q}$ fluctuations, always flattens out asymptotically. With the exception of a four - dimensional(92) Z(2) lattice gauge model, the only models in which the physical confinement problem has been understood reasonably well are two-dimensional continuous gauge theories (12). It has been often stated, that the confinement in two--dimensional gauge models is an automatic consequence of the increasing Coulomb potential. This is certainly true for the static quark confinement. In the QFT including spinor - quarks, only the color - charge neutrality is automatic in two dimensions. The investigations of "Screening versus Confinement" are subtle. They have been carried out within the last years notably by Swieca.

I will explain some of the results in QED_2 model with SU(N) flavours. The only gauge invariant polynomials in ψ are generated by "meson" fields.

$$\bar{\psi}^a \quad \psi^b \tag{54}$$

Hence, according to the standard confinement picture, we expect only "mesons" in the trivial and the adjoint SU(N) representations.

With zero Lagrangian quark mass one can easily exhibit gauge invariant operators which carry the fundamental representation. For their construction one uses bosonization which, as we will show later on, is a special case of the order - disorder duality. These operators are certainly not polynomial in the original ψ's Applied to the vacuum they create "infraparticle" states transforming according to the fundamental representation of SU(N). With a finite Lagrangian quark mass the situation changes drastically: these states carry an infinite energy and are confined. In fact for the θ - vacuum with $\theta = 0$ one can show that there are only states which transform as $\frac{S(N)}{Z_n}$ tensors. Choosing a CP invariant vacumm with $\theta = \frac{\pi}{N}$ one can construct gauge invariant operators which carry the fundamental representation. These operators are not local, they rather have commutation relations similar to the dual algebra. This has the effect, that the usual argumments leading to two-particle scattering states transforming as SU(N) x SU(N) fail. They rather transform as $\overline{SU(N)}$ x SU(N) or SU(N) x $\overline{SU(N)}$ according to whether the first particle moves slower or faster than the second one. These particle states are the quantum field theoretical version of Coleman's quasiclassical "half - asymptotic states"[94] .

We certainly do not want to insinuate that these "exotic" properties have a counter part in actual QCD4. We do however expect that the QCD4 with matter fields lead to a rich and subtle phase structure.

There is one more interesting observation concerning the nature of quark operators in the confining massive QCD_2 model. The quark propagator is a relativistic gauge e.g. the Schwinger gauge is an extremely ill defined object which increases exponentially in x-space. For euclidean distances we have [95]

$$<\psi(x)\bar{\psi}(y)> \sim A\exp BM^2\xi^2 \ln M^2\xi^2$$

where M is the quark mass. This disastrous infrared-behaviour prevents the use of momentum space and dispersion theory for quark propagator and more generally non-gauge-invariant correlation functions of quark fields. The infrared behaviour of the gluon propagator on the other hand is much more descend, it just contains a zero mass pole which is not related to any physical particle and which therefore has been called the "secret long range force". The exponential increasing quark propagator is related to the secret long range form behaviour via the Schwinger Dyson integral equations. In QCD_4 one would expect an exponential behaviour to be related to a zero mass double pole $\frac{1}{(p^2)^2}$ in the gluon propagator. Swieca's opinion about these observations was that although physically non-existing fields as isolated confining quark fields would be expected to have very ill defined mathematical properties, one should avoid to base a confinement philosophy on unphysical quantities.

The remaining topic to be discussed is the functional integral approach to continuous order - disorder fields and kink operators. The relevance of this duality structure for a classification of the different phases had been first recognized by Kadanoff and Ceva [16], in their study of the two-dimensional Ising model. These authors demonstrated that the thermodynamic duality of the Ising model observed decades ago by Kramers and Wannier [96] had a microscopic basis which manifests itself through the existence of a local disorder - variable. Later Mandelstam[97] and in a more explicit fashion 't Hooft[90] and Mack [91] exhibited dual variables in lattice gauge theory and, on a completely formal level, also in the continuous gauge theories[98]. As we will see, the mathematical aspects, in particular the renormalization properties of disorder - variables are very subtle in the continuous QFT.

A comparatively simple illustration of the continuum order - disorder duality is the functional approach to the Mandelstam bosonization in the form discussed by Marino and Swieca[99]. For simplicity we start with a massless free field ϕ and define formally the two component exponential operators

$$\sigma(x) = \exp -ia\gamma^5\phi(x)\begin{pmatrix}1\\1\end{pmatrix}, \quad \mu(x) = \exp - ibl\int_{x,C}^{\infty}\varepsilon^{\mu\nu}\partial_\nu\phi\, dz_\mu\begin{pmatrix}1\\1\end{pmatrix} \tag{55}$$

Assuming for the moment the path independence of μ and choosing the path C parallel to the spatial axis, we may compute the equal time dual algebra between σ and μ from the canonical commutation relations:

$$\mu(x^0,x^1)\;\sigma(x^0,y^1) = \sigma(x^0,y^1)\;\mu(x^0,x^1)\;\; e^{2\pi i\phi\gamma^5\theta(y^1-x^1)} \tag{56}$$

with $\qquad s = \dfrac{ab}{2\pi}$

If μ would be a scalar field under L-transformations (this will be shown later on), this dual algebra has to be valid for space - like distances as well. Formally, the transformation of σ by μ produces a translation to the right of x^1:

$$\phi \rightarrow \begin{cases} \phi + b & y^1 > x^1 \\ \phi & y^1 < x^1 \end{cases} \tag{57}$$

This is the "half-space" version of the global symmetry: $\phi \rightarrow \phi + b$ of the Lagrangian

$$L = \frac{1}{2} \partial_\mu \phi \, \partial_\mu \phi \tag{58}$$

In addition to σ and μ, we define conjugates:

$$\bar{\sigma} = \sigma^{+tr} \gamma^0 = \sigma^{tr}, \quad \mu = \mu^{+tr} \gamma^0 = \mu^{+tr}$$

$$\text{with } \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The euclidean correlation functions of σ, e.g.

$$<\sigma_i(x)\bar{\sigma}_j(y)> = \frac{1}{N}\int [d\phi] \quad \exp - \frac{1}{2} \int (\partial_\mu \phi)^2 + ia \left[\gamma^5{}_{ii}\phi(x) + \gamma^5{}_{jj}\phi(y)\right] \tag{59}$$

have an obvious electrostatic interpretation:

The saturating euclidean configuration ϕ^{cl}

$$- \partial^2 \phi^{cl}(z) = a\gamma^5{}_{ii} \, \delta(z-x) + a\gamma^5{}_{jj} \delta(z-y) \tag{60}$$

inserted into the induced action yields the electrostatic energy of the two (imaginary) charges. The quadratic fluctuations which are independent of x and y as well as the electrostatic selfenergies can be absorbed into the wave function renormalization factor. The result has the form

$$< \sigma(x)\bar{\sigma}(y) > = \exp. - E_{stat.}(x,y) \tag{61}$$

where:

$$E_{stat}(x,y) = -\frac{\alpha^2}{2\pi}\gamma_x^5\gamma_y^5 \ln|x-y| - \frac{\alpha^2}{2\pi}\ln R\left|1+\gamma_x^5\gamma_y^5\right| \tag{62}$$

is the electrostatic interaction energy. The last term in Estat. originates from the Dirichlet boundary condition $\phi(R) = 0$. This term yields in the limit $R \to \infty$ T the well known chiral selection rule of the Thirring model $\sigma_1 \sim \psi_1^+\psi_2$, in Minkowski space:

$$< \sigma_1\bar{\sigma}_2 > = <\sigma_1 (\bar{\sigma}\gamma^0)_1 > = 0$$

For the correlation functions of μ, e.g.

$$<\mu(x)\bar{\mu}(y)> = \frac{1}{N}\int[d\phi]\exp - \frac{1}{2}\int(\partial_\mu\phi)^2 + b\int_{x_1 C}^{y} \varepsilon_{\mu\nu}\partial_\nu\phi dz_\mu \tag{63}$$

we have to observe that the euclidean formalism gives b without an i-factor. The saturating configuration:

$$-\partial^2\phi^{cl}(z) = b\int_{x,c}^{y} \varepsilon_{\mu\nu}\partial_\nu^\eta\, \delta(z-\eta)d\eta_\mu \tag{64}$$

describes electrical dipoles on a string C. The field line picture is the same as that of the magnetic field generated by two currents flowing perpendicular to the z - plane and penetrating this plane at x and y. For this reason we will call the right hand side of (64) a magnetic monopole configuration with a string C. Again renormalization is performed in the language of (magneto) statistics: the monopole self-energies as well as a string self-energy contribution will be absorbed into N. A simple calculation reveals the validity of the following statement.

Statement: the renormalized μ -correlation functions are independent of C.

The proof is based on the observation that a closed contoure Γ :

$$b\int_{x_1 C}^{y} \varepsilon_{\mu\nu}\partial_\nu\phi = b\int_{x_1 C^1}^{y} \varepsilon_{\mu\nu}\partial_\nu\phi + b\int_{\Gamma}\varepsilon_{\mu\nu}\partial_\nu\phi \qquad (65)$$

does not contribute to the path integral, because it can be functionally shifted away:

$$\phi \rightarrow \phi + b\theta_S(x) \qquad (66)$$

$\theta_S(x)$ = characteristic function of region enclosed by

A more careful examination of boundary contribution shows that this shift leaves no residual terms only after the string self energy factors have been absorbed into N. The situation is reminiscen to gauge invariance which also only is valid for the renormalized correlation functions. Note that the independence of μ correlation on the path's is a quantum phenomenon, it happens only in functional integrals and not in the correponding classical quantities.

The really interesting objects in this model are the mixed correlation functions e.g.

$$< \sigma(x^a)\ \mu\ (x^b)\ \bar{\mu}\ (y^b)\ \bar{\sigma}\ (y^a) > \tag{67}$$

In addition to the charge - charge and monopole - monopole interactions there will be now a charge - monopole interaction in the exponent of the correlation function.

As a consequence of this additional contribution, the mixed euclidean correlation functions will be multiple valued. The manifold on which they live is not simply euclidean space but rather a ramified covering with the positions of μ being the ramification points. The number of sheets depends on $s = \frac{ab}{2\pi}$; if this number is rational there will be a finite number of sheets. Everytime the positions of the σ's cross the cuts C and return to their original values we reach another sheet of the function, i.e., the situation is similar to the one in analytic function theory where classes of topologically inequivalent path's give rise to the construction of Riemann surfaces. The independence of the correlation functions on path's within one equivalence class leads to the scalar transformation property of μ .

The multi-sheeted structure of the euclidean domain is a manifestation of duality. Locality, as it is well known, leads to univalent function, in the analyticity domain of general QFT (29) ; this domain includes the euclidean points. Hence the dual structure transcends the Wigthman - Osterwalder-Schrader (100) framework. Of course this lack of univaluedness does not lead to ambiguities in the definition of physical operators.

An interesting feature appears if we were to introduce local "dyon" operators:

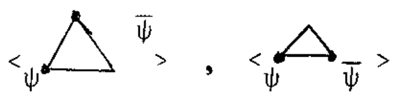
$$\begin{aligned} \psi_{1,2}(x) &= \sigma_{1,2}(x)\ \mu(x) \\ \bar{\psi}_{1,2}(x) &\quad \bar{\sigma}_{1,2}(x)\ \bar{\mu}(x) \end{aligned} \tag{68}$$

Each time a charge crosses a string before the equal point limit is taken, we obtain a discontinuity of the form e^{iab} . In other words there is a phase ambiguity in the definition of euclidean dyon correlation functions The physical boundary values of these correlation functions are precisely those of the massless Thirring-Klaiber model with:(101)

$$s \pm \frac{ab}{2\pi} = \text{(Lorentz) spin} \tag{69a}$$

$$\dim \psi = \frac{a^2 + b^2}{4\pi} \tag{69b}$$

Only in the case of ordinary spin $s = \frac{1}{2}$ one can relate the $\pm$ sign ambiguity with the order of euclidean operators inside correlation functions. For $s \neq \frac{1}{2}$ the phase ambiguity cannot be dependent into a linear operator arrangement. One could think of inventing a "crazy" algebra. For example in case of $s = \frac{1}{N}$ one may place the and on the edges of an N polygon, e.g.



etc...

This would go against the good custom of writing from left to right (or the other way around). Collecting the main result we may say that the products of two-dimensional order and disorder operators lead to spinors with exotic statistics (101)(15) which has its origin in the topologically inequivalent path classes appearing in euclidean functional integrals.(102)

In the case of a Sine - Gordon Lagrangian instead of (58) the b weight in the definition of μ must be related to the κ in

$$L = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{k}{\beta} \cos \beta\phi \tag{70}$$

by: $b = \frac{2\pi}{\beta}$ (71)

Only with (71) one obtains the path independence of μ . It is interesting that the finite energy requirement for the Minkowski - space soliton states is equivalent to the path independence (or the covariant transformation property) of the euclidean formulation.

A rapid glance at QED_2 reveals that the case of this dyon formalism yields the correlation functions in the unitary $\sqrt{\pi}$ gauge in a natural way:

$$<\psi(x)\ \exp^{-ie \int_{x,c}^{y} A^{\mu} dz_{\mu}}\ \overline{\psi}(y)>_{Schwinger} \quad (72)$$

$$= \frac{1}{N} \int (dA_{\mu}[(dy]\ \bar{e}^{S}\ \exp i\sqrt{\pi}\left[\gamma^{5}{}_{x}\phi(x)+\gamma^{5}_{y}\phi(y)-i\int_{x,c}^{y} \varepsilon_{\mu\nu}\partial_{\nu} y dz_{\mu}\right] \quad (73)$$

$$S = \int L d^{2}x, \quad L = \frac{1}{2}\,(\partial_{\mu}\phi)^{2} + \frac{1}{4}\,F^{2}_{\mu\nu} + \frac{ie}{\sqrt{\pi}}\,\varepsilon_{\mu\nu}\partial_{\nu}\phi\ A^{\mu} + ie\,\frac{\theta}{2\pi}\,\varepsilon^{\mu\nu}\partial_{\mu}A_{\nu}$$

Performing first the A_{μ} integration we obtain a mass term for the ϕ as a result of the coupling with A_{μ} . Using the language which is dual to the previous terminology, i.e., calling ϕ a monopole potential and F_{12} a magnetic field, the A_{μ} integration produces a magnetic plasma. As a result the chiral selection rule (monopole - charge selection rule) is lost and the vacuum can be regarded as a chiral condensate. The C - integral now represents a tube of electric flux and different path's C are now no longer equivalent. This picture is very similar to Mandelstam's[97] scenario of 4-dimensional confinement.

The euclidean functional representation of dual variables not related to exponentials of a massless neutral field or a Sine - Gordon field is much more subtle. Consider as an example a complex massive free field:[103]

$$(\Box - m^2)\,\phi(x) = 0$$

A dual U(1) variable is a local scalar field obeying the duality relation:

$$\mu_s(x)\,\phi(y) = \phi(y)\mu_s(x)\; e^{2\pi i s\,\theta(y^1-x^1)} \tag{73}$$

$$\mu_s^+(x)\phi\,(y) = \phi\,(y)\;\mu_s^+(x)\; e^{-2\pi i s\,\theta(y^1-x^1)}$$

i.e. μ induces a "half-space" U(1) - rotation centered at x. The global invariance of the theory is U(1). The formal application of the canonical formalism suggests:

$$\mu_s(x) = \exp\, i2\pi s \int_x^\infty j_\mu(x)\varepsilon^{\mu\nu}dz_\nu \tag{74}$$

but this is not the correct renormalized expression since (74) terms out to be path - dependent. The correct answer is more appealing than (74):

$$<\mu\,(x)\mu^+(y)> = \frac{1}{z}\int[d\phi]\;[d\phi^+]\;\; e^{-\int L\,(A_\mu)d^2x}$$

where $\qquad L(A_\mu) = \overline{D_\mu\phi}D_\mu\phi\, - m^2\overline{\phi}\phi \qquad\qquad (76)$

$$D_\mu = \partial_\mu - i\,A_\mu \tag{77}$$

$$A_\mu(z) = 2\pi s\int_x^y \varepsilon_{\mu\nu}\frac{dx_\nu(s)}{ds}\;ds\;\delta(z-x(s)) \tag{78}$$

i.e. a scalar charged particle in an external string gauge field.

The calculation of the determinant requires regularizations, the use of the Pauli-Villars regularization will again lead to a ζ function formula of the type (5) ., where μ is a Pauli-Villars mass. Hence there exists a multiplicative renormalization. An explicit calculation can be performed for the 1-point function of μ . It is convenient to use the gauge invariance in order to go from the string gauge to the radially symmetric "vortex" gauge

$$A_\mu = s \; \varepsilon_{\mu\nu} \, \partial_\nu \phi \qquad , \nu \neq \phi = \text{polar angle} \tag{79}$$

A relevant quantity whose computation is reasonably simple is the euclidean 3-point function

$$\langle \Phi^+(x)\mu(o)\Phi(y)\rangle = C \nu G\,(x,y\;;\;A_\mu) \tag{80}$$

Up to a normalization constant this is just the euclidean Green function in the string gauge. Again one does the calculation in the simple vortex gauge. There the effect of $A\mu$ just amounts to a quasi-periodic boundary condition in the ϕ angle:

$$\Phi(r,\phi) = e^{2\pi i s}\,\Phi(r,\phi+2\pi) \tag{81}$$

The euclidean Green function, which has a representation in terms of Bessel--functions, has to be gauge transformed back to the string gauge. From the resulting euclidean 3-point function one then obtains via analytic continuation and Fourier-transformation the momentum space kernels:

$$\langle p|\mu(o)\,|p'\rangle \quad , \quad \langle p\bar{p}'\,|\mu\,(o)|o\rangle$$

Since the disorder operator in a form field theory can be written in the form:

$$\mu = :\text{exp. bilinear } (\Phi): \qquad (83)$$

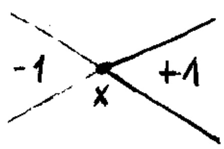
where the double dots indicate the Wick-ordering, the kernels (82) completely determine the bilinear expression in the exponent i.e. the operator μ. The problem of calculating correlation functions can then be in principle done with the help of Wick-contractions. The dual algebra (73) is equivalent to multivaluedness of the mixed euclidean correlation functions which can be traced back to the quasi-periodic property of the basic Green function(80).

Naively one would expect the model to exhibit a periodicity in s, this is however not the case. The higher s - values lead to Green functions which contain as a basic element the reduced functions for s mod 1, but there is no strict periodicity in s.

An interesting functional integral representation of a nontrivial Z_2 gauge-bundle situation emerges if one consider the construction of the disorder variable μ of a real field ϕ . In that case one has no chance to use vector-potentials. Rather one has to phrase the gauge aspect in the form of a "flat" but "nontrivial" Z_2 bundle. In other words the covariant derivative (77) nowagrees with the usual one:

$$D_\mu = \partial_\mu$$

and the nontrivial aspect is contained in nontrivial transition function e.g. for (80):



The imprint of this nontrivial gauge situation on the eigenstates of the euclidean eigenvalue equation for the matter field is an antiperiodic boundary condition. By "doubling" the ϕ field, which is equivalent to the use of a complex field, one achieves a "trivialization" i.e. one is able to use the previous A_μ formalism. It turns out that the vacuum expectative values for μ of the doubled model are precisely the squares of those for a real field:

$$<\mu(x_1)\ldots\mu(x_n')>_{complex} = <\mu(x_1)\ldots\mu(x_2)>^2_{real} \qquad (84)$$

In the zero mass limit, all the correlation functions can be calculated explicitely and they provide a new class of two-dimensional conformally invariant models different from the massless Thirring model.

Again, as in the previous "bosonization" model, the product of order and disorder yields noncanonical "exotic" spinor fields.

A particularly useful order-disorder algebra results if one starts from a massive free Majorana spinor ψ The corresponding flat but nontrivial Z_2 gauge-bundle leads to a scalar operator μ with the corresponding square relationship (84)' to the trivialized complex bundles construction. This terms out to be the disorder variable of the two-dimensional Ising model (for $T \to T_c + \varepsilon$) in the scaling limit. The corresponding scalar order variable may be either constructed by a short-distance expansion of ψ and μ, or directly in terms of a suitable euclidean functional integral.

Again one observes a square relationship with the σ-expectation values of the doubled (complex spinor) model. Naturally in the doubled mixed correlation functions of μ and σ one looses the sign in the duality relation. These doubled fields are simply the local fields

$$\mu = :\cos \sqrt{\pi}\,\Phi: \qquad (85)$$

$$\sigma = :\sin \sqrt{\pi}\,\Phi:$$

where Φ is the Sine Gordon potential for $\beta = \sqrt{4\pi}$ which is bilinear in fermion -creation and-annihilation operators. Thus the euclidean approach unravels the rôle of doubling and demystifies the way in which Truong and I[104] obtained the correlation functions of the Ising model which up to now always appeared to me somewhat artificial as compared with the more logical construction of Sato et al.[105] In fact the euclidean approach leads to a novel method to derive other results of SMJ as for example the field theoretic solution of the Riemann-Hilbert problem. Using formula for the nonabelian fermion determinant[106] one can carry out those calculations explicitly

The euclidean functional approach to the order-disorder duality is not limited to disorder operators in form field theories. For interacting models e.g. the Φ^4 coupling, it works the same way.

In case the interaction terms are not superrenormalizable one may however encounter additional renormalization problems.

Encouraged by these results Swieca and collaborators took a renewed interest in the Z_n generalization of the Ising model. An S-matrix[107] candidate for a suitably defined Z_n model in the scaling limit had been known for some time. The charaterizing property of a Z_n field theory is the Z_n selection rule (in the $T_c+\varepsilon$ phase):

$$\langle\sigma(x_1)\ldots\sigma(x_n)\rangle = \begin{cases} 0 & \text{if } n \neq D \bmod N \\ \neq 0 & \text{otherwise} \end{cases} \tag{86}$$

and similarly for the μ in the $T_c-\varepsilon$ phase. Closely related are the Z_n operator relations:

$$\sigma^{+} \quad \sigma^{N-1} \quad , \quad \mu^{+} - \mu^{N-1} \tag{87}$$

In a free field theory i.e. with Gaussian functional integrals one cannot fulfill these relations, they will be inconsistent with the cluster properties. For this reason the candidates for correlation functions (in the massless limit) proposed by Fradkin and Kadanoff do not constitute a solution of the Z_n problem[108]. One rather expects a new class of conformal invariant correlation function different from exponentials of free fields. The Z_n invariance in the gauge-theory terminology of functional integrals corresponds to strings which are periodic in S. For example for Z_3 the situations

should be equivalent. This can be formally achieved by

$$L_{int} = \phi^{N} + \phi^{+N}$$

The path independence with such an interaction enforces the quantization $\frac{1}{N}$ of the A_{μ} strength. We have not insofar been able to find a Lagrangian which yields the known Z_n S-matrix.

There is one observation which may be interesting if one compares the lattice approach with the continuous euclidean functional integrals. A Z_n lattice field has no formal continuous counter-part; it is not possible to rewrite the functional sums simply into a functional integral for the corresponding continuous theory because there are no Z_n valued fields in a continuous theory. Generalizing from the experience with the Ising model one may expect the existence of other dynamical variables which allow for a formal transition to functional integrals. These variables would be the analog a of the Lieb-Mattis Schultz (109) lattice fermions. Without those variables a continuous functional integral for the scaling limit of the Ising model could not have been written. Part of the problem in the scaling limit construction of Z_n models and lattice gauge theories may be in the construction of the "right" dynamical variables.

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